BASIC ARCHITECTURAL FRAMEWORK

PHYSICAL LAYER

OUTLINES

- Basic Components
- Source Encoding
  - The Efficiency of a Source Encode
  - Pulse Code Modulation and Delta Modulation
- Channel Encoding
  - Types of Channels
  - Information Transmission over a Channel
  - Error Recognition and Correction
- Modulation
  - Modulation Types
  - Quadratic Amplitude Modulation
  - Summary
- Signal Propagation

PHYSICAL LAYER - OVERVIEW

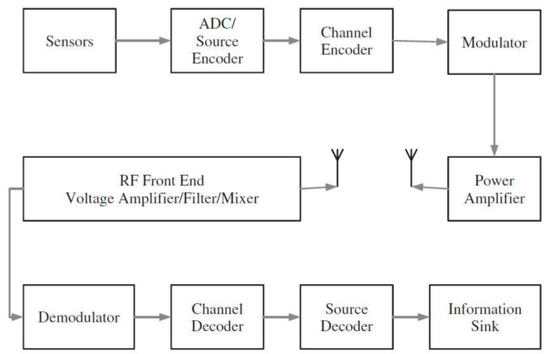
- One of the desirable aspects of WSNs is their ability to communicate over a wireless link, so
  - mobile applications can be supported
  - flexible deployment of nodes is possible
  - the nodes can be placed in areas that are inaccessible to wired nodes
- Once the deployment is carried out, it is possible to
  - rearrange node placement - optimal coverage and connectivity
  - the rearrangement can be made without disrupting the normal operation
- Some formidable *challenges*:
  - limited bandwidth
  - limited transmission range
  - poor packet delivery performance because of interference, attenuation, and multi-path scattering
- therefore, it is vital to understand their properties and some of the mitigation strategies
- In this Chapter, we'll discuss a fundamental introduction to *point-to-point* wireless digital communication

COMMUNICATION SYSTEM – BASIC COMPONENTS

- Communication Systems deals with the flow of some sort of information in some network.
- Information my be electrical signals, words, pictures, music etc.
- The *basic components* of a digital communication system:
  - Transmitter or Source
  - Channel or Transmission Networks, which transfer the communique from transmitter to receiver
  - Receiver or Destination

- In WSN communication, we study the *short range communication* - because nodes are placed close to each other.

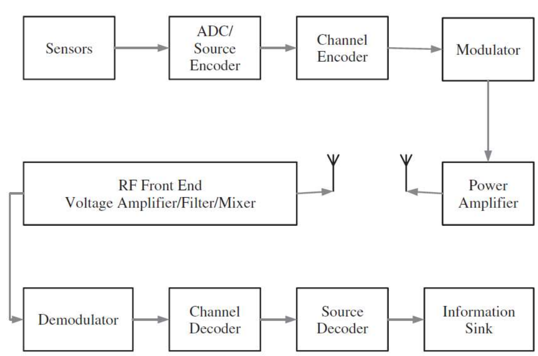
PHYSICAL LAYER – BASIC COMPONENTS



Components of a digital communication system.

- *The communication source* represents one or more sensors and produces *a message signal* - *an analog signal*
  - the signal is a *baseband* signal having dominant frequency components *near zero*
  - the message signal has to be converted to a discrete signal (*discrete* both in *time* and *amplitude*)

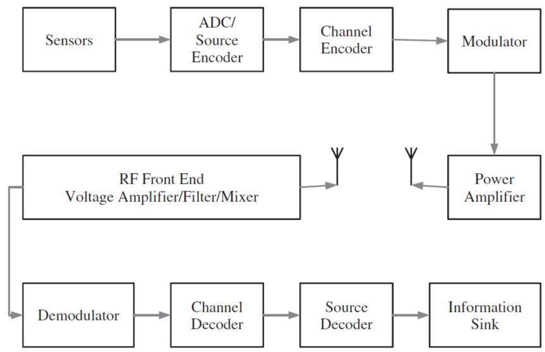
PHYSICAL LAYER – BASIC COMPONENTS



Components of a digital communication system.

- This conversion requires sampling the signal at least at *Nyquist Rate* - no information will be lost
  - The Nyquist rate sets a lower bound on the sampling frequency
  - Hence, the minimum sampling rate should be twice the bandwidth of the signal

PHYSICAL LAYER – BASIC COMPONENTS

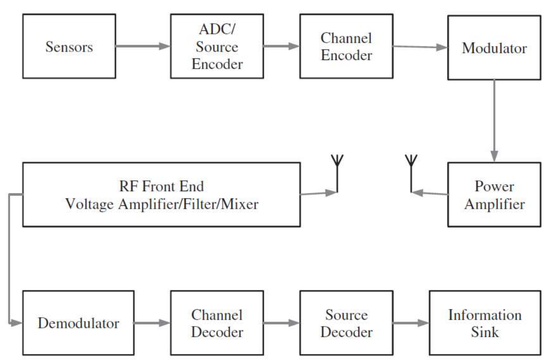


Components of a digital communication system.

Source Encoding: the discrete signal is converted to a binary stream after sampling.

- An efficient source-coding technique can satisfy the channel's bandwidth and signal power requirements.
- It can be achieved by defining a probability model of the information source.
  - The length of each information symbol depends on its probability of occurrence.

PHYSICAL LAYER – BASIC COMPONENTS

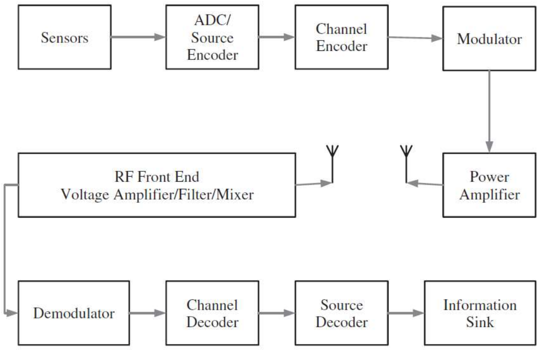


Components of a digital communication system.

Channel Encoding –

- Its aim is to make the transmitted signal robust to noise and interference.
- Moreover, in case of signal corruption, it enables an error to be recognized and the original data to be recovered.
- There are two essential approaches:
  - to transmit symbols from a predetermined codebook, and
  - to transmit redundant symbols.

PHYSICAL LAYER – BASIC COMPONENTS

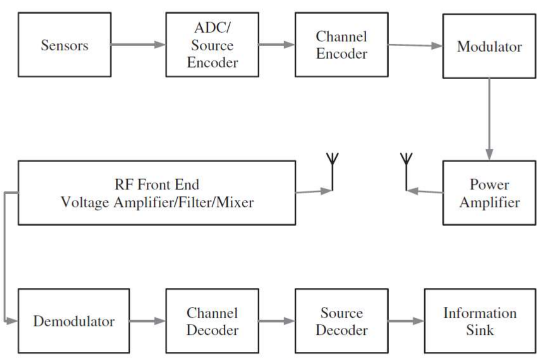


Components of a digital communication system.

Modulation–

- This is a process by which the baseband signal is transformed into a bandpass signal.
- Modulation is useful for various reasons, but the main reason is to transmit and receive signals with short antennas.
- In general, the shorter the wavelength of the transmitted signal, the shorter is the length of the antenna.

PHYSICAL LAYER – BASIC COMPONENTS

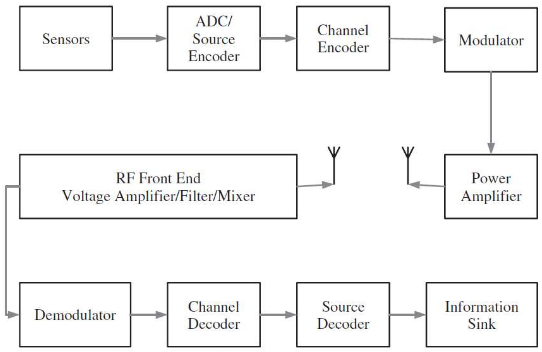


Components of a digital communication system.

Power Amplifier–

- The modulated signal has to be amplified and the electrical energy is converted into electromagnetic energy (electromagnetic radiation) by the transmitter's antenna, and
- The signal is propagated over a wireless link to the desired destination.

PHYSICAL LAYER – BASIC COMPONENTS



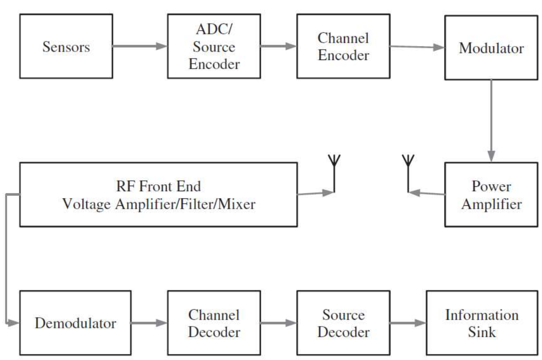
Components of a digital communication system.

Receiver End

- The components of the receiver block carry out the reverse process to retrieve the message signal from the electromagnetic waves. The receiver antenna induces a voltage that is, ideally, similar in shape, frequency, and phase with the modulated signal.

- The magnitude and shape of the signal are changed because of losses and interferences

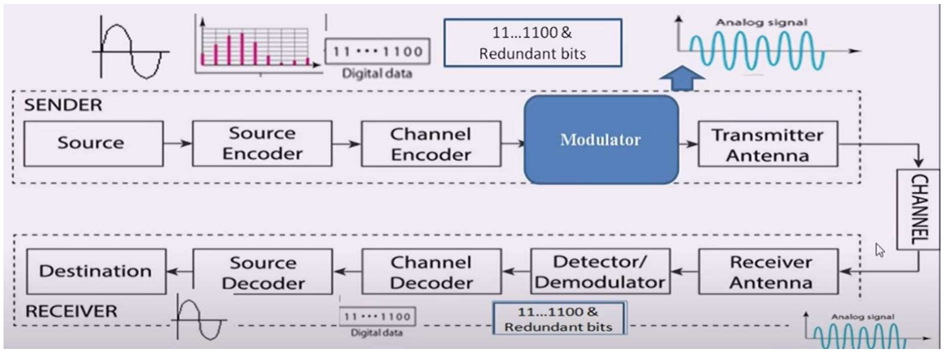
PHYSICAL LAYER – BASIC COMPONENTS



Components of a digital communication system.

- The signal has to pass through a series of *amplification* and *filtering processes*
- It is then transformed back to a baseband signal through the process of *demodulation* and *detection*
- Finally, the baseband signal undergoes *a pulse-shaping process* and *two* stages of *decoding* (channel and source)
  - extract the sequence of symbols - the original analog signal (the message)

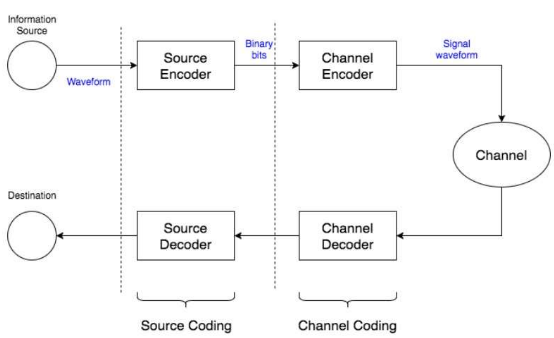
BLOCK DIAGRAM OF DIGITAL COMMUNICATION SYSTEM



BLOCK DIAGRAM OF DIGITAL COMMUNICATION SYSTEM

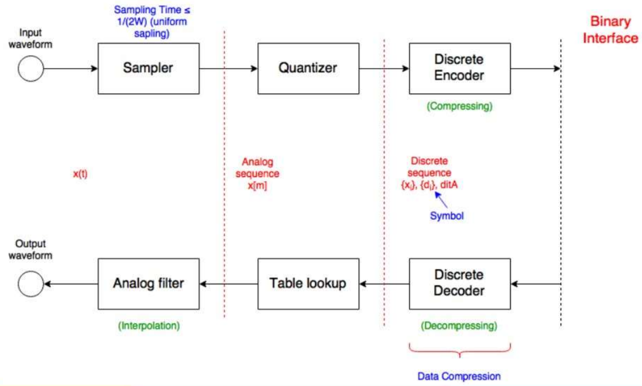
Digital communication systems are communication systems that use digital sequence (typically binary digits, that is, bits) as an interface between the source coding part and the channel coding part.

Source coding: The source encoder converts information waveforms to bits, while the decoder converts bits back to waveforms. The block diagram of source coding is depicted in Figure.



BLOCK DIAGRAM OF DIGITAL COMMUNICATION SYSTEM

Source Coding Source encoding aims to convert information waveforms (text, audio, image, video, etc.) into bits, the universal currency of information in the digital world. The three major steps are:
Sampling: convert the continuous-time analog waveform to discrete-time sequence (but still continuous-valued).
Quantization: convert each continuous-valued symbol to discrete-valued representatives.
Data compression: remove the redundancy in the data and generate roughly i.e. uniformly distributed bits.



PHYSICAL LAYER – SOURCES ENCODING

Source Encoding

- A source encoder transforms an *analog signal* into a *digital sequence*.

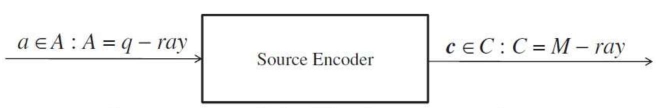
- The process consists of: *sampling, quantizing, encoding*
  - Suppose a sensor produces an analog signal *s(t)*
  - *s(t)* will be sampled and quantized by the analog-to-digital converter (ADC) that has a resolution of *Q* distinct values
  - as a result, a sequence of samples, *S =(s[1], s[2], ..., s[n])* are produced
  - the difference between the sampled *s[j]* and its corresponding analog value at time *tj* is the *quantization error*
  - as the signal varies over time, the quantization error also varies and can be modeled as a random variable with a probability density function, *Ps (t)*

PHYSICAL LAYER – SOURCES ENCODING

Source Encoding

- The *aim* of the source encoder is to map each quantized element, *s[j]* into a corresponding binary symbol of length *r* from a codebook, *C*
- *Block code:* if all the binary symbols (code words) in the codebook are of *equal* length
- Often, the symbol length and the sampling rate are *not uniform*
- It is customary to assign:
    - *short-sized symbols* and *high* sampling rates to the most probable sample values
    - *long-sized symbols* and *low* sampling rates to less probable sample values

PHYSICAL LAYER – SOURCES ENCODING



Input – output relationship of a source encoder.

- A codebook, *C*, can be uniquely decoded, if each sequence of symbols, *(C(1), C(2), ...)* can be mapped back to a corresponding value in *S =*(s[1], s[2], ..., s[n]*)*
- A binary codebook has to satisfy the following equation to be uniquely decoded

$$\sum_{i=1}^{u}\left(\frac{1}{r}\right)^{l_i} \le 1$$

- where *u* is the size of the codebook
- *li* is the size of the codeword *C(i)*
- *r is the Binary Coding System i.e. 2.*

PHYSICAL LAYER – SOURCES ENCODING

- A codebook can be instantaneously decoded
  - if each symbol sequence can be extracted (decoded) from a stream of symbols *without* taking into consideration previously decoded symbols

- This will be possible
  - *if* there does *not exist* a symbol in the codebook, such that the symbol $a = (a1, a2, ..., am)$ is not a prefix of the symbol $b = (b1, b2, ..., bn)$, where $m < n$ and $ai = bi$, $\forall i = 1, 2, ..., m$ within the same codebook

PHYSICAL LAYER – SOURCES ENCODING

Source-encoding techniques

	C^1	C^2	C^3	C^4	C^5	C^6
s_1	0	00	0	0	0	0
s_2	10	01	100	10	01	10
s_3	00	10	110	110	011	110
s_4	01	11	11	1110	111	111
Block code	No	Yes	No	No	No	No
Uniquely decoded	No	Yes	No	Yes	Yes	Yes
$\sum_{i=1}^{n} \left(\frac{1}{2}\right)^{l_i}$	$1\frac{1}{4}$	1	1	$\frac{15}{16} < 1$	1	1
Instantly decoded	No	Yes (block code)	No	Yes (comma code)	No	Yes

INFORMATION THEORY

Why Information Theory?

When the Communication thing is readily measurable, such as an electric current, the study of the communication system is relatively easy. But, when the communication thing is information, the study becomes rather difficult.

- Information Theory answers the following questions:
  - Measure for an amount of information.
  - Measure to improve the communication of information.
- *Unit of information*
  - Communication systems are of *statistical* nature; i.e. the performance of the system can never be described in a deterministic sense.
  - Communication systems are *unpredictable* or *uncertain*.
  - An amount of information is calculated by statistical parameter associated with a *probability scheme*. The parameter should indicate a relative measure of uncertainty relevant to the occurrence of each message in the message ensemble.

INFORMATION THEORY

The amount of information is inversely proportional to the probability of an event. The more the probability of an event, the less is the amount of information associated with it, and vice versa.

$$I(x_j) = f\left[\frac{1}{p(x_j)}\right] \quad \text{..........Eq. 1}$$

Where x_j is an event with a probability $p(xj)$ and the information associated with it is $I(x_j)$.

Now, let there be another event y_k such that x_j and y_k are independent. Hence the probability of the joint event is $p(x_j, y_k) = p(xj)\ p(y_k)$ with associated information content

$$I(x_j, y_k) = f\left[\frac{1}{p(x_j, y_k)}\right] = f\left[\frac{1}{p(xj)\ p(y_k)}\right] \quad \text{..............Eq. 2}$$

The total information $I(x_j, y_k)$ must be equal to the sum of individual information $I(x_j)$ and $I(y_k)$. Where,

$$I(y_k) = f\left[\frac{1}{p(y_k)}\right]$$

Thus, it can be seen that the function on RHS of Eq. 2 must be a function which converts multiplication into addition. Logarithm is one such function. Thus,

INFORMATION THEORY

Thus, it can be seen that the function on RHS of Eq. 2 must be a function which converts multiplication into addition. Logarithm is one such function. Thus,

$$I(x_j, y_k) = \log\left[\frac{1}{p(x_j)\ p(y_k)}\right]$$

$$= \log\left[\frac{1}{p(x_j)}\right] + \log\left[\frac{1}{p(y_k)}\right]$$

$$= I(x_j) + I(y_k)$$

Hence, the basic equation defining the amount of information (or self information) is

$$I(x_j) = \log\left[\frac{1}{p(x_j)}\right] = -\log[p(x_j)]$$

Different units of information can be defined for different bases of logarithms.
Base 2 :*Unit is bit*
Base e :*Unit is nat*
Base 10 :*Unit is decit*

In Information theory, unit of information is bit. So, it is assumed to take base 2.

INFORMATION THEORY

An intuitive and meaningful measure of information should have the following properties:

- Self information should decrease with increasing probability.
- Self information of two independent events should be their sum.
- Self information should be a continuous function of the probability.

The only function satisfying the above conditions is the -log of the probability.

Entropy

When we observe the possibilities of the occurrence of an event, how surprising or uncertain it would be, it means that we are trying to have an idea on the average content of the information from the source of the event.

"Entropy is a measure of uncertainty."

Entropy can be defined as a *measure of the average information content per source symbol*. Claude Shannon, the "father of the Information Theory", provided a formula for it as −

$$H = \sum_{i=1}^{M} p_i \log_2 \frac{1}{p_i}$$

INFORMATION THEORY

If someone is sending us a stream symbols from a source (such as those above), we generally have some uncertainty about the sequence of symbols we will be receiving. If not, there would no need for the transmitter to even bother sending us the data.

A useful and common way to model this uncertainty is to assume that the data is coming randomly according to some probability distribution.

The simplest model is to assume that each the symbols $s_1, \ldots, s_M$ have associated probabilities of occurrence $p_1, \ldots, p_M$, and we see a string of symbols drawn independently according to these probabilities. Since the p_i are probabilities, we have $0 \le p_i \le 1$ for all i.

Also, we assume the only possible symbols in use are the $s_1, \ldots, s_M$, and so

$$p_1 + \cdots + p_M = 1.$$

INFORMATION THEORY

How much information does a source provide?

Consider just one symbol from the source. The probability that we observe the symbol s_i is p_i, so that if we do indeed observe s_i then we get $\log \frac{1}{p_i}$ bits of information. Therefore, the average number of bits of information we get based on observing one symbol is

$$p_1 \log_2 \frac{1}{p_1} + \cdots + p_M \log_2 \frac{1}{p_M} = \sum_{i=1}^{M} p_i \log_2 \frac{1}{p_i}$$

This is an important quantity called the entropy of the source and is denoted by H.

Definition

Given a source that outputs symbols $s_1, \ldots, s_M$ with probabilities $p_1, \ldots, p_M$, respectively, the entropy of the source, denoted H, is defined as

$$H = \sum_{i=1}^{M} p_i \log_2 \frac{1}{p_i}$$

INFORMATION THEORY

Case I:

If there is only one possible message, i.e. M=1 and p_k =P =1, then

$$H = p1 \log[1/p_1] = 1 \log [1/1] = 0$$

In this case of single possible message, the reception of total message conveys no information.

Case 2:

Only one message out of M having a probability 1 and others have 0.

$$H = \sum_{k=1}^{M} p_k \log_2 \frac{1}{p_k}$$

$$H = p_1 \log \frac{1}{p1} + \lim_{p \to 0} \left[p \log \frac{1}{p} + p \log \frac{1}{p} + \cdots \ldots \ldots . . \right]$$

$$H = 1 \log [1/1] + 0$$

$$H = 0$$

In this case, entropy is zero.

INFORMATION THEORY

Case 3:

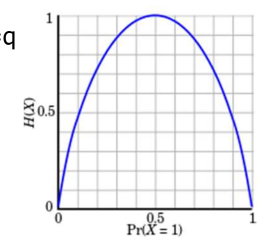
For Binary System (M=2), the entropy is

$$H = p_1 \log \frac{1}{p1} + p_2 \log \frac{1}{p2}$$

Let p1 = p then p2 = 1 − p1 = 1 − p = q

$$H = p \log \frac{1}{p} + (1-p) \log \frac{1}{(1-p)}$$

$$H = p \log \frac{1}{p} + q \log \frac{1}{q}$$



The maximum value of H can be found by putting p = 1/2

$$H_{max} = 1/2 \log \frac{1}{1/2} + 1/2 \log \frac{1}{1/2} = 1 \text{ bit/message}$$

In this case, we have seen that the entropy is maximum when p = 1/2, i.e. both the message are equally probable. In this case, the maximum entropy is then

$$H_{max} = \sum_{k=1}^{M} p_k \log_2 \frac{1}{p_k} = \sum_{k=1}^{M} \frac{1}{M} \log M = \log M \text{ bits/message}$$

INFORMATION THEORY

Rate of Information

If a message source generates messages at the rate of r messages per second, the rate of information R is defined as the average number of bits of information per second. Now, H is the average number of bits of information per message. Hence

$$R = r\,H \text{ bits/sec}$$

INFORMATION THEORY

- The importance of the entropy of a source lies in its operational significance concerning coding the source. Since *H* represents *the average number of bits of information per symbol from the source,*
- We might expect that we need to use at least *H* bits per symbol to represent the source with a uniquely decodable code.
- This is in fact the case, and moreover, if we wish to code longer and longer strings of symbols, we can find codes whose performance (average number of bits per symbol) gets closer to H. This result is called the *Source Coding Theorem* and was discovered by *Shannon* in 1948.
- *Source Coding Theorem*
  a) The average number of bits/symbol of any uniquely decodable source must be greater than or equal to the entropy H of the source.

  a) If the string of symbols is sufficiently large, there exists a uniquely decodable code for the source such that the average number of bits/symbol of the code is as close to H as desired.

PHYSICAL LAYER – SOURCES ENCODING

The Efficiency of a Source Encoder

- Quantity that expresses the average length
- Sampled analog signal: *L(C) =E [li(C)]*
- Suppose the probability of a m-ary source
  - i.e., it has *q* distinct symbols
  - producing the symbol *si* is *Pi* and the symbol *Ci* in a codebook is used to encode *si*
  - the expected length of the codebook is given by:

$$L(C) = \sum_{i=1}^{m} P_i * l_i(C)$$

PHYSICAL LAYER – SOURCES ENCODING

The Efficiency of a Source Encoder

- To express efficiency in terms of the information entropy or *Shannon's entropy*
  - defined as *the minimum message length* necessary to communicate information
  - related to the *uncertainty* associated with the information
  - if the symbol *si* can be expressed by a binary symbol of *n* bits, the information content of *si* is:

$$I(s_i) = -\log_2 P_i = \log_2 \frac{1}{P_i}$$

  - Logarithmic Measure of information possesses the desired additive property when a number of source outputs is considered as a block.
  - the entropy (in bits) of a *m*-ary memoryless source encoder is expressed as:

$$H = \sum_{i=1}^{M} p_i \log_2 \frac{1}{p_i}$$

PHYSICAL LAYER – SOURCES ENCODING

The Efficiency of a Source Encoder

- The efficiency of a source encoder in terms of entropy reveals the unnecessary redundancy in the encoding process. This can be expressed by
- Average information per message/information actually transmitted per message

$$\eta = \frac{\text{Actual Transinformation}}{\text{Maximum Transinformation}}$$

$$\eta = \frac{I(X;Y)}{C} = \frac{I(X;Y)}{Max\ I(X;Y)}$$

$$\eta(C) = \frac{H(S)}{L(C)}$$

Where,
H(S) is the Entropy of the Samples (Actual Transinformation)
L(C) is the Entropy of the Code Block (Maximum Transinformation)

- Transinformation is a measure of the average information per symbol transmitted in the system. i.e. the rate of transmission of information.
- The redundancy of the encoder is: $\frac{L - H(S)}{L} = 1 - \eta$

PHYSICAL LAYER – SOURCES ENCODING

Example 1

Message	Probability	Code	Length of Code
m1	1/2	C1 = 00	l1=2
m2	1/4	C2= 01	l2=2
m3	1/8	C3= 10	l3=2
m4	1/8	C4=11	l4=2

- It is quantized into four distinct values, 0, *1*, *2*, *3* .
- if the *probability* of occurrence of these values is *P(1)* = 0.5, *P(2)* = 0.25, *P*(3) = 0.125, *P(4)* = 0.125, then, it is possible to compute the efficiency of the codebooks given in Table
- *li has been taken equal for* each symbol. Hence:

$L(C) = \sum_{i=1}^{4} l_i * P_i$ = (2 * 1/2) + (2 * 1/4) + (2 * 1/8) + (2 * 1/8) = 2 bits/message

PHYSICAL LAYER – SOURCES ENCODING

Example 1

- Using Equation, the entropy of *Codebook* is calculated as:

$$H(C) = \frac{1}{2}\log\frac{1}{2} + \frac{1}{4}\log\frac{1}{4} + \frac{1}{8}\log\frac{1}{8} + \frac{1}{8}\log\frac{1}{8} = 7/4 = 1.75$$

$$\eta(C) = \frac{H(C)}{L(C)} = \frac{1.75}{2} = 87.5\ \%$$

- Therefore, the encoding efficiency of the codebook, *C2* (see Table) is:

$$\eta(C) = 87.5\ \%$$

- The redundancy in *C2* is:

$$R(C) = 1 - \eta = 1 - 0.875 = 0.125 = 12.5\%$$

- In terms of energy efficiency, this implies that *12.5% of the transmitted bits are unnecessarily redundant*, because *C* is not compact enough.

PHYSICAL LAYER – SOURCES ENCODING

Example 1

Message	Probability	Code	Length of Code
m1	1/2	C1 = 0	l1=1
m2	1/4	C2= 10	l2=2
m3	1/8	C3= 110	l3=3
m4	1/8	C4=111	l4=3

- It is quantized into four distinct values, 0, *1, 2, 3* .
- if the *probability* of occurrence of these values is *P(1)* = 0.5, *P(2)* = 0.25, *P(3)* = 0.125, *P(4)* = 0.125, then, it is possible to compute the efficiency of the codebooks given in Table
- *li has been taken different for* each symbol. Hence:

$$L(C) = \sum_{i=1}^{4} l_i * P_i = (1 * 1/2) + (2 * 1/4) + (3 * 1/8) + (3 * 1/8) = 1.75 \text{ bits/message}$$

PHYSICAL LAYER – SOURCES ENCODING

Example 2

- Using Equation, the entropy of *Codebook* is calculated as:

$$H(C) = \frac{1}{2}\log\frac{1}{2} + \frac{1}{4}\log\frac{1}{4} + \frac{1}{8}\log\frac{1}{8} + \frac{1}{8}\log\frac{1}{8} = 7/4 = 1.75$$

$$\eta(C) = \frac{H(C)}{L(C)} = \frac{1.75}{1.75} = 100\ \%$$

- Therefore, the encoding efficiency of the codebook, *C2* (see Table) is:

$$\eta(C) = 87.5\ \%$$

- The redundancy in *C2* is:

$$R(C) = 1 - \eta = 1 - 1 = 0 = 0\%$$

- In terms of energy efficiency, this implies that no bits are *transmitted as redundant bits*, because *C* is perfect enough and efficient codebook.

PHYSICAL LAYER – SOURCES ENCODING

Source Coding Techniques

1. Huffman Code
2. Lempel-Ziv Code
3. Shannon-Fano Code
4. Arithmetic Code

Pulse Code Modulation & Delta Modulation

PCM and DM are the two predominantly employed *source encoding* techniques

- *In digital pulse code modulation*
  - the signal is quantized first
  - each sample is represented by a binary word from a finite set of words

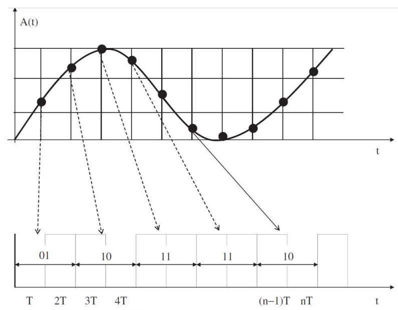
- The resolution of a PCM technique and the source encoder bit rate are determined by
  - the size of the individual words
  - the number of words in the set

PHYSICAL LAYER – SOURCES ENCODING

Pulse Code Modulation

In PCM, information is conveyed in the presence or absence of *pulses*

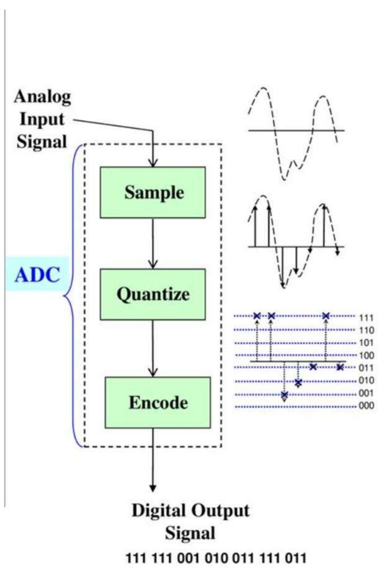
- greatly enhances the transmission and regeneration of *binary words*
- the associated cost with this form of source encoding is
  - the quantization error, the energy and bandwidth required to transmit the multiple bits for each sampled output!
- Figure illustrates a PCM technique that uses two bits to encode a single sample
- Four distinct levels are permissible during sampling



A PCM based source encoding.

PHYSICAL LAYER – SOURCES ENCODING

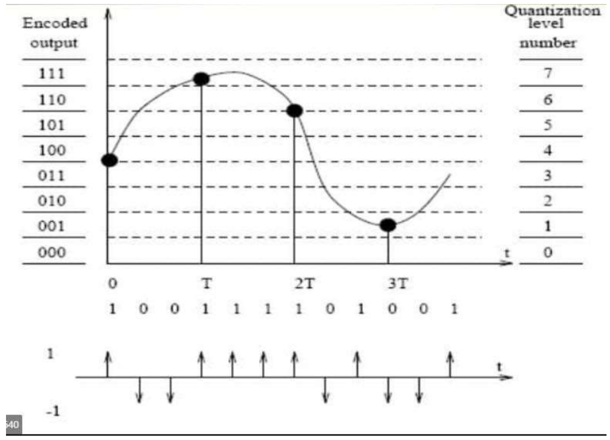
Pulse Code Modulation



- **The *Analog-to-digital Converter* (ADC)** performs three functions:
  - **Sampling**
    - Makes the signal discrete in time.
    - If the analog input has a bandwidth of *W* Hz, then the ***minimum sample frequency*** such that the signal can be reconstructed without distortion is 2W or more.
  - **Quantization**
    - Makes the signal discrete in amplitude.
    - Round off to one of q discrete levels.
  - **Encode**
    - Maps the quantized values to digital words that are ν bits long.
- If the (Nyquist) ***Sampling Theorem*** is satisfied, then only quantization introduces distortion to the system.

PHYSICAL LAYER – SOURCES ENCODING

Pulse Code Modulation



PHYSICAL LAYER – SOURCES ENCODING

Pulse Code Modulation

Level	Code word
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Levels are encoded using this table

Table: Quantization levels with belonging code words

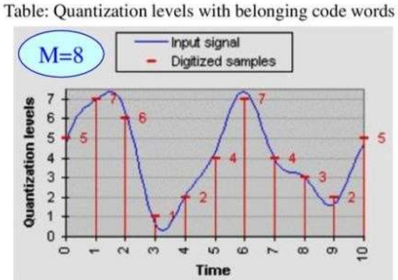


Chart 1. Quantization and digitalization of a signal.

Signal is quantized in 11 time points & 8 quantization segments.

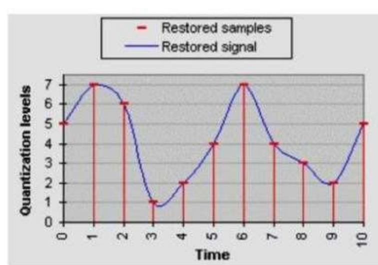


Chart 2. Process of restoring a signal.
PCM encoded signal in binary form:
101 111 110 001 010 100 111 100 011 010 101
Total of **33** bits were used to encode a signal

PHYSICAL LAYER – SOURCES ENCODING

Delta Modulation

- Linear PCM (LPCM) is a linear quantization PCM.
- Differential PCM (DPCM) encodes PCM values as the difference between the current value and the expected value. The algorithm predicts the next sample based on the previous samples, and the encoder only stores the differentiation between this projection and the actual value. If the projection is reasonable, less bits can be used to represent the same information. For audio, this type of encoding decreases the number of bits required per sample by about 25% compared to PCM.
- Adaptive DPCM (ADPCM) is a variation of DPCM that changes the size of the quantization steps to allow further reduction in the bandwidth required for a given signal-to-noise ratio.
- Delta modulation is a form of DPCM which uses one bit per sample to indicate whether the signal is increasing or decreasing from the previous sample.

PHYSICAL LAYER – SOURCES ENCODING

Delta Modulation

Delta modulation is a *digital pulse modulation* technique

- it has found widespread acceptance *in low bit rate* digital systems
- it is a *differential* encoder and transmits bits of information
- the information describes *the difference between successive* signal values, as opposed to the actual values of a time-series sequence
- the difference signal, *V*d*(t)*, is produced by first estimating the signal's magnitude based on previous samples (*Vi (t*0)) and comparing this value with the actual input signal, *V*in(*t*0)
- The polarity of the difference value indicates the polarity of the pulse transmitted

PHYSICAL LAYER – SOURCES ENCODING

Delta Modulation

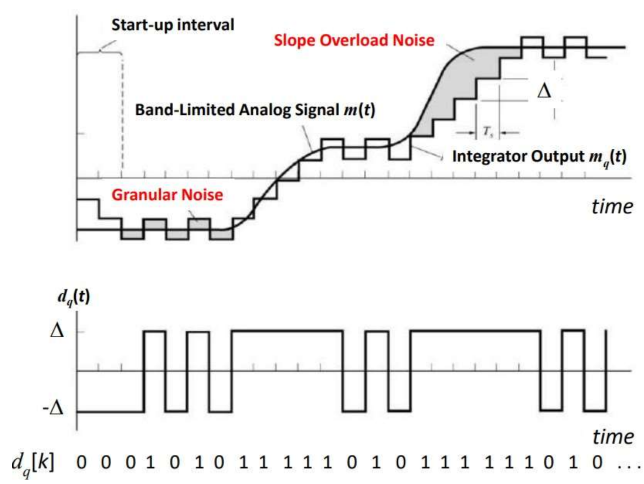
Delta modulation is a *digital pulse modulation* technique

- Delta modulation (DM) is the simplest method for analog-to-digital conversion (ADC).
- Delta modulation uses 1-bit per sampling period (TS) – it is a 1-bit ADC.
-
- Delta modulation requires a sampling rate much greater than the Nyquist rate (commonly four or five times the Nyquist rate).
- DM is closely related to DPCM.
- In DM we use a first-order predictor (one time delay TS is the predictor).
- DM uses very simple hardware and is low cost for that reason.
- The transmitted output is a binary stream of $\pm\Delta$ pulses at f_S. It gives a stepwise approximation m_q (t) to m(t).

PHYSICAL LAYER – SOURCES ENCODING

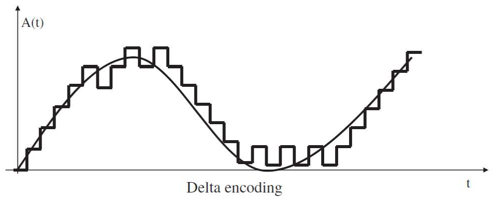
Delta Modulation

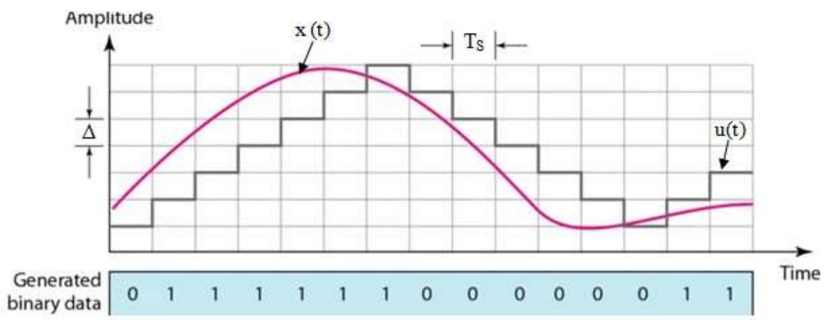
- The difference signal is a *measure* of the *slope of the signal*
   - first, *sampling* the analog signal
   - then, *varying* the amplitude, width, or the position of the digital signal in accordance with the amplitude of the sampled signal



PHYSICAL LAYER – SOURCES ENCODING

Delta Modulation





PHYSICAL LAYER – CHANNEL ENCODING

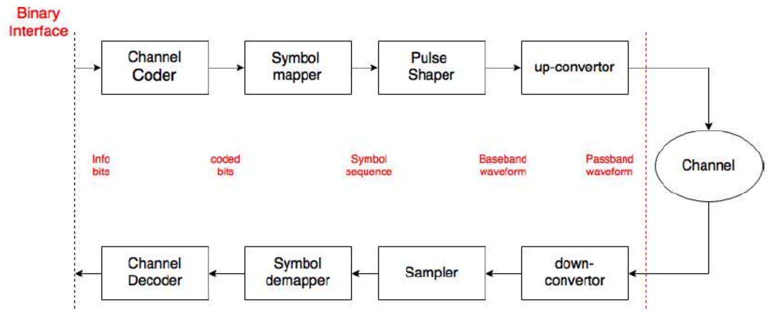
Channel Encoding

- To produce a sequence of data that is robust to noise.
- To provide error detection and forward error correction mechanisms.
- In simple and cheap transceivers, forward error correction is costly and, therefore, the task of channel encoding is limited to the detection of errors in packet transmission.
- The noise present in a channel creates unwanted errors between the input and the output sequences of a digital communication system. The error probability should be very low, nearly $\leq 10^{-6}$ for a reliable communication.
- The channel coding in a communication system, introduces redundancy with a control, so as to improve the reliability of the system.
- The source coding reduces redundancy to improve the efficiency of the system.

PHYSICAL LAYER – CHANNEL ENCODING

Channel Encoding

- Channel coding consists of two parts of action.
  1. Mapping incoming data sequence into a channel input sequence. The mapping is done by the transmitter, with the help of an encoder.
  2. Inverse Mapping the channel output sequence into an output data sequence. The inverse mapping is done by the decoder in the receiver.
- The final target is that the overall effect of the channel noise should be minimized.



PHYSICAL LAYER – CHANNEL ENCODING

Channel Encoding

- The four major steps are:
  1. Error correcting codes: introduce redundancy into the information bits and produce longer coded bits.
  2. Symbol mapping: map the coded bits to *constellation points*, each of which is a complex symbol.
  3. Pulse shaping: modulate the symbol to suitable *baseband* waveforms.
  4. Up conversion: convert the baseband waveform to *passband* waveform, so that the effective frequency band follows the constraints from the physical world.
- Channel decoding does the reverse of encoding.
- These Error Correcting Techniques are also called Error-Control Coding:
  1. Block Code (Parity Check Code, Binary Code Space, Linear Block Code)
  2. Hamming Code
  3. Cyclic Codes
  4. Convolutional Codes

PHYSICAL LAYER – CHANNEL ENCODING

Shannon Channel Coding Theorem

- It is related with the rate of transmission over a communication channel.
- The communication channel may be noisy or limited bandwidth.
- Shannon's Coding Theorem says that

  "It is possible to device a mean whereby a communication system will transmit information with an arbitrarily small possibility of error provided that the information rate R is less than equal to C, Channel Capacity."

- The statements of Shannon's Theorem is:
  - Given a source of M equally likely messages, with M>>1, which is generating information at a rate R. Given a Channel Capacity C. Then, if R<<C, there exists a coding technique such that the output of the source may be transmitted over the channel with a probability of error of receiving the message which may be made arbitrarily small.
  - It indicates that for R ≪ C. error free transmission is possible in the presence of noise.

PHYSICAL LAYER – CHANNEL ENCODING

Shannon Hartley Theorem

- According to the Shannon–Hartley theorem, the capacity of a channel to transmit a message without an error is given as:

$$C = B \cdot \log_2 \left(1 + \frac{S}{N}\right)$$

  where *C* is the channel capacity in bits per second;
  B is the bandwidth of the channel in hertz;
  S is the average signal power over the entire bandwidth, measured in watts; and
  N is the average noise power over the entire bandwidth, measured in watts.

- This equation states that
  - For data transmission with free of errors, its transmission rate should be below the channel's capacity.
  - It also indicates how the signal-to-noise (SNR) ratio, can improve the channel's capacity.

PHYSICAL LAYER – CHANNEL ENCODING

Shannon Hartley Theorem

- The equation reveals two independent reasons why errors can be introduced during transmission:
  1. Information will be lost if the message is transmitted at a rate higher than the channel's capacity. This type of error is called *equivocation* in information theory. It is characterized as a subtractive error.
  2. Information will be lost because of noise, which adds irrelevant information into the signal.
- It is possible to describe the impact of irrelevance and equivocation as well as the percentage of information that can be transmitted over the channel without an error, which is also called Transinformation or Mutual Information.
- Question is How?
  Answer: Entropy, Joint Entropy & Conditional Entropy

PHYSICAL LAYER – CHANNEL ENCODING

Joint Entropy & Conditional Entropy

- In a single probability scheme, we may study the behavior of either transmitter or receiver by calculating the associated Entropy.
- To study of Communication System, we must simultaneously study the behavior of transmitter and receiver. This will rise the concept of two dimensional probability scheme.
- Let there be two finite discrete sample space S_1 and S_2 and we can assume their product as $S= S_1S_2$, Let

  $[X] =[x_1 x_2 x_3 .....x_m]$

  $[Y] =[y_1 y_2 y_3 ....y_n]$
- Be the sets of events in S_1 and S_2 respectively. Each event x_j of S_1 may occur in conjunction with any event y_k in S_2. Hence, the complete set of events in $S=S_1S_2$ is

$$[XY] = \begin{bmatrix} x_1y_1 & \cdots & x_1y_n \\ \vdots & \ddots & \vdots \\ x_my_1 & \cdots & x_my_n \end{bmatrix}$$

Thus, we have three sets of complete probability schemes

$$P(X) =[P(x_j)]$$
$$P(Y) =[P(y_k)]$$
$$P(XY) =[P (x_j, y_k)]$$

PHYSICAL LAYER – CHANNEL ENCODING

Joint Entropy & Conditional Entropy

- We have three complete probability schemes and naturally there will be three associative entropies.

$$H(X) = -\sum_{j=1}^{M} p(xj)\log_2 p(xj)$$

$$P(x_j) = \sum_{k=1}^{n} p(xj, yk)$$

$$H(Y) = -\sum_{k=1}^{n} p(y_k)\log_2 p(y_k)$$

$$P(y_k) = \sum_{j=1}^{m} p(xj, yk)$$

$$H(XY) = -\sum_{j=1}^{m}\sum_{k=1}^{n} p(xj, yk)\log_2 p(xj, yk)$$

H(X) and H(Y) are marginal entropies of X & Y respectively, and H(XY) is the joint entropy of X & Y.
The conditional probability p(X/Y) is given by

$$P(X/Y) = \frac{P(X,Y)}{P(Y)}$$

We know that the y_K may occur in conjunction with $x_1, x_2, \ldots, x_m$.

PHYSICAL LAYER – CHANNEL ENCODING

Joint Entropy & Conditional Entropy

- The conditional entropy may be defined as

$$H(X/Y) = -\sum_{j=1}^{m}\sum_{k=1}^{n} p(xj, yk)\, log_2\, p(xj/yk)$$

- Similarly, it can be shown that

$$H(Y/X) = -\sum_{j=1}^{m}\sum_{k=1}^{n} p(xj, yk)\, log_2\, p(yk/xj)$$

H(X/Y) and H(Y/X) are average conditional entropies or simply conditional entropies.

PHYSICAL LAYER – CHANNEL ENCODING

Joint Entropy & Conditional Entropy

- There are five Entropies associated with a two-dimensional probability scheme. They are: H(X), H(Y), H(X,Y), H(X/Y), and H(Y/X). Where X represents a transmitter and Y a receiver. Therefore, the following interpretations of the different entropies for a communication system can be derived:
- H(X): Average information per character at the transmitter, or entropy of the transmitter. *It is the probabilistic nature of the transmitter.*

$$H(X) = -\sum_{j=1}^{M} p(xj)\log_2 p(xj)$$

- H(Y): Average information per character at the receiver, or entropy of the receiver. *It is the probabilistic nature of the receiver.*

$$H(Y) = -\sum_{k=1}^{n} p(yk)\log_2 p(yk)$$

- H(X, Y): Average information per pair of the transmitted and received characters or, *the average uncertainty of the communication system as a whole.*

$$H(XY) = -\sum_{j=1}^{m}\sum_{k=1}^{n} p(xj, yk)\log_2 p(xj, yk)$$

PHYSICAL LAYER – CHANNEL ENCODING

Joint Entropy & Conditional Entropy

- H(X/Y): A received character y_k may be the result of the transmission of one of the x_j's with a given probability. The entropy associated with this probability scheme, when y_k covers all received symbols. The conditional entropy is a measure of information about the transmitter, where it is known that Y is received.
- The content of information that can be lost because of the channel's inherent constraints can be quantified by observing the input *x* given that the output *y* is known:

$$H(X/Y) = -\sum_{j=1}^{m}\sum_{k=1}^{n} p(xj, yk)\, log_2\, p(xj/yk)$$

A good channel encoding scheme is one that has a high inference probability. This can be achieved by introducing redundancy during channel encoding.

H(X/Y) indicates how well one can recover the transmitted symbols, from the received symbols; i.e. It gives the information lost in the channel. This is also known as equivocation.

PHYSICAL LAYER – CHANNEL ENCODING

Joint Entropy & Conditional Entropy

- H(Y/X): A transmitted character x_j may be the result of the reception of one of the y_k's with a given probability. The entropy associated with this probability scheme, when x_j covers all transmitted symbols. The conditional entropy is a measure of information about the receiver, where it is known that X is transmitted.

  The content of information that can be introduced into the channel due to noise is described as the conditional information content, I(*Y/X*). It is the information content of *y* that can be observed provided that *x* is known. The conditional entropy is given as:

$$H(Y/X) = -\sum_{j=1}^{m}\sum_{k=1}^{n} p(xj, yk)\, log_2\, p(yk/xj)$$

  A good channel encoder is one that reduces the irrelevance entropy.

  H(Y/X) indicates how well one can recover the received symbols, from the transmitted symbols; i.e. It can be considered as the noise entropy added in the channel. Thus, it is a measure of an error or noise due to channel.

PHYSICAL LAYER – CHANNEL ENCODING

Joint Entropy & Conditional Entropy

- The relationship between the different entropies is found as follows:

$$H(XY) = H(X/Y) + H(Y)$$

  Similarly, it can be shown that

$$H(XY) = H(Y/X) + H(X)$$

PHYSICAL LAYER – CHANNEL ENCODING

Mutual Information

- The information content $I(X;Y)$ that overcomes the channel's constraints to reach the destination (the receiver) is called transinformation. The mutual information I(X; Y) indicates a measure of average information per symbol transmitted in the system. Given the input entropy, $H(X)$, and equivocation, $H(X|Y)$, the transinformation is computed as:

$$I(X;Y) = H(X) - H(X/Y) \quad \text{------- } 1$$

$$I(X;Y) = H(Y) - H(Y/X) \quad \text{------- } 2$$

$$I(X;Y) = H(X) + H(Y) - H(X,Y) \quad \text{------ } 3$$

Since H(X) represents the uncertainty of the channel input (or source) before the channel output is observed, and

H(X/Y) is the uncertainty of the channel input (or Source) after the channel output is observed.

The mutual information I(X; Y) represents the uncertainty of the channel input (or Source) that is resolved by observing the channel output. *Since, I(X; Y) indicates a measure of the information transferred through the channel. It is also known as transferred information and Transinformation.*

PHYSICAL LAYER – CHANNEL ENCODING

Mutual Information

$$I(X;Y) = H(X) - H(X/Y) \quad \text{------- } 1$$

The transinformation is equal to the average source information minus the uncertainty that still remains about the message.

In other words, H(X/Y) is the additional information needed at the receiver after the reception in order to completely specify the message sent.

Thus, H(X/Y) gives the information lost in the channel. This is also known as equivocation.

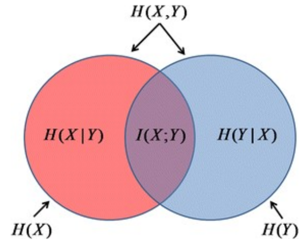
$$I(X;Y) = H(Y) - H(Y/X) \quad \text{------- } 2$$

The transinformation is equal to the receiver entropy minus that part of the receiver entropy which is not the information about the source.

Thus, H(Y/X) is considered as the noise entropy added in the channel. Thus, it is a measure of an error or noise due to channel.

PHYSICAL LAYER – CHANNEL ENCODING

Equivocation, Irrelevance & Transinformation



Channel Capacity

The mutual information I(X; Y) indicates a measure of average information per symbol transmitted in the system. A suitable measure for the efficiency of transmission of information may be introduced by comparing the actual rate and the upper bound of the rate of the information transmission of the channel.

Shannon's has introduced the concept of the Channel Capacity:

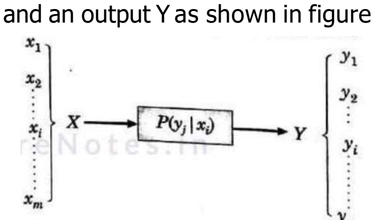
Channel Capacity is defined as the maximum of mutual information. Thus the channel capacity C is given by

$$C = \max I(X;Y) = \max [\, H(X) - H(X/Y) \,]$$

PHYSICAL LAYER – SOURCES ENCODING

Channel Representation

A communication channel may be defined as the path or medium through which the symbols flow to the receiver end. A discrete memory channel (DMC) is a statistical model with an input X and an output Y as shown in figure:



During each unit of time (signaling interval, the channel accepts an input symbol from X, and in response it generates an output symbol from Y. The channel is said to be "discrete" when the alphabets of X and Y are both finite.

Also, it is said to be "memoryless" when the current output depends on only the current input and not on any of the previous inputs.

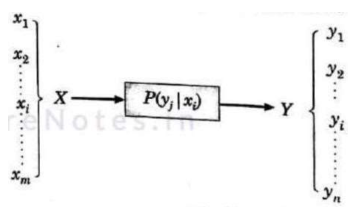
PHYSICAL LAYER – SOURCES ENCODING

Channel Representation

A diagram of DMC with m inputs and n outputs has been given in the figure. The input X consists of input symbols $x_1, x_2, \ldots\ldots, x_m$.

The a priori probabilities of these source symbols $P(x_i)$ are assumed to be known. The output Y consists of output symbols $y_1, y_2, \ldots., y_n$.

Each possible input-output path is indicated along with probability $P(y_j/x_i)$, where $P(y_j/x_i)$ is called probability of obtaining output y_j given that input x_i and is called a channel transition probability.



PHYSICAL LAYER – SOURCES ENCODING

Channel Matrix

A Channel is completely specified by the complete set of transition probabilities. Accordingly, the channel is often specified by the matrix of transition probabilities [P(Y/X)]. This matrix is given by

$$[P(Y|X)] = \begin{bmatrix} P(y_1|x_1) & P(y_2|x_1) & \cdots & P(y_n|x_1) \\ P(y_1|x_2) & P(y_2|x_2) & \cdots & P(y_n|x_2) \\ \cdots & \cdots & \cdots & \cdots \\ P(y_1|x_m) & P(y_2|x_m) & \ldots & P(y_n|x_m) \end{bmatrix}$$

This matrix is called a Channel Matrix. Since each input to the channel results some output, each row of the channel matrix must be sum to unity. This means that

$$\sum_{j=1}^{n} P(y_j|x_i) = 1 \text{ for all } i$$

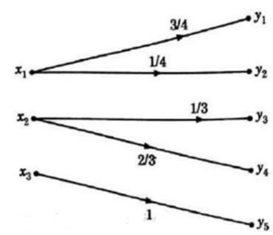
PHYSICAL LAYER – SOURCES ENCODING

Lossless Channel

A Channel is described by a channel matrix with only one non-zero element in each column is called a lossless channel. An example of lossless channel has been shown in figure, and the corresponding channel matrix is given in the matrix below:

$$P[Y/X] = \begin{pmatrix} 3/4 & 1/4 & 0 & 0 & 0 \\ 0 & 0 & 1/3 & 2/3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

It can be shown in the lossless channel, no source information is lost in the transmission.



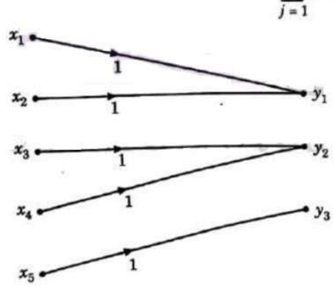
PHYSICAL LAYER – SOURCES ENCODING

Deterministic Channel

A Channel is described by a channel matrix with only one non-zero element in each row is called a deterministic channel. An example of deterministic channel has been shown and the corresponding channel matrix is given below:

It may be noted that since each row has only one non-zero element, therefore, this element must be unity by equation given below. Thus, when a given source symbols is sent in the deterministic channel, it is clear which output symbol will be received.

$$\sum_{j=1}^{n} P(y_j \mid x_i) = 1 \text{ for all } i$$

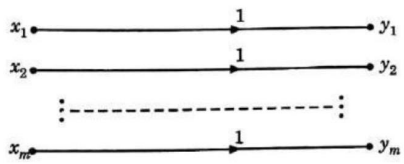


$$[P(Y \mid X)] = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

PHYSICAL LAYER – SOURCES ENCODING

Noiseless Channel

A Channel is called noiseless if it is both lossless and deterministic. A noiseless channel has been shown in the figure. The channel matrix has only one element in each row and each column, and this element is unity. Note that the input and output symbols are of the same size, i.e., m=n for the noiseless channel.



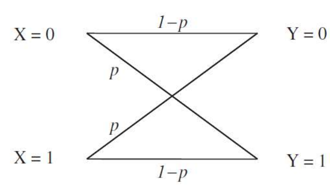
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

PHYSICAL LAYER – SOURCES ENCODING

Binary Symmetry Channel (BSC)

A Binary Symmetry Channel is defined by the channel diagram and its channel matrix given below:

$$P_{BSC} = \begin{bmatrix} (1-p) & p \\ p & (1-p) \end{bmatrix}$$



A BSC channel has two inputs (x1=0, x2=1) and two outputs (y1=0, y2=1). This channel is symmetry because the probability of receiving a 1 if a O is sent is the same as the probability of receiving a 0 if a 1 is sent,