Module-6

Subject: Network Theory

Content: Two Port Network

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6.1 The different type of two port network as follows

1. Z-Parameters (Open circuit impedance parameters)
2. Y-Parameters (Short Circuit admittance parameters)
3. h-Parameters (hybrid parameters)
4. ABCD-Parameters (Transmission line parameters)

Z-Parameters: In the two-port network the input voltage (V_1) and output voltage(V_2) can expressed in terms of input current (I_1) and output current (I_2)

$$[V] = [Z][I]$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

Let us assume that port-2 is open circuited then $I_2=0$

$$Z_{11} = \frac{V_1}{V_2}|_{I_2=0} = \text{Input impedance}$$

$$Z_{21} = \frac{V_2}{I_1}|_{I_2=0} = \text{Forword transfer impedance}$$

Similarly, when port-1 is open circuited then $I_1=0$

$$Z_{12} = \frac{V_1}{I_2}|_{I_1=0} = \text{Reverse transfer impedance}$$

$$Z_{22} = \frac{V_2}{I_2}|_{I_1=0} = \text{Output impedance}$$

Y-Parameters: In a two-port network, the input current I_1 and I_2 can be expressed in terms of input and output voltage V_1 and V_2 respectively as

[I] = [Y] [V], where Y is admittance matrix

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

Assuming that port 2 (o/p) is short circuited ($V_2=0$)

$$\left(Y_{11} = \frac{I_1}{V_1} \middle| V_2 = 0 \right) = \text{Input driving point admittance}$$

$$\left(Y_{21} = \frac{I_2}{V_1} \middle| V_2 = 0 \right) = \text{forward transfer admittance}$$

Similarly, the port 1 (Input port) is short circuit ($V_1 = 0$)

$$\left(Y_{12} = \frac{I_1}{V_2} \middle| V_1 = 0 \right) = \text{reverse transfer admittance}$$

$$\left(Y_{22} = \frac{I_1}{V_2} \right|_{V_1 = 0}) = \text{output driving point admittance}$$

h-Parameter: h-parameter representation in widely used in modelling of electronic components and circuits, particularly transistors. as both open circuit and short circuit terminal conditions and utilized hence, this type of parameter is known as hybrid parameter**.**

Hence input voltage and output current are expressed in terms of input current and output voltages,

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Assuming short circuit conditions at the output ($V_2 = 0$)

$$h_{11} = \frac{V_1}{I_1}| V_{2=0} = \text{input impedance}$$

$$h_{21} = = \frac{I_2}{I_1}| V_{2=0} = \text{forward current gain}$$

Similarly for this input open circuited ($I_1 = 0$)

$$h_{12} = \frac{V_1}{V_2}| I_{1=0} = \text{reverse voltage gain}$$

$$h_{22} = \frac{I_1}{V_2}| I_{1=0} = \text{output admittance}$$

ABCD Parameter:

The ABCD parameter can be expressed as the following set of equation

$V_1 = AV_2 – BI_2$

$I_2 = CV_2 – DI_2$

These are useful for defining transmission lines expressing sending end voltage and current in terms of receiving end voltage and current.

Assuming that point 2 is open circuited, $A = V_1/V_2 |I_2=0$ =reverse voltage ratio

$C = I_1 / V_2 |I_2=0$ =transfer Impedance

Similarly, Port 2 is short circuited, $B = -V_1/I_2 |V_{2=0}$ =transfer admittance

$D = I_1/I_2 |V_{2=0}$ =reverse current ratio

6.2 Conversion of Parameters:

(a) Z-Parameters in Terms of h-Parameters

Z- parameters equation can be represented as

$V_1 = Z_{11}I_1 + Z_{12}I_2$ (6.1)

$V_2 = Z_{21}I_1 + Z_{22}I_2$ (6.2)

Similarly, h- parameter can be represented as

$V_1 = h_{11}I_1 + h_{12}V_2$ (6.3)

$I_2 = h_{21}I_1 + h_{22}V_2$ (6.4)

From (6.4) $V_2 = \frac{I_2 - h_{21}I_1}{h_{22}}$

$V_1 = h_{11}I_1 + h_{12} (\frac{I_2 - h_{21}I_1}{h_{22}})$

$V_1 = (h_{11} - \frac{h_{12}h_{21}}{h_{22}})I_1 + \frac{h_{12}}{h_{22}}$ (6.5)

Comparing equation 6.5 and 6.1

We get,

$Z_{11} = \frac{h_{22}h_{11} - h_{21}h_{12}}{h_{22}}$, $Z_{12} = h_{12} / h_{22}$

From equation 6.2

$V_2 = -\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2$ (6.6)

Comparing equation 6.3 and 6.6, it can obtain as

$Z_{21} = -\frac{h_{21}}{h_{22}}$, $Z_{22} = \frac{1}{h_{22}}$

(b) ABCD parameter in terms of Z parameters

The ABCD parameter can be represented as

$V_1 = AV_2 – VI_2$ (6.7)

$I_1 = CV_2 – DI_2$ (6.8)

The Z parameter can be represented as

$V_1 = Z_{11}I_1 + Z_{12}I_2$ (6.9)

$V_2 = Z_{21}I_1 + Z_{22}I_2$ (6.10)

From (6.10), $I_1 = \frac{V_2 - Z_{22}I_2}{Z_{21}}$

Putting I_1 in equation 6.9, we get

$V_1 = Z_{11}(\frac{V_2 - Z_{22}I_2}{Z_{21}}) + Z_{12}I_2$

$V_1 = \frac{Z_{11}}{Z_{21}} V_2 - \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}} I_2$ (6.11)

Comparing equation 6.7 and 6.11, we get

$A = \frac{Z_{11}}{Z_{21}}$, $B = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}$

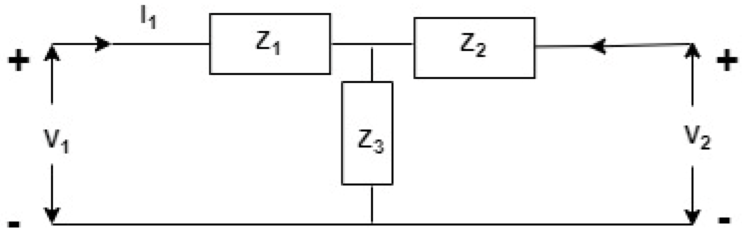
From 6.10, $I_1 = \frac{V_2}{Z_{21}} - \frac{Z_{22}}{Z_{21}} I_2$ (6.12)

Comparing 6.12 & 6.8, we get

$$C = \frac{1}{Z_{21}}$$

$$D = \frac{Z_{22}}{Z_{21}}$$

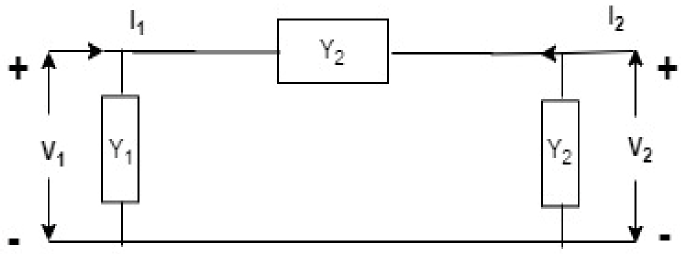
6.3 **Conversion T → π and π – T network**



T network

We know that

$$Z_{11} = Z_1 + Z_3$$
$$Z_{21} = Z_{12} = Z_3$$
$$Z_{22} = Z_2 + Z_3$$



$Y_{11} = Y_1 + Y_2$

$Y_{21} = Y_{12} = - Y_2$

$Y_{22} = Y_2 + Y_3$

$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$ $Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$

$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} [Y^{-1}]$

$= \frac{1}{\Delta Y} \begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}$

Where $\Delta Y = Y_{11}Y_{22} - Y_{12}Y_{21}$

$$\Delta Y = Y_1Y_2 + Y_2Y_3 + Y_3Y_1$$

$$\Rightarrow \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{Y_{22}}{\Delta Y} & \frac{-Y_{12}}{\Delta Y} \\ -\frac{Y_{21}}{\Delta Y} & \frac{Y_{11}}{\Delta Y} \end{bmatrix}$$

Comparing the above equation, $Z_{11} = \frac{Y_{22}}{\Delta Y} = Z_1 + Z_3$

$Z_{12} = -\frac{Y_{12}}{\Delta Y} = Z_3$

$Z_{21} = -\frac{Y_{21}}{\Delta Y} = Z_3$

$Z_{22} = \frac{Y_{11}}{\Delta Y} = Z_2 + Z_3$

$$Z_1 + Z_3 = \frac{Y_2 + Y_3}{\Delta Y} \qquad (6.13)$$

$$Z_3 = \frac{-Y_{12}}{\Delta Y} = \frac{Y_2}{\Delta Y} \qquad (6.14)$$

$$Z_2 + Z_3 = \frac{Y_{11}}{\Delta Y} = Y_1 Y_2 \qquad (6.15)$$

Solving equation 6.13,6.14 & 6.15

$Z_2 = \frac{Y_1}{\Delta Y} = \frac{Y_1}{Y_1 Y_2 + Y_2 Y_3 + Y_3 Y_1}$

$Z_1 = \frac{Y_3}{Y_1 Y_2 + Y_2 Y_3 + Y_3 Y_1}$

$Z_3 = \frac{Y_2}{Y_1 Y_2 + Y_2 Y_3 + Y_3 Y_1}$

Similarly,

$Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = Z^{-1}$

$= \frac{1}{\Delta Z} \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$

$Y_{11} = \frac{Z_{22}}{\Delta Z}$ $\qquad \Delta z = Z_{11} Z_{22} - Z_{12} Z_{21}$

$Y_{12} = -\frac{Z_{12}}{\Delta Z}$ $\qquad = Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1$

$Y_{21} = -\frac{Z_{21}}{\Delta Z}$

$Y_{22} = \frac{Z_{11}}{\Delta Z}$

Therefore, $Y_{11} = \frac{Z_{22}}{\Delta Z} = Y_1 + Y_2$ or $Y_1 + Y_2 = \frac{Z_2 Z_3}{\Delta Z}$ (6.16)

$Y_{12} = -\frac{Z_{12}}{\Delta Z} = -Y_2$ or $Y_2 = \frac{Z_3}{\Delta Z}$ (6.17)

$Y_{21} = -\frac{Z_{21}}{\Delta Z} = -Y_2$ $\quad Y_2 = \frac{Z_3}{\Delta Z}$ (6.18)

$Y_{22} = \frac{Z_{11}}{\Delta Z} = Y_2 + Y_3$ $\quad Y_2 + Y_3 = \frac{Z_2 Z_3}{\Delta Z}$ (6.19)

On solving equation 6.16, 6.17, 6.18 & 6.19

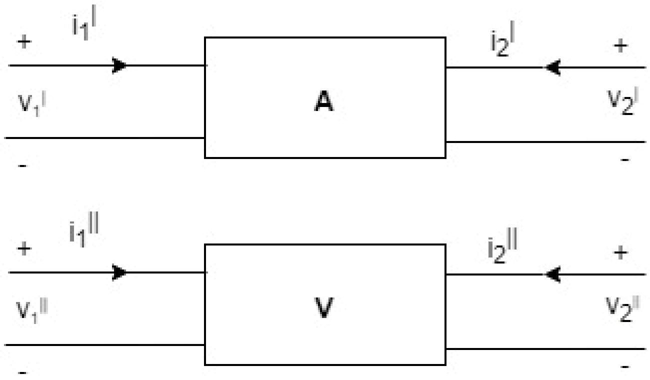
$$Y_1 = \frac{Z_1}{\Delta Z} = \frac{Z_1}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1} \quad (6.20)$$

$$Y_2 = \frac{Z_3}{\Delta Z} = \frac{Z_3}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1} \quad (6.21)$$

$$Y_3 = \frac{Z_2}{Z_1Z_2 + Z_2Z_3 + Z_3Z_1} \quad (6.22)$$

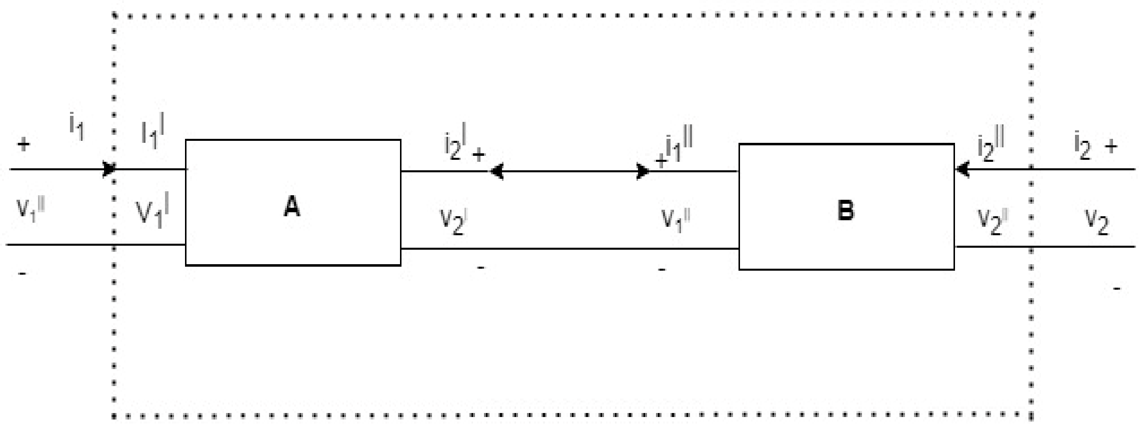
6.4 Inter-connection of two ports:

Let us consider two ports A and B where voltages and currents are shown.



Suppose these two networks are connected together in a certain manner ,so that resulting network(N) is also a two port network .Then N can also characterized by two port parameters.

6.4.1 Cascade connection;



The connection of A and B is known as cascade connection. Let V_1, $V_2, I_1, and\ I_2$ be the port voltages and currents of N.

Here $\quad I_1 = I_1^{|} \qquad V_1 = V_1^{|}$

$$I_2^{|} = -I_1^{||} \qquad V_2^{I} = V_1^{||}$$

$$I_2 = I_2^{||} \qquad V_2 = V_2^{||} \qquad (6.23)$$

For network A,

$V_1^{|}$ = $A_1 V_2^{|} - B_1 I_2^{|}$

$I_1^{|}$ = $C_1 V_2^{|} - D_1 I_2^{|}$

$$\begin{bmatrix} V_1^{|} \\ I_1^{|} \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_2^{|} \\ -I_2^{|} \end{bmatrix} \qquad (6.24)$$

Similarly for network B,

$$\begin{bmatrix} V_1^{||} \\ I_1^{||} \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_2^{||} \\ -I_2^{||} \end{bmatrix} \qquad (6.25)$$

Since $\begin{bmatrix} V_1^{||} \\ I_1^{||} \end{bmatrix} = \begin{bmatrix} V_2^{|} \\ -I_2^{||} \end{bmatrix}$

From (6.23)

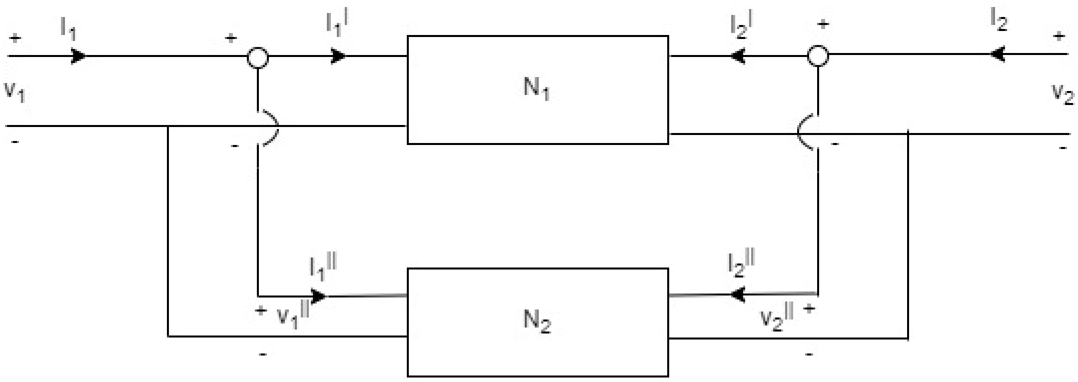
$$\begin{bmatrix} V_1^{|} \\ I_1^{|} \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_2^{|} \\ -I_2^{|} \end{bmatrix}$$

$$\begin{bmatrix} \frac{V_1}{I_1} \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_2^{||} \\ -I_2^{||} \end{bmatrix}$$

$$\begin{bmatrix} \frac{V_1}{I_1} \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \qquad (6.26)$$

Thus, we see that in the cascade connection ABCD parameter have to be multiplied.

6.4.2 Parallel connection: -



The connection of N_1 and N_2 as shown is called parallel connection.

Let V_1, V_2, I_1 and I_2 be the port voltages and currents of the combination.

Then equations of N_1 and N_2 in terms of y-parameters are

$$I_1{}' = y_{11}{}'V_1{}' + y_{12}{}'V_2{}'$$

$$I_2{}' = y_{21}{}'V_1{}' + y_{22}{}'V_2{}'$$

and

$$I_1'' = y_{11}''V_1'' + y_{12}''V_2''$$

$$I_2'' = y_{21}''V_1'' + y_{22}''V_2''$$

The inter connection two two-port network it is clear that

$I_1 = I_1' + I_1''$, $I_2 = I_2' + I_2''$

And $V_1 = V_1' = V_1''$, $V_2 = V_2' = V_2''$

Therefore , $I_1 = I_1' + I_1''$

$$=y_{11}'V_1' + y_{12}'V_2' + y_{11}''V_1'' + y_{12}''V_2''$$

$$I_1 \ =(y_{11}' + y_{11}'')\, V_1 + (y_{12}' + y_{12}'')\, V_2 \qquad (6.27)$$

$I_2 = I_2' + I_2''$

$$=y_{21}'V_1' + y_{22}'V_2' + y_{21}''V_1'' + y_{22}''V_2''$$

$$=(y_{21}' + y_{21}'')\, V_1 + (y_{21}' + y_{22}'')\, V_2 \qquad (6.28)$$

But equations for combination in terms of y-parameters will be

$$I_1 = y_{11}V_1 + y_{12}V_2 \qquad (6.29)$$

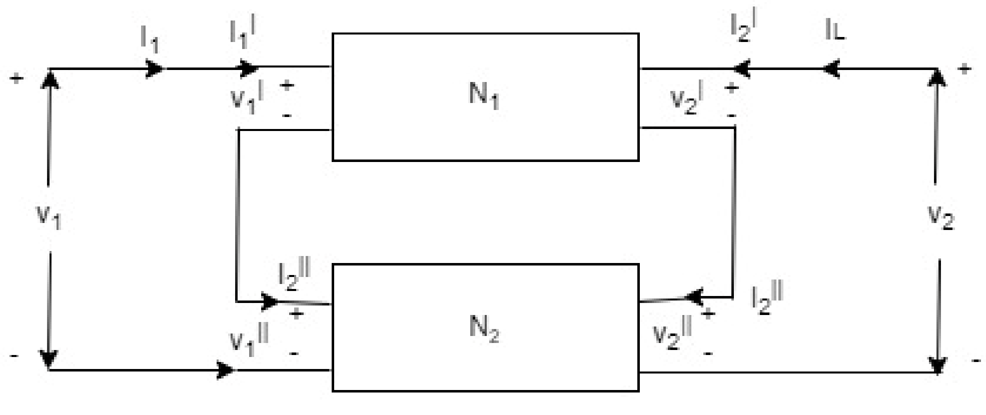
$$I_2 = y_{21}V_1 + y_{22}V_2 \qquad (6.30)$$

Comparing the above equations we have,

$y_{11} = y_{11}' + y_{11}''$, $y_{12} = y_{12}' + y_{12}''$

$y_{21} = y_{21}' + y_{21}''$, $y_{22} = y_{22}' + y_{22}''$

6.4.3 Series Connection:



The series connection of two two-port networks are $N_1\ and N_2$.

Let the Z-parameters of the network N_1 are $Z_{11}', Z_{12}', Z_{21}', Z_{22}'$ *and* that *of network* N_2 is $Z_{11}', Z_{12}', Z_{21}', Z_{22}'$.

The Z-parameters equation for network N1 are

$$V_1' = Z_{11}'I_1' + Z_{12}'I_2'$$

$$V_2' = Z_{21}'I_1' + Z_{22}'I_2'$$

Similarly, for network N_2,

$$V_1'' = Z_{11}''I_1'' + Z_{12}''I_2''$$

$$V_2'' = Z_{21}''I_1'' + Z_{22}''I_2''$$

We have , $V_1 = V_1' + V_1''$

$$=Z_{11}'I_1' + Z_{12}'I_2' + Z_{11}''I_1'' + Z_{12}''I_2''$$

$$=Z_{11}'I_1 + Z_{12}'I_2 + Z_{11}''I_1 + Z_{12}''I_2$$

$V_1 = (Z_{11}' + Z_{11}'')I_1 + (Z_{12}' + Z_{12}'')I_2$ (6.31)

$V_2 = V_2' + V_2''$

$=Z_{21}'I_1' + Z_{22}'I_2' + Z_{21}''I_1'' + Z_{22}''I_2''$

$V_2 = (Z_{21}' + Z_{21}'')I_1 + (Z_{21}'' + Z_{22}'')\ I_2$ (6.32)

$V_1 = Z_{11}I_1 + Z_{12}I_2$ (6.33)

$V_2 = Z_{21}V_1 + Z_{22}V_2$ (6.34)

Comparing the equation 6.31 to 6.33 and 6.32 to 6.34

The equivalent Z-parameter of series connection are

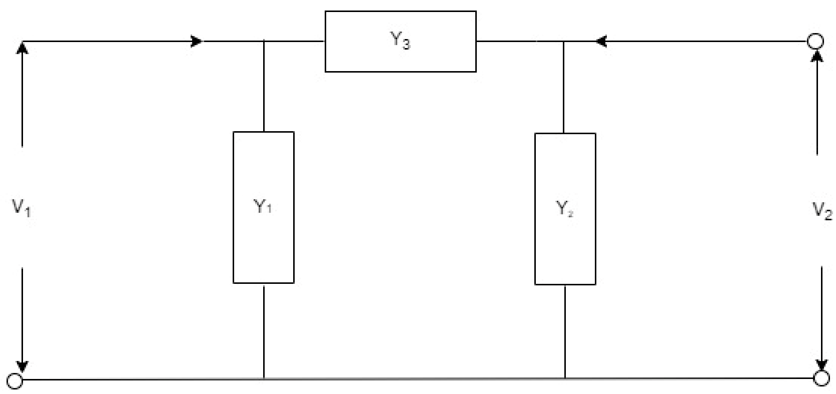
$Z_{11} = Z_{11}' + Z_{11}''$, $Z_{12} = Z_{12}' + Z_{12}''$

$Z_{21} = Z_{21}' + Z_{21}''$, $Z_{22} = Z_{22}' + Z_{22}''$

6.5 Equivalent T and π representation.

T and π network are the two network which are used frequently to represent transmission lines, filter etc.

The π network is shown below.



We find out Y_1 , Y_2 and Y_3 is terms of y-parameters,

When we short circuited at port 2,

Then $y_{11} = \frac{V_1}{I_1}|V_2 = 0$

$y_{11} = Y_1 + Y_3$ (6.35)

$Y_{21} = \frac{I_2}{V_1}|\ V_2 = 0$ As $-I_2 = V_1 Y_3$

$= \frac{-V_1 Y_3}{V_1} = -Y_3$

$\therefore\ Y_{21} = -Y_3$ (6.36)

Similarly, we can find out Y_{12} and Y_{22} by short circuited by port 4

$y_{22} = Y_2 + Y_3$ (6.37)

$y_{12} = -Y_3$ (6.38)

we get $\therefore\ y_{11} = Y_1 + Y_3$, $y_{12} = -Y_3$

$y_{21} = -Y_3$ $y_{22} = Y_2 + Y_3$

on solving the above equation, we get,

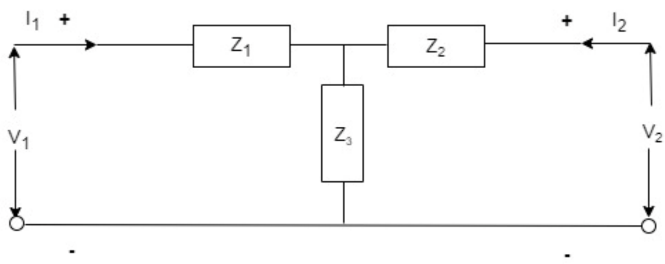
$Y_1 = y_{11} + y_{12}$

$Y_2 = y_{22} + y_{12}$

$Y_3 = -y_{12}$

T – Network

If Z- parameter of a network are known then it is can equivalent to construct an equivalent T-model.



When open circuit at port 2, we have

$Z_{11} = \frac{V_1}{I_1}|\ I_{2=0} = Z_1 + Z_3$ (6.39)

$Z_{21} = \frac{V_2}{I_1}|\ I_2 = 0$ As $V_2 = Z_3 I_1$

$= \frac{Z_3 I_1}{I_1} = Z_3$

$\therefore Z_{21} = Z_3$ (6.40)

Similarly, by open circuit at port-1

$Z_{22} = Z_2 + Z_3$ (6.41)

$Z_{12} = Z_3$ (6.42)

Now solving equation (6.39), (6.40), (6.41), & (6.42),

we obtained as
$Z_1 = Z_{11} - Z_{12}$
$Z_2 = Z_{22} - Z_{12}$
$Z_3 = Z_{12}$

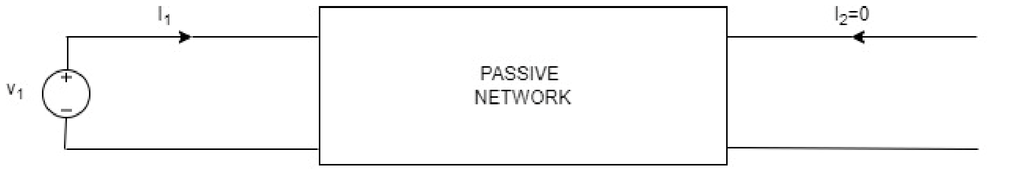
6.6 Reciprocity and symmetrical:

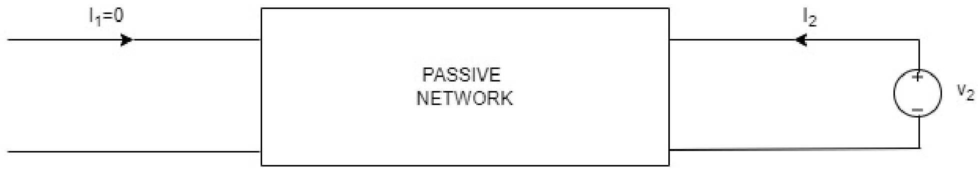
A two port network is said to be symmetrical if the Input and output ports can be interchanged without changing the port voltage and current.

Mathematically,

For symmetrical network,

$$\frac{V_1}{I_1}\ at\ (I_2 = 0) = \frac{V_2}{I_2}\ at\ (I_1 = 0) \qquad (6.43)$$





If the network is reciprocal the ratio of response transform to the excitation remains same if we interchange the position of the excitation and response in the network.

$$\frac{V_1}{I_2}\ at\ (V_2 = 0) = \frac{V_2}{I_1}\ at\ (V_1 = 0) \qquad (6.44)$$

Z-parameter

$V_1 = Z_{11}I_1 + Z_{12}I_2$

$V_2 = Z_{21}I_1 + Z_{22}I_2$

$I_2 = 0$, $\frac{V_1}{I_1}$ at $(I_2 = 0) = Z_{11}$, , $\frac{V_2}{I_2}\ at (I_1 = 0) = Z_{22}$

$Z_{11} = Z_{22}$

For reciprocity,

$\frac{V_1}{I_2}\ at\ (V_2 = 0) = \frac{V_2}{I_1}\ at\ (V_1 = 0)$

$V_1 = Z_{11}I_1 + Z_{12}I_2$

$0 = Z_{21}I_1 + Z_{22}I_2$

$\Rightarrow I_1 = -\frac{Z_{22}}{Z_{21}} I_2$

$V_1 = -\frac{Z_{22}}{Z_{21}} I_2 \times Z_{11} + I_2 Z_{12}$

$V_1 = I_2\left[Z_{12} - \frac{Z_{11}Z_{22}}{Z_{21}}\right] = I_2\left[\frac{Z_{12}Z_{21} - Z_{11}Z_{22}}{Z_{21}}\right]$

$\frac{V_1}{I_2} = \left[\frac{Z_{12}Z_{21} - Z_{11}Z_{22}}{Z_{21}}\right]$ (6.45)

Similarly,

$0 = Z_{11}I_1 + Z_{12}I_2$

$\Rightarrow I_2 = -\frac{Z_{11}}{Z_{12}}I_1$

$V_2 = Z_{21}I_1 + Z_{22}I_2$

$= Z_{21}I_1 - Z_{22}\frac{Z_{11}}{Z_{12}}I_1$

$V_2 = I_1\left[\frac{Z_{12}Z_{21} - Z_{11}Z_{22}}{Z_{12}}\right]$

$\frac{V_2}{I_1} = \left[\frac{Z_{12}Z_{21} - Z_{11}Z_{22}}{Z_{12}}\right]$ (6.46)

From equation (6.45) and (6.46),

$Z_{12} = Z_{21}$

ABCD Parameter:

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

Symmetrical:

$\frac{V_1}{I_1}\ at(I_2 = 0)$

$\frac{V_1}{I_1} = \frac{A}{C}$ (6.47)

$\frac{V_2}{I_2}$ at $(I_1 = 0)$

$0 = CV_2 - DI_2$

$CV_2 = DI_2$

$\frac{V_2}{I_2} = \frac{D}{C}$ (6.48)

From equation (6.47) and (6.48),

$\frac{A}{C} = \frac{D}{C}$

A=D

Reciprocal:

$V_1 = 0, \quad 0 = AV_2 - BI_2$

$AV_2 - BI_2 \Rightarrow I_2 = \frac{A}{B} V_2$

$I_1 = CV_2 - DI_2 = CV_2 - D\frac{A}{B}V_2 \Rightarrow \frac{V_2}{I_1} = \frac{B}{BC-AD}$

$\frac{V_2}{I_1}\ at(V_1 = 0) = \frac{B}{BC-AD}$ (6.49)

$\frac{V_1}{I_2}\ at\ (V_2 = 0),\quad V_1 = -B\ I_2 \Rightarrow \frac{V_1}{I_2}$=-B

$I_1 = -DI_2$

$\frac{V_1}{I_2}\ at\ (V_2 = 0)$=-B (6.50)

From equation (6.49) and (6.50)

$\frac{B}{BC-AD} = -B$

BC-AD=-1

AD-BC=1

h-parameter:

$V_1 = h_{11}I_1 + h_{12}V_2$

$I_2 = h_{21}I_1 + h_{22}V_2$

$\frac{V_1}{I_1}\ at\ (I_2 = 0) = \frac{V_2}{I_2}\ at(I_1 = 0)$, $I_2 = 0$,

$0 = h_{21}I_1 + h_{22}V_2$

$V_2 = -\frac{h_{21}}{h_{22}}I_1$

$V_1 = h_{11}I_1 + h_{12}\left[-\frac{h_{21}}{h_{22}}I_1\right]$

$V_1 = I_1[\frac{h_{11}h_{22}-h_{12}h_{21}}{h_{22}}] \Rightarrow \frac{V_1}{I_1}\ at\ (\ I_2 = 0) = [\ \frac{h_{11}h_{22}-h_{12}h_{21}}{h_{22}}]$ (6.51)

$I_1 = 0,\quad I_2 = h_{22}V_2 \Rightarrow \frac{V_2}{I_2} = \frac{1}{h_{22}}$ (6.52)

$h_{11}h_{22} - h_{12}h_{21} = 1$, Δh=1(symmetrical)

$\frac{V_1}{I_2}\ at(V_2 = 0) = \frac{V_2}{I_1}\ at\ (V_1 = 0)$

$V_2 = 0\ ,\ \ V_1 = h_{11}I_1$

$I_2 = h_{21}I_1 \Rightarrow I_1 = \frac{I_2}{h_{21}}$

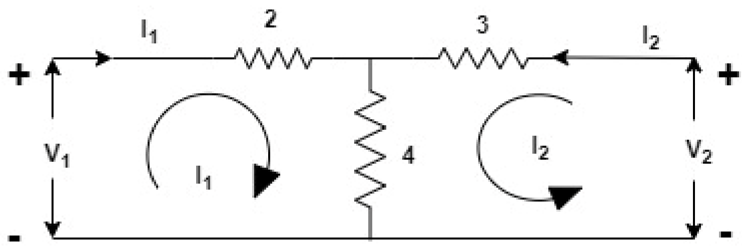
$V_1 = h_{11}\frac{I_2}{h_{21}} \Rightarrow \frac{V_1}{I_2}\ at(V_2 = 0) = \frac{h_{11}}{h_{21}}$ (6.53)

$0 = h_{11}I_1 + h_{12}V_2 \Rightarrow V_2h_{12} = -h_{11}I_1 \Rightarrow \frac{V_2}{I_1} = -\frac{h_{11}}{h_{12}}$ (6.54)

$\frac{h_{11}}{h_{21}} = -\frac{h_{11}}{h_{12}}$

$h_{12} = -h_{21}$

Q.1 Find out the Z, Y, ABCD parameter of the given out



$V_1 - 2I_1 - 4 (I_1 + I_2) = 0$

$V_1 = 6I_1 + 4I_2$ (1)

$V_2 - 3I_2 - 4 (I_1 + I_2) = 0$

$V_2 = 4I_1 + 7I_2$ (2)

$V_1 = Z_{11}I_1 + Z_{12}I_2$ (3)

$V_2 = Z_{21}I_1 + Z_{22}I_2$ (4)

Comparing the equation (1) & (3), (2) & (4)

$Z_{11} = 6\Omega$, $Z_{12} = 4\Omega$

$Z_{21} = 4\Omega$, $Z_{22} = 7\Omega$

$I_1 = y_{11}V_1 + y_{12}V_2$ (5)

$I_2 = y_{21}V_1 + y_{22}V_2$.(6)

From equation (2)

$I_2 = \frac{V_2 - 4I_1}{7}$

$V_1 = 6I_1 + 4\left(\frac{V_2 - 4I_1}{7}\right)$

$V_1 = 6I_1 + \frac{4}{7}V_2 - \frac{16}{7}I_1$

$I_1 - \frac{4}{7}V_2 = \left(\frac{42-16}{7}\right)I_1 = \left(\frac{26}{7}\right)I_1$

$V_1 - \left(\frac{4}{7}\right)V_2 = \left(\frac{42-16}{7}\right)I_1 = \frac{26}{7}I_1$

$I_1 = \frac{7}{26}V_1 - \frac{4}{26}V_2$.(7)

Comparing the equation 5 and 7

$Y_{11} = \frac{7}{26}$ ℧ , $y_{12} = \frac{-4}{26}$ ℧

From equation (1)

$I_1 = \frac{V_1 - 4_2}{6}$

$V_2 = 4\left(\frac{V_1 - 4I_2}{6}\right) + 7I_2$

$V_2 = \frac{4}{6} V_1 - \frac{16}{6} I_2 + 7I_2$

$7I_2 - \frac{16}{6} I_2 = - \frac{4}{6} V_2 + V_2$

$I_2\left(\frac{42-16}{6}\right) = - \frac{4}{6} V_1 + V_2$

$I_2\frac{26}{6} = - \frac{4}{6} V_1 + V_2$

$I_2 = - \frac{4}{26} V_1 + \frac{6}{26} V_2$ (8)

Comparing 6 and 8

$Y_{21} = {-4}/{26}$, $y_{22} = {6}/{26}$