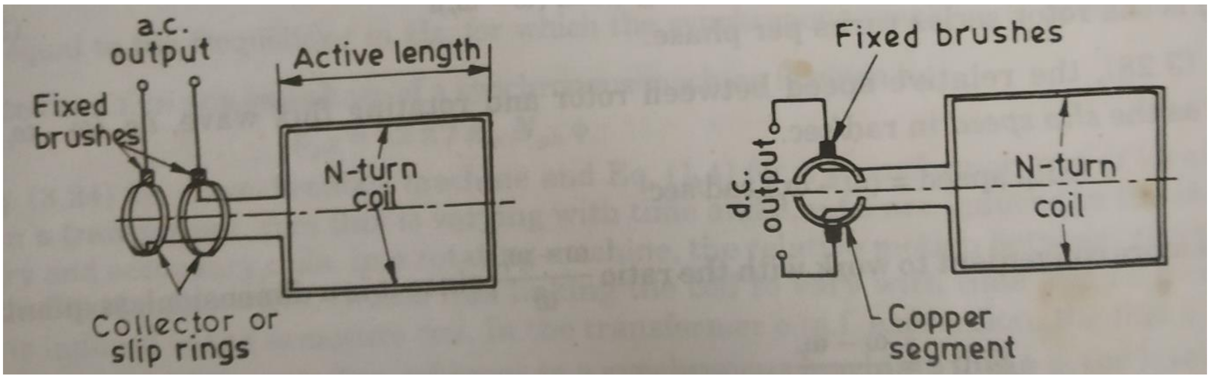
DC Machines

DC Machine is considered to be an electromechanical energy coversion device.

DC Machines:
- DC Generator (Converts Mechanical Energy to Electrical Energy)
- DC Motor(Converts Electrical Energy to Mechanical Energy)

DC Generator

Working Principle of DC Generator:

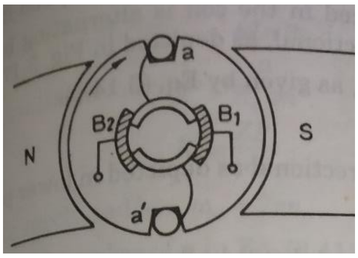


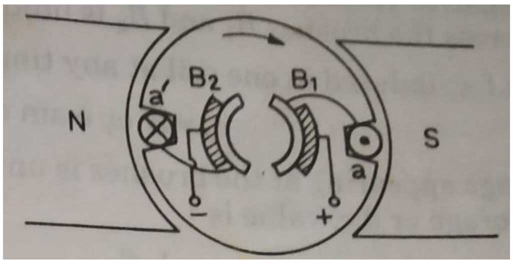
(a) (b)

Fig. 1.1 Elementary forms of (a) A C Generator (b) DC Generator

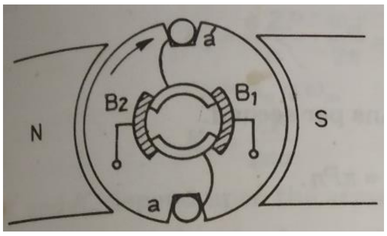
- Elementary form of generator consists of one full pitched N-turn coil with 2 poles as shown in fig. 1.1.
- Alternating current is generated in the N-turn coil of both fig. 1.1 (a) and (b).
- In fig. 1.1 (a), it is directly collected by the sliprings and brushes in AC form and delivered to the external load circuit.
- In fig. 1.1 (b) it is converted from AC to DC by means of a commutator and is then collected by the fixed carbon brushes for onward transmission to load circuit.
- In fig. 1.1 (b), a copper ring is split into two halves. But, in an actual DC machine there are large numbers of coils and accordingly a copper segment is cut into a large number of segments. These copper segments as a whole are called as commutator.

Rectification Process:

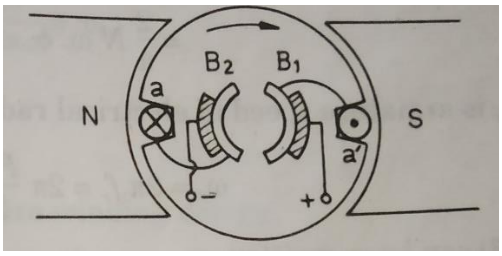




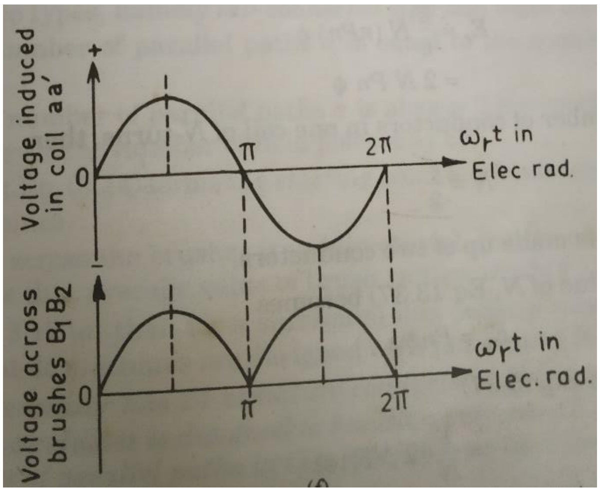
(a) (b)



(c)



(d)



(e)

Fig. 1.2 Rectification of AC to DC by commutator

- Due to the commutator action brush B_1 is always in contact with that coil-side which is under the influence of south pole and brush B_2 is always in contact with that coil-side which is under the influence of north pole for which the polarities of the brushes B_1 and B_2 remain positive and negative respectively.
- Hence, the emf induced in the coil is alternating but the voltage available across the brushes B_1 and B_2 is unidirectional.

Construction of DC Machines

- All the rotating electrical machines have many essential features from the construction point of view. For example, every rotating every electrical machine including DC Machine has (i) Stator (stationary parts), (ii) rotor (rotating parts), and (iii) air-gap separating the stator and rotor.
- In DC machine, stator consists of (i) Yoke, (ii) Field Poles, (iii) bearings, (iv) brush holders, (v) end covers etc. and the rotor consists of (i) armature (ii) commutator, (iii) shaft.

Stator:

(i) Yoke:

- The outer frame of the machine is known as yoke.

- The two functions of the yoke are: (i) it provides path for the pole flux φ and carries half of it i.e. φ/2. (ii) it acts as a protecting cover for the whole machine and provides mechanical support to the whole machine and the poles.
- Cast iron is used for the construction of yoke in small DC machines and fabricated steel, cast steel or rolled steel is used for the construction of yoke in large DC machines.
- Yoke is laminated to reduce the eddy current loss if the machine operated through a power electronics converter.

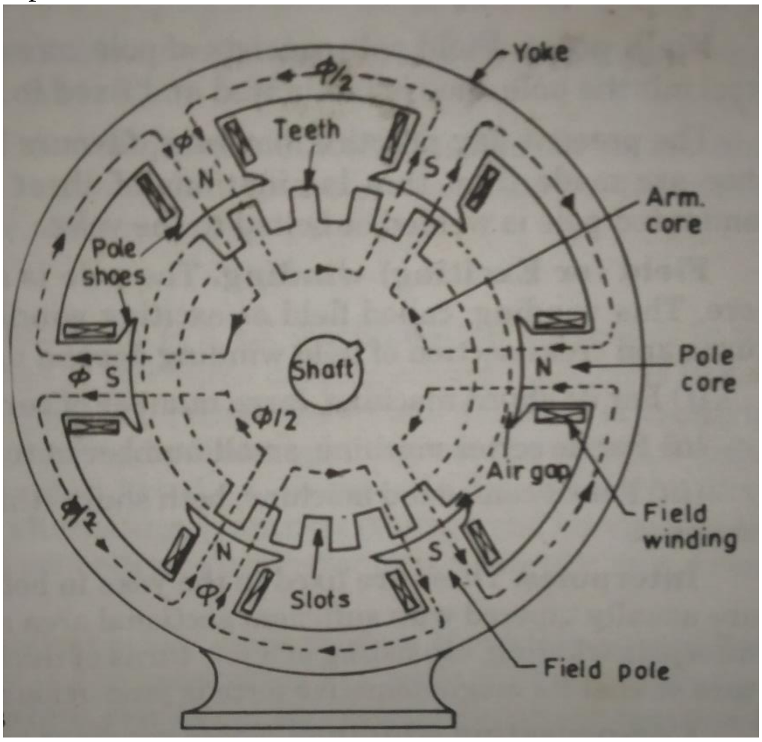


Fig. 1.3 Constructional features of DC Machines

(ii) Field Poles:

- Field pole or field magnet consists of (i) pole core and (ii) pole shoe. Pole core is made from cast iron or cast steel and pole shoe is laminated and fixed to the pole core by means of riveting.
- The pole core is of smaller cross sections than the pole shoe and therefore requires less copper for the field winding.
- The pole shoe serves two purposes: (i) it spreads out the flux in the airgap and being larger cross sections, reduces the reluctance of the magnetic path, (ii) it provides mechanical support and strength to the exciting coils or field windings.

(iii) Field (or Exciting) winding:

- The winding wound around the pole core which is used to excite the pole is known as field winding or exciting winding.
- The field winding consists of copper wire.
- When current is passed through the field winding, the poles are electromagnetised.

(iv) Brushes:

- Brushes are used to collect current from the commutator and are usually made up of carbon or graphite. Brushes are in the shape of a rectangular block and they are housed in the box-type brush holders attached to the stator end cover or the stator yoke.

Rotor:

(i) Armature:

Armature consists of armature core and armature winding.

(a) Armature Core:

- It serves two purposes: (i) it houses the armature coils in the slots (ii) it provides the low reluctance path to the magnetic flux φ/2.
- It is cylindrical in structure and is made from 0.35 to 0.50 mm thick laminations of silicon steel to reduce the iron loss.

(b) Armature Winding:

- Armature windings are made from copper and housed in the armature slots.
- Two types of armature winding are: (i) lap winding & (ii) wave winding.
- Lap winding is suitable for low-voltage high-current machines and wave winding is used for high-voltage low current machines.

(ii) Commutator:

- The main function of the commutator is to collect the alternating current induced in the armature coil and to convert this alternating current into direct current for delivering to the external load circuit.
- It is of cylindrical structure and built of wedge-shaped segments of high conductivity hard drawn or drop forged copper. These segments are insulated from each other by thin layer of mica of thickness 0.8 mm.
- The number of commutator segments is equal to the number of armature coils.

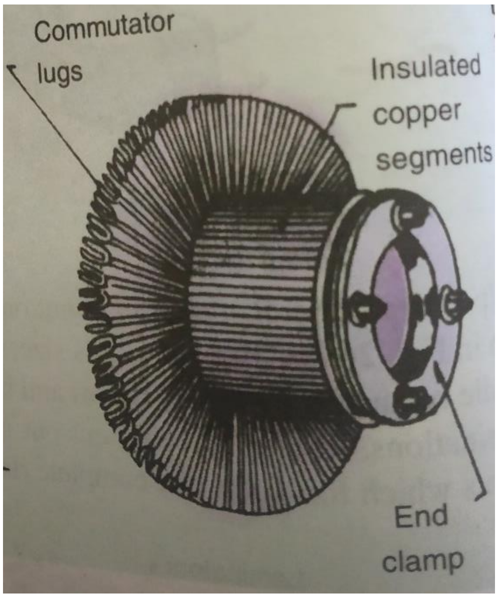


Fig. 1.4 Commutator

(iii) Shaft:

- Armature and commutators are mounted over the shaft.

EMF Equation of DC Generator:

Let, $\phi =$ flux/pole in weber

$Z =$ Total number of armature conductors

$P =$ Nos. of generator poles

$N =$ Armature rotations in revolutions per minute (rpm)

$E_a =$ Generated emf across the armature terminals

According to Farday's Laws of electromagnetic induction,

Average emf generated/conductor $= d\phi / dt$ volts.

Flux cut/conductor in one revolution, $d\phi = \phi P$ Wb.

Time taken for one revolution, $dt = 60 / N$ second.

Hence, Average emf generated/conductor $= d\phi / dt = \phi PN / 60$ volts.

In a DC generator, Z nos. of conductors are arranged in A parallel paths.

Hence, nos. of conductors per parallel path $= Z / A$.

Generated emf across the armature terminals, $E_a =$ Emf generated per parallel path $= (\phi PN / 60)(Z / A)$ volts $= (\frac{\phi ZN}{60} \times \frac{P}{A})$ volts.

$= (ZP / A)(\phi N / 60) = K_e (\phi N / 60)$, where Ke is known as emf constant & $K_e = \frac{ZP}{A}$

Armature speed in mechanical radian/sec $\omega_m = \frac{2\pi N}{60}$

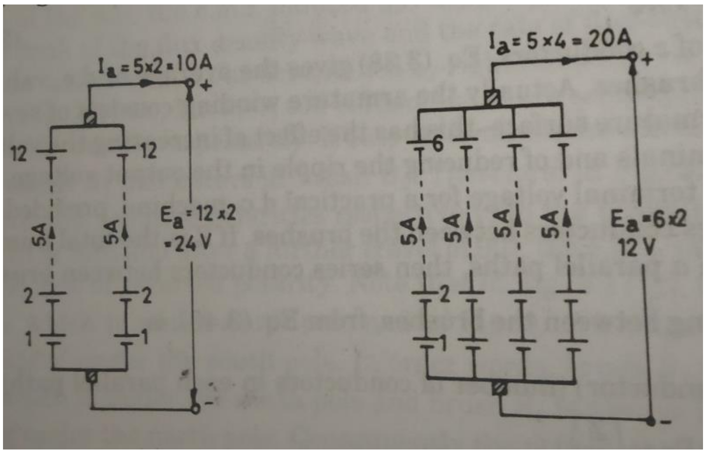
$$\Rightarrow N = 60\omega_m / 2\pi$$

Putting this value in the emf equation, $Ea = (ZP / 2\pi A)\ \phi\ \omega_m = K_a \phi \omega_m$

Where Ka is known as armature constant & $Ka = \frac{ZP}{2\pi A}$.

Ke and Ka depend upon the armature winding design.

For lap winding $A = P$ and for wave winding $A = 2$.



(a) (b)

Fig. 1.5 Armature winding of a 4 pole 24 conductors machine for (a) wave connection, (b) lap connection. (Current carrying capacity of each conductor is assumed to be 5A)

Circuit Model of Armature:

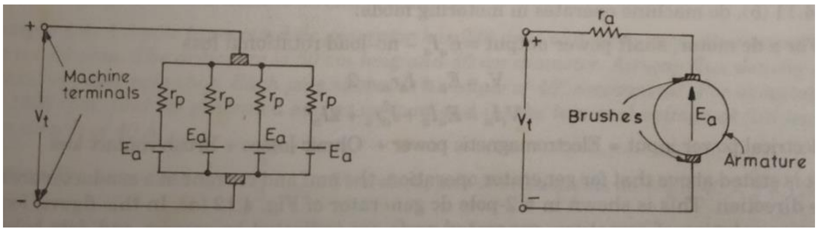


Fig. 1.6 Armature Circuit of a DC machine

Methods of Excitation and Types of DC Machine:

- A DC machine works as an electromechanical energy conversion device only its field winding is excited with DC.
- There are two methods of excitation (i) separate excitation and (ii) self-excitation.
- DC Machines with separate excitation are known as separately excited DC machines.
- DC Machines with self-excitation are known as self-excited DC Machines.

Self-Excitation:

- In case of separate excitation, the field winding of the DC Machine is excited from a separate source of DC supply.
- In case of self-excitation, the field winding of the DC Machine is excited from its armature voltage in case of generator and from the supply voltage in case of motor.

Types of self-excited DC Machines:

(i) Series, (ii) Shunt and (iii) Compound Machines

Compound machines are again two types (i) long shunt and (ii) short shunt.

A long shunt compound machine may be cumulatively compounded or differentially compounded and similarly a short shunt compound machine may be cumulatively compounded or differentially compounded. `

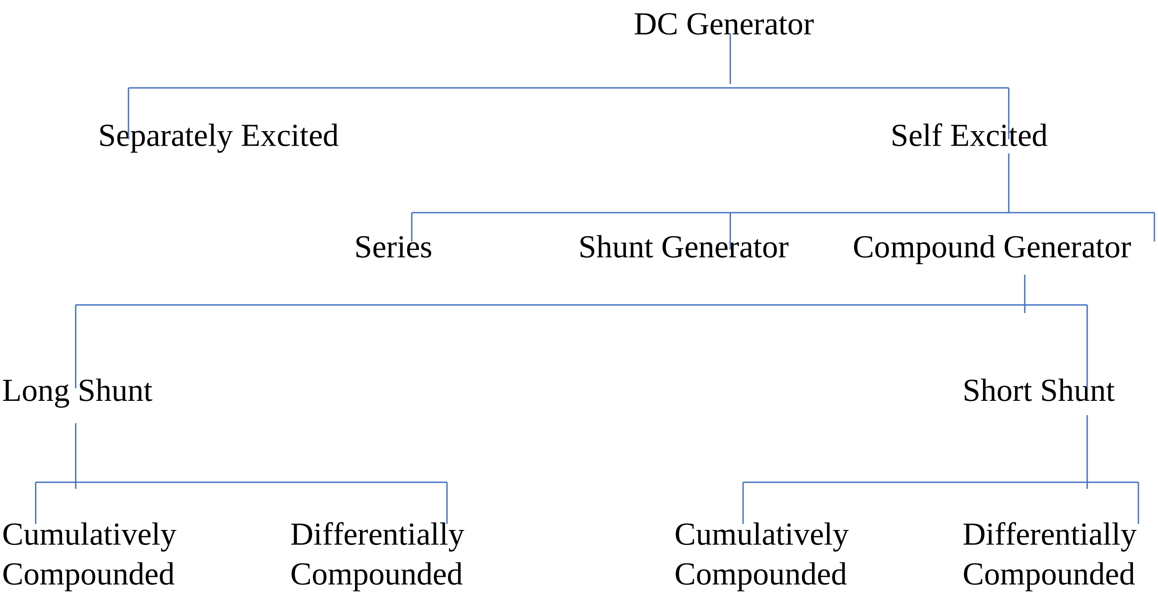
(i) Series Excitation: Field winding consists of a few turns of thick wire to keep the resistance less and it is connected in series with the armature. In this case, the series field current and armature current are same and therefore, the series field is called as current operated field.

(ii) Shunt Excitation: Field winding consists of a large nos. of turns of thin wire to keep the resistance high and it is connected in parallel with the armature. In this case, the armature voltage and shunt field voltage are same and therefore, the shunt field is called as voltage operated field.

(iii) Compound Excitation: A compound excitation consists of both the series field and shunt field winding.

- If the series field flux aids the shunt field flux so that the resultant flux per pole is increased then the machine is known as cumulatively compounded DC Machine
- If the series field flux opposes the shunt field flux so that the resultant flux per pole is decreased then the machine is known as differentially compounded DC Machine.

Classifications of DC Generator:



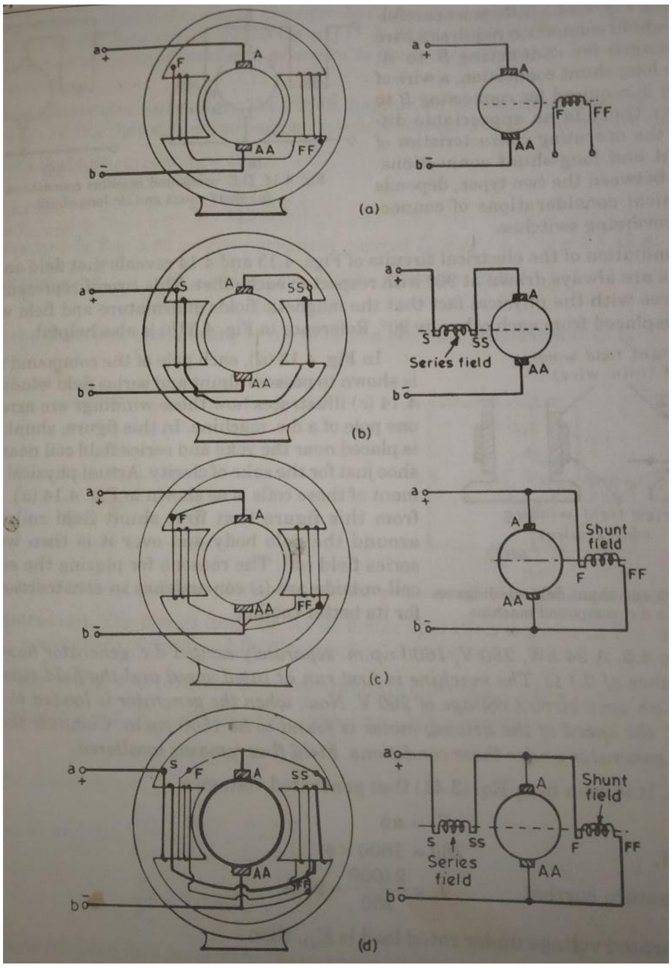
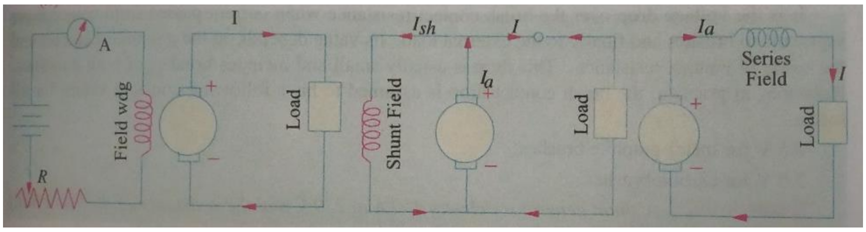
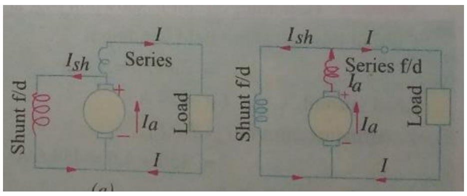


Fig. 1.8 (a) Separately Excited DC Machine, (b) Series Excited DC Machine (c) Shunt Excited DC Machine (d) Compound Excited DC Machine



(a) (b) (c)



(d) (e)

Fig. 1.9 Schematic diagram of (a) Self Excited Generator, (b) Shunt Generator, (c) Series Generator, (d) Short-shunt Generator, (e) Log-shunt Generator.

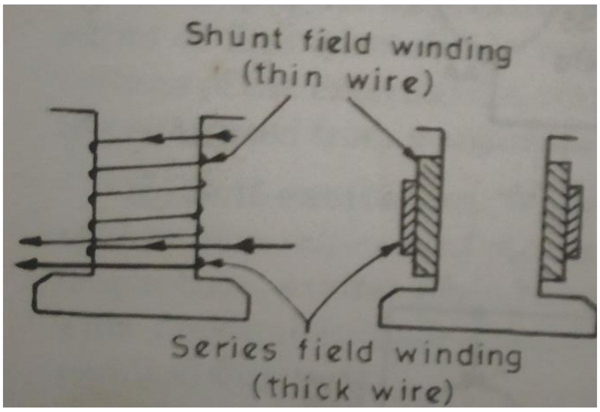


Fig. 1.7 Pole of a Compound DC machine

Problem:

An 8-pole DC Generator has 500 armature conductors, and a useful flux of 0.05 Wb per pole. What will be the emf generated if it is lap-connected and runs at 1200 rpm? What must be the speed at which it is to be driven to produce the same emf if it is wave wound?

Solution:

Given that, $P = 8,\ Z = 500,\ \phi = 0.05\ Wb$.

For lap wound:

$$A = P,\ Hence,\ (P/A) = 1.$$
$$N = 1200\ rpm.$$
$$Ea = (\phi ZN/60)\ (P/A) = \frac{(0.05 \times 500 \times 1200)}{60} = 500\ volts.$$

For wave wound:

$$A = 2,$$
$$Ea\ is\ given\ now\ and\ Ea = 500\ volts.$$
$$Hence,\ Ea = (\phi ZN/60)\ (P/A)$$
$$\Rightarrow 500 = (\frac{0.05 \times 500 \times N}{60}) \times (\frac{8}{2})$$
$$\Rightarrow N = 300\ rpm$$

Problem:

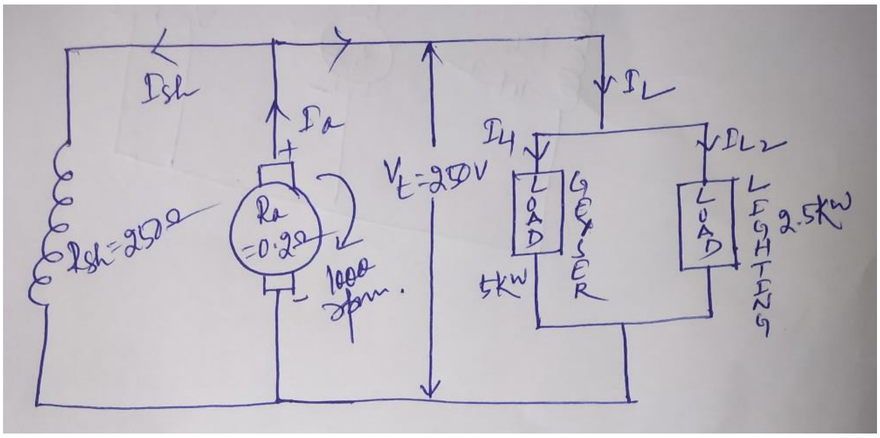
A 4-pole lap-connected armature of a dc shunt generator is required to supply the loads connected in parallel: (1) 5 KW Geyser at 250 V, and (2) 2.5 KW lighting load also at 250 V.

The generator has an armature resistance of 0.2 Ω and a field resistance of 250 Ω. The armature has 120 conductors in the slots and runs at 1000 rpm. Allowing 1 V per brush for contact drops and neglecting friction, find (i) flux per pole, (ii) Armature-current per parallel path.

Solution:

Using the formula, $P = V_t I$ we have,

Current through the Geyser $I_{L1} = P_1 / V_t = 5000/250 = 20\ A$.



Current through the lighting load, $I_{L2} = P_2 / V_t = (2.5 \times 1000)/250 = 10\ A$.

Hence, total load current, $I_L = I_{L1} + I_{L2} = 20 + 10 = 30\ A$.

$$I_{sh} = V_t / R_{sh} = 250/250 = 1A.$$
$$I_a = I_L + I_{sh} = 30 + 1 = 31\ A.$$
$$Ea = V_t + I_a R_a + BD = 250 + (31 \times 0.2) + 2 = 258.2\ V.$$
$$Now,\ Ea = (\phi ZN/60)\ (P/A)$$
$$\Rightarrow 258.2 = (\phi \times 120 \times 1000/60)$$
$$\Rightarrow \phi = 0.1291\ wb.$$

Armature current per parallel path $= 31/4 = 7.75\ A$.

Problem:

The armature winding of a 4-pole lap wound DC shunt generator consists of 220 turns each of 0.004 Ω resistance. Calculate its armature resistance.

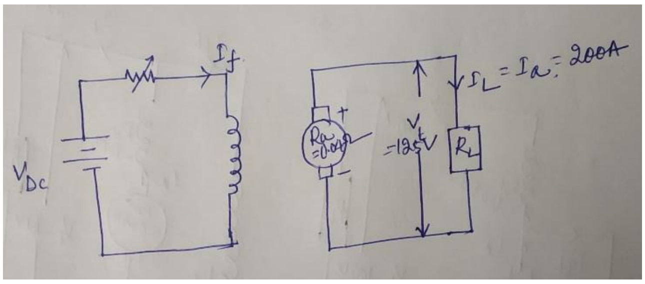
Solution:

$$Nos.\ of\ turns\ per\ parallel\ path = 220/4 = 55$$
$$Resistance\ of\ each\ parallel\ path,\ R_p = 55 \times 0.004 = 0.22\ \Omega$$
$$Resistance\ of\ the\ armature\ circuit,\ R_a\ is\ given\ by:$$
$$1/R_a = (1/R_p) + (1/R_p) + (1/R_p) + (1/R_p)$$
$$\Rightarrow 1/R_a = 4/R_p$$
$$\Rightarrow R_a = R_p/4 = 0.22/4 = 0.055\ \Omega.$$

Problem:

A separately excited DC Generator, when running at 1200 rpm supplies 200 A at 125 V to a circuit of constant resistance. What will be the generated emf when the speed is reduced dropped to 1000 rpm and the field current is reduced to 80%? Armature resistance is 0.04 Ω and total drop at brushes 2 V. Ignore saturation and armature reaction.

Solution:



$$Ea = V_t + I_a R_a + BD = 125 + (200 \times 0.04) + 2 = 135\ V$$
$$Ea = (\phi ZN / 60)\ (P / A)$$

Hence, $Ea \propto \Phi N$

$$Ea_1 \propto \Phi_1 N_1$$

and $Ea_2 \propto \Phi_2 N2$

$$\frac{Ea_2}{Ea_1} = \frac{\phi_2 N_2}{\phi_1 N_1}$$

$$\Rightarrow Ea_2 = \frac{\phi_2 N_2}{\phi_1 N_1} Ea_1$$

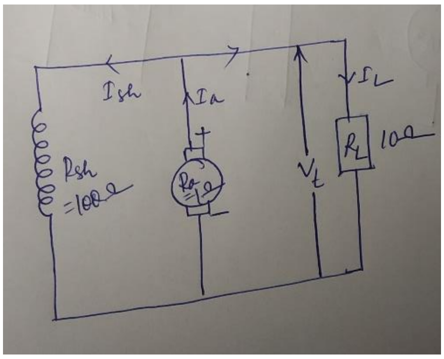
$$\therefore Ea_2 = \frac{0.8\phi_1 \times 1000 \times 135}{\phi_1 \times 1200} = 90V.$$

Problem:

A 4-pole DC shunt generator with a shunt field resistance of 100 Ω and an armature resistance of 1 Ω has 378 wave-connected conductors in its armature. The flux per pole is 0.02 wb. If a load resistance of 10 Ω is connected across the armature terminals and the generator is driven at 1000 rpm, calculate the power absorbed by the load.

Solution:

$$Ea = (\phi ZN / 60)\ (P / A) = (0.02 \times 378 \times 1000 / 60)\ (4 / 2) = 252\ volts$$



If V_t *is the terminal voltage of the generator then*

$$Ia = (V_t / 10) + (V_t / 100) = (11\, V_t) / 100$$

$$Now,\ V_t = E_a - I_a R_a$$

$$V_t = 252 - (11\, V_t) / 100$$

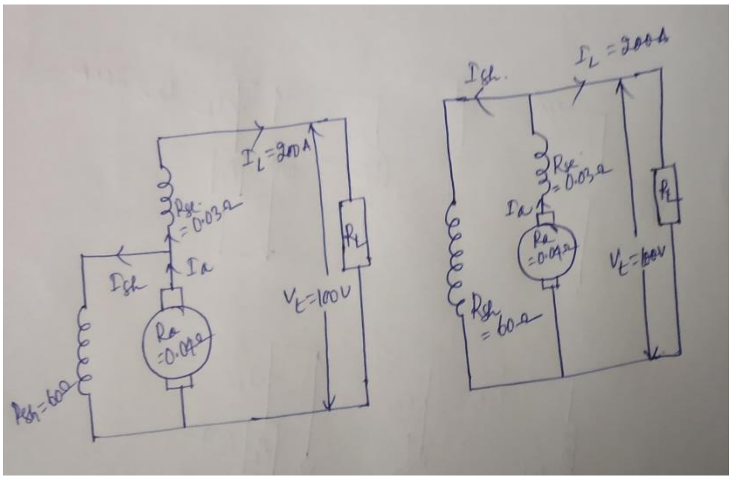
$$V_t = 227\ V$$

$$Power\ absorbed\ by\ the\ load = V_t^2 / R_L = 227^2 / 10 = 5153\ W$$

Problem:

A short-shunt DC compound generator supplies 200 A at 100 V. The resistance of armature, series field and shunt field windings are 0.04, 0.03 and 60 ohms respectively. Find the emf generated. Also find the emf generated if same machine is connected as a long-shunt machine.

Solution:



(a) Short shunt (b) long shunt

For short shunt:

$$Ea = V_t + I_L R_{se} + I_a R_a$$

$$V_{sh} = V_t + I_L R_{se} = 100 + (200 \times 0.03) = 106\ V$$

$I_{sh} = V_{sh}/R_{sh} = 106/60 = 1.76\ A$

$I_a = I_L + I_{sh} = 200 + 1.76 = 201.76$

$E_a = V_t + I_L R_{se} + I_a R_a$

$= 100 + (200 \times 0.03) + (201.76 \times 0.04)$

$= 114.07\ V$

For long-shunt:

$I_{sh} = V_t/R_{sh} = 100/60 = 1.67\ A$

$I_a = I_L + I_{sh} = 200 + 1.67 = 201.67$

$E_a = V_t + I_a (R_a + R_{se})$

$= 100 + 201.67(0.04 + 0.03)$

$= 114.12\ V$

Armature Reaction:

- The effect of armature flux on the main-field flux distribution in the air gap is called armature reaction.

- The armature flux produces two undesirable effects on the main field flux:

(i) Net reduction of the main field flux per pole which in turn reduces the generated voltage and torque

(ii) Distortion of the main field flux wave along the air-gap periphery which in turn reduces the limits of successful commutation.

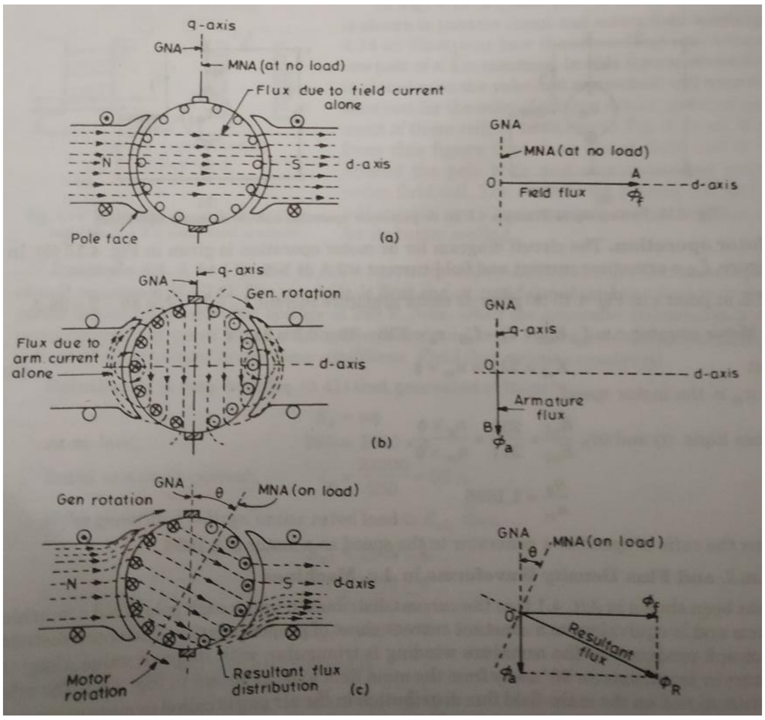


Fig. 1.10 Space distribution of (a) main field flux, (b) armature flux and (c) resultant of both main field and armature fluxes.

- The path of armature flux φ_a is perpendicular to the main field flux φ_f. In other words, the path of the armature flux *crosses* the path of the main field flux. Thus, the effect of armature flux on the main field flux is entirely cross-magnetising and it is for this reason that the armature flux is known as *cross-flux*.
- The net flux under each pole is reduced from its no load value which is known as *demagnetising* effect of armature flux.
- A geometric neutral axis (GNA) is along the quadrature axis of the DC Machine and magnetic neutral axis (MNA) is always perpendicular to the direction of resultant field flux.
- The effects of armature reaction are:

(i) To distort the main-field flux thereby causing non-uniform distribution of flux under the main poles.

(ii) To shift the MNA from GNA in the direction of rotation for a generator and against the direction of rotation for a motor. This shift is dependent upon the magnitude of armature current. Greater is the magnitude of armature (or load) current, greater is the shift of MNA from GNA.

(iii) To reduce the main-field flux from its no load value due to magnetic saturation.

<u>Methods of limiting the effects of armature reaction:</u>

Various methods of mitigating the effects of armature reaction are given below.

(i) High-reluctance pole tips:

This can be achieved by

- Using chamfered or eccentric pole shoes which results in short air-gap length at the pole centre and longer air-gap lengths under the pole tips, i.e. the profile of the pole shoe is not concentric with the armature core as shown in fig. 1.11 (a).
- This type of construction also produces sine wave flux density wave in the air-gap. But in DC machine the flux density wave in the air-gap need not be a sine wave whereas in synchronous machines it is required to have sine wave flux density wave in the air-gap. Hence, this type of construction for pole shoe is used to reduce the effect of cross-flux in DC machines and to obtain sine wave flux density wave in the air-gap in salient pole synchronous machines.
- Assembling alternatively the pole laminations as shown in fig. 1.11 (b). Due to this the iron area under the pole is halved and hence the reluctance of the pole tips is considerably increased.
- It should be noted that the influence of increased pole-tip reluctance is more pronounced on the cross flux than on the main-field flux because the main field mmf can be increased to maintain constant main field flux.

(ii) Reduction in armature flux:

- This is achieved by using field-pole laminations having several rectangular holes punched in the pole core as shown in fig. 1.12.
- In this type of construction of pole core, the main-field flux remains almost unaffected.

(iii) Strong main-field flux:

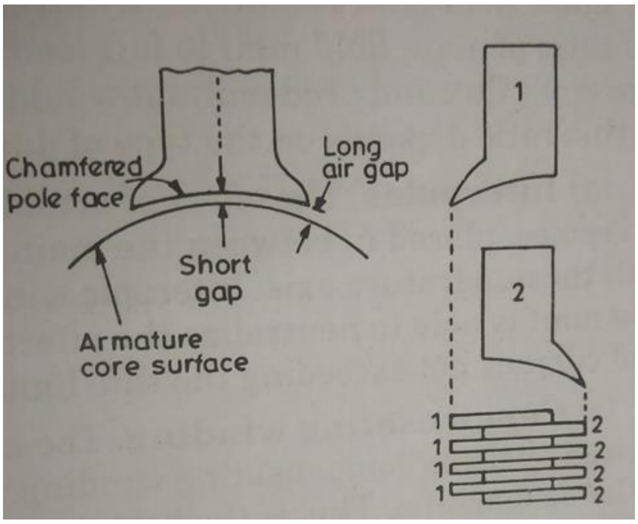
- During the design, it should be ensured that the main field mmf is sufficiently strong in comparison to full load armature mmf.
- Greater is the ratio of main field mmf to full load armature mmf, less is the distortion produced by armature cross flux.

(iv) Interpoles:

- The effect of armature reaction in the interpolar zone can be overcome by interpoles, placed between the main poles.
- Interpole windings are connected in series with the armature winding to neutralise the effect of armature mmf in the interpolar zone.

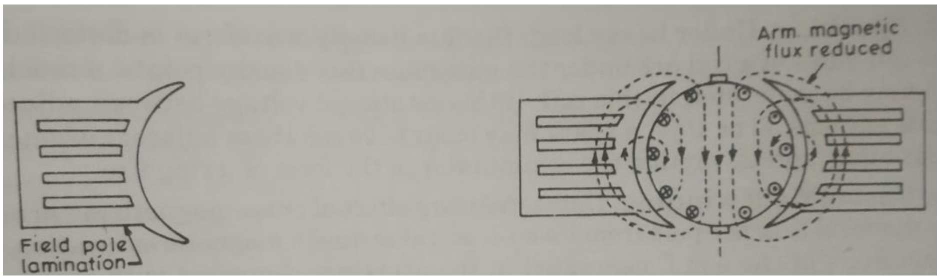
(v) Compensating winding:

- This winding is embedded in slots cut in the pole faces and connected in series with the armature windings to reduce the effect of armature reaction under the poles.



(a) (b)

Fig.1.11 (a) Chamfered or ecentric pole, (b) Laminations 1 and 2 are stacked alternatively



(a) (b)

Fig. 1.12 (a) One field-pole lamination with four punched holes (b) Two-pole DC Machine having punched field-pole laminations.

Commutation:

- The reversal of current in the armature coil by means of brush and commutator bars is called commutation.
- Good commutation means no sparking at the brushes and the commutator surface remains unaffected during continuous operation of the DC machine.

Losses and Efficiency of DC Generators:

- In accordance with the law of conservation of power (or energy), it can be stated in electrical machines that Power input = Power output+losses
- Power loss in a machine does not perform any useful work. It only helps in heating of the electrical machine.
- Power losses in electrical machines is essential from the following viewpoints:
  - (i) Losses determine the operating cost of the electrical machines. A machine with lower efficiency has increased losses and hence increased operating costs.
  - (ii) Losses cause heating and therefore temperature rise of the machine which in turn determine the rated power output and life span of the electrical machines.
  - (iii) Voltage drop IR is associated with the ohmic losses while current component like core-loss current is associated with the iron loss of the electrical machines and therefore, these losses caused due to voltage drops and current components must be appropriately taken into consideration in the equivalent circuit of an electrical machine for proper analysis of the electrical machine.
- Efficiency of an electrical machine is defined as:

$$Efficiency, \eta = \frac{P_{out}}{P_{in}} = \frac{P_{in} - P_{out}}{P_{in}} = 1 - \frac{losses}{P_{in}}$$

- There are two methods of computing the efficiency of an electrical machine, i.e. (i) direct method, and (ii) indirect method.
- In direct method, the machine is loaded and the efficiency is directly computed by measuring the input and output of the machine.
- In indirect method, the efficiency is computed by measuring the losses of the machine without actually loading the machine.
- Indirect method of calculating the efficiency is more accurate and convenient.
- Indirect method of calculating the efficiency is more convenient because losses are a small percentage of output or input and therefore these can be measured conveniently and economically.
- Indirect method of calculating the efficiency is more accurate because a small percentage of error in the measurement of losses, results in a still smaller percentage of error in the computation of efficiency.

- Direct method of calculating the efficiency suffers from following disadvantages.
  - (i) It is difficult to perform because of the cost of providing large inputs and difficulty of dissipating the large outputs. It is often difficult to find a suitable loads and supply to actually load the machines in the labs. Also, it is difficult to find measuring instruments of such high rating to measure the input and output power when it is loaded.

(ii) It does not give accurate result due to the errors in the measuring instruments. In this case, the input and output are high and a small error in the measurement of either input or output produces almost the same amount of error in the computed efficiency which could be very significant.

- Accuracy of the indirect method over direct method for calculating the efficiency is established with the help of the following example.
  Let us consider an electrical machine with input of 1000 W and output of 900 W. Hence, the losses are 100 W and the efficiency is 90%.
  Direct Method:
  If there is an error of 10% less in the measurement of output, then the efficiency is $(810/1000)\times 100 = 81\%$. Thus, a given percentage of error (10% here) is producing almost the same amount of error in the computed efficiency.
  Indirect Method:
  But, if there is an error of 10% less in the measurement of losses, then the efficiency is $\eta = (1 - \frac{90}{1000}) \times 100 = 91\%$. Thus, a given percentage of error (1% here) is producing only one tenth (1% here) of that percentage error in the computed efficiency.
  In view of the above, it is always preferable to calculate the efficiency by indirect method rather than the direct method.
- A machine with large efficiency has more fixed charges but it is more reliable, delivers better performance with less breakdowns, has less running charges and maintenance cost as compared to less efficient machine.

Losses in DC Generator:

The various losses taking place in a DC generator are described below.

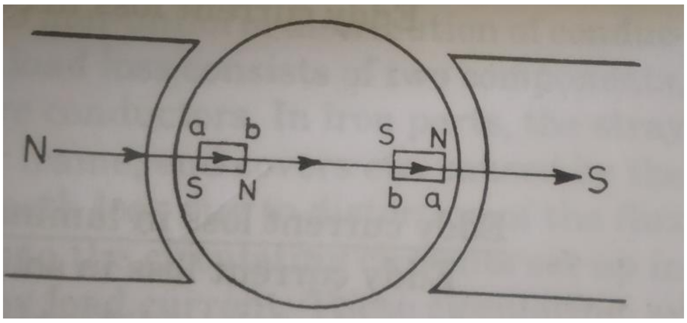
(i) Copper loss or as I^2R loss or ohmic loss: This loss takes place in the armature winding and field winding and is caused due to resistance of the windings.

(a) *Armature copper loss* $= I_a^2 r_a$.
It includes the brush contact loss occuring due to brush contact resistance.
(b) *Field copper loss*
Shunt field copper loss $= I_{sh}^2 r_{sh}$
Series field copper loss $= I_{se}^2 r_{se}$

(ii) Magnetic loss or Iron loss or Core loss: This loss takes place in the armature core of the DC generator. These losses are divided into two types, i.e. (a) hysteresis loss and (b) eddy current loss

(a) Hysteresis loss:

- When the armature rotates, there are continuous magnetic reversals of small iron pieces like 'ab' as shown in the figure 1.24. For instance, when the iron piece 'ab' is under the influence of north pole, main field flux passes through it from 'a' to 'b' causing the appearance of south pole at 'a' and north pole at 'b'. After half revolution, when the iron piece 'ab' is under the south pole, main field flux passes through it from 'b' to 'a' causing the appearance of south pole at 'b' and north pole at 'a'.



- The armature core is made up of numbers of small iron pieces like ‘ab’ and the power required for the magnetic reversal of these iron pieces is called speed.
- Hysteresis loss is directly proportional to the frequency of magnetic reversals or the speed of rotation of the armature. The mathematical expression for hysteresis loss is given below.

$$\textbf{Hysteresis loss, } P_h \propto B_m^{1.6} f$$
$$\Rightarrow P_h = K_h B_m^{1.6} fV$$

Where,

K_h = *Proportionality constant which depends upon the quality of the core material.*

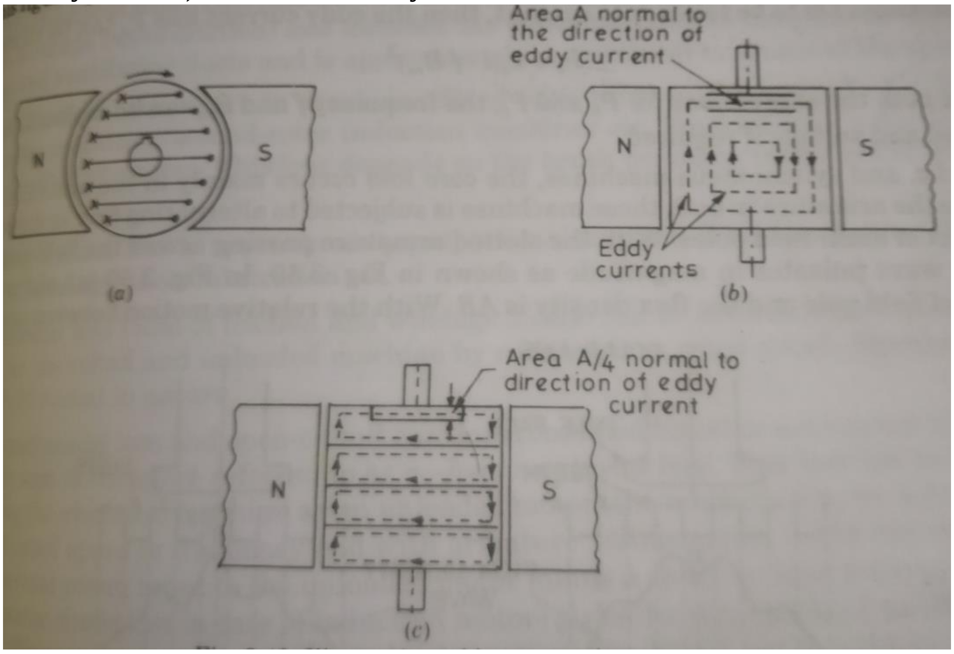
B_m = *Maximum flux density*

f = *frequency of magnetic reversals*

V = *Volume of armature core*

(b) Eddy current loss:

- When the armature rotates, armature core also cuts the magnetic flux and hence an emf is induced in the armature core. In fig. 1.25 (a), these emfs are indicated by dots and crosses. The emfs in the solid iron give rise to circulating currents called as eddy currents as shown in the fig. 1.25 (b) which is another view called the plan of fig. 1.25 (a).
- The power loss equal to (eddy current)^2(resistance offered by the armature core to the flow of eddy current) is known as eddy current loss.



- The eddy current losses can be minimised by laminating the iron core of the armature as explained below.
- Referring to fig. 1.25 (c), there are four laminations of the armature core.

$$\begin{aligned}
&Let,\\
&A = Cross\text{-}sectional\ area\ of\ the\ armature\ core\\
&R_{sld} = Resistance\ offered\ by\ the\ solid\ iron\ to\ the\ flow\ of\ eddy\ cuurent\\
&R_{lam} = Resistance\ offered\ by\ the\ laminated\ iron\ to\ the\ flow\ of\ eddy\ cuurent\\
&R_{sld} \propto 1/A\\
&R_{lam} \propto \frac{1}{(A/4)}\\
&\therefore \frac{R_{sld}}{R_{lam}} = \frac{1}{A}\times\frac{A}{4} = 1/4\\
&If, the\ emf\ in\ solid\ armature\ core = E,\\
&then, emf\ per\ lamination\ will\ be\ E/4\ as\ the\ flux\ cut\ by\ each\ lamination\ is\ one\ fourth\ of\ the\ total\ flux.\\
&\Rightarrow \frac{emf\ per\ lamination}{emf\ in\ solid\ armature\ core} = \frac{E/4}{E} = 1/4\\
&\frac{Eddy\ current\ loss\ per\ lamination}{Eddy\ current\ loss\ in\ solid\ iron} = \frac{(emf\ per\ lamination)^2/R_{lam}}{(emf\ in\ solid\ armature\ core)^2/R_{sld}}\\
&\Rightarrow \frac{Eddy\ current\ loss\ per\ lamination}{Eddy\ current\ loss\ in\ solid\ iron} = \frac{(emf\ per\ lamination)^2}{(emf\ in\ solid\ armature\ core)^2}\times\frac{R_{sld}}{R_{lam}}\\
&\Rightarrow \frac{Eddy\ current\ loss\ per\ lamination}{Eddy\ current\ loss\ in\ solid\ iron} = (1/4)^2\times(1/4) = \frac{1}{64}\\
&\therefore \frac{Eddy\ current\ loss\ in\ laminated\ core}{Eddy\ current\ loss\ in\ solid\ core} = \frac{1}{64}\times\frac{4}{1} = (1/4)^2
\end{aligned}$$

- From the above, it is concluded that eddy current loss is proportional to the square of the lamination thickness.
- More is the laminations for a given axial length of the core, less is the thickness and hence, less is the eddy current loss. The mathematical expression for eddy current loss is given below.

$$\begin{aligned}
&\mathbf{Eddy\ current\ loss}, \boldsymbol{P_e \propto B_m^2 f^2}\\
&\Rightarrow \boldsymbol{P_e = K_e B_m^2 f^2 t^2 V^2}\\
&Where,\\
&K_e = Proportionality\ constant\ which\ depends\ upon\ the\ resistivity\ of\ the\ core\ material\\
&B_m = Maximum\ flux\ density\\
&f = frequency\ of\ magnetic\ reversals\\
&t = thickness\ of\ lamination\\
&V = Volume\ of\ armature\ core
\end{aligned}$$

- For both the expressions for P_h and P_e, the frequency f and B_m can be replaced by speed and voltage respectively if needed.

(iii) Mechanical loss or friction and windage loss: These losses take place in the rotating parts of the electrical machines due to friction in the rotating parts of the machine and air-friction.

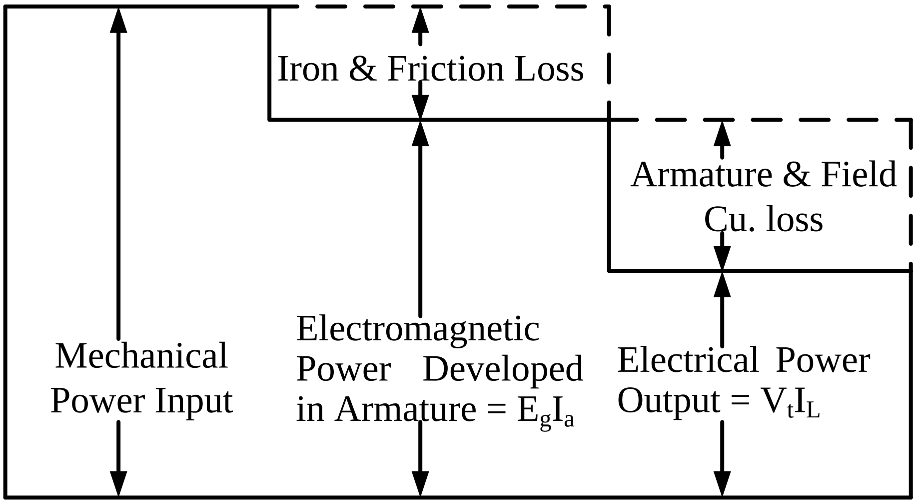
Stray loss or no-load rotational loss or rotational loss:

- It is the summation of magnetic and mechanical loss and is considered to be constant loss in a DC machine.

Constant loss and Variable loss:

- Constant loss: These losses are independent of the load current and remains constant at all loads. Constant loss includes rotational loss and if it is a shunt generator then it also includes shunt field copper loss.
- Variable loss: These losses are dependent of the load current and varies with the loads. Variable loss includes armature copper loss and series field copper loss.

Power Stages:



Condition for maximum efficiency:

- The load current for which the maximum efficiency occurs can be found as follows.

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + losses} = \frac{V_t I_L}{V_t I_L + I_a^2 r_a + constant\ loss}$$

For maximum efficiency,

$$\frac{d\eta}{dI_L} = 0$$

$$Now, I_a = I_L + I_{sh}$$

$$Since, I_{sh}\ is\ very\ less, I_L\ can\ be\ taken\ equal\ to\ I_a$$

$$\therefore \frac{d\eta}{dI_L} = \frac{d\eta}{dI_a} = \frac{d}{dI_a}\left(\frac{V_t I_a}{V_t I_a + I_a^2 r_a + constant\ loss}\right) = 0$$

$$\Rightarrow \frac{V_t(V_t I_a + I_a^2 r_a + constant\ loss) - V_t I_a (V_t + 2I_a r_a)}{(V_t I_a + I_a^2 r_a + constant\ loss)^2} = 0$$

$$\Rightarrow V_t(V_t I_a + I_a^2 r_a + constant\ loss) = V_t I_a (V_t + 2I_a r_a)$$

$$\Rightarrow Constant\ loss = I_a^2 r_a$$

Hence the condition for maximum efficiency is

Constant loss = Variable loss

The armature current corresponding to maximum efficiency is given by,

$$\boldsymbol{I_a = \sqrt{\frac{Constant\ loss}{r_a}}}$$

Problem:

A 10 KW, 250 V, DC shunt generator has total no-load rotational loss of 400 watts. The armature circuit and shunt field resistances are 0.5 Ω and 250 Ω respectively. Determine the shaft power input and the efficiency at rated load. Also, calculate the maximum efficiency and the corresponding power output.

Solution:

$$Constant\ shunt\ field\ current, I_{sh} = \frac{V_t}{R_{sh}} = \frac{250}{250} = 1\,A$$

Case - I

Rated Load Condition :

$Shunt\ field\ copper\ loss = I_{sh}^2 R_{sh} = 1^2 \times 250 = 250\ W$

Aletrnatively,

$Shunt\ field\ copper\ loss = V_t I_{sh} = 250 \times 1 = 250\ W$

$\therefore Constant\ loss = Noload\ rotational\ loss + Shunt\ field\ copper\ loss = 400 + 250 = 650\ W$

$$Rated\ load\ current, I_L = \frac{P_{out}}{V_t} = \frac{10,000}{250} = 40\,A$$

$\therefore\ Armature\ current, I_a = I_L + I_{sh} = 40 + 1 = 41\,A$

$Armature\ copper\ loss = I_a^2 R_a = 41^2 \times 0.5 = 840.5\ W$

$\therefore Total\ losses\ at\ rated\ load = Constant\ loss + Armature\ copper\ loss = 650 + 840.5 = 1490.5\ W$

$Shaft\ power\ input\ at\ rated\ load, P_{in} = P_{out} + Total\ losses\ at\ rated\ load = 10,000 + 1490.5 = 11490.5\ W\ (Ans.)$

$$Efficiency\ at\ rated\ load, \eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{10,000}{11490.5} \times 100 = 87.03\%\ (Ans.)$$

Case - II

Maximum Efficiency Condition :

$At\ maximum\ efficiency, Variable\ loss\ i.e.\ Armature\ copper\ loss = Constant\ loss$

$\Rightarrow Armature\ copper\ loss = 650$

$\Rightarrow I_a^2 R_a = 650$

$$\Rightarrow I_a = \sqrt{\frac{650}{0.5}} = 36.05\,A$$

$I_L = I_a - I_{sh} = 36.05 - 1 = 35.05\,A$

$P_{out} = V_t \times I_L = 250 \times 35.05 = 8762.5\ W\ (Ans.)$

$Total\ losses = 2 \times Constant\ loss = 2 \times 650 = 1300\ W$

$P_{in} = P_{out} + Total\ losses = 8762.5 + 1300 = 10062.5\ W$

$$Maximum\ Efficiency, \eta_{\max} = \frac{P_{out}}{P_{in}} \times 100 = \frac{8762.5}{10062.5} \times 100 = 87.08\%\ (Ans.)$$

Problem:

In a DC machine, the total iron loss is 8 KW at its rated speed and excitation. If excitation remains the same, but speed is reduced by 25%, the total iron loss is found to be 5 KW. Calculate the hysteresis and eddy current losses at (i) rated speed and (ii) half the rated speed.

Solution:

$Let,$

$N = Rated\ speed$

$P_{h1} = hysteresis\ loss\ at\ rated\ speed$

$P_{h2} = hysteresis\ loss\ when\ the\ speed\ is\ reduced\ by\ 25\%\ of\ rated\ speed$

$\quad i.e.\ hysteresis\ loss\ at\ 75\%\ of\ rated\ speed$

$P_{h3} = hysteresis\ loss\ at\ half\ the\ rated\ speed$

$P_{e1} = eddy\ current\ loss\ at\ rated\ speed$

$P_{e2} = eddy\ current\ loss\ when\ the\ speed\ is\ reduced\ by\ 25\%\ of\ rated\ speed$

$\quad i.e.\ eddy\ current\ loss\ at\ 75\%\ of\ rated\ speed$

$P_{e3} = eddy\ current\ loss\ at\ half\ the\ rated\ speed$

$P_{h1} \propto N$

$P_{h2} \propto 0.75N$

$\therefore \frac{P_{h2}}{P_{h1}} = 0.75$

$\Rightarrow P_{h2} = 0.75P_{h1}\ (1)$

$P_{e1} \propto N^2$

$P_{e2} \propto (0.75N)^2$

$\therefore \frac{P_{e2}}{P_{e1}} = \frac{(0.75N)^2}{N^2} = 0.5625$

$\Rightarrow P_{e2} = 0.5625P_{e1}\ (2)$

$At\ rated\ speed, the\ total\ iron\ loss\ is\ 8\,KW$

$\therefore P_{h1} + P_{e1} = 8\ (3)$

$When\ the\ speed\ is\ reduced\ by\ 25\%\ of\ rated\ speed\ means\ at\ 75\%\ of\ rated\ speed, the\ total\ iron\ loss\ is\ 5\,KW$

$\therefore P_{h2} + P_{e2} = 5$

$Putting\ the\ value\ of\ P_{h2}\ and\ P_{e2}\ from\ (1)\ \&\ (2)\ respectively, we\ have,$

$0.75P_{h1} + 0.5625P_{e1} = 5\ \ (4)$

$Multiplying\ 0.75\ in\ (3), we\ have,$

$0.75P_{h1} + 0.75P_{e1} = 6\ (5)$

$Subtracting\ (4)\ from\ (5), we\ have,$

$0.1875P_{e1} = 1$

$\Rightarrow P_{e1} = 5.34\,KW$

$Putting\ the\ value\ of\ P_{e1}\ in\ (3), we\ have,$

$P_{h1} = 2.66\,KW$

$\therefore The\ hysteresis\ loss\ and\ eddy\ current\ loss\ at\ rated\ speed\ are\ 2.66\,KW\ and\ 5.34\,KW\ respectively.$

$P_{h1} \propto N$

$P_{h3} \propto 0.5N$

$\therefore \frac{P_{h3}}{P_{h1}} = 0.5$

$\Rightarrow P_{h3} = 0.5P_{h1} = 0.5 \times 2.66 = 1.33\,KW$

$P_{e1} \propto N^2$

$P_{e3} \propto (0.5N)^2$

$\therefore \frac{P_{e3}}{P_{e1}} = \frac{(0.5N)^2}{N^2} = 0.25$

$\Rightarrow P_{e3} = 0.25P_{e1} = 0.25 \times 5.34 = 1.335\,KW$

$\therefore The\ hysteresis\ loss\ and\ eddy\ current\ loss\ at\ half\ the\ rated\ rated\ speed\ are$

$1.33\,KW\ and\ 1.335\,KW\ respectively.$

Problem:

The hysteresis loss and eddy current loss in a DC Machine running at 1000 rpm are 250 W and 100 W respectively. If the flux remains constant, at what speed will be total iron losses be halved and at this speed, find the hysteresis loss and eddy current loss.

Solution:

Let,

$250 \propto 1000$

$\Rightarrow 250 = K_h 1000$

$\Rightarrow K_h = 0.25$

$100 \propto 1000^2$

$\Rightarrow 100 = K_e 1000^2$

$\Rightarrow K_e = 0.0001$

Let N be the speed at which the total iron losses are halved with flux rmaining unchanged.

$\therefore K_h N + K_e N^2 = 175$

$0.25N + 0.0001N^2 = 175$

$\Rightarrow 0.0001N^2 + 0.25N - 175 = 0$

$$\Rightarrow N = \frac{-0.25 \pm \sqrt{[0.25^2 - \{4 \times 0.0001 \times (-175)\}]}}{2 \times 0.0001} = 570\,rpm$$

Negative value of N is not considered as speed cannot be negative in this case.

Hysteresis loss at this speed of $570\,rpm = K_h 570 = 0.25 \times 570 = 142.5\ W$

Eddy current loss at this speed of $570\,rpm = K_e 570^2 = 0.0001 \times 570^2 = 32.5\ W$

Alternately,

If, P_{h1} *is the hysteresis loss at 1000 rpm and*

P_{h2} *is the hysteresis loss at 570 rpm, then*

$P_{h1} \propto 1000$

$P_{h2} \propto 570$

$$\Rightarrow \frac{P_{h2}}{P_{h1}} = \frac{570}{1000}$$

$$\Rightarrow P_{h2} = P_{h1} \times \frac{570}{1000} = 250 \times \frac{570}{1000} = 142.5\ W$$

$\therefore$ *Hysteresis loss at this speed of* $570\,rpm$ *is* $142.5\ W$

If, P_{e1} *is the eddy cuurent loss at* $1000\,rpm$ *and*

P_{e2} *is the eddy cuurent loss at* $1000\,rpm$*, then*

$P_{e1} \propto 1000^2$

$P_{e2} \propto 570^2$

$$\Rightarrow \frac{P_{e2}}{P_{e1}} = (570/1000)^2$$

$\Rightarrow P_{e2} = P_{e1} \times (570/1000)^2 = 100 \times (570/1000)^2 = 32.5\ W$

$\therefore$ *Eddy current loss at this speed of* $570\,rpm$ *is* $32.5\ W$

The total iron losses occurring at the speed of $1000\,rpm = 250 + 100 = 350W$

Total iron losses at the speed of $570\,rpm = 142.5 + 32.5 = 175W$ *which is half of the total iron losses occurring at speed* $1000\,rpm$.

DC Motor

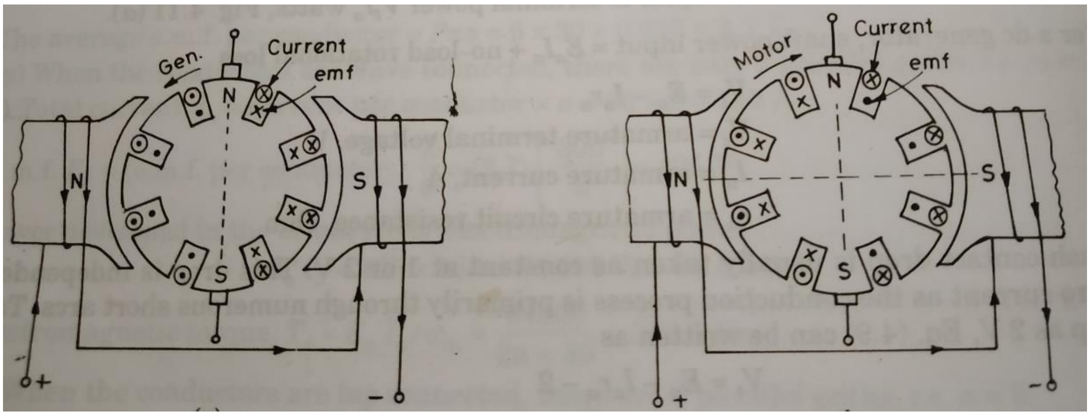
- A DC Motor is an electromechanical energy conversion device which converts electrical energy to mechanical energy.
- The action of a DC Motor is based on the principle that whenever a current carrying conductor is placed in a magnetic field, it experiences a mechanical force.

Construction:

- There is no difference in construction between a DC generator and DC motor. In fact, the same machine can be used as generator as well as motor.

Working Principle:

- The working principle of a DC Motor is explained with the help of fig. 2.1(b).
- The field poles and armature of a DC Motor are supplied with DC current from the supply mains. The field poles produce the necessary magnetic flux and the current carrying armature conductors are now placed inside this magnetic field. Hence, the armature conductors experience a unidirectional force tending to rotate the armature in clockwise direction.
- The armature conductors under the influence of N pole carry current upwards indicated by dot sign and the armature conductors under the influence of S pole carry current downwards indicated by cross sign as shown in the fig. 2.1 (b). These currents establish a magnetic field in the armature and the N and S poles of this magnetic field are indicated in fig. 2.1 (b).
- The N and S poles of the armature magnetic field interacts with the N and S poles of the magnetic field produced by the field poles thereby causing the armature to rotate in clockwise direction as shown in the fig. 2.1 (b). The direction of rotation of the armature can also be verified by applying Fleming's left-hand rule.
- The motor will continue to rotate in the clockwise direction if a unidirectional clockwise is developed in the armature all the time for which it is required that the direction of current in the armature conductor must change when it passes from one pole to another pole. By reversing the current in each armature conductor as it passes from one pole to another, the motor develops a continuous and directional torque. This reversal of current in the armature conductor is performed by the commutator. Hence, in a DC motor, the function of the commutator is to change the direction of current in the armature conductor when the armature conductor passes from one pole to another pole and the commutator acts as a mechanical inverter in DC Motor.



(a) (b)

Fig. 2.1 DC Machine (a) Generating action and (b) Motoring action

Back emf:

- When the armature of a DC motor rotates, the armature conductors also cut the main field flux and hence emf is generated in the armature conductors. The emf so generated in the armature is called the counter emf as it opposes the flow of armature current. The emf is also known as back emf as it is the voltage at the back of armature resistance. The direction of this back emf can be found by applying Fleming's right-hand rule and it is seen that the back emf is in direct opposition to the applied voltage to oppose the flow of armature current as shown in the fig. 2.2. The applied voltage is always greater than the back emf so that it can be able to drive the armature current against the opposition of the back emf and the work done in overcoming this opposition is converted to mechanical energy. The back emf is denoted by E_a or E_b. The power required to overcome this opposition is $E_a I_a$. The magnitude of the back emf is given by $E_a = \frac{\Phi ZN}{60} \times \frac{P}{A}$ volts. The back emf depends upon the speed. As shown in fig. 2.2, the armature current is given by $I_a = \frac{V_t - E_b}{r_a}$. As the armature speed increases, back emf increases and the armature current decreases. With decrease in armature speed, back emf decreases and armature current increases. Therefore, it can be stated that, the back emf acts like a governor i.e. it makes the DC motor self-regulating so that it draws as much current as is just necessary.

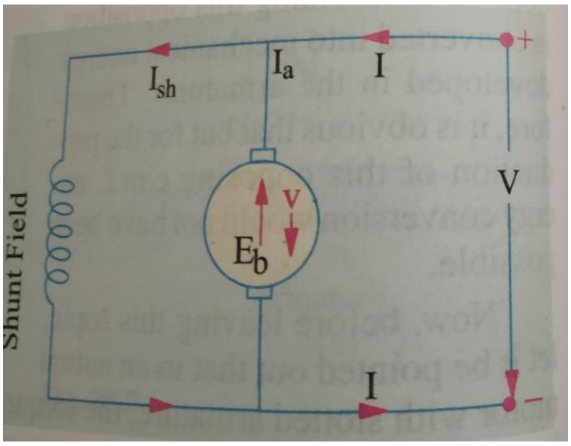


Fig. 2.2 DC motor

Comparison of Generator and Motor action:

- Energy conversion is not possible unless there is some opposition whose overcoming provides the necessary means for such conversion. In case of a generator, this opposition is provided by the torque developed in the armature and in case of a motor the opposition is provided by the emf generated in the armature known as back emf as explained below.

Generating action:

- Fig. 2.1 (a) explains the generating action of a DC machine and in this fig., for anticlockwise rotation of armature, it is found by applying Fleming's right hand rule that generated emfs in armature conductors under N pole have dot signs and generated emfs in armature conductors under S pole have cross signs. Due to this generated emf, current flows in the armature conductors and the armature current establishes a magnetic field in the armature.

- Now, current carrying armature is placed inside the magnetic field and therefore, the armature experiences a mechanical torque which provides the necessary opposition for energy conversion. The applied torque has to overcome this armature torque and the mechanical work done in overcoming this opposition is converted into electrical energy in generator operation.
- For the given anticlockwise rotation of armature, N and S poles in armature must be created by the armature currents as shown in the fig. 2.1 (a) to provide the necessary opposition against the mechanical input for energy conversion. This magnetic polarity demands the current directions in the armature conductors to be indicated by dots under N pole and by crosses under S pole and these current directions in the armature slots are shown by crosses and dots within the circles of fig. 2.1 (a).
- Hence, the emf and current in the armature conductors are in the same direction for generator operation.
- In DC generator, $E_a > V_t$.

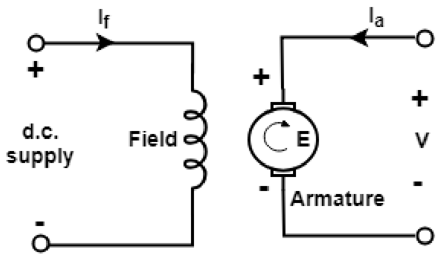
Motoring action:

- Fig. 2.1 (b) explains the motoring action of a DC machine and in this fig., for anticlockwise rotation of armature, the direction of current in the armature conductors is indicated by crosses and dots within the circles in the armature slots.
- As the armature rotates, the armature conductors cut the same flux that gives rise to motor torque and hence, emf is generated in the armature conductors due to the flux cutting action. The emf so generated in the armature provides the necessary opposition against the electrical input for energy conversion in DC motor. This demands the direction of the generated emf to be in opposite to the direction of armature current and accordingly the direction of the emf is indicated by crosses and dots below the circles in the slots. The direction of the emf can also be found by applying Fleming's left-hand rule. Since the emf generated in the armature opposes the flow of armature current, it is called as the counter emf. The generated emf is also called as back emf because it is the voltage at the back of armature resistance. The back emf is denoted by E_a or E_b. The applied voltage has to overcome this back emf and the electrical work done in overcoming this opposition is converted into mechanical energy during motor operation. Hence, the back emf is always less than the applied voltage and can never be equal to applied voltage.
- Hence, the emf and current in the armature conductors are in opposite direction for motor operation.
- In DC Motor, $E_a < V_t$.

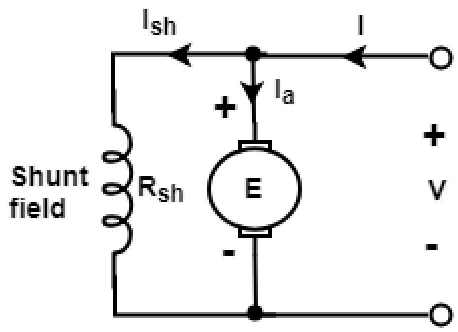
Classifications of DC Motor:

- DC motors can be classified according to the methods of excitation of the field poles as given below.

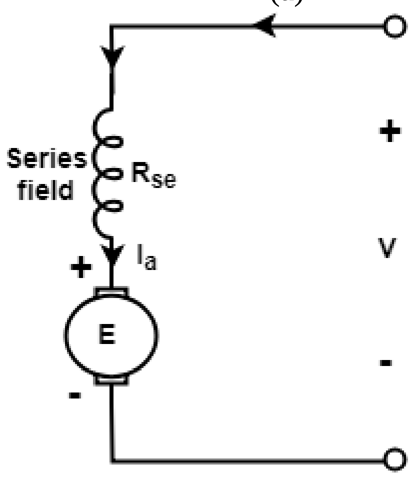
- DC Motor
  - Separately Excited
  - Self Excited
    - Series Motor
    - Shunt Motor
    - Compound Motor
      - Long Shunt
        - Cumulatively Compounded
        - Differentially Compounded
      - Short Shunt
        - Cumulatively Compounded
        - Differentially Compounded



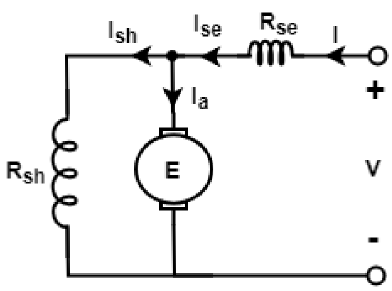
(a)



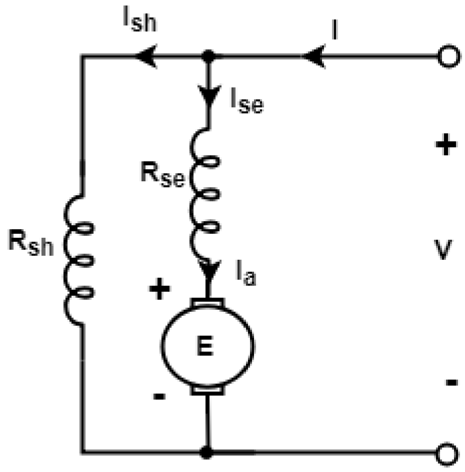
(b)



(c)



(d)



(e)

Fig. 2.3 Schematic diagram of (a) Self Excited Motor, (b) Shunt Motor, (c) Series Motor, (d) Short-shunt Motor, (e) Log-shunt Motor.

Motor efficiency and condition for maximum power:

- Let us consider a shunt motor as shown in the fig. 2.2. The applied voltage has to overcome the back emf as well armature resistance drop. Hence, the voltage equation of the motor is given by $V_t = E_a + I_a r_a$. Multiplying both sides by I_a in this equation, we have, $V_t I_a = E_a I_a + I_a^2 r_a$.
- In the above equation,
  $V_t I_a$ = *Electrical input to the armature*,
  $E_a I_a$ = *Electromagnetic power developed in the armature which is the electrical equivalent of the mechanical power developed in the armature*,
  $I_a^2 r_a$ = *Armature copper loss*

  Hence, out of the total armature input, some power is wasted in the form of armature copper loss and rest of the power is available as mechanical power within the armature.
- *Motor efficiency*, $\eta = \frac{E_a I_a}{V_t I_a} = \frac{E_a}{V_t}$. From this equation, it is obvious that higher the value of E_a in comparison to V_t, higher is the motor efficiency.
- $V_t I_a = E_a I_a + I_a^2 r_a$. In this equation, $E_a I_a$ can be called as the gross mechanical power developed in the armature and denoted by P_m.
- $\therefore P_m = V_t I_a - I_a^2 r_a$

  The value of armature current, for which maximum mechanical power is developed in the armature can be determined as follows.

$$\therefore P_m = V_t I_a - I_a^2 r_a$$
$$For\ maximum\ mechanical\ power, \frac{dP_m}{dI_a} = 0$$
$$\Rightarrow V_t - 2I_a r_a = 0$$
$$\Rightarrow I_a r_a = \frac{V_t}{2}$$
$$Now, E_a = V_t - I_a r_a$$
$$\Rightarrow E_a = V_t - \frac{V_t}{2}$$
$$\Rightarrow E_a = \frac{V_t}{2}$$

Hence, maximum gross mechanical power is developed in the motor when the back emf equals half of the applied voltage. However, this condition is not realized in practice because the current would be much higher than the normal current of the motor causing a huge copper loss. Again, in this condition, the motor efficiency will be $\eta = \frac{E_a}{V_t} \times 100 = \frac{V_t / 2}{V_t} \times 100 = 50\%$. Further, if the iron & friction losses are taken into account then the efficiency will be well below 50%.

Torque:

- Torque is the turning or twisting moment of a force about an axis. It is given by the product of the force and the radius at which this force acts.
- Let us consider a pulley of radius r meter as shown in the fig. 2.4. A circumferential force F is acting upon the pulley to rotate the pulley at a speed of N revolutions per second (rps).

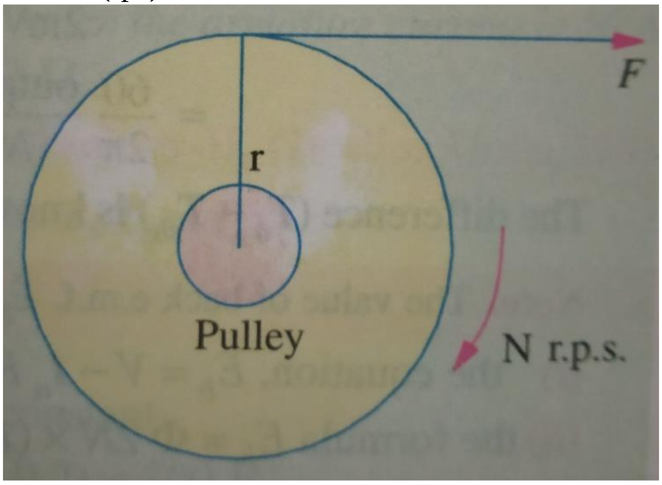


Fig. 2.4 Torque exerted on a pulley

$$Torque\ exerted\ on\ the\ pulley, T = (F \times r)\ Nm$$
$$Wordone\ by\ the\ force\ in\ one\ revolution = Force \times distance = (F \times 2\pi r)\ Nm\ or\ joule$$
$$Power\ developed\ in\ the\ pulley, P = \frac{Wordone}{Time\ taken\ to\ complete\ one\ revolution}$$
$$= \frac{F \times 2\pi r}{1/N} = (2\pi N)(F \times r) = 2\pi NT\ watt$$
$$\therefore P = 2\pi NT\ (where\ N\ is\ the\ speed\ in\ rps)$$
$$But, the\ angular\ speed, \omega = 2\pi N\ (If\ N\ is\ in\ rps)$$
$$\therefore P = \omega T$$

$$P = \frac{2\pi NT}{60} (where\ N\ is\ the\ speed\ in\ rpm)$$

$$But,\ the\ angular\ speed,\ \omega = \frac{2\pi N}{60} (If\ N\ is\ in\ rpm)$$

$$\therefore P = \omega T$$

$$The\ torque\ is\ given\ by,\ T = \frac{P}{\omega}$$

$$(Here,\ \omega = 2\pi N,\ if\ N\ is\ in\ rps\ and\ \omega = \frac{2\pi N}{60}\ if\ N\ is\ in\ rpm)$$

Electromagnetic torque developed in the armature (T_e)**:**

- The electromagnetic torque developed in the armature is given by,

$$T_e = \frac{P_e}{\omega}$$

$$where,\ P_e\ is\ the\ electromagnetic\ power\ developed\ in\ the\ armature\ and\ is\ given\ by,\ P_e = E_a I_a$$

$$\therefore T_e = \frac{E_a I_a}{\omega}$$

$$\Rightarrow T_e = \frac{E_a I_a}{2\pi N} (if\ N\ is\ in\ rps)$$

$$and,\ T_e = \frac{E_a I_a}{(2\pi N / 60)} (if\ N\ is\ in\ rpm)$$

$$T_e = \frac{E_a I_a}{(2\pi N / 60)} = \frac{I_a (\Phi ZN / 60) \times (P / A)}{(2\pi N / 60)} = \frac{\Phi ZP I_a}{2\pi A}$$

$$\Rightarrow T_e = K_a \Phi I_a,\ where,\ K_a\ is\ armature\ constant\ and\ K_a = \frac{ZP}{2\pi A}$$

$$\therefore T_e \propto \Phi I_a$$

Neglecting saturation and armature reaction,

For series motor, $\Phi \propto I_a$

$$\therefore T_e \propto I_a^2$$

For shunt motor, Φ *is constant because field current* I_{sh} *is constant.*

$$\therefore T_e \propto I_a$$

The electromagnetic torque developed in the armature is also known as gross torque of the motor.

Shaft Torque (T_{sh})**:**

- The electromagnetic torque developed in the armature is not available for doing the useful work. Out of this torque, some torque is lost due to iron and friction losses and the remaining torque is available at the shaft for doing the useful work. The torque available at the shaft is known as shaft torque. The shaft torque is given by, π

$$T_{sh} = \frac{P_{out}}{2\pi N} \quad (if\ N\ is\ in\ rps)$$

$$and,\ T_{sh} = \frac{P_{out}}{(2\pi N / 60)} \quad (if\ N\ is\ in\ rpm)$$

The shaft torque is also known as the net torque of the motor.

- The difference $(T_e - T_{sh})$ is known as lost torque and is due to the iron and friction losses of the motor.

Speed of DC Motor:

- The back emf of a DC motor is given by,

$$E_a = \frac{\Phi Z N}{60} \times \frac{P}{A}$$
$$\Rightarrow N = \frac{60 A E_a}{\Phi Z P}$$
$$\therefore N \propto \frac{E_a}{\Phi}$$

- For series motor,

  Let,
  E_{a1}, I_{a1}, N_1 *and* Φ_1 *be the back emf, armature current, speed and flux respectively in the* 1*st case, and*
  E_{a2}, I_{a2}, N_2 *and* Φ_2 *be the back emf, armature current, speed and flux respectively in the* 2*nd case.*
  Then,
  $$N_1 \propto \frac{E_{a1}}{\Phi_1}, \text{where } E_{a1} = V_t - I_{a1} r_a, \text{and}$$
  $$N_2 \propto \frac{E_{a2}}{\Phi_2}, \text{where } E_{a2} = V_t - I_{a2} r_a$$
  $$\therefore \frac{N_2}{N_1} = \frac{E_{a2}}{E_{a1}} \times \frac{\Phi_1}{\Phi_2}$$
  Neglecting saturation and armature reaction,
  $$\Phi_1 \propto I_{a1}, \text{and}$$
  $$\Phi_2 \propto I_{a2}$$
  $$\therefore \frac{N_2}{N_1} = \frac{E_{a2}}{E_{a1}} \times \frac{I_{a1}}{I_{a2}}$$

- For shunt motor,

  Let,
  E_{a1}, I_{a1}, N_1 *and* Φ_1 *be the back emf, armature current, speed and flux respectively in the* 1*st case, and*
  E_{a2}, I_{a2}, N_2 *and* Φ_2 *be the back emf, armature current, speed and flux respectively in the* 2*nd case.*
  Then,
  $$N_1 \propto \frac{E_{a1}}{\Phi_1}, \text{where } E_{a1} = V_t - I_{a1} r_a, \text{and}$$
  $$N_2 \propto \frac{E_{a2}}{\Phi_2}, \text{where } E_{a2} = V_t - I_{a2} r_a$$
  $$\therefore \frac{N_2}{N_1} = \frac{E_{a2}}{E_{a1}} \times \frac{\Phi_1}{\Phi_2}$$
  Neglecting saturation and armature reaction, Φ *remains constant*, i.e. $\Phi_1 = \Phi_2$.
  $$\therefore \frac{N_2}{N_1} = \frac{E_{a2}}{E_{a1}}$$

Speed Regulation:

- The speed regulation is defined as the change in speed expressed as a percentage of the rated speed when load on the motor is reduced from its rated value to zero.
  $$\therefore \text{Percentage speed regulation} = \frac{\text{No load speed} - \text{Full load speed}}{\text{Full load speed}} \times 100$$
- The term speed regulation refers to the change in motor speed due to change in applied load with other conditions remaining unchanged. It refers to the speed change happening due to inherent properties of the motor because of load change and it does

not refer to the speed change due to the manipulation of speed controlling devices like rheostats etc.

Power Stages:

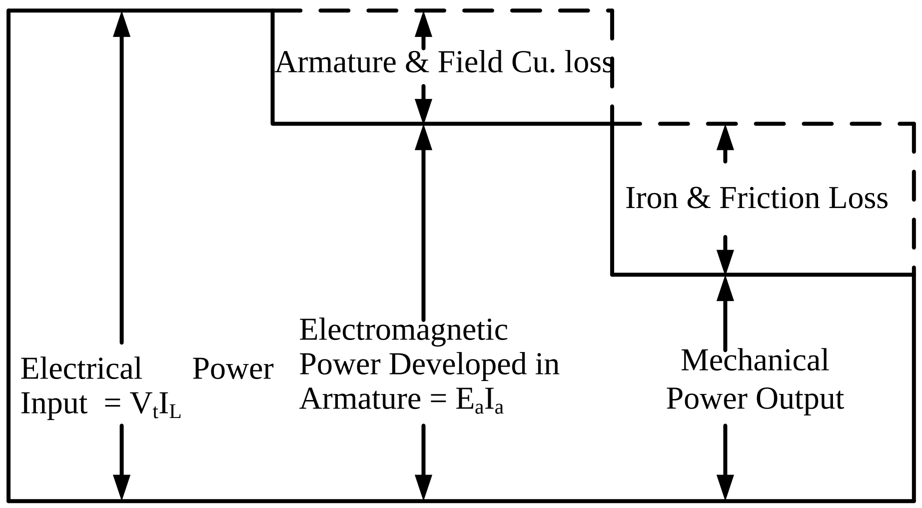


Fig. 2.5 Power stage diagram of DC motor

Condition for maximum efficiency:

- The load current for which the maximum efficiency occurs can be found as follows.

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{in} - losses}{P_{in}} = \frac{V_t I_L - I_a^2 r_a \text{ - constant loss}}{V_t I_L}$$

For maximum efficiency,

$$\frac{d\eta}{dI_L} = 0$$

Now, $I_L = I_a + I_{sh}$

Since, I_{sh} *is very less,* I_L *can be taken equal to* I_a

$$\therefore \frac{d\eta}{dI_L} = \frac{d\eta}{dI_a} = \frac{d}{dI_a}\left(\frac{V_t I_a - I_a^2 r_a \text{ - constant loss}}{V_t I_a}\right) = 0$$

$$\Rightarrow \frac{V_t I_a (V_t - 2I_a r_a) - V_t(V_t I_a - I_a^2 r_a \text{ - constant loss})}{(V_t I_a)^2} = 0$$

$$\Rightarrow V_t(V_t I_a - I_a^2 r_a \text{ - constant loss}) = V_t I_a (V_t - 2I_a r_a)$$

$$\Rightarrow \text{Constant loss} = I_a^2 r_a$$

Hence the condition for maximum efficiency is

Constant loss = Variable loss

The armature current corresponding to maximum efficiency is given by,

$$I_a = \sqrt{\frac{\text{Constant loss}}{r_a}}$$

Problem:

A 250 V, 15 KW, shunt motor runs at has a maximum efficiency of 88% when running at a speed of 700 rpm and delivering 80% of its rated output. The resistance of its shunt field is 100 Ω. Determine the efficiency and speed when the motor draws a current of 78 A from the mains.

Solution:

Case - I Maximum efficiency condition : .

$Motor\ output, P_{out} = 0.8 \times 15 = 12\,KW = 12{,}000\,W$

$Motor\ input, P_{in} = \frac{P_{out}}{\eta} = \frac{12{,}000}{0.88} = 13636\,W.$

$Losses = P_{in} - P_{out} = 13636 - 12{,}000 = 1636\,W.$

$Constant\ losses = Variable\ losses = \frac{1636}{2} = 818\,W.$

$Input\ current\ of\ motor, I_{L1} = \frac{P_{in}}{V_t} = \frac{13636}{250} = 54.544\,A.$

$Shunt\ field\ current, I_{sh} = \frac{V_t}{R_{sh}} = \frac{250}{100} = 2.5\,A.$

$Armature\ current, I_{a1} = I_{L1} - I_{sh} = 54.544 - 2.5 = 52.044\,A.$

$Variable\ losses\ i.e.\ armature\ copper\ loss = I_a^2 r_a = 818,$

$Armature\ resistance, r_a = \frac{818}{52.044^2} = 0.302\ \Omega.$

$Back\ emf, E_{a1} = V_t - I_{a1} r_a = 250 - (52.044 \times 0.302) = 234.3\,V.$

$N_1 = 700\,rpm.$

Case - II, When the motor draws a current of 78 A, i.e., I_{L2} = 78A :

$Shunt\ field\ current, I_{sh} = \frac{V_t}{R_{sh}} = \frac{250}{100} = 2.5\,A$

$Armature\ current, I_{a2} = I_{L2} - I_{sh} = 78 - 2.5 = 75.5\,A$

$Variable\ losses\ i.e. Armature\ copper\ loss = I_{a2}^2 r_a = (75.5)^2 \times 0.302 = 1721.5\,W.$

$Total\ losses = Constant\ losses + Variable\ losses = 818 + 1721.5 = 2539.5\,W.$

$Motor\ input, P_{in} = V_t \times I_{L2} = 250 \times 78 = 19500\,W.$

$Motor\ output, P_{out} = P_{in} - Total\ losses = 19500 - 2539.5 = 16960.5\,W$

$\therefore Motor\ efficiency, \eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{16960.5}{19500} \times 100 = 87\%$ ***(Ans.)***

$Back\ emf, E_{a2} = V_t - I_{a2} r_a = 250 - (75.5 \times 0.302) = 227.2\,V.$

$$\frac{E_{a2}}{E_{a1}} = \frac{N_2}{N_1}$$

$\Rightarrow N_2 = \frac{E_{a2}}{E_{a1}} \times N_1 = \frac{227.2}{234.3} \times 700 = 678\,rpm.$ ***(Ans.)***

Problem:

A 10 KW, 900 rpm, 400 V DC shunt motor has armature circuit resistance (including brushes) of 1 Ω and shunt field resistance of 400 Ω. If efficiency at rated speed is 85%, then determine:

(a) The no-load armature current,
(b) The speed when the motor draws 20 A from the mains and
(c) The armature current, when the total (or internal) torque developed is 98.5 Nm.

Assume the flux to remain constant.

Solution:

Case - I Rated load condition :

$Motor\ output, P_{out} = 10\,KW = 10{,}000W.$

$$Motor\ input, P_{in} = \frac{P_{out}}{\eta} = \frac{10{,}000}{0.85} = 11765W.$$

$Total\ Losses = P_{in} - P_{out} = 11765 - 10{,}000 = 1765W.$

$$Input\ current\ of\ motor, I_L = \frac{P_{in}}{V_t} = \frac{11765}{400} = 29.5A.$$

$$Shunt\ field\ current, I_{sh} = \frac{V_t}{R_{sh}} = \frac{400}{400} = 1A.$$

$Armature\ current = I_L - I_{sh} = 29.5 - 1 = 28.5\,A.$

$\therefore I_{a1} = 28.5\,A.$

$Variable\ losses\ i.e.\ armature\ copper\ loss = I_{a1}^2 r_a = (28.5)^2 \times 1 = 812W.$

$\therefore Constant\ losses = Total\ Losses - Variable\ losses\ i.e.\ armature\ copper\ loss$

$= 1765 - 812 = 953W.$

$Back\ emf, E_{a1} = V_t - I_{a1} r_a = 400 - (28.5 \times 1) = 371.5V.$

$N_1 = 900\,rpm.$

$$T_{e1} = \frac{E_{a1} I_{a1}}{(2\pi N_1 / 60)} = \frac{371.5 \times 28.5}{(2 \times \pi \times 900 / 60)} = 112.34\,Nm.$$

(i) Case - II, When the motor runs at no - load condition :

$P_{in} - losses = P_{out}$

$At\ no\ load, P_{out} = 0,$

$\therefore P_{in} - losses = 0.$

$Let\ I_{a0}\ be\ the\ no-load\ armature\ current.$

$\therefore V_t I_{L0} - (Constant\ losses + Variable\ losses) = 0$

$\Rightarrow V_t (I_{a0} + I_{sh}) - (Constant\ losses + I_{a0}^2 r_a) = 0 \qquad (1)$

$\Rightarrow V_t (I_{a0} + 1) - (953 + I_{a0}^2) = 0$

$\Rightarrow V_t I_{a0} + V_t - 953 - I_{a0}^2 = 0$

$\Rightarrow 400 I_{a0} + 400 - 953 - I_{a0}^2 = 0$

$\Rightarrow 400 I_{a0} - 553 - I_{a0}^2 = 0$

$\Rightarrow I_{a0}^2 - 400 I_{a0} + 553 = 0$

$$\Rightarrow I_{a0} = \frac{400 \pm \sqrt{400^2 - 4 \times 553}}{2} = 398.61\,A\ or\ 1.39\,A.\ \textbf{\textit{(Ans.)}}$$

$Higher\ value\ of\ I_{a0}\ is\ neglected\ as\ it\ is\ much\ higher\ than\ the\ rated\ current.$

$\therefore I_{a0} = 1.39\,A.$(A

$Alternatively,$

$Since\ I_{a0}\ is\ very\ small, I_{a0}^2 r_a\ can\ be\ neglected\ without\ causing\ appreciable\ error.$

$Hence, equation\ (1)\ can\ be\ written\ as:$

$V_t (I_{a0} + I_{sh}) - (Constant\ losses) = 0 \qquad (1)$

$\Rightarrow V_t (I_{a0} + 1) - (953) = 0$

$\Rightarrow V_t I_{a0} + V_t - 953 = 0$

$\Rightarrow 400 I_{a0} + 400 - 953 = 0$

$\Rightarrow 400 I_{a0} - 553 = 0$

$\Rightarrow 400 I_{a0} = 553$

$\Rightarrow I_{a0} = 1.3825\,A.$

(ii) Case - III, When the motor draws 20A from the mains :

$Armature\ current, I_{a2} = I_L - I_{sh} = 20 - 1 = 19\,A$

$Back\ emf, E_{a2} = V_t - I_{a2} r_a = 400 - (19 \times 1) = 381V.$

$$\frac{E_{a2}}{E_{a1}} = \frac{N_2}{N_1}$$

$$\Rightarrow N_2 = \frac{E_{a2}}{E_{a1}} \times N_1 = \frac{381}{371.5} \times 900 = 923\,rpm.\ \textbf{(Ans.)}$$

(iii) Case - IV, When the total torque developed is 98.5 Nm :

$$\frac{T_{e2}}{T_{e1}} = \frac{I_{a2}}{I_{a1}}$$

$$\Rightarrow I_{a2} = \frac{T_{e2}}{T_{e1}} \times I_{a1}$$

$$\therefore I_{a2} = \frac{98.5}{112.34} \times 28.5 = 25\,A.\ \textbf{(Ans.)}$$

Problem:

A 10 KW, 240 V DC shunt motor draws a line current of 5.2 A while running at no-load speed of 1200 rpm from a 240 V DC supply. It has an armature resistance of 0.25 Ω and a field resistance of 160 Ω. Estimate the efficiency of the motor when it delivers rated load.

Solution:

Case - I No - load condition :

$$Shunt\ field\ current, I_{sh} = \frac{V_t}{R_{sh}} = \frac{240}{160} = 1.5\,A$$

$No-load\ armature\ current, I_{a0} = I_{L0} - I_{sh} = 5.2 - 1.5 = 3.7\,A$

$Armature\ copper\ loss = I_{a0}^2 r_a = (3.7)^2 \times 0.25 = 3.4225W$

$P_{in} - Total\ losses = P_{out}$

$At\ no-load, P_{out} = 0$

$\therefore P_{in} = Total\ losses$

$\Rightarrow V_t I_{L0} = Constant\ losses + Variable losses\ i.e. armature\ copper\ losses$

$Constant\ losses = (240 \times 5.2) - 3.4225 = 1244.5775W$

Case - II Rated load condition :

$P_{out} = P_{in} - Total\ losses$

$\Rightarrow P_{out} = P_{in} - (Constant\ losses + Armature\ copper\ loss)$

$\Rightarrow 10000 = V_t I_L - (1244.5775 + I_a^2 r_a)$

$\Rightarrow 10000 = V_t (I_a + I_{sh}) - (1244.5775 + I_a^2 r_a)$

$\Rightarrow 10000 = 240(I_a + 1.5) - (1244.5775 + 0.25 I_a^2)$

$\Rightarrow 10000 = 240 I_a + 360 - 1244.5775 - 0.25 I_a^2$

$\Rightarrow 10000 = 240 I_a - 884.5775 - 0.25 I_a^2$

$\Rightarrow 10884.5775 = 240 I_a - 0.25 I_a^2$

$\Rightarrow 0.25 I_a^2 - 240 I_a + 10884.5775 = 0$

$$\Rightarrow I_a = \frac{240 \pm \sqrt{240^2 - 4 \times 0.25 \times 10884.5775}}{2 \times 0.25} = 912.27\,A\ or\ 47.725\,A$$

$Higher\ value\ of\ I_a\ is\ not\ considered.$

$\therefore I_a = 47.725\,A$

$Armature\ copper\ loss = I_a^2 r_a = (47.725)^2 \times 0.25 = 569.42W$

$Total\ losses = Constant\ losses + Armature\ copper\ loss = 1244.5775 + 569.42 = 1814W$

$$\therefore Efficiency, \eta_{fl} = \frac{P_{out}}{P_{in}} \times 100 = \frac{P_{out}}{P_{out} + Total\ losses} \times 100 = \frac{10000}{10000 + 1814} \times 100$$

$$= \frac{10000}{11814} \times 100 = 84.64\%\ \textbf{(Ans.)}$$

Transformer

Principle of Transformer Action

- A transformer works on the principle of electromagnetic induction between two (or more) coils.
- According to this principle, an emf is induced in a coil if it links a changing flux.
- Thus the transformer action requires the existence of alternating mutual flux linking the various windings on a common magnetic core.

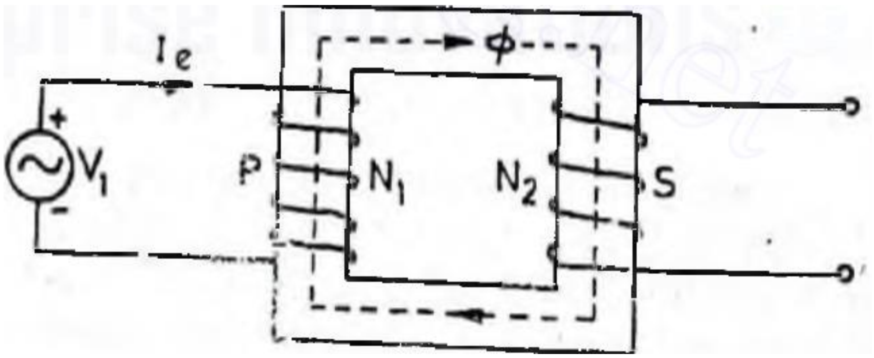


Fig. 1 Working principle of transformer

Ideal Transformer

- In the beginning, a transformer is assumed to be an ideal one, merely for obtaining an easier explanation of what happens in a transformer.

Assumptions for Ideal Transformer

1. Winding resistance are negligible.
2. All the flux set up by the primary links the secondary windings, i.e. all the flux is confined to the magnetic core and there is no leakage flux.
3. The core losses (hysteresis and eddy current losses are negligible).
4. The core has constant permeability, i.e. the magnetization curve of the core is linear.

At a later stage the effect of these assumptions will be taken up one by one to understand the behaviour of an actual transformer.

Ideal Transformer under no load

- Let V_1 be the voltage applied to the primary, with secondary open circuited be, sinusoidal.
- Then the current I_e due to the applied voltage V_1 will also be sinusoidal.
- The mmf $N_1 I_e$ and therefore the core flux ϕ will follow the variations of I_e very closely, i.e. the flux ϕ is in time phase with the current I_e and varies sinusoidally.
- Let the sinusoidal variation of flux ϕ be represented as $\phi = \phi_{max}\, Sin\omega t$ (1)
- The emf e_1 in volts induced in the primary winding by the alternating flux ϕ is given by:

$$e_1 = -N_1 \frac{d\phi}{dt} = -N_1 \frac{d}{dt}(\phi_{max} \sin \omega t) = -N_1 \omega \phi_{max} \cos \omega t = N_1 \omega \phi_{max} \sin\left(\omega t - \frac{\pi}{2}\right) \quad (2)$$

- Its maximum value, E_{1max} occurs when $sin\left(\omega t - \frac{\pi}{2}\right)$ is equal to 1.

$$\therefore E_{1max} = N_1 \omega \phi_{max}$$

$$Now, \quad e_1 = E_{1max} sin\left(\omega t - \frac{\pi}{2}\right) \quad (3)$$

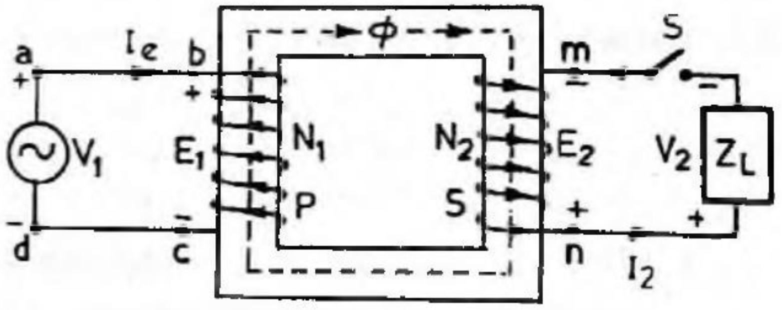


Fig. 2 Ideal transformer under no load

- RMS value of emf E_1 induced in primary winding is $E_1 = \frac{E_{1\max}}{\sqrt{2}} = \frac{N_1\omega\phi_{\max}}{\sqrt{2}} = \frac{2\pi fN_1\phi_{\max}}{\sqrt{2}} = \sqrt{2}\pi fN_1\phi_{\max}$ (4)

- As per Lenz's law emf induced in primary winding e_1 opposes the applied voltage v_1
- Since the primary winding has no resistance, e_1 at every instant must be equal and opposite to v_1 .

$$\therefore v_1 = -e_1 = N_1\frac{d\phi}{dt} \Rightarrow V_1 = -E_1 \quad (5)$$

- The emf induced in secondary is

$$e_2 = -N_2\frac{d\phi}{dt} = -N_2\frac{d}{dt}(\phi_{max}\sin\omega t) = -N_2\omega\phi_{max}\cos\omega t = N_2\omega\phi_{max}\sin\left(\omega t - \frac{\pi}{2}\right) = E_{2max}\sin\left(\omega t - \frac{\pi}{2}\right) \quad (6)$$

- RMS value of emf E_2 induced in secondary winding is given by

$$E_2 = \frac{E_{2max}}{\sqrt{2}} = \frac{N_2\omega\phi_{\max}}{\sqrt{2}} = \frac{2\pi fN_2\phi_{\max}}{\sqrt{2}} = \sqrt{2}\pi fN_2\phi_{max} \quad (7)$$

From equation (4) & (7)

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} \text{ or } \frac{E_1}{N_1} = \frac{E_2}{N_2} = \sqrt{2}\pi f\phi_{max} \quad (8)$$

i.e. emf per turn in primary =emf per turn in secondary.

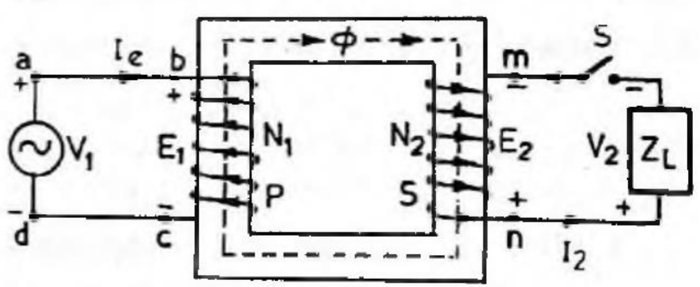


Fig. 2 Ideal transformer under no load

Waveforms and Phasor Diagram

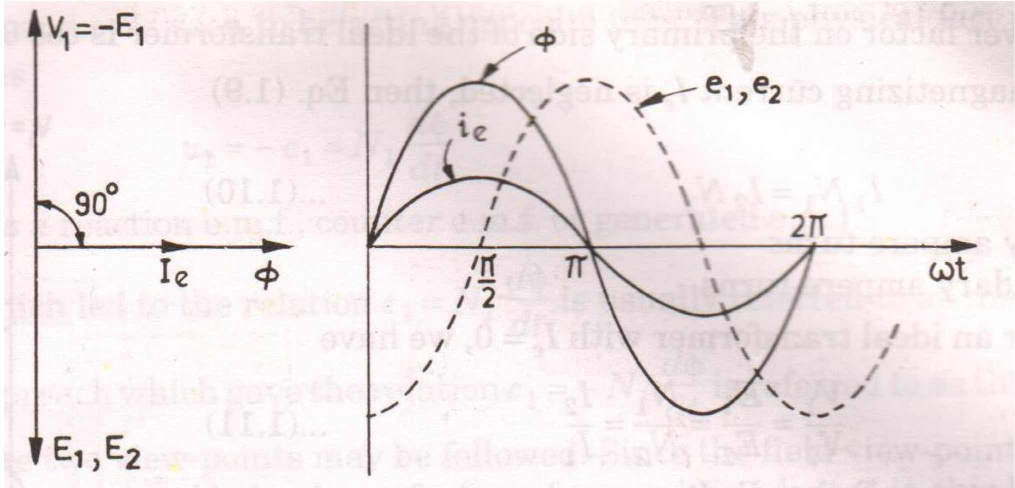
- The waveforms and phasor diagram can be drawn with the help of the equation(1), (3), (5) & (6) which are depicted below.
- Here, N_1 *and* N_2 are assumed equal for convenience and therefore, $E_1 = E_2$

$$\phi = \phi_{\max} \, Sin\omega t \quad (1)$$

$$e_1 = E_{1maax} \sin\left(\omega t - \frac{\pi}{2}\right) \quad (3)$$

$$v_1 = -e_1 \quad (5)$$

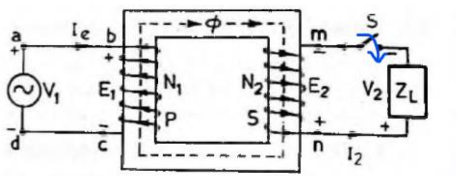
$$e_2 = E_{2\max} \sin\left(\omega t - \frac{\pi}{2}\right) \quad (6)$$



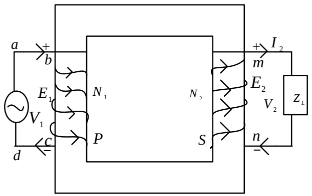
(a) (b)

Fig. 3. Ideal transformer under no load (a) Phasor diagram, (b) time diagram

Ideal Transformer under load



(a)

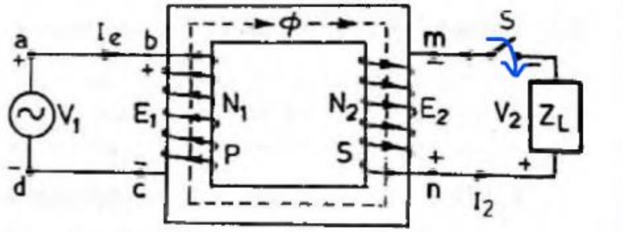


(b)

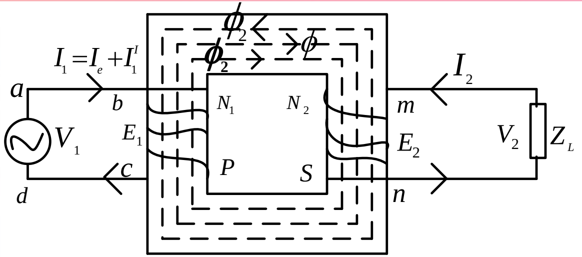
(Fig. 4 Ideal transformer under load (a) with polarity ‘n’ +ve and ‘m’ -ve and (b) with polarity ‘m’ +ve and ‘n’ -ve)

- If the switch S is closed, the load impedance Z_L gets connected across the secondary terminals.
- Since the secondary winding resistance is zero, $V_2 = E_2$.
- The direction of secondary current I_2 can be found by applying Lenz’s law and therefore the current will leave the terminal ‘n’ and enter in the terminal ‘m’.
- The secondary winding behaves like a voltage source. Terminal ‘n’ is treated as positive and terminal ‘m’ as negative as shown in Fig. 4(a).
- When terminal ‘b’ is treated as positive with respect to terminal ‘c’ in primary side, at the same time terminal ‘n’ is treated as positive with respect to terminal ‘m’ in the secondary side. This forms the basis of polarity marking in transformers.
- If secondary winding is in opposite manner then terminal ‘m’ would be positive with respect to terminal ‘n’ as shown in Fig. 4 (b).
- Polarity markings of the transformer depends upon the manner in which the windings are wound around the legs with respect to each other.

Ideal Transformer under load



(a)



(b)

Fig. 5 (a) and (b), Ideal transformer under load

- In Fig. 5, the secondary mmf F_2 being opposite to ϕ tends to reduce the alternating mutual flux ϕ.
- Any reduction in ϕ would reduce E_1. For an ideal transformer $V_1 = -E_1$. Since V_1 is constant, E_1 and therefore, ϕ will remain constant as per the equation $E_1 = \sqrt{2}\pi fN_1\phi_{max}$.
- Thus the primary draws more current I_1^I from the source to neutralize the demagnetizing effect of F_2 such that $I_1^1 N_1 = I_2 N_2$ (9) i.e. compensating primary mmf F_1= secondary mmf F_2
- The total primary current is given by: $I_1 = I_e + I_1^I$ (10). Here I_1^1 is known as load component of the primary current.
- If the magnetizing current I_e is neglected, then $I_1 N_1 = I_2 N_2$, i.e. primary amp-turns=secondary amp-

turns. Thus, $\frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = K$ (11). Here, K is known as the transformation ratio.

Phasor Diagram

- It is thus seen that core flux in an ideal transformer remains constant and is independent of the load current.
- Assuming I_2 to lag behind V_2 by an angle θ_2, the phasor diagram of an ideal transformer under load can be drawn as shown in Fig. (6).
- The power factor on the primary side of transformer is $\cos\theta_1$.

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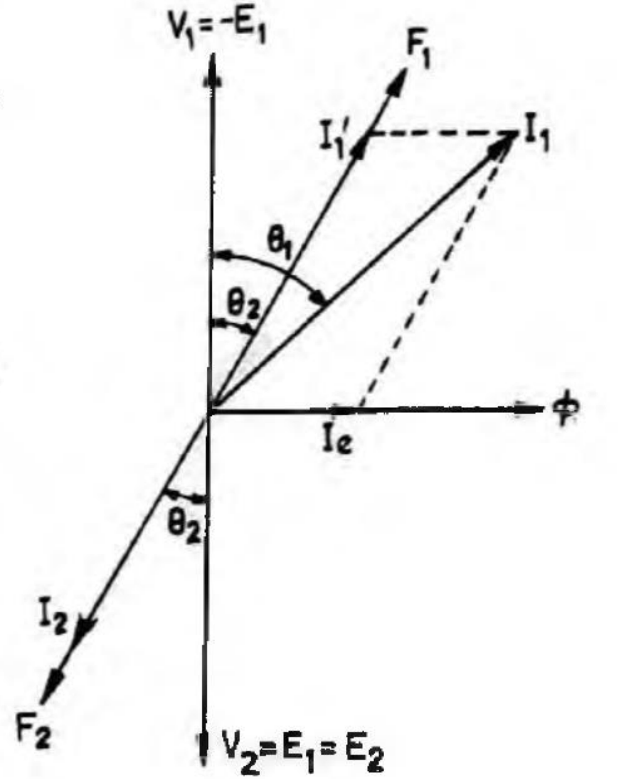


Fig. 6 Phasor diagram of an ideal transformer under load for lagging pf

Phasor Diagram of Actual Transformer

- The purpose of first considering an ideal transformer is merely to highlight the most important aspects of transformer action. Such a transformer never exists and now the phasor diagram of actual transformer with various imperfections will be considered.
- Magnetization curve of the actual transformer core is nonlinear and its effect is to introduce higher order harmonics in the magnetizing current.
- Since all the quantities in a phasor diagram must be of the same frequency, these higher order harmonics can't be represented in the phasor diagram.
- In view of the above, linear magnetization curve for the transformer core will continue to be assumed.

Phasor Diagram of Actual Transformer under no load

- The magnetic flux ϕ being common to both primary and secondary is drawn first. The induced emfs E_1 and E_2 lags behind ϕ by 90^0 and are shown accordingly in Fig. 7 (c).
- The voltage $-E_1$ being replaced by V_1^I just for convenience.
- Alternatively, V_1^I be considered as a voltage drop on the primary, in the direction of flow of primary current. Now the various imperfections in a real transformer will be considered one by one.

Effect of transformer core loss

- The current in the primary is alternating, therefore, the magnetizing force H is cyclically varying from one positive value say H_1 to a corresponding negative value $-H_1$ as shown in Fig. 7 (a).
- From Fig. 7 (a) & (b) it is evident that, the exciting current i_e leads the magnetic flux ϕ (or ϕ lags i_e) by some time angle α called hysteretic angle.

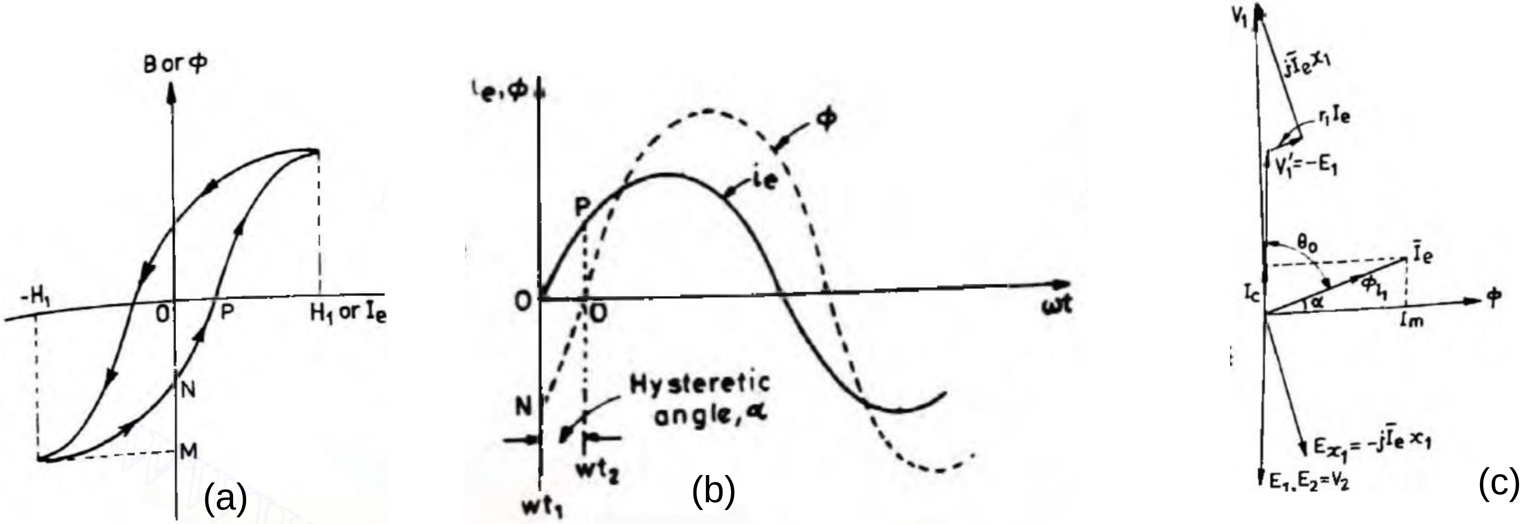


Fig. 7 (a) Hysteresis loop for transformer core , (b) exciting current and core flux waveforms and (c) no load phasor diagram of an actual transformer

- The no load primary current I_e is called the exciting current of the transformer and can be resolved into two components.
- The component I_m along ϕ is called the reactive or magnetizing current since its function is to produce the required magnetic flux in the core.
- The component along V_1^I is I_c and this component is called the core loss component or power component of I_e since it is responsible for the core loss in the transformer. The product of V_1^I and I_c gives the core loss P_c of the transformer.
- From Fig. 7(c), it is seen that, $I_e = \sqrt{I_m^2 + I_c^2}$ (12)
- In an ideal transformer, core loss current $I_c = 0$ and therefore, exciting current, I_e = magnetizing current, I_m

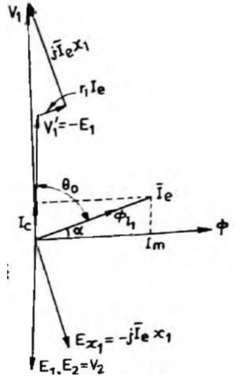


Fig. 7 (c) no load phasor diagram of an actual transformer

Effect of transformer resistance

- The effect of primary resistance r_1 can be accounted for, by adding to V_1^I, a voltage drop equal to $r_1 I_e$ as shown in Fig. 7 (c) such that $r_1 I_e$ is in phase with I_e and therefore drawn parallel to I_e in the phasor diagram.

Effect of leakage flux

- As shown in Fig. 8, the magnetic potential difference between point ‘A’ and ‘B’ establishes: (i) the mutual flux ϕ linking both the cores and (ii) the primary leakage flux ϕ_{l1} which links only the primary winding.
- The distinctive behaviour of the mutual flux ϕ and the primary leakage flux ϕ_{l1} must be carefully understood.
- The mutual flux ϕ exists entirely in the ferromagnetic core and, therefore, involves hysteresis loop. Therefore, I_e leads ϕ by an angle α.
- The primary leakage flux ϕ_{l1} does not involve hysteresis loop and hence it is in phase with I_e that produces it.

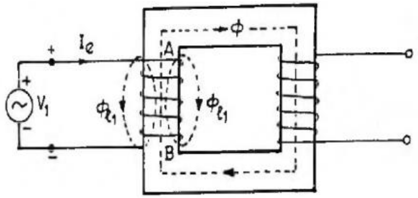


Fig. 8 Actual Transformer under load

- In the primary winding ϕ induces an emf E_1 lagging it by 90^0. Similarly, ϕ_{l1} induces an emf E_{x1} in the primary winding lagging it by 90^0. Since I_e leads E_{x1} by 90^0, it is possible to write $\boldsymbol{E_{x1} = -jI_e x_1}$. The primary applied voltage V_1 must have a component $jI_e x_1$ equal and opposite to E_{x1}.
- Here, x_1 has the nature of reactance and is referred to as primary leakage reactance in ohms.

- The total voltage drop in primary at no load is $I_e(r_1 + jx_1) = I_e z_1$, where z_1 is the primary leakage impedance.
- The primary voltage equation at no load is $V_1 = V_1^I + I_e(r_1 + jx_1) \quad (13)$.
- The phasor diagram of actual transformer at no load is depicted below.

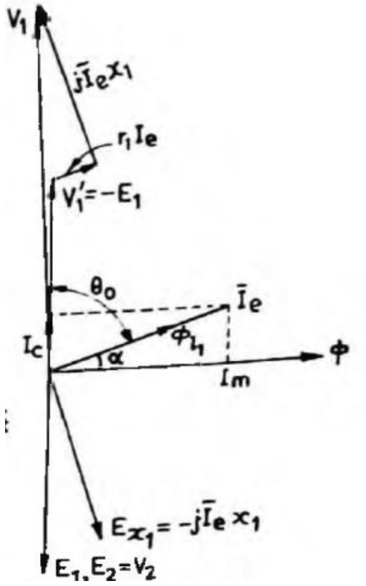


Fig. 7 (c) No load phasor diagram of an actual transformer

Phasor Diagram of Actual Transformer under load

- The secondary circuit of the transformer is considered first and then the primary circuit, for developing the phasor diagram of actual transformer under load.
- The secondary side voltage equation is given by

$$E_2 = V_2 + I_2(r_2 + jx_2) = V_2 + I_2 z_2 \quad (14)$$

- The primary side voltage equation is given by

$$V_1 = V_1^I + I_1(r_1 + jx_1) \quad (15)$$

- Phasor diagram of actual transformer under load is depicted in Fig. 10.

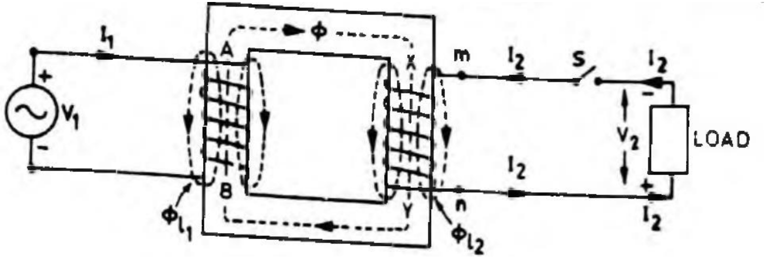


Fig. 9 Actual transformer under load

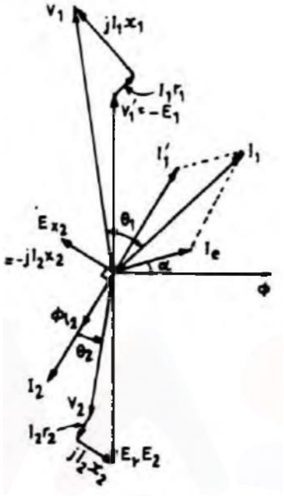


Fig. 10 Phasor diagram of actual transformer under load

Important formulas and derivations of Induction Motor

Magnitude of resultant Rotating Magnetic Field/Revolving Magnetic Field (RMF) $= 1.5\phi_m$, where ϕ_m is the maximum value of the flux in any phase.

Speed at which the RMF revolves round the stator or airgap is called synchronous speed (n_s)

$$\boldsymbol{n_s = \frac{120f_1}{P}} \textbf{ in RPM}$$

$$\boldsymbol{n_s = \frac{2f_1}{P}} \textbf{ in RPS}$$

Where, $f_1 =$ Stator or Supply frequency in Hz and $P =$ Nos. of Stator Poles

Slip Speed $= n_s - n_r$, Where $n_r =$ Rotor speed

Slip, $\boldsymbol{s = \frac{n_s - n_r}{n_s}}$

% Slip, $\boldsymbol{\% \ s = \frac{n_s - n_r}{n_s} \times 100}$

During standstill condition, $N_r = 0$ and hence, $s = \frac{n_s - 0}{n_s} = \frac{n_s}{n_s} = 1$

At standstill, speed of rotor field with respect to rotor structure is n_s

During running condition, speed of rotor field with respect to rotor structure is sn_s

Speed of rotor field with respect to stator is $=$ rotor speed $+$ speed of rotor field with respect to rotor structure $= n_r + sn_s = n_s(1-s) + sn_s = n_s$

The stator and rotor fields are stationary with respect to each other at all possible speeds.

Rotor frequency, $\boldsymbol{f_2 = sf_1}$

During standstill condition, $\boldsymbol{s = 1}$ **and hence,** $\boldsymbol{f_2 = f_1}$

$\boldsymbol{f_2}$ **is also called as the slip frequency.**

Rotor emf/phase at standstill $= \boldsymbol{E_2}$

Rotor emf/phase during running condition $= \boldsymbol{sE_2}$

Rotor leakage reactance/phase at standstill $= \boldsymbol{x_2}$

Rotor leakage reactance/phase during running condition $= \boldsymbol{sx_2}$

Rotor resistance/phase $= r_2$ at standstill as well as during running condition. It is independent of rotor frequency.

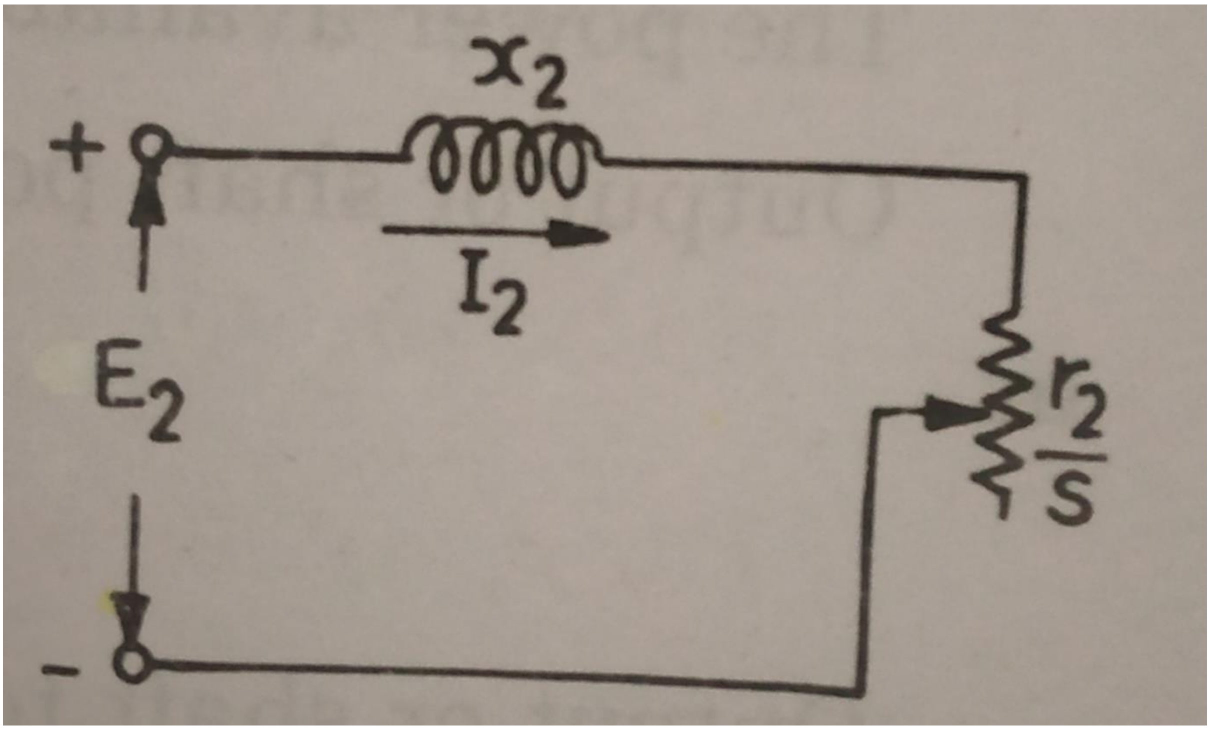
Rotor leakage impedance/phase at standstill $= \sqrt{r_2^2 + x_2^2}$

Rotor leakage impedance/phase during running condition $= \sqrt{r_2^2 + (sx_2)^2}$

Rotor current/phase at standstill $= \dfrac{E_2}{\sqrt{r_2^2 + x_2^2}}$

Rotor current/phase during running condition, $I_2 = \dfrac{sE_2}{\sqrt{r_2^2 + (sx_2)^2}} = \dfrac{E_2}{\sqrt{\left(\dfrac{r_2}{s}\right)^2 + x_2^2}}$

Rotor equivalent circuit:



(Rotor equivalent circuit of an induction motor)

Rotor power factor:

$$Rotor\ power\ factor, cos\theta_2 = \frac{Rotor\ resistance\ /\ phase}{Rotor\ impedance\ /\ phase}$$

$$\therefore cos\theta_2 = \frac{r_2}{\sqrt{r_2^2 + (sx_2)^2}} = \frac{r_2\ /\ s}{\sqrt{(r_2\ /\ s)^2 + x_2^2}}$$

Rotor input power/phase (P_2) or Airgap power/phase (P_g):

$$\boldsymbol{P_g = E_2 I_2 \cos\theta_2}$$
$$= \frac{E_2}{\sqrt{\left(\frac{r_2}{s}\right)^2 + x_2^2}} \times I_2 \times \frac{r_2}{s}$$
$$= \boldsymbol{I_2^2 \frac{r_2}{s}}$$
$$= I_2^2 r_2 \left(\frac{1}{s} + 1 - 1\right)$$
$$= I_2^2 r_2 \left[1 + \left(\frac{1}{s} - 1\right)\right]$$
$$= I_2^2 r_2 \left[1 + \left(\frac{1-s}{s}\right)\right]$$
$$= \boldsymbol{I_2^2 r_2 + I_2^2 r_2 \left(\frac{1-s}{s}\right)}$$

$\therefore$ ***Airgap power*** (P_g) ***= Rotor ohmic loss + Internal or gross mechanical power developed in rotor*** (P_m)

$$\boldsymbol{P_g = sP_g + (1-s)P_g}$$

Rotor ohmic loss $= \boldsymbol{sP_g = \left(\frac{s}{1-s}\right)P_m}$

<u>Internal or gross or electromagnetic torque developed/phase</u>:

$$T_e = \frac{\text{Internal or gross mechanical power developed in rotor}}{\text{Rotor speed in mechanical radian per second}}$$

$$\Rightarrow T_e = \frac{P_m}{\omega_r} = \frac{(1-s)P_g}{(1-s)\omega_s}$$
$$\therefore \boldsymbol{T_e = \frac{P_g}{\omega_s} = \frac{1}{\omega_s} \times \frac{I_2^2 r_2}{s} = \frac{\text{Rotor ohmic loss / phase}}{s\omega_s}}$$

Alternatively,

$$T_e = \frac{1}{\omega_s} \times \frac{I_2^2 r_2}{s} = \frac{1}{\omega_s} \times \left(\frac{sE_2}{\sqrt{r_2^2 + (sx_2)^2}}\right)^2 \times \frac{r_2}{s}$$
$$\Rightarrow T_e = \frac{1}{\omega_s} \times \frac{sE_2^2 r_2}{r_2^2 + (sx_2)^2}$$
$$\Rightarrow \boldsymbol{T_e = \frac{KsE_2^2 r_2}{r_2^2 + s^2 x_2^2}}$$
$$\textit{Where, } \boldsymbol{K = \frac{1}{\omega_s}}$$

At, standstill, $s = 1$ *and hence, starting torque,* $\boldsymbol{T_{st} = \frac{KE_2^2 r_2}{r_2^2 + x_2^2}}$

Where, $\boldsymbol{\omega_s = 2\pi n_s = \frac{4\pi f_1}{P}}$ ***is the synchronous speed in mechanical radian per second***

Condition for maximum starting torque:

$$T_{st} = \frac{KE_2^2 r_2}{r_2^2 + x_2^2}$$

For maximum starting torque,

$$\frac{d}{dr_2}\left(T_{est}\right) = 0$$

$$\Rightarrow \frac{d}{dr_2}\left(\frac{KE_2^2 r_2}{r_2^2 + x_2^2}\right) = 0$$

$$\Rightarrow \frac{KE_2^2\left(r_2^2 + x_2^2\right) - 2r_2\left(KE_2^2 r_2\right)}{\left(r_2^2 + x_2^2\right)^2} = 0$$

$$\Rightarrow KE_2^2\left(r_2^2 + x_2^2\right) = 2r_2\left(KE_2^2 r_2\right)$$

$$\Rightarrow r_2^2 = x_2^2$$

$$\therefore \boldsymbol{r_2 = x_2}$$

Condition for maximum torque under running condition:

$$T_e = \frac{KsE_2^2 r_2}{r_2^2 + s^2 x_2^2}$$

For maximum torque under running condition

$$\frac{d}{ds}\left(T_e\right) = 0$$

$$\Rightarrow \frac{d}{ds}\left(\frac{KsE_2^2 r_2}{r_2^2 + s^2 x_2^2}\right) = 0$$

$$\Rightarrow \frac{KE_2^2 r_2\left(r_2^2 + s^2 x_2^2\right) - 2sx_2^2\left(KsE_2^2 r_2\right)}{\left(r_2^2 + s^2 x_2^2\right)^2} = 0$$

$$\Rightarrow KE_2^2 r_2\left(r_2^2 + s^2 x_2^2\right) = 2sx_2^2\left(KsE_2^2 r_2\right)$$

$$\Rightarrow s^2 x_2^2 = r_2^2$$

$$\therefore \boldsymbol{r_2 = sx_2}$$

$$\Rightarrow s = \frac{r_2}{x_2}$$

$\therefore$ *Slip corresponding to maximum torque,* $\boldsymbol{s_m = \frac{r_2}{x_2}}$

Maximum torque under running condition, $T_{max} = \frac{Ks_m E_2^2 r_2}{r_2^2 + s_m^2 x_2^2} = \frac{Ks_m^2 E_2^2 x_2}{2s_m^2 x_2^2}$

$$\therefore \boldsymbol{T_{max} = \frac{KE_2^2}{2x_2}}$$

Maximum torque under running condition (T_{max}) *also known as pullout torque or breakdown torque* (T_b)
Slip corresponding to maximum torque or breakdown torque is also denoted by $\boldsymbol{s_b}$

Full load Torque $\left(T_f\right)$**:**

Full load torque, $\boldsymbol{T_f = \dfrac{K s_f E_2^2 r_2}{r_2^2 + s_f^2 x_2^2}}$

Where, s_f is the slip at full load.

Relation between starting torque and full load torque:

$$\frac{T_{st}}{T_f} = \frac{\left(\dfrac{KE_2^2 r_2}{r_2^2 + x_2^2}\right)}{\left(\dfrac{K s_f E_2^2 r_2}{r_2^2 + s_f^2 x_2^2}\right)} = \frac{KE_2^2 r_2}{r_2^2 + x_2^2} \times \frac{r_2^2 + s_f^2 x_2^2}{K s_f E_2^2 r_2} = \frac{r_2^2 + s_f^2 x_2^2}{s_f\left(r_2^2 + x_2^2\right)} = \frac{\left(r_2^2 + s_f^2 x_2^2\right)/x_2^2}{s_f\left(r_2^2 + x_2^2\right)/x_2^2} = \frac{\left(r_2/x_2\right)^2 + s_f^2}{s_f\left[\left(r_2/x_2\right)^2 + 1\right]}$$

$$\therefore \boldsymbol{\frac{T_{st}}{T_f} = \frac{s_m^2 + s_f^2}{s_f\left(s_m^2 + 1\right)}}$$

Relation between full load torque and maximum torque:

$$\frac{T_f}{T_{max}} = \frac{\left(\dfrac{K s_f E_2^2 r_2}{r_2^2 + s_f^2 x_2^2}\right)}{\left(\dfrac{KE_2^2}{2x_2}\right)} = \frac{K s_f E_2^2 r_2}{r_2^2 + s_f^2 x_2^2} \times \frac{2x_2}{KE_2^2} = \frac{2 s_f r_2 x_2}{r_2^2 + s_f^2 x_2^2} = \frac{2 s_f r_2 x_2 / x_2^2}{\left(r_2^2 + s_f^2 x_2^2\right)/x_2^2} = \frac{2 s_f \left(r_2/x_2\right)}{\left(r_2/x_2\right)^2 + s_f^2}$$

$$\therefore \boldsymbol{\frac{T_f}{T_{max}} = \frac{2 s_m s_f}{s_m^2 + s_f^2}}$$

Relation between starting torque and maximum torque:

$$\frac{T_{st}}{T_{\max}} = \frac{\left(\dfrac{KE_2^2 r_2}{r_2^2 + x_2^2}\right)}{\left(\dfrac{KE_2^2}{2x_2}\right)} = \frac{KE_2^2 r_2}{r_2^2 + x_2^2} \times \frac{2x_2}{KE_2^2} = \frac{2 r_2 x_2}{r_2^2 + x_2^2} = \frac{2 r_2 x_2 / x_2^2}{\left(r_2^2 + x_2^2\right)/x_2^2} = \frac{2\left(r_2/x_2\right)}{\left(r_2/x_2\right)^2 + 1}$$

$$\therefore \boldsymbol{\frac{T_{st}}{T_{max}} = \frac{2 s_m}{s_m^2 + 1}}$$

Torque at any slip in terms of maximum torque:

Torque at any slip, $T_e = \dfrac{K s E_2^2 r_2}{r_2^2 + s^2 x_2^2}$

Breakdown torque, $T_b = \dfrac{K s_m E_2^2 r_2}{r_2^2 + s_m^2 x_2^2}$

$$\frac{T_e}{T_m} = \frac{\left(\frac{KsE_2^2 r_2}{r_2^2 + s^2 x_2^2}\right)}{\left(\frac{KE_2^2}{2x_2}\right)} = \left(\frac{KsE_2^2 r_2}{r_2^2 + s^2 x_2^2}\right) \times \left(\frac{2x_2}{KE_2^2}\right) = \frac{2sr_2 x_2}{r_2^2 + s^2 x_2^2} = \frac{2}{\left(r_2 / sx_2\right) + \left(sx_2 / r_2\right)}$$

$$\therefore \boldsymbol{T_e = T_m \left[\frac{2}{(s_m / s) + (s / s_m)}\right]}$$

Net mechanical power output or net power output (P_{out}) or shaft power (P_{sh}) :

$$= P_m - Mechanical\ losses\ (friction\ and\ windage\ losses)$$
$$= (P_g - Rotor\ ohmic\ loss) - friction\ and\ windage\ losses$$
$$= (P_{in} - Stotor\ ohmic\ loss - Stator\ iron\ loss) - Rotor\ ohmic\ loss - friction\ and\ windage\ losses$$

Shaft torque (T_{sh}) :

$$T_{sh} = \frac{Shaft\ Power\ (P_{sh})}{Rotor\ speed\ in\ mechanical\ radian\ per\ second\ (\omega_r)}$$

$$\therefore \boldsymbol{T_{sh} = \frac{P_{sh}}{\omega_s (1 - s)}}$$

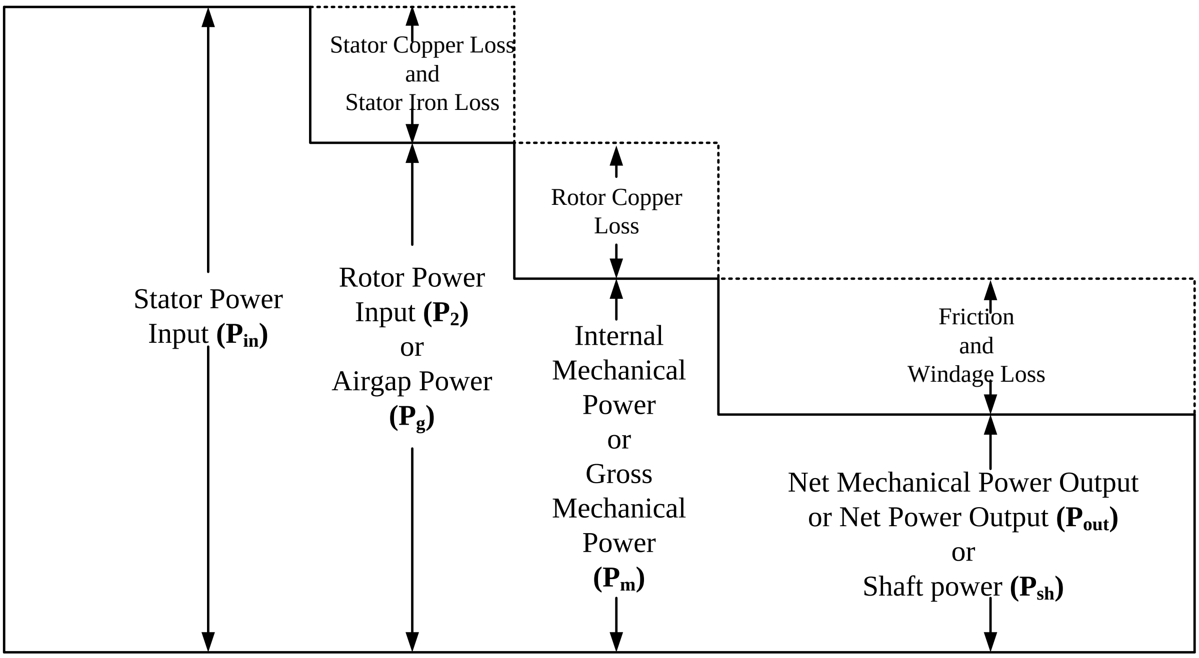
Efficiency:

Efficiency of IM, $\boldsymbol{\eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{P_{out}}{P_{out} + P_f + P_{oh}} \times 100}$

Where,

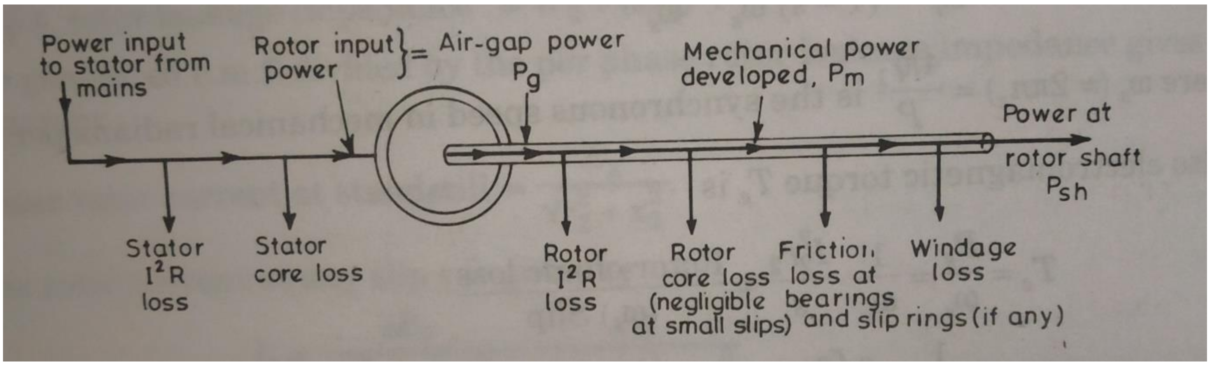
$P_{in} = Power\ input\ to\ the\ stator$
$P_{out} = Net\ mechanical\ power\ output$
$P_f = fixed\ losses\ or\ rotational\ losses = stator\ core\ loss + friction\ \&\ windage\ loss$
$P_{oh} = variable\ losses = stator\ \&\ rotor\ ohmic\ losses + brush\ contact\ loss\ (in\ case\ of\ SRIM)$

Power Stages:



(Power stages of an induction motor)

Power flow diagram:



(Power flow diagram of an induction motor)