Module-2

Subject: Network Theory

Content: Single Phase & Three Phase Circuit

Prepared by

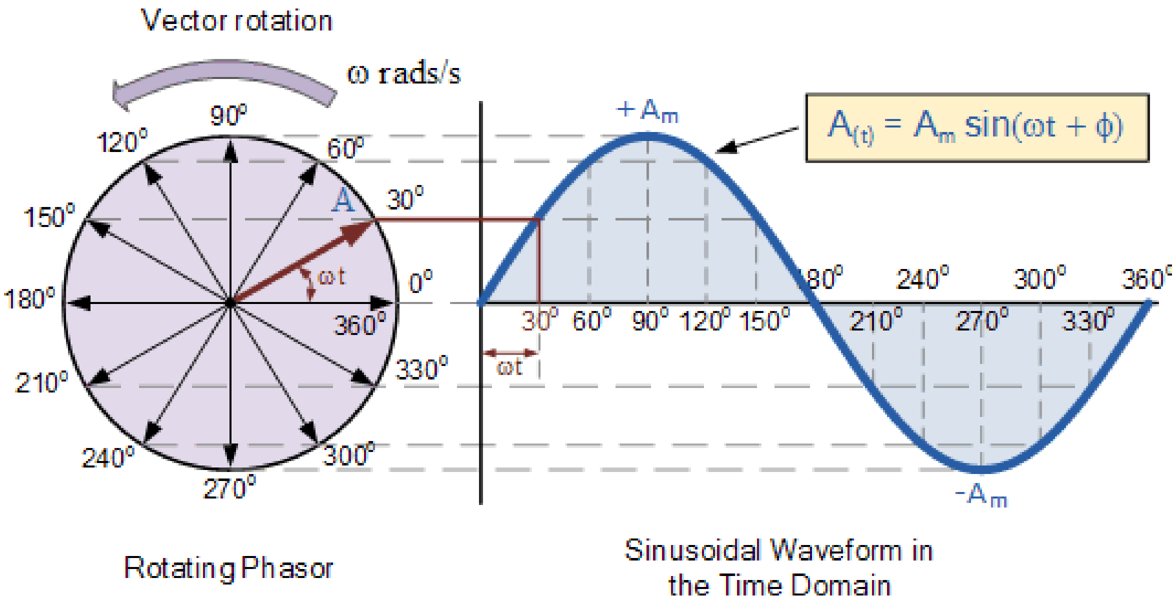
Dr. Sangram Keshori Mohapatra, Associate professor

Department of Electrical Engineering,
Government College of Engineering, keonjhar, Odisha, India

Single phase Circuit:

2. Phasor representation of an Alternating Quantity

As we know that the alternating quantities of voltage or current are vector quantities having both magnitude and direction. But in case of instantaneous values are continuously changing so it can be represented by a rotating vector or phasor.so we can define a phasor is a vector rotating at constant angular velocity.



Here at time t1, $\omega t_1 = 30^0$, $v_1 = v_m \sin \omega t_1 = OA\sin \omega t_1 = OA\sin 30^0$

$t_2, \omega t_2 = 60^0$, $v_1 = v_m \sin \omega t_2 = OA\sin \omega t_2 = OA\sin 60^0$

$t_3, \omega t_3 = 90^0$, $v_1 = v_m \sin \omega t_3 = OA\sin \omega t_3 = OA\sin 90^0$

$t_4, \omega t_4 = 120^0$, $v_1 = v_m \sin \omega t_4 = OA\sin \omega t_4 = OA\sin 120^0$

$t_5, \omega t_5 = 150^0$, $v_1 = v_m \sin \omega t_5 = OA\sin \omega t_5 = OA\sin 150^0$

$t_6 . \omega t_6 = 180^0$, $v_1 = v_m \sin \omega t_6 = OA\sin \omega t_6 = OA\sin 180^0$

And so on.

Consequently, the phasor having magnitude v_m and rotating in anticlockwise direction at an angular velocity ωt represents the sinusoidal voltage $v = v_m \sin \omega t$

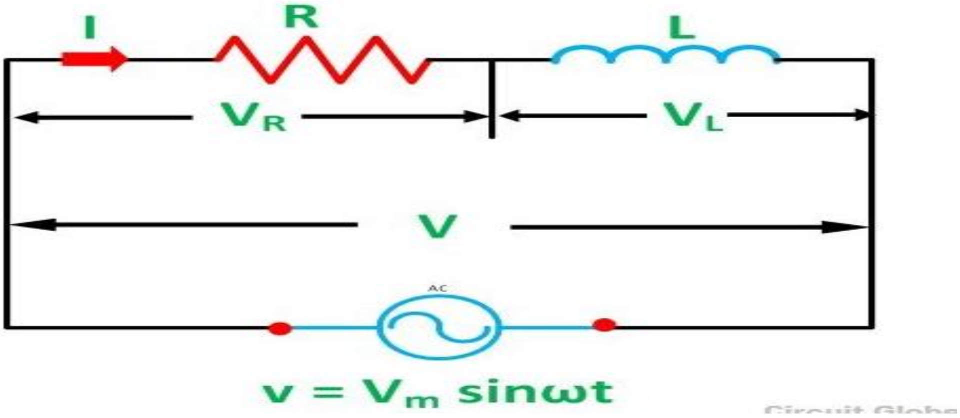
Phasor diagram:

The phasor diagram is one in which different alternating quantities of the same frequency are represented by phasor with their correct relationship.

The phasor representing two or more alternating quantities of the same frequency rotate in counter-clock wise direction with the same angular velocity, thereby maintaining a fixed position with respect to one another.

2.1 A.C Through Pure Resistance and Inductance in series Circuit

Let V and I be the rms value of voltage and current respectively in the given circuit.



$$\overline{V} = \overline{V_R} + \overline{V_L}$$

$$V = \sqrt{{V_R}^2 + {V_L}^2}$$

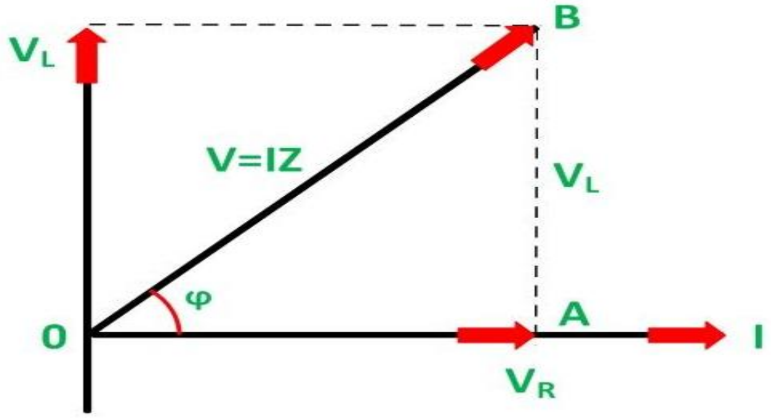
$$V = \sqrt{(IR)^2 + (IX_L)^2}$$

$$V = I\sqrt{R^2 + {X_L}^2}$$

$$I = \frac{V}{\sqrt{R^2 + {X_L}^2}} = \frac{V}{Z_{RL}}$$

Where $Z_{RL} = \sqrt{R^2 + {X_L}^2}$ =Impedance in R-L circuit

The phasor diagram of R-L circuit

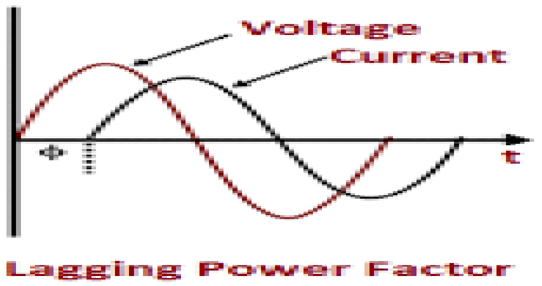


$$\tan\phi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R} = \frac{\omega L}{R}$$

$\phi = \tan^{-1}(\frac{\omega L}{R})$

From the phasor diagram, Current I lag behind the supplied voltage V by an angle ϕ

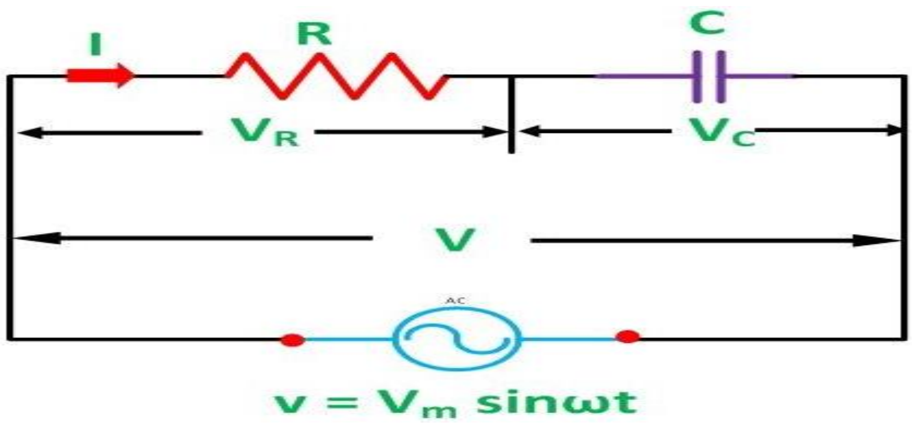
If voltage $V = V_m \sin \omega t$ then the resulting current in the circuit $I = I_m \sin(\omega t - \phi)$



Power factor: $\cos\phi = \frac{V_R}{V} = \frac{IR}{IZ_{RL}} = \frac{R}{Z_{RL}}$ =power factor of this circuit

2.2 A.C Through Pure Resistance and Capacitance in series Circuit

Let V and I be the rms value of voltage and current respectively in the given circuit.



$$\overline{V} = \overline{V_R} + \overline{V_C}$$

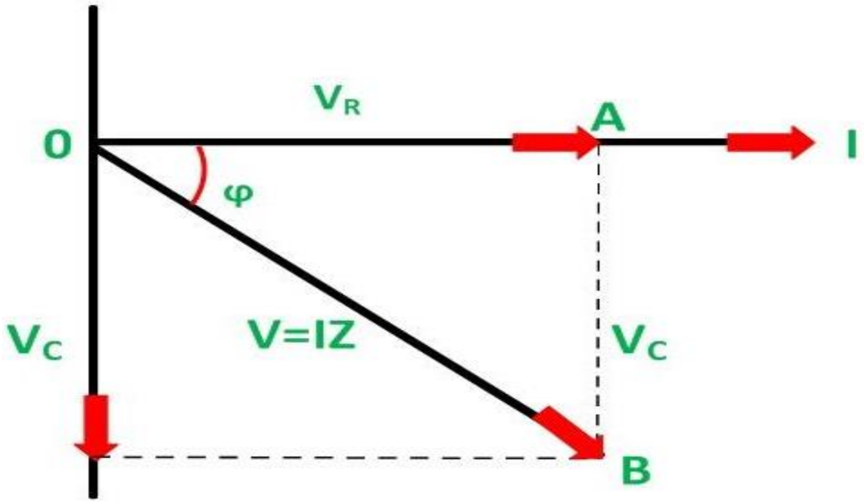
$$V = \sqrt{V_R{}^2 + V_C{}^2}$$

$$V = \sqrt{(IR)^2 + (-IX_C)^2}$$

$$V = I\sqrt{R^2 + X_C{}^2}$$

$$I = \frac{V}{\sqrt{R^2 + X_C{}^2}} = \frac{V}{Z_{RC}}$$

Where $Z_{RC} = \sqrt{R^2 + X_C{}^2}$ =Impedance in R-C circuit



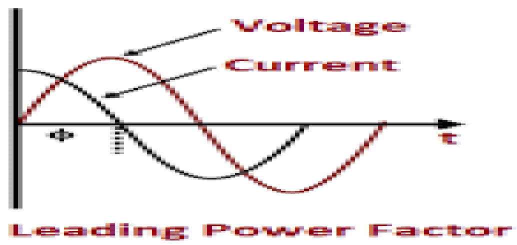
The phasor diagram of R-C circuit

$$\tan\phi = \frac{V_C}{V_R} = \frac{-IX_C}{IR} = \frac{-X_C}{R} = \frac{-1}{\omega RC}$$

$\phi = \tan^{-1}(\frac{-1}{\omega RC})$ where $X_C = \frac{1}{\omega C}$

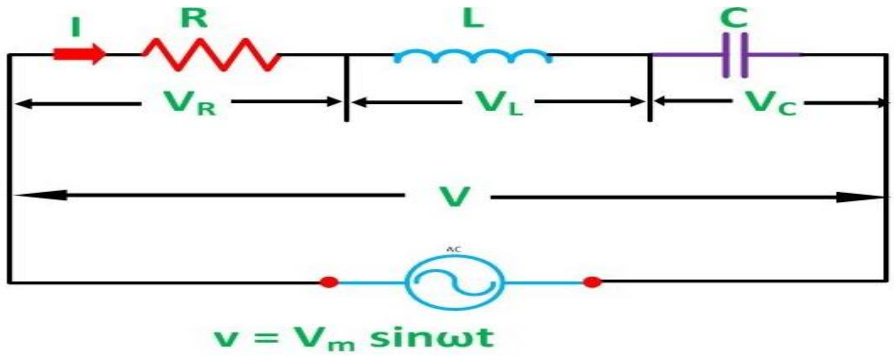
If voltage $V = V_m \sin \omega t$ then the resulting current in the circuit $I = I_m \sin(\omega t + \phi)$

Power factor: $\cos\phi = \frac{V_R}{V} = \frac{IR}{IZ_{RC}} = \frac{R}{Z_{RC}}$ =power factor of this circuit



2.3 A.C Through Pure Resistance, Inductance and Capacitance in series Circuit

Let V and I be the rms value of voltage and current respectively to the R-L-C series circuit.



Then

$$\overline{V} = \overline{V_R} + \overline{V_L} + \overline{V_C}$$

$$V = \sqrt{V_R^{\,2} + V_L^{\,2} + V_C^{\,2}}$$

$$V = \sqrt{(IR)^2 + (IX_L)^2 + (IX_C)^2}$$

$$V = I\sqrt{R^2 + X_L^{\,2} + X_C^{\,2}}$$

$$I = \frac{V}{\sqrt{R^2 + X_L^{\,2} + X_C^{\,2}}} = \frac{V}{Z_{RLC}}$$

Where $Z_{RLC} = \sqrt{R^2 + X_L^{\,2} + X_C^{\,2}}$ =Impedance in R-L-C circuit

Phasor diagram:

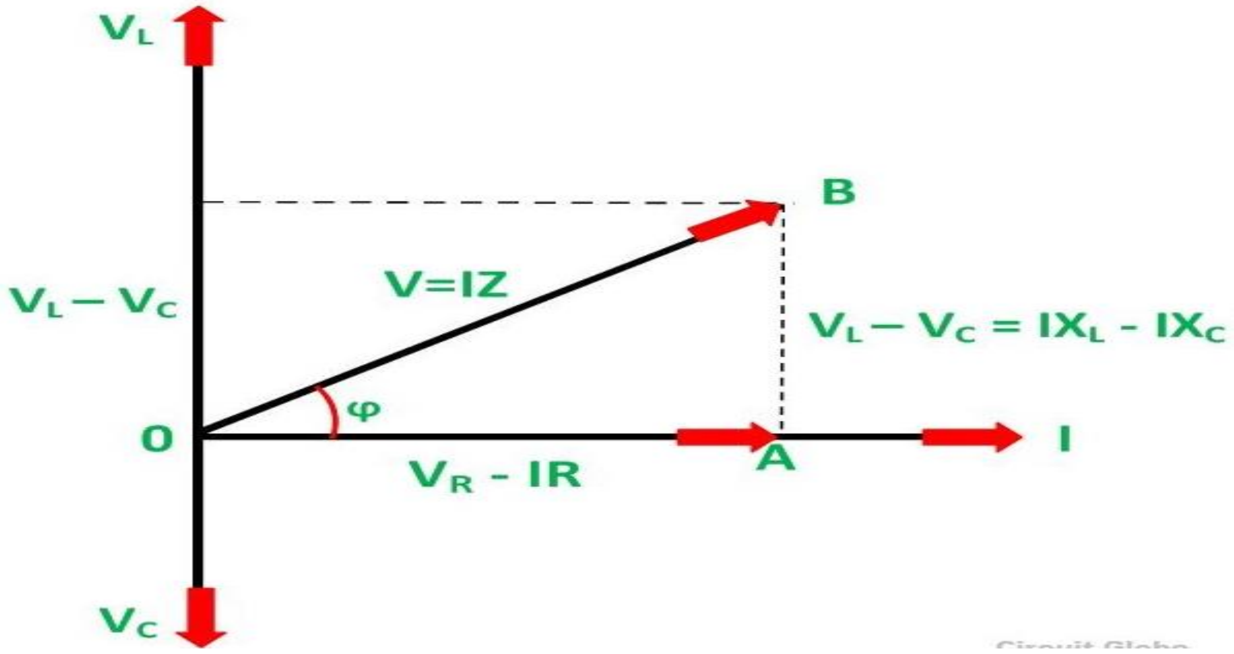
As V_L and V_C are two vector 180^0 out of phase with each other ie opposite direction of vectors.

Here the phasor diagram can be obtained in two different cases

Case-1: if $X_L > X_C$, $IX_L > IX_C$ that means $V_L > V_C$ the net voltage is $V_L - V_C$

Case -2: if $X_L < X_C$, $IX_L < IX_C$ that means $V_L < V_C$ the net voltage is $V_C - V_L$

Case-1 phasor diagram as shown in the Figure



$$V = I\sqrt{R^2 + (X_L - X_C)^2}$$

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{Z_{RLC}}$$

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{IX_L - IX_C}{IR} = \frac{X_L - X_C}{R}$$

$$\phi = \tan^{-1}[\frac{X_L - X_C}{R}]$$

If $V = V_m \sin \omega t$ then the resulting current in the circuit $I = I_m \sin(\omega t - \phi)$

Similarly, in the case-2

$$I = \frac{V}{\sqrt{R^2 + (X_C - X_L)^2}} = \frac{V}{Z_{RLC}}$$

$$\tan\phi = \frac{V_C - V_L}{V_R} = \frac{IX_C - IX_L}{IR} = \frac{X_C - X_L}{R}$$

$$\phi = \tan^{-1}[\frac{X_C - X_L}{R}]$$

If $V = V_m \sin \omega t$ then the resulting current in the circuit $I = I_m \sin(\omega t + \phi)$

$$\cos\phi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z}$$ =power factor of the circuit

Admittance of A.C circuit:

Admittance Y is the reciprocal of impedance Z of the A.C circuit.

$$Y = \frac{1}{Z} = \frac{1}{V/I} = \frac{I}{V}$$

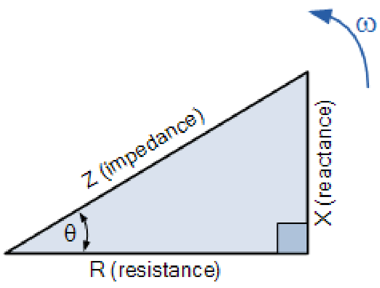
$Y = \frac{1}{Z} = G \pm jB$ where G=real part of the admittance =Conductance

B=Imaginary part of the admittance = Susceptance

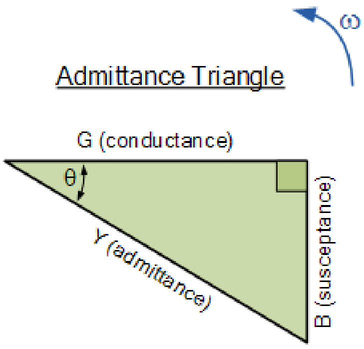
$G = Y\cos\theta$ =conductance $= \frac{1}{Z}\frac{R}{Z} = \frac{R}{Z^2}$

$B = Y\sin\theta$ =susceptance $= \frac{1}{Z}\frac{X}{Z} = \frac{X}{Z^2}$

$$Y = \sqrt{G^2 + B^2}$$



Impedance Triangle



2.5 RMS Value:

The average value of this square wave I $= \frac{i_1^2 + i_2^2 + i_3^2 + \dots\dots\dots + i_n^2}{n}$

The root of this expression is the root mean square (RMS)

$$I_{rms} = \sqrt{\frac{i_1^2 + i_2^2 + \dots\dots\dots + i_n^2}{n}}$$

The General expression of the rms value of the any periodic function f(t) with a period T is

$$G_{rms} = \sqrt{\frac{1}{T}\int_0^T [f(t)]^2 dt}$$

Ex. The rms value of sinusoidal current $I_{rms} = I_m \sin \omega t$ can be obtained as

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T [I_m \sin \omega t]^2 dt}$$

$$I_{rms} = \sqrt{\frac{I_m^{\ 2}}{2T}\int_0^T [1-\cos 2\omega t]dt}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

Similarly, for the non-sinusoidal wave of current or any function can be obtained in rms value as follows

$$I = I_0 + I_{m1}\sin \omega t + I_{m2}\sin 2\omega t + I_{m3}\sin 3\omega t + ... + I_{mn}\sin n\omega t$$

$$I_{rms} = \sqrt{I_0^{\ 2} + I_{m1}^{\ 2} + I_{m2}^{\ 2} + ... + I_{mn}^{\ 2}}$$

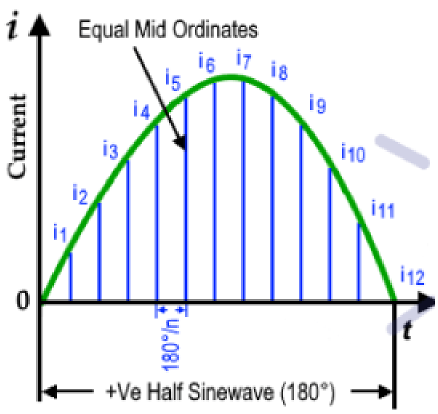
$$I_{rms} = \sqrt{I_0^{\ 2} + I_{1rms}^{\ 2} + I_{2rms}^{\ 2} + ... + I_{nrms}^{\ 2}}$$

$$I_{rms} = \sqrt{I_0^{\ 2} + \frac{I_{m1}^{\ 2}}{2} + \frac{I_{m2}^{\ 2}}{2} + ... + \frac{I_{mn}^{\ 2}}{2}}$$

2.6 Average Value:

The average value of an A.C current or voltage is the average of all the instantaneous values during one alteration. They are actually DC value.

$$I_{avg} = \frac{i_1 + i_2 + i_3 + ... + i_n}{n}$$



$$I_{avg} = \frac{i_1 + i_2 + i_3 + ... + i_{12}}{12}$$

The general expression for average value of any function f(t) having a periodic function with period T

$$G_{avg} = \frac{1}{T}\int_0^T f(t)dt$$

EX. The average value of sinusoidal current $I_{rms} = I_m \sin \omega t$ can be obtained as

$$I_{avg} = \frac{1}{T}\int_0^T I_m \sin \omega t dt = \frac{2I_m}{\Pi} = 0.637 I_m$$

$$Form\ Factor(FF) = \frac{rms value}{average value}$$

Average Power

Let the instantaneous voltage and current can be taken as

$$v(t) = v_m \cos(\omega t + \theta)$$

$i(t) = i_m \cos(\omega t + \phi)$ then the instantaneous power $p(t) = v(t) * i(t)$

$$p(t) = v_m \cos(\omega t + \theta) * I_m \cos(\omega t + \phi)$$

$$p(t) = v_m i_m [\frac{1}{2}\cos(\theta - \phi) + \frac{1}{2}\cos(2\omega t + \theta + \phi)]$$

The average power $P_{avg} = \frac{1}{T}\int_0^T p(t)dt$

$$P_{avg} = \frac{1}{T}\int_0^T p(t)dt = \frac{1}{T}\int_0^T v_m i_m [\frac{1}{2}\cos(\theta - \phi) + \frac{1}{2}\cos(2\omega t + \theta + \phi)dt \qquad (1)$$

$$P_{avg} = \frac{1}{T}\int_0^T p(t)dt = \frac{v_m i_m}{2}\cos(\theta - \phi) = v_{rms} i_{rms} \cos(\theta - \phi)$$

$$P_{avg} = v_{rms} i_{rms} \cos(\theta - \phi)$$

In the equation (1) having two terms, 1st term is the independent of time function and 2nd term is sinusoid having frequency twice the frequency of original voltage and current waveform. The average power for the 2nd term for a full cycle is zero.

2.7 Complex Power:

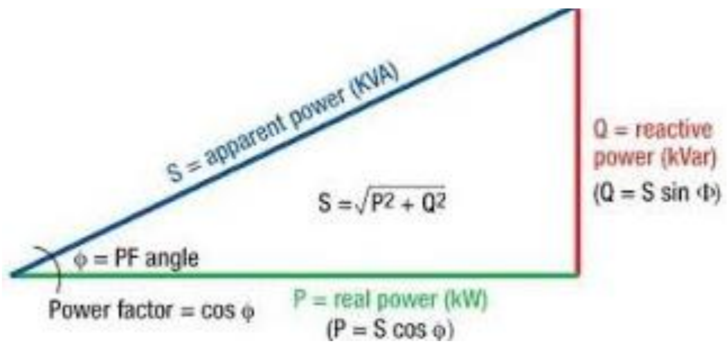
If the current and voltage of a circuit are given in phasor form then the complex power is

$$S = VI^* = P \pm jQ$$

Where $S = P + jQ$ for net inductive circuit

$S = P - jQ$ for net capacitive circuit

S is the total power or apparent power, P is active power and Q is reactive power



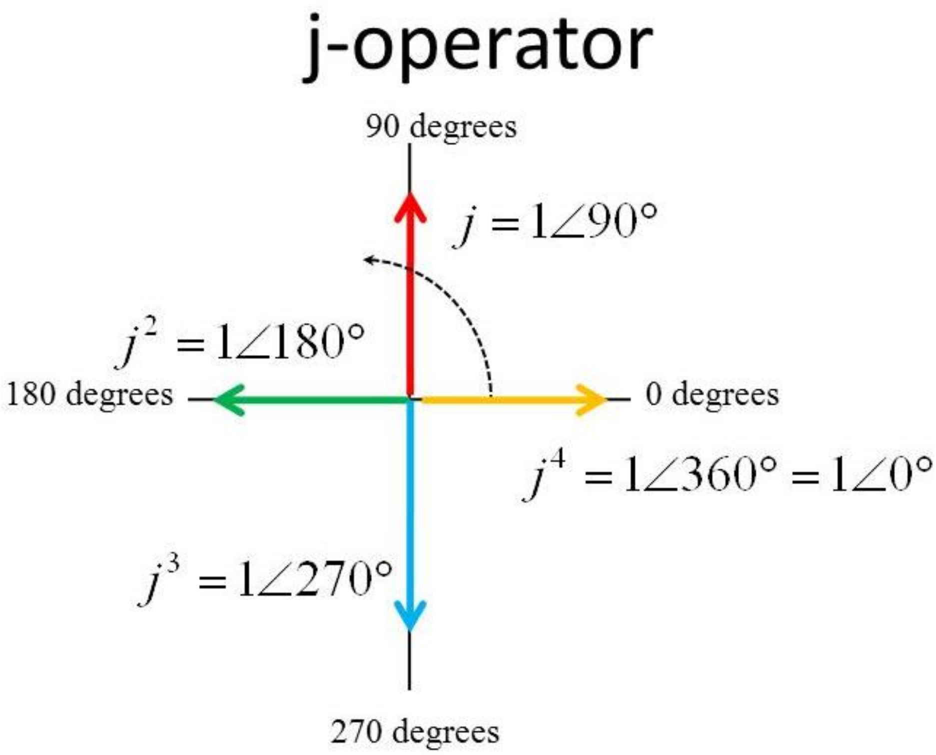
$\cos\phi = \frac{P}{S} = \frac{kW}{KVA}$ =power factor of the circuit

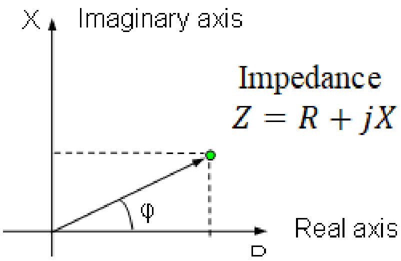
$P = S\cos\phi = V_{rms} I_{rms} \cos\phi$

$Q = S\sin\phi = V_{rms} I_{rms} \sin\phi$

Complex Notation of A.C circuit:

2.8 J Operator





A vector can be written in different form

(a) Cartesian or rectangular form
(b) Trigonometrical form
(c) Polar form
(d) Exponential form

Let us consider a vector in **cartesian form** $Z = R + jX$

$|Z| = \sqrt{R^2 + X^2}$ = magnitude of the vector, $\varphi = \tan^{-1}[\frac{X}{R}]$ =phase angle, this phase angle can be measured from the positive real axis to the vector and taken as positive in counter clockwise direction and taken in negative in clockwise direction.

In this vector j is a operator which operate 90^0 , In counter clock wise direction taken as positive value.

Ex In cartesian vector can be addition or subtraction of two or more vector as follows

If $Z_1 = R_1 + jX_1$ $Z_2 = R_2 + jX_2$

$$Z_1 + Z_2 = R_1 + jX_1 + R_2 + jX_2 = (R_1 + R_2) + j(X_1 + X_2)$$

$$Z_1 - Z_2 = R_1 + jX_1 - (R_2 + jX_2) = (R_1 - R_2) + j(X_1 - X_2)$$

The same vector can be written in **Trigonometrical form** as

$$\overline{Z} = |Z|\cos\varphi + j|Z|\sin\varphi$$

$$\overline{Z} = |Z|(\cos\varphi + j\sin\varphi)$$

Then this vector can write in **polar form** as

$$\overline{Z} = |Z|\angle\varphi$$

In polar form of vectors can be multiplied or divided by two or more vector as follows

$$\overline{Z_1} = |Z_1|\angle\varphi_1 \quad \overline{Z_2} = |Z_2|\angle\varphi_2$$

Multiplication of two polar vector $\overline{Z} = (|Z_1|\angle\varphi_1)(|Z_2|\angle\varphi_2) = |Z_1||Z_2|\angle\varphi_1 + \varphi_2$

Division of two polar vector $\frac{\overline{Z_1}}{\overline{Z_2}} = \frac{|Z_1|\angle\varphi_1}{|Z_2|\angle\varphi_2} = \frac{|Z_1|}{Z_2}\angle\varphi_1 - \varphi_2$

This vector also written in **exponential form** as

$$\overline{Z} = |Z|e^{j\varphi}$$

2.9 Representation of voltage, current and impedance in complex Notation

Let the voltage and current in an A.C circuit can be expressed in the form

$\overline{V} = |V|\angle\alpha$ and $\overline{I} = |I|\angle\beta$ where α and β are the angular displacement of $\overline{V}$ and $\overline{I}$ respectively from a reference direction.

(a) A.C Through pure Resistance

Let voltage phasor $\overline{V}$ be taken as reference phasor so $\overline{V} = |V|\angle 0^0$

But in case of pure resistive circuit, the current and voltage are in phase, so current taken as $\overline{I} = |I|\angle 0^0$

$$Z_R = \frac{\overline{V}}{\overline{I}} = \frac{|V|\angle 0^0}{|I|\angle 0^0} = \frac{V}{I}\angle 0^o = R\angle 0^0 = R + j0 = R$$

(b) A.C Through pure Inductance

In pure inductance, the current lags the voltage by 90^0

$$\overline{I} = |I|\angle -90^0$$

$$Z_L = \frac{\overline{V}}{\overline{I}} = \frac{|V|\angle 0^0}{|I|\angle -90^0} = \frac{V}{I}\angle 90^o = X_L\angle 90^0 = 0 + jX_L = j\omega L$$

(c) A.C Through pure capacitance

In pure capacitance, current leads the voltage by 90^0

$$\overline{I} = |I|\angle 90^0$$

$$Z_C = \frac{\overline{V}}{\overline{I}} = \frac{|V|\angle 0^0}{|I|\angle 90^0} = \frac{V}{I}\angle -90^o = X_C\angle -90^0 = 0 - jX_C = \frac{-j}{\omega C}$$

(d) A.C Through R-L circuit

The total impedance of a series RL A.C circuit is given by

$$Z_{RL} = Z_R + Z_L = (R + j0) + (0 + jX_L) = R + jX_L$$

(e) A.C Through R-C circuit

The total impedance of a series R-C A.C circuit is given by

$$Z_{RC} = Z_R + Z_C = (R + j0) + (0 - jX_C) = R - jX_C$$

(f) A.C Through R-L-C circuit

The total impedance of a series R-L-C A.C circuit is given by

$$Z_{RLC} = Z_R + Z_L + Z_C = (R + j0) + (0 + jX_L) + (0 - jX_C) = R + j(X_L - X_C)$$
$$Z_{RLC} = R + j(X_L - X_C) = R + jX$$

If two or more impedance are connected in series than total impedance of a series circuit is the phasor sum of the impedances of the circuit

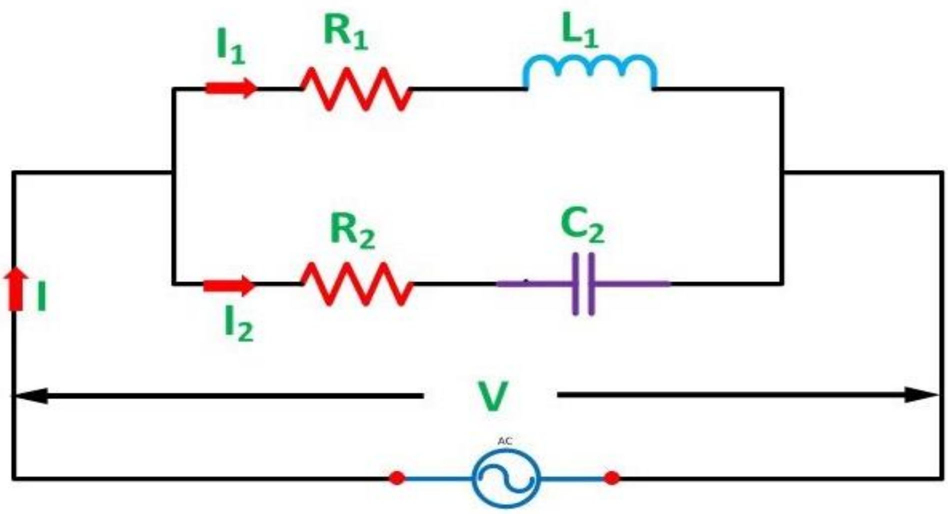
$$Z_T = Z_1 + Z_2 + Z_3 + \dots\dots\dots\dots + Z_n$$

If two or more impedance are connected in parallel than the total impedance can be obtained as

$$\frac{1}{Z_T} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \dots\dots\dots + \frac{1}{Z_n}$$
$$Y_T = Y_1 + Y_2 + Y_3 + \dots\dots\dots\dots + Y_n$$

Where Y_1, Y_2 and Y_3 are the admittances corresponding to Z_1, Z_2 and Z_3 respectively and so on

2.10 Parallel A.C Circuit:



Let us consider that two branch impedance $Z_1 = R_1 + jX_1$ and $Z_2 = R_2 - jX_2$

Are connected in parallel through an A.C supply voltage $V = V_m \sin \omega t$

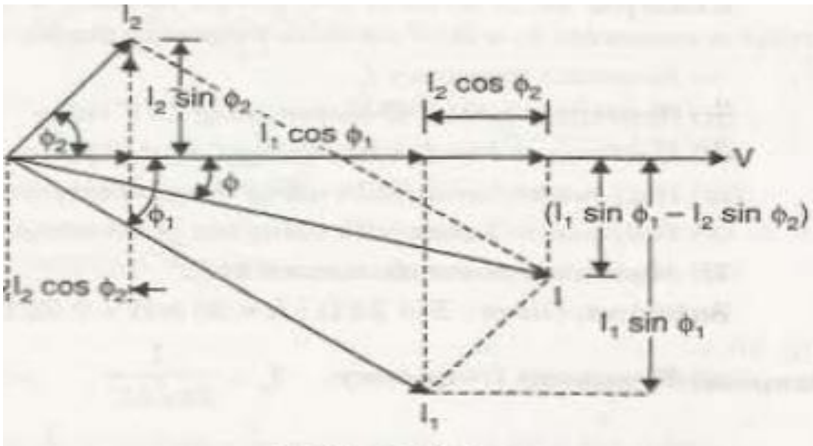
$$|Z_1| = \sqrt{R_1^{\ 2} + X_1^{\ 2}}, I_1 = \frac{V}{Z_1}, \cos\phi_1 = \frac{R_1}{Z_1}, \phi_1 = \cos^{-1}\frac{R_1}{Z_1}$$

The current I_1 is lagging the applied voltage V by an angle ϕ_1 in Z_1 branch

$$|Z_2| = \sqrt{R_2^{\ 2} + X_2^{\ 2}}\ I_2 = \frac{V}{Z_2}, \cos\phi_2 = \frac{R_2}{Z_2}, \phi_2 = \cos^{-1}\frac{R_2}{Z_2}$$

The current I_2 is lagging the applied voltage V by an angle ϕ_2 in Z_2 branch

The resultant current $\overline{I} = \overline{I_1} + \overline{I_2}$



(b) Phasor diagram

Q.1.A coils takes 2.5A, when connected across 200V,5oHz main. The power consumed by the coil is found to be 400W. Find the inductance, and power factor of the coil.

Sol: $P = I^2R = (2.5)^2 R = 400W$

$$R = \frac{400}{6.25} = 64\,\Omega$$

$$Z = \frac{V}{I} = \frac{200}{2.5} = 80\,\Omega\,,\ X_L = \sqrt{80^2 - 64^2} = 48\,\Omega$$

$$X_L = 2\pi * 50 * L = 48\,,\ L = 0.153H$$

Power factor= $\cos\phi = \frac{R}{Z} = \frac{64}{80} = 0.8$ (lagging)

Q.2. An inductive coil, when connected across a 200V,50Hz supply. draws a current of 6.25A, and a power of 1000W. Another coil, connected across the same supply, draws a current of 10.75A, and a power of 1155W.Find the current drawn, and the power in net, when the two coils are connected in series across the same supply.

Sol:

For coil-1: $P_1 = 1000W = {I_1}^2 R_1 = (6.25)^2 R_1$

$$R_1 = 25.6\,\Omega,\ Z_1 = \frac{V}{I_1} = \frac{200}{6.25} = 32\,\Omega,\ X_{L1} = \sqrt{32^2 - 25.6^2} = 19.2\,\Omega$$

For Coil-2: $P_2 = 1155W = {I_2}^2 R_2 = (10.75)^2 R_2\,,\ R_2 = 10\,\Omega$

$$Z_2 = \frac{V}{I_2} = \frac{200}{10.75} = 18.6\,\Omega,\ X_{L2} = \sqrt{18.6^2 - 10^2} = 15.7\,\Omega$$

When two coils are connected in series , $R = 25.6 + 10 = 35.6\,\Omega,\ X_L = 19.2 + 15.7 = 34.9\,\Omega$

$Z = \sqrt{35.6^2 + 34.9^2} = 49.85\,\Omega$

$I = \frac{V}{Z} = \frac{200}{49.85} = 4.01$ A

Power= $P = I^2 R = (4.01)^2 * 35.6 = 573$ W

Q.3. Find the average power in a resistance R=10Ω if the current is

$i = 20\sin\omega t + 10\sin 3\omega t + 5\sin 5\omega t$ A.

Sol: $I_{rms} = \sqrt{\frac{1}{2}*20^2 + \frac{1}{2}*10^2 + \frac{1}{2}*5^2} = 16.2$ A

The average power $P = I^2 R = (16.2)^2 * 10 = 2624.4$ W

Q.4.A two element series circuit of R=5Ω and XL=10Ω has an effective applied voltage 100V. Determine the P, Q, S and Power factor.

$R = 5\,\Omega$ and $X_L = 10\,\Omega$

$Z = 5 + j10 = 11.18\angle 63.43^0$

$I = \frac{V}{Z} = \frac{100}{11.18\angle 63.43^0} = 8.944\angle -63.43^0$

$P = I^2 R = (8.944)^2 5 = 400$ W

$Q = I^2 X_L = (8.944)^2 10 = 800$ VA (lagging

$S = I^2 Z = (8.944)^2 11.18 = 894.4$ VA

Power factor= $\cos\phi = \frac{R}{Z} = \frac{5}{11.18} = 0.447$

2.11 3-Phase circuit:

A system with more than one phase is called polyphase system. A polyphase system contains two or more A.C voltage sources of the same frequency. These source voltages have a fixed phase angle difference between them. The most extensively used polyphase system is the 3-phase system. The three phase systems are in common use for generation, transmission, distribution and utilization of electric energy.

Advantage of 3-phase system:

1. A 3-phase machine has a smaller size as compared to 1-phase machine of the same power output.
2. The 3-phase system is used in almost all commercial electric generation.
3. The conductor material required to transmit a given power at a particular distance by 3-phase system is required less than that by a an equivalent 1-phase system.
4. For a particular size of frame, the output of a 3-phase machine is greater than of a 1-phase motor.

2.12 Generation of 3-phase supply:

The three-phase generation has three identical coils placed with their axis 120^0 apart from each other and rotated in a uniform magnetic field, a sinusoidal voltage is generated across each coil.

The sinusoidal induce voltage are

$$v_1 = v_m \sin \omega t$$

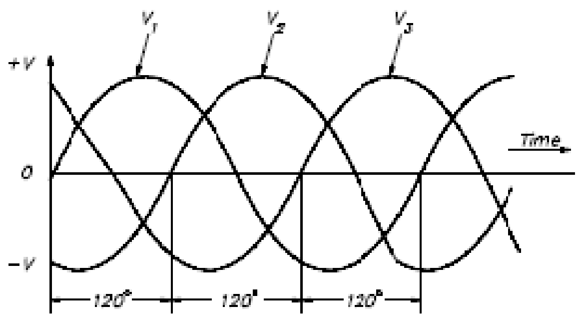
$$v_2 = v_m \sin(\omega t - 120^0)$$

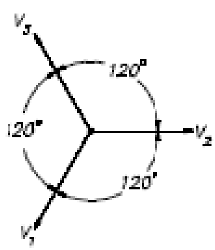
$$v_3 = v_m \sin(\omega t - 240^0)$$

$$v = v_1 + v_2 + v_3$$

$$v = v_m \sin \omega t + v_m \sin(\omega t - 120^0) + v_m \sin(\omega t - 240^0) = 0$$

Here at any instant of time, the algebraic sum of three voltage is zero.





Phase: The phase of alternating quantity implies the phase is nothing but a traction of time period that has started from reference position. The two alternating quantities are said to be in phase if they reach their zero position or reference value and maximum value at the same time. If not, they are said to be out of phase.

Phase difference:

When two alternating quantities don't reach their zero and maximum value at the same time they are said to be out of phase.

Balanced system:

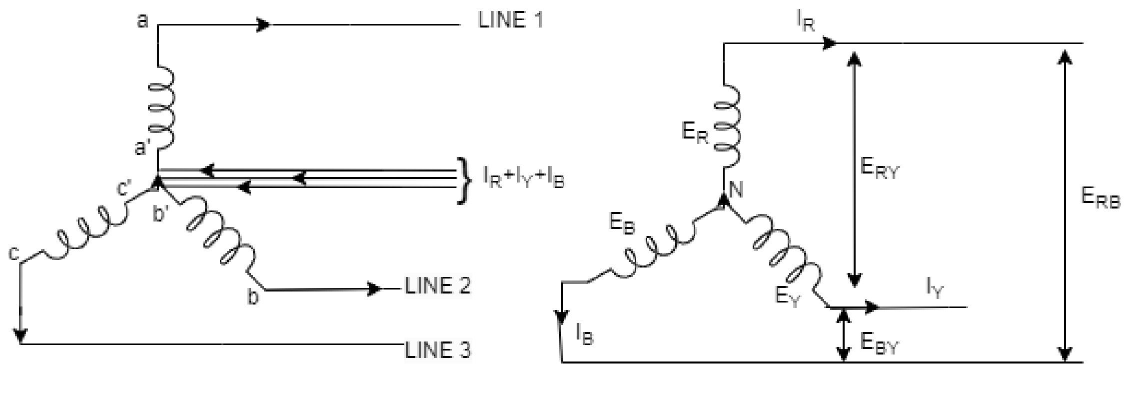
A system is balanced if various voltages are equal in magnitude, the various currents are equal in magnitude and the phase are the same for each phase.

Phase Sequence:

The phase sequence is the order or sequence in which the current or voltage in different phases attain their maximum value one after the other.

2.13 STAR CONNECTION:

As any coil having one starting end and other is finish end. In this STAR connection the three similar ends of the coils are joined together. The start ends are joined at point N and the finish ends of three windings connected to the line. The currents in each winding returns through neutral wire through the point N which is known as star point or neutral point. This type of connection is also known as 3-phase,4 wire system. The current in each winding are known as phase current but through the line is known as line current. Similarly, the voltage across each winding is known as phase voltage and voltage measured between any pair of lines or terminal are known as line voltage. The three conductors of the neutral point can be replaced by a single wire.so the summation of currents at neutral is Zero.

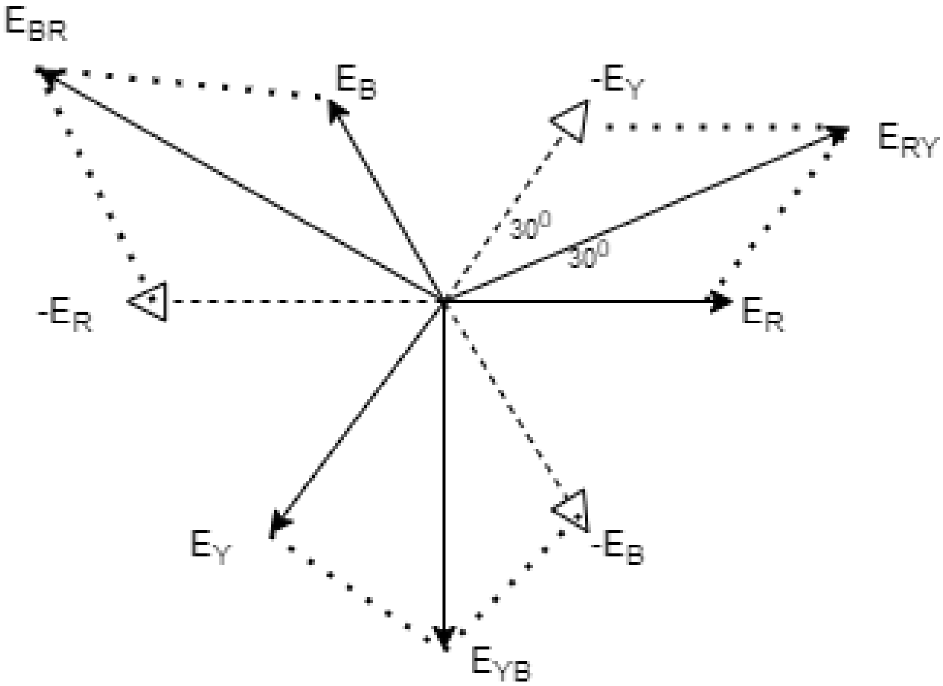


Relation between line and phase voltage.

Here the phase voltage in the Three winding are E_R , E_Y and E_B . All the phase voltages are same in magnitude but 120^0 apart as the three coils are same in all respect such type of condition is called balanced system.

$E_R = E_Y = E_B = E_{ph}$ (phase voltage)

Similarly the voltage available between any pair of terminals is called line voltage (E_{RY} , E_{BR} , E_{YB}) the current flowing in each lines are I_R , I_Y and I_B , it is called line current.



The line voltage between 1 and 2 or line voltage E_{RY} is the vector difference of phase voltages E_R and E_Y .

$E_{RY} = E_R - E_Y$ (Vector difference)

$E_{RY} = E_R + (-E_Y)$ (Vector sum)

Since the phase angle between vectors E_R and ($-E_Y$) is 60^0

From the vector diagram $E_{RY} = \sqrt{|E_R|^2 + |E_Y|^2 + 2|E_R||-E_Y|\cos 60^0}$

But ($E_R = E_Y = E_B = E_{ph}$)

$$E_{RY} = \sqrt{E^2{}_{ph} + E^2{}_{ph} + 2E_{ph}E_{ph}\frac{1}{2}}$$

$$E_{RY} = \sqrt{3}E_{ph}$$

Similarly, line voltages between 2 and 3

$$E_{YB} = E_Y - E_B = \sqrt{3}E_{ph}$$

And line voltage between 3 and 1

$E_{BR} = E_B - E_R = \sqrt{3}E_{ph}$

Line voltage $E_L = \sqrt{3}E_{ph}$

Relation between line current and phase current.

$I_R = I_Y = I_B = I_{ph}$

Line current $I_L = I_{ph}$ (phase current)

Power:

If the three-phase current has a phase difference between the phase voltage is ϕ

Then power output per phase= $E_{ph}I_{ph}\cos\phi$

Total power output $P = 3E_{ph}I_{ph}\cos\phi$

$$P = 3\frac{E_L}{\sqrt{3}}I_L\cos\phi$$

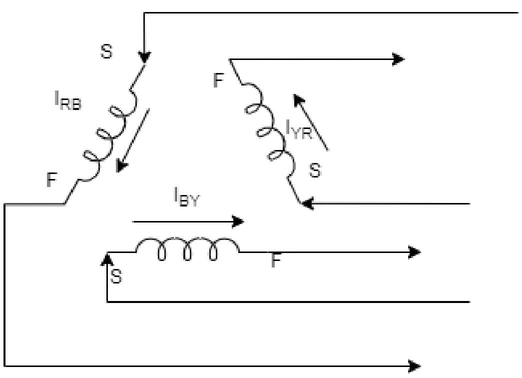
$P = \sqrt{3}E_LI_L\cos\phi$ = Total active power

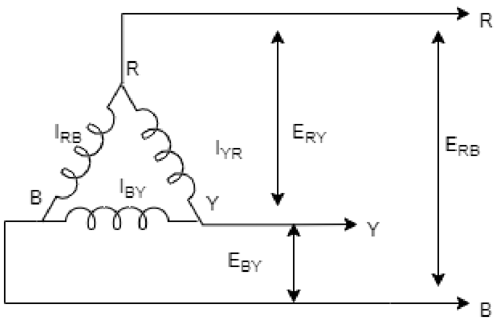
Total reactive power $Q = \sqrt{3}E_LI_L\sin\phi$

Total apparent power $S = \sqrt{3}E_LI_L$

2.14 DELTA CONNECTION:

In Delta connection, the dissimilar ends of the coil are joined together in triangular form that means the starting end of one is connected with the finishing end of another. The three coils are connected in series and formed a closed path. It is clear that the summation of voltages is Zero in that closed path as the system become balanced. Here outward direction taken as positive.





Relation between line current and phase current

It is clear that line current is vector difference of phase current of two-phase current.

Line current in line R is $I_R = I_{YR} - I_{RB}$ (Vector difference)

$$I_R = I_{YR} + (-I_{RB}) \text{ (Vector Sum)}$$

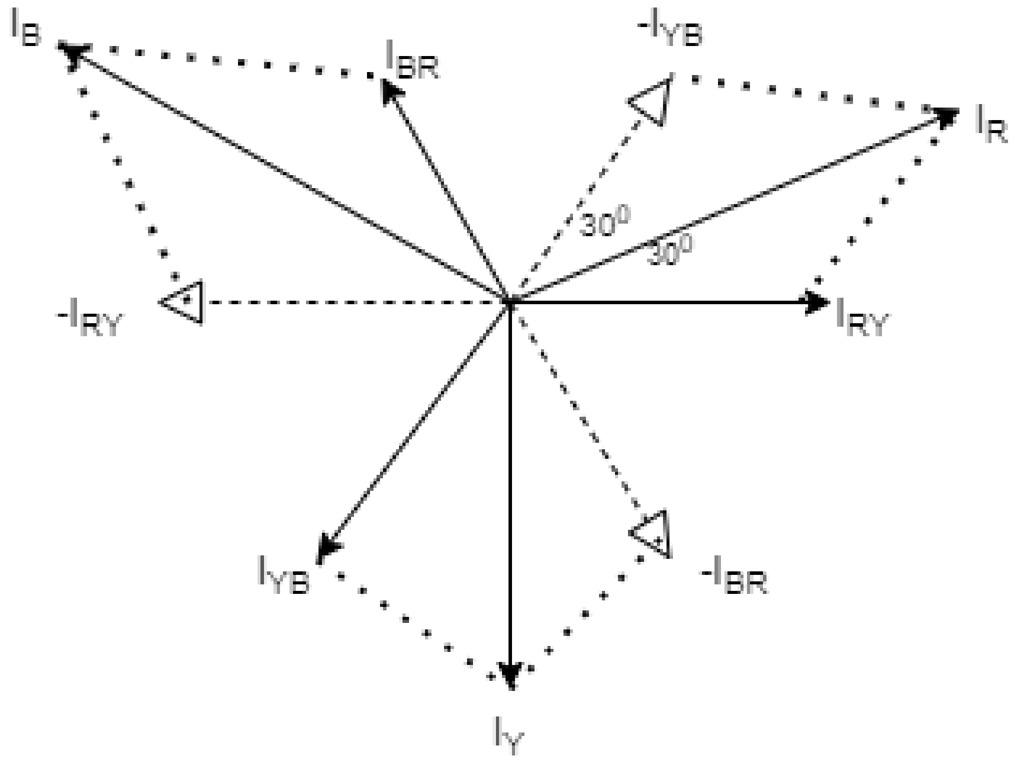
Where I_{RB} and I_{YR} are phase current.

Similarly line current in line Y is $I_Y = I_{BY} - I_{YR}$

$$I_Y = I_{BY} + (-I_{YR})$$

And line current in line B is $I_B = I_{RB} - I_{BY}$

$$I_B = I_{RB} + (-I_{BY})$$



Vector diagram

Since phase angle between phase current I_{YR} and $-I_{RB}$ is 60^0

$$I_R = \sqrt{|I_{YR}|^2 + |I_{RB}|^2 + 2|I_{YR}||-I_{RB}|\cos 60^0}$$

$$I_R = \sqrt{I^2{}_{ph} + I^2{}_{ph} + 2I_{ph}I_{ph}\frac{1}{2}} \text{ as } (I_{YR} = I_{RB} = I_{BY} = I_{ph})$$

$$I_R = \sqrt{3}I_{ph}$$

Similarly, $I_Y = I_{BY} - I_{YR} = \sqrt{3} I_{ph}$ and $I_B = I_{RB} - I_{BY} = \sqrt{3} I_{ph}$

$$I_R = I_Y = I_B = I_L$$

$$I_L = \sqrt{3} I_{ph}$$

Relation between line voltage and phase voltage

The voltage between any pair of line is equal to the phase voltage of the phase winding connected between the two lines considered.

Line voltage (E_L)= Phase voltage (E_{ph})

Power:

Then power output per phase= $E_{ph} I_{ph} \cos\phi$

Total power output $P = 3E_{ph} I_{ph} \cos\phi$

In term of line voltage and current

Total power $P = 3E_L \dfrac{I_L}{\sqrt{3}} \cos\phi$

$$P = \sqrt{3} E_L I_L \cos\phi$$

Q.1.A balanced star connected load of (8+j6) Ω per phase is connected to a three phase 230V supply. Find the line current, power factor, active power and total volt-amps.

$Z_{ph} = (8 + j6)\,\Omega = 10\angle 36.87^0, V_L = 230$ V

For star connection, $E_L = \sqrt{3} E_{ph}, V_{ph} = \dfrac{230}{\sqrt{3}} = 132.79$ V

(i) $I_L = I_{ph} = \dfrac{V_{ph}}{Z_{ph}} = \dfrac{132.79}{10\angle 36.87} = 13.28\angle -36.87^0$ A

(ii) power factor=cos36.87^0=0.8(lagging)

(iii)Active power= $P = \sqrt{3} E_L I_L \cos\phi = \sqrt{3} * 230 * 13.28 * 0.8 = 4232$W

(iv) Reactive power= $Q = \sqrt{3} E_L I_L \sin\phi = \sqrt{3} * 230 * 13.28 * 0.6 = 3174$Var

(v)Total VA= $S = \sqrt{3} E_L I_L = \sqrt{3} * 230 * 13.28 = 5290$VA

Q.2. The three similar coils each of resistance 20Ω and inductance 0.5H are connected (i) in star (ii) in delta to a 3-phase ,50Hz,400V supply, Calculate the line current and the total power absorbed.

Sol. Rph=20Ω, XL=ωL=2π×50×0.5=157Ω

$$Z_{ph=}\sqrt{R^2{}_{ph}+\ X_L}^2=\sqrt{20^2+157^2}=158.3\ \Omega$$

$$Cos\Phi=\frac{R_{ph}}{Z_{ph}}=\frac{20}{158.3}=0.1264$$

(i) Star connection

$V_{ph}=\frac{V_L}{\sqrt{3}}=\frac{400}{\sqrt{3}}=230.9$V

$$I_{ph}=\frac{V_{ph}}{Z_{ph}}=\frac{230.8}{158.3}=1.457=I_L$$

$$P=\sqrt{3}V_L I_L Cos\Phi=\sqrt{3}\times 400\times 1.459\times 0.1264=127.8W$$

(ii) Delta connection $V_{ph}=V_L=400V$

$$I_{ph}=\frac{V_{ph}}{Z_{ph}}=\frac{400}{158.3}=2.528A$$

$$I_L=\sqrt{3}\times I_{ph}=\sqrt{3}\ \times 2.528=4.38\ A$$

$P=\sqrt{3}\times V_L I_L\ \ Cos\Phi=\sqrt{3}$ ×400×4.38×0.1264=383.6 W

Q.3. Three identical coils connected in delta to a 440 V,3-phase supply take a total power of 50 kW and a line current of 90A. Find: (a) the phase current (b) the power factor (c) total reactive power (d) total apparent power taken by the coils

Sol: $I_{ph}=\frac{I_L}{\sqrt{3}}=\frac{30}{\sqrt{3}}=51.96\ A$

$P=\sqrt{3}\,V_L\ \ I_L Cos\Phi=\sqrt{3}\times 400\times 90\times cos\Phi$=50,000W

$Cos\Phi=0.8$=power factor

$$V_{ph}=V_L=400\ V$$

$Q=\sqrt{3}\,V_L I_L Sin\Phi=\ \sqrt{3}\ \times 400\times 90\times$ =

$$S=\sqrt{3}\ V_L I_L=\sqrt{3}\ \times 400\times 90=63354\ W$$