Module-3

Subject: Network Theory

Content: Coupled and Resonance circuit

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3.0 Coupled circuit:

The coupled circuit associated with magnetic circuit. The best example is 1-phase transformer. In single phase transformer having two circuit or winding they are electrically isolated with each other but magnetically coupled with each other. In this circuit both self and mutual inductance present for the operation of transformer by supplying an alternating voltage to the circuit/winding.

3.1 Self-Inductance:

As per Faraday's Law, when a current change in a circuit, the magnetic flux linking the same circuit changes and an emf is induced in the circuit.

This induced emf in the circuit is proportional to the rate of change of current.

VL=L $\frac{dI}{dt}$ where

L is the self-inductance of the circuit

Also, L= $\frac{N\phi}{I}$

N=Number of turns in the circuit, φ=flux linkage of the circuit

VL=L $\frac{d\frac{N\phi}{L}}{dt}$ =N $\frac{d\phi}{dt}$

3.2 Mutual Inductance:

When two coils are electrically isolated but magnetically coupled with each other than there will be mutual inductance present between these coils.

Suppose I_1 and I_2 are the two-current carrying in the coil-1 and coil-2 respectively than due to these current produces leakage flux as well as linkage flux or mutual flux.

Let ϕ_{11} and ϕ_{22} are the leakage flus in coil-1 and coil-2

ϕ_{21} and ϕ_{12} are linkage or mutual fluxes in coil-1 and coil-2 respectively.

When current i1 is passes through the coil-1 than voltage is induced in the coil-2 which is given by

VL2=N2 $\frac{d\phi 12}{dt}$ (3.1)

As I1 is directly proportional to φ12N2

Where N2=No. of turn in coil-2

ϕ_{12} N2=MI1 where M is constant of proportionality called mutual inductance between the two coil.

VL2= $\frac{d(N2\phi 12)}{dt} = \frac{d(MI1)}{dt}$

VL2=M $\frac{dI1}{dt}$ (3.2)

From equation (1) and (2),

N2 $\frac{d\phi 12}{dt}$ =M $\frac{dI1}{dt}$

M=N2 $\frac{d\phi 12}{dI1}$ (3.3)

Similarly, when current passes through the coil-2 than mutual inductance is given by

M=N1 $\frac{d\phi 21}{dI2}$

3.3 Co-efficient of coupling (k):

The fraction of total flus which links the coil is called the co-efficient of coupling(k)

K= $\frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2}$ where $\phi_1 = \phi_{11} + \phi_{12}$ $\phi_2 = \phi_{22} + \phi_{21}$ $\phi 1 \geq \phi_{12}, \phi_2 \geq \phi_{21}$

As M=N2 $\frac{d\phi 12}{dI1}$ (3.4)

M=N1 $\frac{d\phi 21}{dI2}$ (3.5)

Multiplying equation (3.4) and (3.5)

$$M^2 = \frac{N_1 N_2 \phi_{12} \phi_{21}}{I_1 I_2}$$

$$M^2 = \frac{N_1 N_2 k\phi_1 k\phi_2}{I_1 I_2} = k^2 \frac{N_1\phi_1}{I_1} \frac{N_2\phi_2}{I_2} = k^2 L_1 L_2 \quad \text{As } L_1 = \frac{N_1\phi_1}{I_1} \text{ and } L_2 = \frac{N_2\phi_2}{I_2}$$

$$M^2 = k^2 L_1 L_2$$

$$M = k\sqrt{L_1 L_2}$$

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

This constant k is called the co-efficient of coupling.

It is also defined as the ratio of mutual inductance actually present between the two coils to the maximum possible value of mutual inductance.

The maximum possible value of mutual inductance can be obtained in three different cases

Case-1: If the flux due to one coil completely links with other than k=1(Coil tightly coupled)

Case-2: If the flux due to one coil dose not link with other than k=0(magnetically isolated from each other)

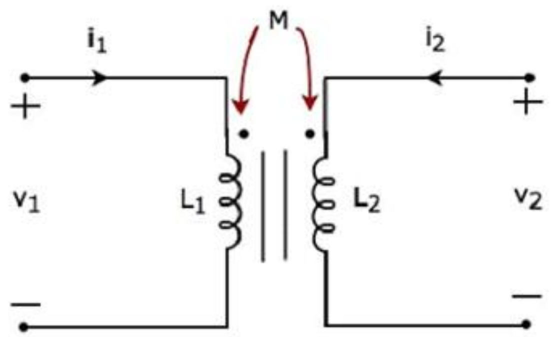
Case-3: If flux due to one coil slightly /partially links with other coil than k has finite value(0<K>1)

3.4 DOT Convention:

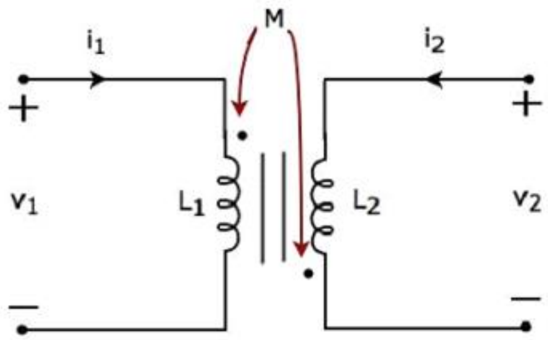
Supposed in the given coupled circuit coil-1 and coil-2 having N1 and N1 turns carrying current i1 and i2 produces two voltages drop in each coil (one voltage produces due to self-inductance and other voltage produces due to mutual inductance. The voltage induced due to mutual inductance is either positive or negative sign depend upon the direction of flow of fluxes or can be determined by use of DOT convention. For this we can apply DOT Rule

DOT Rule:

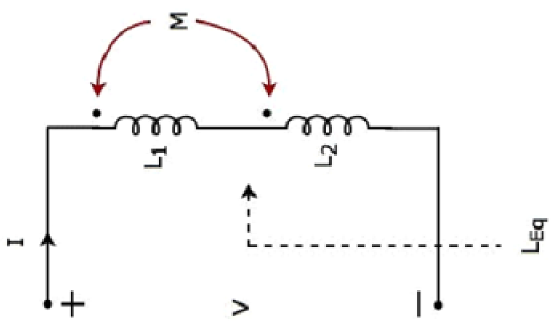
(1) When both assumed currents enter or leave a pair of coupled coils at the dotted terminals, the sign of the M terms will be the same as the sign of the L terms
(2) When one current enters at a dotted terminal and leaves by a dotted terminal, the sign of the M terms is opposite to the sign of the L terms.



(+M)



(-M)



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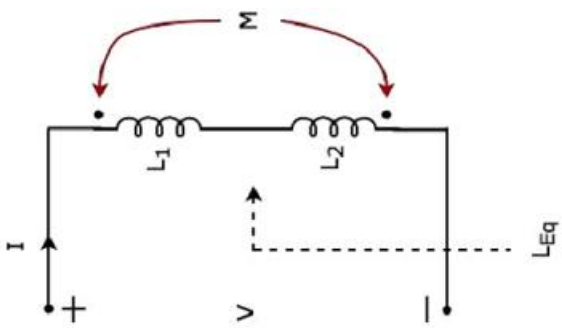
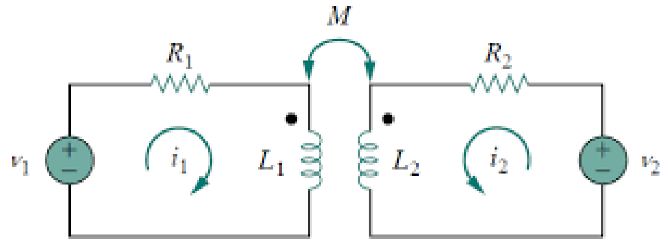


-M

3.5 Voltage Equation of two coupled coils (time domain and frequency domain)



The above coupled circuit we can express the voltage equation in term of time domain and frequency domain

$$V_1 - i_1R_1 - L_1\frac{di_1}{dt} - [+M\frac{di_2}{dt}] = 0 \quad (3.6)$$

$$V_2 - i_2R_2 - L_2\frac{di_2}{dt} - [+M\frac{di_1}{dt}] = 0 \quad (3.7)$$

The equation-1 and 2 known as time domain voltage equation of coupled circuit.

The voltage equation-3.6 and equation-3.7 can be expresses in term of frequency domain as follows

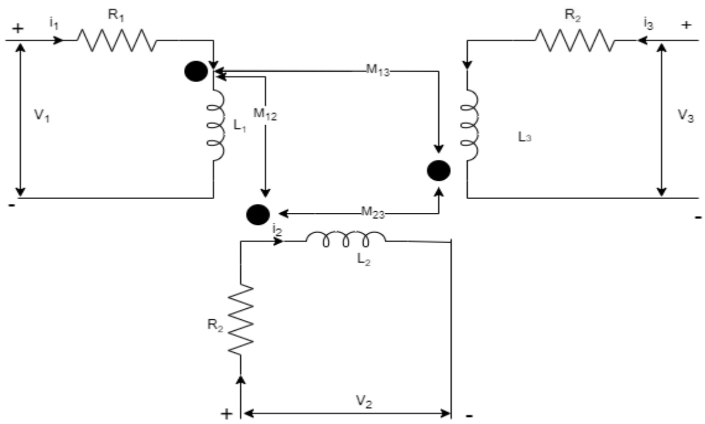
$V_{1=}\ I_1R_1 + j\omega L_1\ I_1 - j\omega MI_2$ (3.8)

$V_{2=}\ I_2R_2 + j\omega L_2\ I_2 - j\omega MI_1$ (3.9)

$V_{1=}\ I_1[R_1 + j\omega L_{1\,]}\, I_1 - [j\omega M]I_2$ (3.10)

$V_{2=}\ I_2[R_2 + j\omega L_2\ ]I_2 - [j\omega M]I_1$ (3.11)

Voltage equation of three coupled coils:



$$V_1 - i_1R_1 - L_1\frac{di_1}{dt} - \left[+M_{12}\frac{di_2}{dt}\right] - \left[-M_{13}\frac{di_3}{dt}\right] = 0$$

$$V_2 - i_2R_2 - L_2\frac{di_2}{dt} - \left[+M_{12}\frac{di_1}{dt}\right] - \left[-M_{23}\frac{di_1}{dt}\right] = 0$$

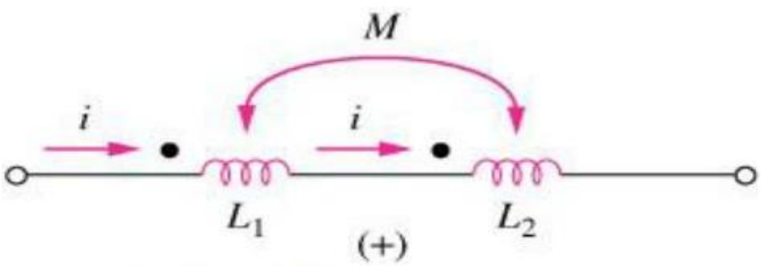
$$V_3 - i_3R_3 - L_3\frac{di_3}{dt} - \left[-M_{13}\frac{di_1}{dt}\right] - \left[-M_{23}\frac{di_2}{dt}\right] = 0$$

Voltage equations in term of time domain

3.6 Different connection of coupled coils:

(a)Series connection of coupled coils:(Series additive)

When two coupled coils are connected in series with dot is given entering point of both the coils.



Then from the above circuit total voltage V=VL1+VL2

$$V_1 = L_1\frac{di}{dt} + M\frac{di}{dt}$$

$$V_2 = L_1\frac{di}{dt} + M\frac{di}{dt}$$

$$V = L_1\frac{di}{dt} + M\frac{di}{dt} + L_2\frac{di}{dt} + M\frac{di}{dt}$$

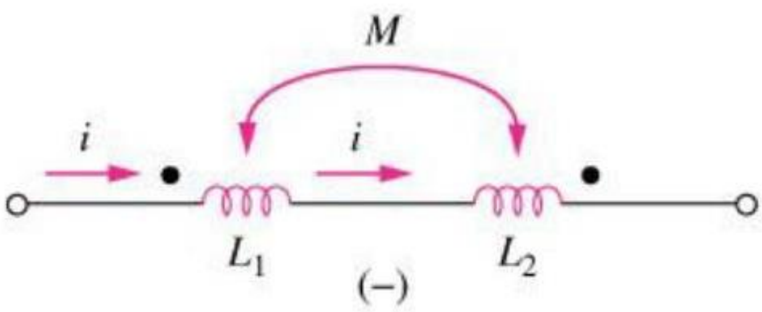
$$V = (L_1 + L_2 + 2M)\frac{di}{dt}$$

$$L_{eq}\frac{di}{dt}=(L_1+L_2+2M)\frac{di}{dt}$$

$$L_{eq}=(L_1+L_2+2M) \qquad (3.12)$$

Equivalent inductance in series connection of two coupled coil in series additive

Csae-2: Series connection of coupled coil (series opposing)



V=VL1+VL2

$$V_1=L_1\frac{di}{dt}-M\frac{di}{dt}$$

$$V_2=L_1\frac{di}{dt}-M\frac{di}{dt}$$

$$V=L_1\frac{di}{dt}-M\frac{di}{dt}+L_2\frac{di}{dt}-M\frac{di}{dt}$$

$$V=(L_1+L_2-2M)\frac{di}{dt}$$

$$L_{eq}\frac{di}{dt}=(L_1+L_2-2M)\frac{di}{dt}$$

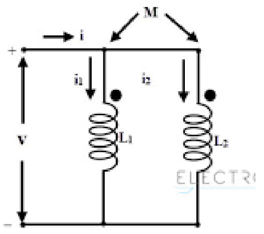
$$L_{eq}=(L_1+L_2-2M) \qquad (3.13)$$

Equivalent inductance in series connection of two coupled coil in series opposing

Case-3: Parallel connection of coupled coils

$i = i_1 + i_2$

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} \quad (3.14)$$



$$V = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \quad (3.15)$$

$$V = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \quad (3.16)$$

$$L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

$$\frac{di_1}{dt} = \left(\frac{L_2 - M}{L_1 - M}\right) \frac{di_2}{dt} \quad (3.17)$$

Putting the value of $\frac{di_1}{dt}$ in equation (3.14)

$$\frac{di}{dt} = \left(\frac{L_2 - M}{L_1 - M}\right) \frac{di_2}{dt} + \frac{di_2}{dt} \, ,$$

$$\frac{di}{dt} = \left[\frac{L_2 - M}{L_1 - M} + 1\right] \frac{di_2}{dt} \quad (3.18)$$

From equation-2

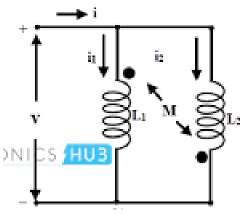
$$L \frac{di}{dt} = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$\frac{di}{dt} = \frac{1}{L}[L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$[\frac{L_2 - M}{L_1 - M} + 1]\frac{di_2}{dt} = \frac{1}{L}[L_1(\frac{L_2 - M}{L_1 - M}) + M]\frac{di_2}{dt}$$

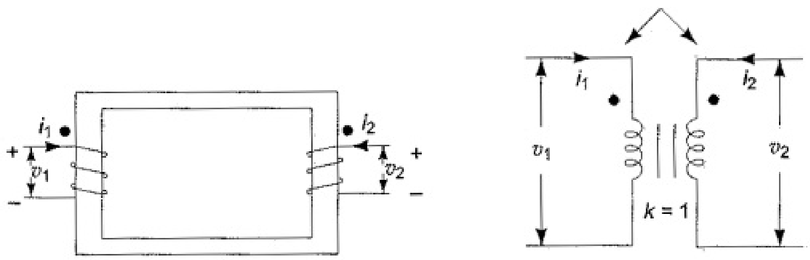
$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} \qquad (3.19) \text{ (Parallel adding)}$$

Similarly, in the same brochure for parallel opposing the equivalent inductance



$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} \qquad (3.20)$$

3.7 Ideal Transformer



(a) (b) $N_1 : N_2$

The magnitude of self induced emf $V = L\frac{di}{dt} = N\frac{d\phi}{dt}$

$$\Rightarrow L = N\frac{d\phi}{di}$$

But $\phi = \frac{Ni}{S}$,S= reluctance

$$L = N\frac{d}{di}(\frac{Ni}{S})$$

$$L = N\frac{d}{di}(\frac{Ni}{S}) = \frac{N^2}{S}$$

From the above $\frac{L_2}{L_1} = \frac{N_2{}^2}{N_1{}^2} = a^2$

The ideal transformer is very useful model for circuit calculations. The input impedance of the transformer can be determined by using the voltage equation in (b) by inserting a load impedance Z_L in secondary side coil across v_2.

$$V_1 = j\omega L_1 I_1 - j\omega M I_2 \qquad (3.21)$$

$$0 = -j\omega M I_1 + (Z_L + j\omega L_2) I_2 \qquad (3.22)$$

Where V1 and V2 are thr voltage phasors and I1 ,I2 are the current phasors

From equation (3.222) , $I_2 = \frac{j\omega M I_1}{(Z_L + j\omega L_2)}$

Put I2 in equation (3.21)

$$V_1 = j\omega L_1 I_1 + \frac{I_1 \omega^2 M^2}{Z_L + j\omega L_2}$$

The input impedance $Z_{in} = \frac{V_1}{I_1} = j\omega L_1 + \frac{\omega^2 M^2}{(Z_L + j\omega L_2)}$

Let us assumed k=1, $M = \sqrt{L_1 L_2}$

$$Z_{in} = \frac{V_1}{I_1} = j\omega L_1 + \frac{\omega^2 L_1 L_2}{(Z_L + j\omega L_2)}$$

We have already known the relation, $\frac{L_2}{L_1} = a^2$ where $a = \frac{N_2}{N_1}$ =turn ratio

$$Z_{in} = j\omega L_1 + \frac{\omega^2 L_1{}^2 a^2}{(Z_L + j\omega L_2)}$$

$$Z_{in} = \frac{(Z_L + j\omega L_2)\, j\omega L_1 + \omega^2 L_1{}^2 a^2}{Z_L + j\omega L_2}$$

$$Z_{in} = \frac{j\omega L_1 Z_L}{Z_L + j\omega L_2}$$

As L_2 is assumed to be infinitely large compared to Z_L

$$Z_{in} = \frac{j\omega L_1 Z_L}{j\omega a^2 L_1} = \frac{Z_L}{a^2} = (\frac{N_1}{N_2})^2 Z_L \qquad (3.23)$$

Q.1.In a pair of coupled coils coil 1 has a continuous current of 2A, and the corresponding fluxes ϕ_{11} and ϕ_{21} are 0.3 and 0.6mwb respectively. If the turns are N_1=500 and N_2 =1500,find L_1, L_2, M and k.

Sol:

Total flux $\phi_1 = \phi_{11} + \phi_{21} = 0.3 + 0.6 = 0.9$ mwb

$$L_1 = \frac{N_1 \phi}{i_1} = \frac{(500)(0.9 \times 10^{-3})}{2} = 0.225H$$

$$k = \frac{\phi_{21}}{\phi_1} = \frac{0.6}{0.9} = 0.667$$

$$M = \frac{N_2 \phi_{21}}{i_1} = \frac{(1500)(0.6 \times 10^{-3})}{2} = 0.45H$$

$$M = k\sqrt{L_1 L_2}$$

$$0.45 = 0.667\sqrt{(0.225)L_2}$$

$$L_2 = 2.023H$$

Q.2. Two coupled coils with L1=0.01H and L2=0.04H and k=0.6 are connected in four different ways, series aiding, series opposing, parallel aiding and parallel opposing. Find the equivalent inductances in each case.

Sol: L_1 =0.01H, L_2 =0.04H and k =0.6

$$M = k\sqrt{L_1 L_2} = 0.6\sqrt{(0.01)(0.04)} = 0.012H$$

(i) Series aiding $L_{eq} = (L_1 + L_2 + 2M) = 0.01 + 0.04 + 2(0.012) = 0.074H$

(ii) Series opposing $L_{eq} = (L_1 + L_2 - 2M) = 0.01 + 0.04 - 2(0.012) = 0.026H$

(iii) Parallel aiding

$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = \frac{(0.01)(0.04) - (0.012)^2}{(0.01) + (0.04) - 2(0.012)} = 9.846mH$$

(iv)Parallel opposing $L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} = \frac{(0.01)(0.04) - (0.012)^2}{0.01 + 0.04 + 2(0.012)} = 3.459mH$

Q.3. Two coils when connected in series have a combined inductance of 0.8H or0.6H depending on the mode of connection, one of the coils when isolated from the other has inductance of 0.3H. Find

(a) The mutual inductance between the two coils

(b) the inductance of the other coil

(c) the coupling co-efficient

Sol: $0.8 = (L_1 + L_2 + 2M)$ (1)

$0.6 = (L_1 + L_2 - 2M)$ (2)

Adding equation (1) and (2)

$$1.4 = 2(L_1 + L_2)$$

$$L_2 = 0.7 - 0.3 = 0.4H$$

From equation (1), $0.8 = (0.3 + 0.4 + 2M)$

$M = 0.05H$

$M = k\sqrt{L_1 L_2}$

$0.05 = k\sqrt{0.3*0.4}$, $k = 0.144$

Q.4. The coupled coils have self-inductances L1=10mH,L2=20mH and k=co-efficient of coupling=0.75. Find the Mutual induce (M) , voltage in the second coil and the flux of first coil provided the second coil has 500 turns and the current is given by i1=2sin314t A.

$$M = k\sqrt{L}L2$$

Q2. The two coupled coils having self-inductances L1=0.1H and L2=0.02H and k=0.75 are connected in series aiding, series opposing, parallel aiding and parallel opposing. Find the equivalent inductance in each case.

Sol. L1=0.1H, L2=0.02H, k=0.75

$M = k\sqrt{L}L2$=0.75$\sqrt{0.1 * 0.02}$=

Single Tuned and Double Tuned Circuits:

The single tuned circuit can be expressed that if in a coupled circuit the second coil contains a variable capacitor with the help of secondary circuit makes tuned to resonance.

Similarly, the double tuned circuit can be expressed that if in a coupled circuit both first and second coil contains a variable capacitor to makes a tuned to resonance. The tune circuit uses various application such as broad casting receivers, TV receivers, isolation of input from output circuit. The singe tuned circuit can

Resonance:

Resonance in R-L-C circuit:

We know that from R-L-C

Series circuit, the net reactance

X=X_L-X_C, $Z = \sqrt{R^2 + (X_L - X_c)^2}$

$Z = \sqrt{R^2 + X^2}$

If for same frequency If the applied voltage $|X_L| = |X_C|$

Then X=0

So $Z = \sqrt{R^2 + 0} = R$

The phasor diagram for X=0, i.e. series resonance

Since from the phasor diagram, V_L=0 and V_C=1X_C and two are equal in magnitude but opposite in phase. The two reactance taken to get the act as short cut since no voltage developed across them. The applied voltage V drops entirely across R. So that V=V_R. The cut impedance Z=R.

The frequency at which net reactance is zero is given by

$X_L - X_C = 0$

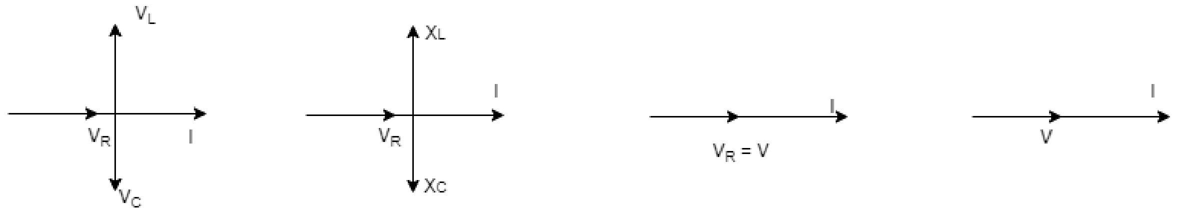
$X_L = X_C$

$\omega_L = \frac{1}{\omega_C}$ or $\omega^2 = \frac{1}{L_c}$

$\omega = \frac{1}{\sqrt{L_c}}$ $2\pi f_0 = \frac{1}{\sqrt{L_c}}$

$$f_0 = \frac{1}{2\pi\sqrt{L_c}}$$

We have seen that under these conditions. The impedance of the circuit equal to the ohmic resistance R. So, the current is maximum and it is value $I_0 = \frac{V}{R}$ is in phase with V. This condition is known as series resonance and the frequency at which it occurs is called resonant frequency ‘f_0’. Hence a series R-L-C circuit is said to be in resonance may be described as a condition of maximum admittance and minimum impedance.



Graphical representation of resonance:

Suppose an alternating current with unit magnitude but od varying frequency is applied to an R-L-C circuit. The variation of resistance and inductive reactance X_L and capacitive reactance X_C with frequency as shown.

R since resistance does not change in frequency. So the curve is straight horizontal line with a magnitude of R above the frequency axis.

$X_L = \omega_L = 2\pi f L$

$X_L \propto f$

Inductive reactance X_L is directly proportional to f. So the graph is a straight line through origin. So X_L is increase linearly with f.

$X_C = \frac{1}{\omega_c} = \frac{1}{2\pi f L}$

So $X_C \propto \frac{1}{f}$

Capacitive reactance is inversely proportional to f. this graph is a rectangular hyperbola which is drawn in 4th quadrant because X_C is negative.

Net resonance X= X_C+X_L

The graph is hyperbola (not rectangular) and cross the X axis at a point A. The value of frequency at point A is called resonance frequency.

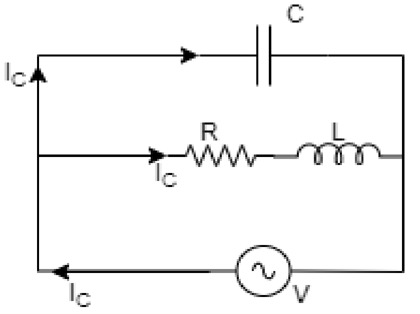
$Z = \sqrt{R^2 + (X_L - X_c)^2} = \sqrt{R^2 + X^2}$

At low frequency, Z is large. But as $X_C > X_L$, impedance is capacitive and the p.f. is leading. At frequency, Z is again large but it is inductive because $X_C > X_L$ and the p.f. is lagging, its minimum value is R, When $X_C = X_L$.

Parallel resonance:

A parallel A.C. cut is said to be resonance when its susceptance is zero. Since at parallel resonance the applied voltage and resulting current will be in phase as a result of whice power factor will be unity.

Consider a general parallel R-L-C cut



It represents a parallel resulting cut where a coil connected across a AC voltage source of variable frequency.

Here $Y_L = \frac{1}{R+j\omega_L} = \frac{R-j\omega L}{R^2+(\omega_L)^2}$

And $Y_C = \frac{1}{X_c} = \frac{1}{1/j\omega_L} = j\omega_L$

$$Y_T = Y_L + Y_C = \frac{R-j\omega L}{R^2+(\omega_L)^2} + j\omega_L = \frac{R-j\omega L}{R^2+(\omega_L)^2} + j\omega(C - \frac{L}{R^2+\omega^2 L^2})$$

For resonance susceptance will be zero.

$$W_\circ(e - \frac{L}{R^2+W_\circ^2 L^2}) = 0$$

$$e = \frac{L}{R^2+W_\circ^2 L^2}$$

$$R^2 + W_\circ^2 L^2 = \frac{L}{e}$$

$$W_\circ^2 L^2 = \frac{L}{e} - R^2$$

$$W_\circ^2 = \frac{1}{Le} - \frac{R^2}{L^2}$$

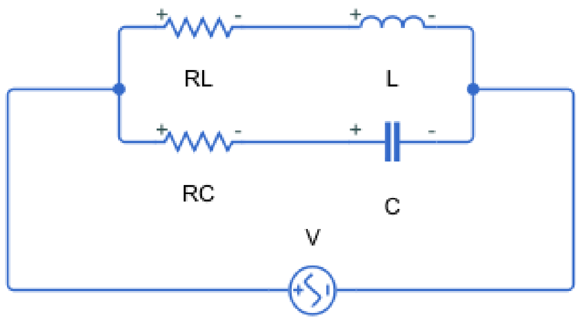
$$W_\circ = \sqrt{\frac{1}{Le} - \frac{R^2}{L^2}}$$

$$f_\circ = \frac{1}{2\pi}\sqrt{\frac{1}{Le} - \frac{R^2}{L^2}}$$

If R is negligible,

$f_\circ = \frac{1}{2\pi\sqrt{Le}}$, which is universal as for series resonance cut.

2. The admittance YL of branch containing L and R ,



$$YL = \frac{1}{ZL} = \frac{1}{R_L + iXL} = \frac{R_{L - JXL}}{{R_L}^2 + XL^2}$$

Simultaneously, admittance Yc of branch containing capacitance C and resistance R is given by,

$$Y_C = \frac{1}{2c} = \frac{1}{R_L - JX_c} = \frac{R_{c - JXc}}{{R_c}^2 + Xc^2}$$

Therefore, the admittance YT will be,

YT = YL + YC

$$\frac{R_{L - JXL}}{{R_L}^2 + XL^2} + \frac{R_{c - JXc}}{{R_c}^2 + Xc^2}$$

$$\left(\frac{R_L}{{R_L}^2 + XL^2} + \frac{R_c}{{R_c}^2 + Xc^2}\right) + \mathrm{i}\left(\frac{X_c}{{R_c}^2 + Xc^2} - \frac{X_L}{{R_L}^2 + XL^2}\right)$$

At resonance, susceptance will be Zero i.e., complex admittance will be a real number.

$$\frac{X_c}{{R_c}^2 + Xc^2} - \frac{X_L}{{R_L}^2 + XL^2} = 0$$

$$\frac{X_c}{{R_c}^2 + Xc^2} = \frac{X_L}{{R_L}^2 + XL^2}$$

$$\frac{\frac{1}{W_\circ c}}{R_c{}^2 + \frac{1}{W_\circ{}^2 c^2}} = \frac{W_\circ L}{R_l{}^2 + \frac{1}{W_\circ{}^2 l^2}}$$

$$R_l{}^2 + W_\circ{}^2 l^2 = W_\circ{}^2 CL \, (R_c{}^2 + \frac{1}{W_\circ{}^2 c^2})$$

$$W_\circ{}^2 LC \left(\frac{L}{C - R_c{}^2}\right) = \frac{L}{C} - R_l{}^2$$

$$W_\circ = \frac{1}{\sqrt{LC}} \sqrt{\frac{\frac{L}{C} - R_l{}^2}{\frac{L}{C} - R_c{}^2}} = \frac{1}{\sqrt{LC}} \sqrt{\frac{R_l{}^2 - \frac{L}{C}}{R_c{}^2 - \frac{L}{C}}}$$

$$f_\circ = \frac{1}{2\pi\sqrt{LC}} \sqrt{\frac{R_l{}^2 - \frac{L}{C}}{R_c{}^2 - \frac{L}{C}}}$$

$\therefore$ *Admittance at resonance will be,*

$Y_\circ = (\frac{R_L}{R_L{}^2 + XL^2} + \frac{R_c}{R_c{}^2 + Xc^2})$, Scince, imagining root equation to be zero. And, Current resonance will be $I_\circ = Y_\circ V = v(\frac{R_L}{R_L{}^2 + XL^2} + \frac{R_c}{R_c{}^2 + Xc^2})$

Properties of resonance of parallel RLC Circuit

1. Power factor is unity
2. Current at resonance and is in phase with the applied voltage. The value of current at resonance is minimum.
3. Net impedance at resonance of the parallel circuit is maximum
4. The admittance is minimum and net sum acceptance is zero at resonance.
5. The resonance frequency of this circuit is given by

$$f_\circ = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

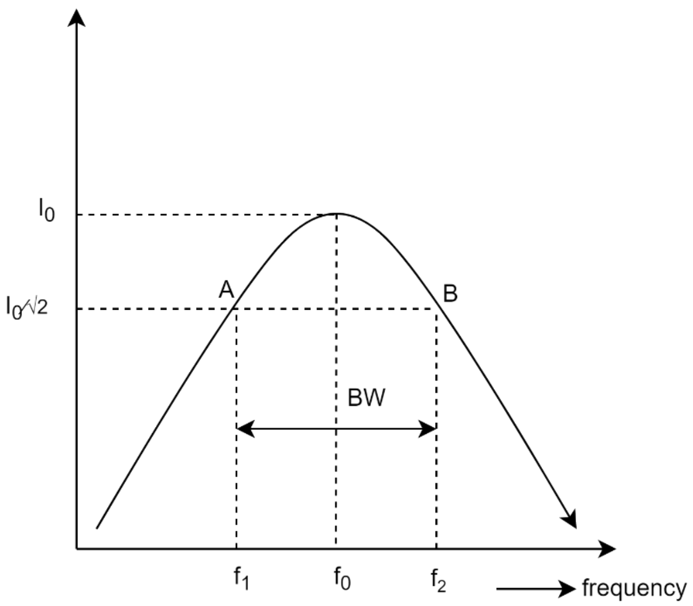
Properties of Resonance of RLC series Circuit

1. The applied voltage and the line resulting current are in phase which also means that the P.F.A of the RLC series resonant circuit is unity.
2. The net reactance is zero at resonance and the impedance does not have the respective potential.
3. The current in the circuit is maximum.
4. At resonance this circuit has got minimum impedance and maximum admittance.
5. Frequency of resonance is given by

$$f_\circ = \frac{1}{2\pi\sqrt{LC}}\ Hz$$

Bandwidth of series resonance circuit

Bandwidth of a circuit is given by the band frequencies which lie between two ports on either side of the resonance frequency where current falls $1/\sqrt{2}$ of its maximum value at resonance. The ratio of the resonant frequency is defined as the selectivity $f_\circ/_{Bw}$



The bandwidth AB is given by

Δf = $f_2 - f_1$ or, Δw = $w_2 - w_1$, Science at the point A and B at which current falls to $1/\sqrt{2}$ of its value at resonance where $I_\circ = \frac{V}{R}$ the maximum current at resonance. So, power at the ports A and B is equal to I^2R .

$$(\frac{I_0}{\sqrt{2}})^2\ R = \frac{1}{2} I_0{}^2 R = \frac{1}{2}\ \times Power\ at\ Resonance$$

That is why these ports A and B on the response curve are known as half power ports.

Let us now find out the values of w_1 and w_2.

$I_0 = \frac{V}{R}$ at Resonance.

$I = \frac{V}{\sqrt{R^2 + (wl - \frac{1}{wl})^2}}$ at any frequency. But, at port A and B, I = $\frac{I_0}{\sqrt{2}} = \frac{1}{\sqrt{2}}\ \frac{V}{R}$

$$1/\sqrt{2} \times \frac{V}{R} = \frac{V}{\sqrt{R^2 + (wl - \frac{1}{wl})^2}}$$

$$\sqrt{2}\, R = \sqrt{R^2 + (wl - \frac{1}{wl})^2}$$

$$2R^2 = R^2 + (wl - \frac{1}{wl})^2$$

$$R^2 = (wl - \frac{1}{wl})^2 = X^2$$

$$X^2 = R^2 \rightarrow X = \pm R$$

It shows that at half power points the net reactance is equal to the resistance. The current at frequency $f_2$is given by:

$$If_2 = \frac{V}{\sqrt{R^2 + (Xl - XC)^2}}$$

But, at f_2, $Xl - XC = R$

$$\therefore If_2 = \frac{V}{\sqrt{R^2 + R^2}} = \frac{V}{\sqrt{2}\, R} = \frac{1}{\sqrt{2}} I_0$$

Where $I_0 = \frac{V}{R}$, the current resonance, Similarly the current at frequency $f_1$is given by :

$If_1 = \frac{V}{\sqrt{R^2 + (X_l - X_c)^2}}$ But, at f_1 $(X_l - X_c = -R)$

$$= \frac{V}{\sqrt{R^2 + (-R)^2}} = \frac{V}{\sqrt{2}\, R} = \frac{1}{\sqrt{2}} I_0$$

These two frequencies f_1 *and* f_2 are called half power frequency because the power dissipated in the circuit of these frequencies is half the power dissipated at resonance frequencies (f_0).

Now, we can find net w_1 *and* w_2. Since

$$X_l - X_c = -R$$

$$W_1 L - \frac{1}{W_1 C} = \text{-R}$$

$$W_1{}^2 LC + W_1 RC - 1 = 0$$

$$W_1{}^2 + W_1 \frac{R}{L} - \frac{1}{LC} = 0$$

$$W_1 = \text{-} \frac{\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} + \frac{4}{LC}}}{2}$$

$$Since, \frac{R^2}{4L^2} \text{ is much less than } \frac{1}{LC}$$

$w_{0=}1/\sqrt{LC}$

$$w_1 = -\frac{R}{2L} \pm \frac{1}{\sqrt{LC}} = -\frac{R}{2L} \pm \omega_0$$

Since only positive value of ω_0 are considered

$$\omega_1 = -\frac{R}{2L} \pm \omega_0 = \omega_0 - \frac{R}{2L}$$

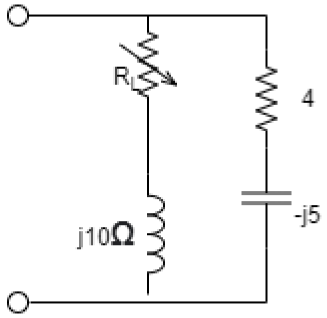
$$f_1 = f_0 - \frac{R}{4\pi L}$$

Similarly,

$\omega_2 = \omega_0 - \frac{R}{2L}$ or

$$f_2 = f_0 - \frac{R}{4\pi L}$$

Q.5. Show that no value of RL in the circuit will make it resonant.



Solution: $Y_L = \frac{1}{R_L + j10} = \frac{R_L - j10}{{R_L}^2 + 100}$

$Y_C = \frac{1}{4 - j5} = \frac{4 + j5}{16 + 25}$,

$Y = Y_L + Y_C = \left[\frac{R_L}{{R_L}^2 + 100} + \frac{4}{41}\right] + j\left[\frac{5}{41} - \frac{10}{{R_L}^2 + 100}\right]$

At resonance,

$\frac{5}{41} = \frac{10}{{R_L}^2 + 100}$ so

${R_L}^2 = -18$

$R_L = j4.24$

6. A series consisting of a capacitor and coil takes a maximum current of 0.314A at 200v , 50Hz. If the voltage across the capacitor is 300v, Determine the value of L, R and Q factor of the coil

Solution – maximum current occurs only under resonance condition

I = V/R = V/Z= 200/R

= 0.314 = 2ω/R, R= 636.94

f_0=50Hz

$$V_C = IX_C = I_0 \times \frac{1}{\omega c} = I_0 \times \frac{1}{2\pi f_0 c} = \frac{0.314}{2\pi \times 50 \times c}$$

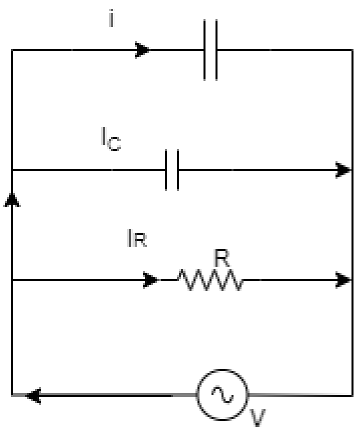
$$300 = \frac{0.314}{2\pi \times 50 \times c}$$

$c = 21\mu F$

$f_0 = \frac{1}{2\pi\sqrt{LC}}$, $\quad {f_0}^2 = \frac{1}{4\pi f_0}$, $L = \frac{1}{{f_0}^2 4\pi^2 c} = 0.48H$

$Q = \frac{\omega_0 L}{R} = \frac{2\pi \times 50 \times 0.48}{636.94}$

10. Find the factor of the parallel resonating circuit?



Y = Y_R+ Y_L+Y_C

= 1/ZC +1/ ZL + 1/ZC

= 1/R + i(ω_C- 1/ωC) The condition for resonance is $\omega_C - \frac{1}{\omega_C} = 0$

ω_0^2 =1/$\sqrt{ZC}$, f_0= $\frac{1}{2\pi\sqrt{LC}}$

Q = R/ωC