

Module II

Modes of occurrence of water in soil: →

Soil water is broadly classified into, two categories

- (1) Free water.
- (2) Held water.

Free water, moves in the pores of the soil, under the influence of gravity.

Held water is retained in the pores of the soil, and it can not move under the influence of gravitational force. Held water is further divided into, '3' types -

- (i) Structural water.
- (ii) Adsorbed water.
- (iii) Capillary water.

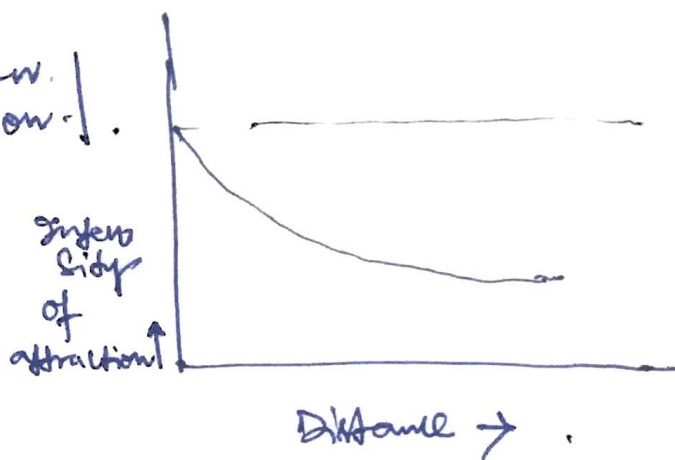
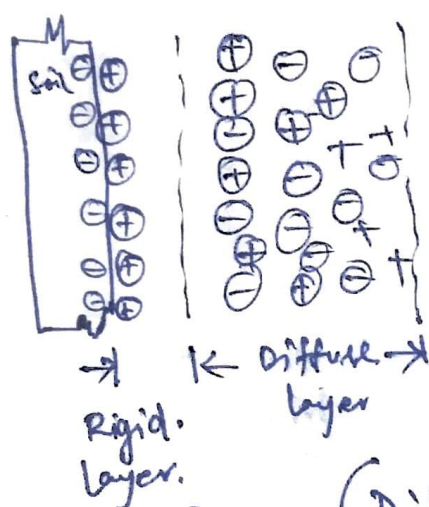
(i) Structural water: → It is chemically combined water - in the crystal structure of the mineral of the soil. The water can not be removed, without breaking the structure of the mineral. A temp^r of more than 300°C is required, for removing the structural water. Structural water is considered as an integral part of the soil solid.

(ii) Adsorbed water: → Due to net negative (-ve) charge on the surface, the clay particles attract cations (such as potassium, calcium and sodium) from the moisture present in the soil, to reach an electrically-balanced equilibrium.

Hence it is believed that there is thin, very tight held layer of water, about 10 Å, is present around the particle. Beyond this thickness, a second layer in which water is more mobile. This second layer extends to the limit of attraction, and is known as, diffuse layer.

Both the layers combined is called, diffuse double layer.

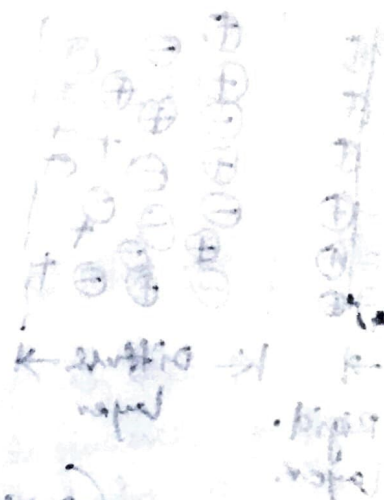
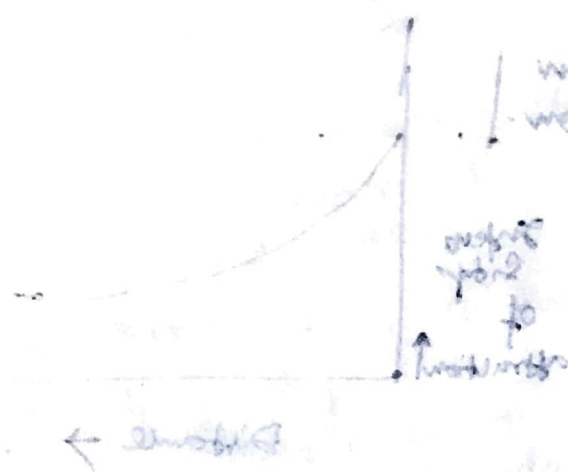
Total thickness of the diffuse double layer is about $400 \text{ \AA}$.



(Diffuse Double layer)

The attractive forces between the adsorbed water, and the soil surface, decrease exponentially with the distance. Until the double layer merges into normal water. It is more viscous and surface tension is also more than normal water. Boiling point is higher, but freezing point is lower than that of normal water. Adsorbed water imparts plasticity characteristic to soils.

(55) Capillary water: $\rightarrow$ It is that water, which is lifted up by surface tension, above the free ground water surface. This water is in suspended condition within the interstices and pores of capillary size of the soil. The capillary water fills all the pores in the soil to a distance above ground table which is known as zone of capillary saturation.



(Effective stress)

When a soil is subjected to a stress, the particles of the soil are pushed together and the voids are compressed. This results in a decrease in the volume of the soil and an increase in the density. The effective stress is the stress that is actually carried by the soil particles. It is the total stress minus the pore water pressure.

Stress conditions in soil:

The effective stress (σ') at a point in the soil mass is equal to the total stress minus the pore water pressure.

$$\sigma' = \sigma - u$$

$$\text{Where } \sigma = \frac{P}{A} = \frac{\gamma \cdot h \cdot A}{A} = \gamma h$$

(Total stress)

pore water pressure (u) is the pressure due to pore water filling the voids of the soil.

$$\therefore \text{Thus, } u = \gamma_w \times h$$

pore water pressure is also known as, neutral pressure or neutral stress.

If the soil is in saturated conditions,

$$\sigma' = \sigma - u$$

$$= \gamma_{\text{sat}} h - \gamma_w h = (\gamma_{\text{sat}} - \gamma_w) h$$

$$\text{Where, } \sigma' = \text{Submerged unit weight} = \gamma' h$$

Q.1 A Sand deposit, 10m thick, and overlies a bed of soft clay. The ground water table is, 3m below the ground surface. If the sand above the ground water table has a degree of saturation of 45%, Plot the diagram showing the variation of the total stress, pore water pressure and the effective stress. The void ratio of the sand is 0.70. Take $G_s = 2.65$

Ans. → Bulk density, $\gamma = \frac{(G_s + Se) \rho_w}{1 + e}$

$$= \frac{(2.65 + 0.45 \times 0.70)}{1 + 0.70} \times 1000$$

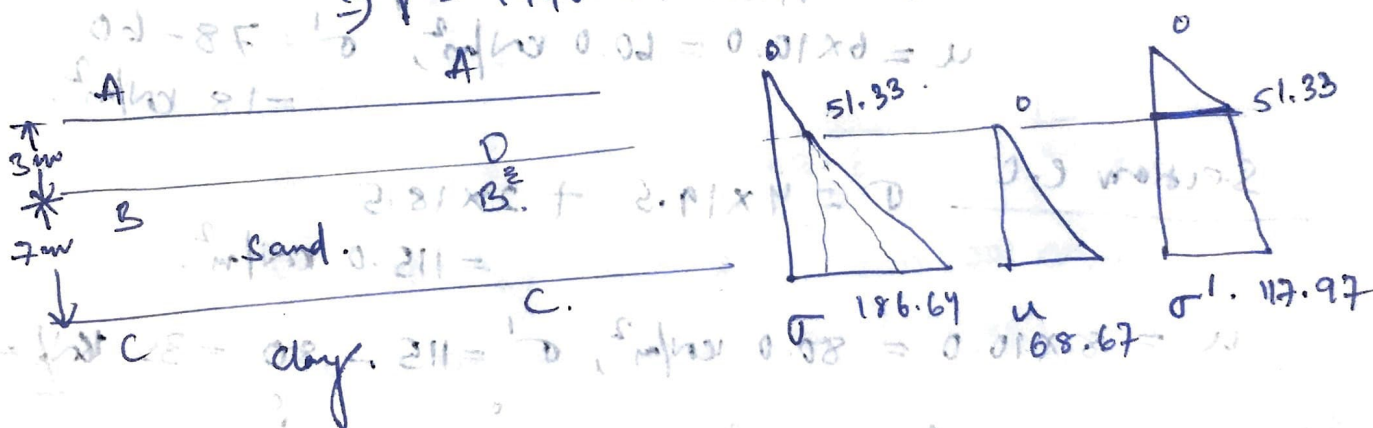
$$= 1744.12 \text{ kg/m}^3$$

$$\Rightarrow \gamma = 1744.12 \times 9.81 \times 10^{-3} = 17.11 \text{ kN/m}^3$$

for saturated soils, $S = 1.0$; and

$$\gamma = \frac{(2.65 + 0.70)}{1 + 0.70} \times 1000 = 1970.59 \text{ kg/m}^3$$

$$\Rightarrow \gamma = 1970.59 \times 9.81 \times 10^{-3} = 19.33 \text{ kN/m}^3$$



Section B-B.

$$\sigma = 17.11 \times 3 = 51.33 \text{ kN/m}^2, u = 0.$$

$$\sigma' = 51.33 - 0 = 51.33 \text{ kN/m}^2$$

Section C-C.

$$\sigma = 17.11 \times 3 + 19.33 \times 7 = 186.64 \text{ kN/m}^2$$

$$u = 7 \times 9.81 = 68.67 \text{ kN/m}^2$$

$$\sigma' = 186.64 - 68.67 = 117.97 \text{ kN/m}^2$$

Q.2 A soil profile consists of a surface layer of a clay 4m thick ($\gamma = 19.5 \text{ kN/m}^3$) and a sand layer 2m thick ($\gamma = 18.5 \text{ kN/m}^3$) overlying an impermeable rock. The water table is at the ground surface. If the water level in a stand pipe driven into the sand layer rises 2m, above the ground surface, draw the plot showing the variation of σ , u and σ' . Take $\gamma_w = 10 \text{ kN/m}^3$.

(b) Determine the increase in effective stress at the top of the rock when the artesian head in the sand is reduced by 1m.

Ans: → Section B-B (clay)

$$\sigma = 19.5 \times 4 = 78.0 \text{ kN/m}^2, u = 4 \times 10.0 = 40 \text{ kN/m}^2$$

$$\sigma' = 78 - 40 = 38 \text{ kN/m}^2$$

Section B-B (sand)

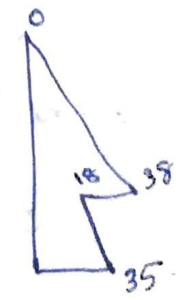
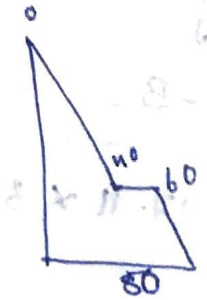
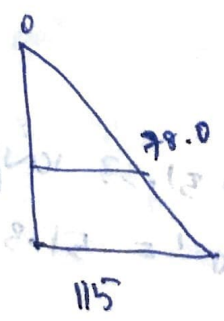
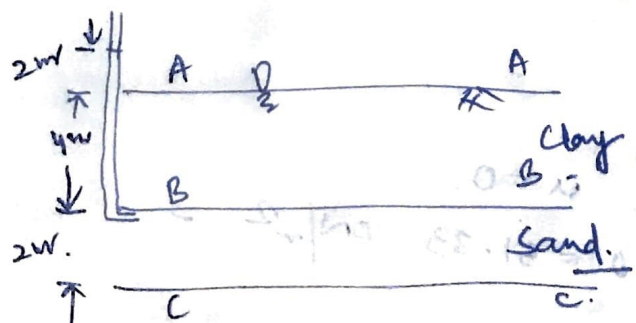
$$\sigma = 19.5 \times 4 = 78.0 \text{ kN/m}^2$$

$$u = 6 \times 10.0 = 60.0 \text{ kN/m}^2, \sigma' = 78 - 60 = 18 \text{ kN/m}^2$$

Section C-C

$$\sigma = 4 \times 19.5 + 2 \times 18.5 = 115.0 \text{ kN/m}^2$$

$$u = 8 \times 10.0 = 80.0 \text{ kN/m}^2, \sigma' = 115 - 80 = 35 \text{ kN/m}^2$$



(b) When, artesian. pressure is reduced by 1m.

Section C-C.

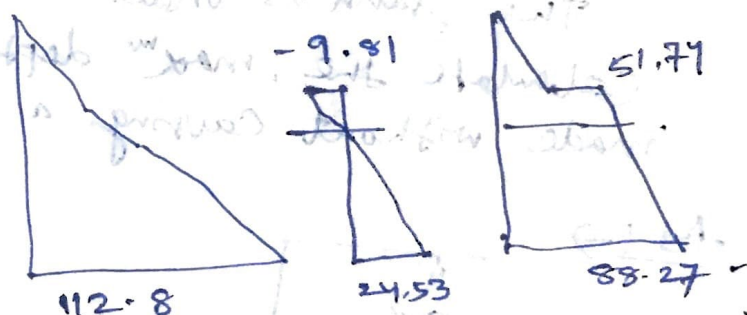
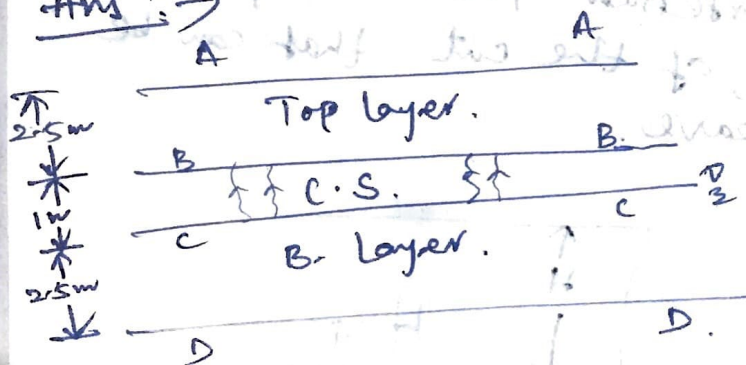
$$\sigma = 1 \times 19.5 + 2 \times 18.5 = 115.0 \text{ kN/m}^2$$

$$u = 7 \times 10.0 = 70 \text{ kN/m}^2, \quad \sigma' = 115 - 70 = 45 \text{ kN/m}^2$$

$$\text{Increase in stress} = 45.0 - 35.0 = 10 \text{ kN/m}^2 \quad \checkmark$$

Q.3 A sand deposit consists of a two layers. The top layer is 2.5 m thick ($\rho = 1709.67 \text{ kg/m}^3$) and the bottom layer is, 3.5 m thick ($\rho_{\text{sat}} = 2064.52 \text{ kg/m}^3$). The water table is at a depth of, 3.5 m from the surface and the zone of capillary saturation is, 1m above the water table. Draw the diagrams, showing the variation of total stress, neutral stress, and effective stress.

Ans: →



$$\gamma = 1709.67 \times 9.81 \times 10^{-3} = 16.77 \text{ kN/m}^3$$

$$\gamma_{\text{sat}} = 2064.52 \times 9.81 \times 10^{-3} = 20.25 \text{ kN/m}^3$$

Level - A-A

$$\sigma' = \sigma - u = 0 - 0 = 0$$

Level B-B

$$\sigma = 16.77 \times 2.5 = 41.93 \text{ kN/m}^2, \quad u = 0$$

top layer

$$\sigma' = 41.93 - 0 = 41.93 \text{ kN/m}^2$$

Level B-B

(Bottom layer)

$$\sigma = 16.77 \times 2.5 = 41.93 \text{ kN/m}^2$$

$$u = 9.81 \times (-1) = -9.81 \text{ kN/m}^2$$

$$\sigma' = 41.93 - (-9.81) = 51.74 \text{ kN/m}^2$$

Level C-C.

$$\sigma = (16.77 \times 2.5) + (20.25 \times 1) = 62.18 \text{ kN/m}^2$$

$u = 0$ $\therefore \sigma' = 62.18 - 0 = 62.18 \text{ kN/m}^2$

Level D-D.

$$\sigma = (16.77 \times 2.5) + (20.25 \times 3.5) = 112.8 \text{ kN/m}^2$$

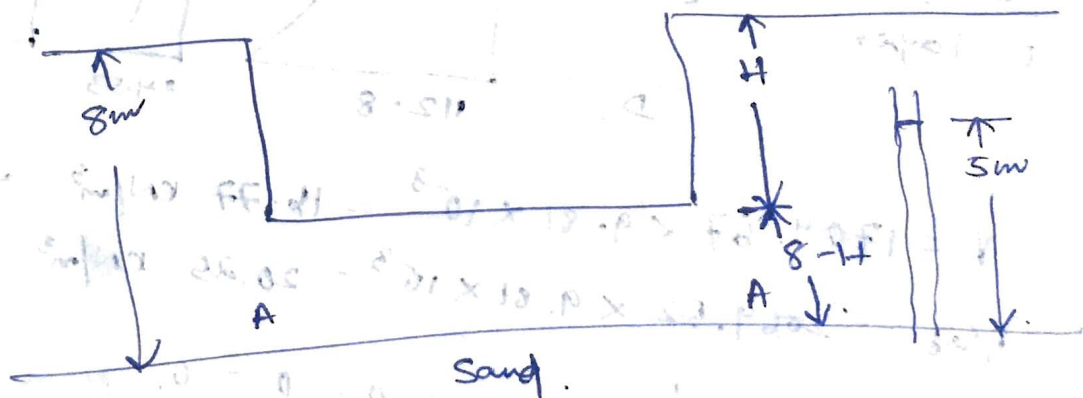
$$u = 2.5 \times 9.81$$

$$= 24.53 \text{ kN/m}^2$$

$$\sigma' = 112.8 - 24.53 = 88.27 \text{ kN/m}^2$$

Q.4 A 8 m thick layer of stiff saturated clay, ($\gamma = 19.0 \text{ kN/m}^3$) is underlain by a layer of sand. The sand is under an artesian pressure of 5 m. Calculate the max^m depth, of the cut that can be made without causing a heave.

Ans: $\rightarrow$



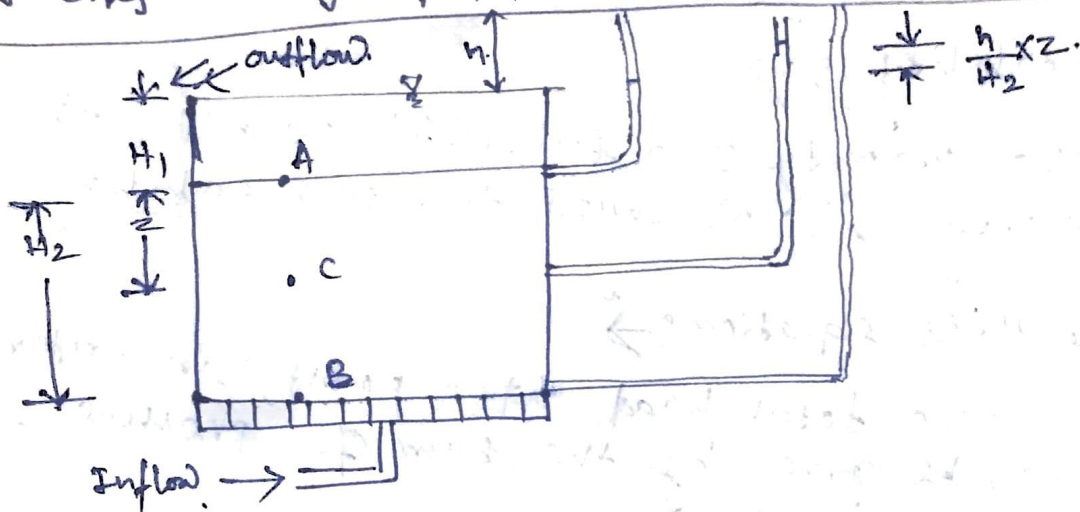
$$\sigma = 19.0 \times (8-H) = 152.0 - 19H$$

$$u = 5 \times 9.81 = 49.05$$

$$\sigma' = 152.0 - 19H - 49.05 = 102.95 - 19H = 0$$

$$\Rightarrow H = 5.42 \text{ m}$$

Stresses in saturated soil with upward seepage: $\rightarrow$



Level-A-A $\sigma' = \sigma - u$ 
 $= H_1 r_w - H_1 r_w = 0.$

level - E-E. $\sigma^1 = \sigma - u$
 $= (H_1 r_w + 2r_{\text{sat}}) - (H_1 + z + iz) r_w$
 $= z r^1 - iz r_w$

$$\underline{\underline{\text{level-B-B}}} \quad \sigma' = \sigma - u$$

$$= (H_1 r_w + H_2 r_{sat}) - (H_g + H_2 + h) r_w$$

$$= H_2 r' - h r_w$$

* For critical condition.

$$\Rightarrow 2\gamma' - i\gamma\omega = 0$$

$$\Rightarrow i_{cr} = \frac{r'}{r\omega} = \frac{C_m}{1 - \dots}$$

(this situation, is generally referred to as boiling or quick conditions).

*. As the water flows through a soil, it exerts a force, on the soil. The force acts in the direction of flow in the case of isotropic soils. This force is known as seepage force.

seepage force per unit volume = $\frac{i \gamma_w \times A}{\cancel{A}} = i \gamma_w$



permeability :>

The property of a soil, which permits flow of water, through it is called the permeability.

Bernoulli's equation :>

The total head at a point, in water, under motion can be given by the sum of pressure, velocity and elevation head.

$$h = \frac{u}{\gamma_w} + \frac{v^2}{2g} + z$$

$\frac{u}{\gamma_w}$: pressure head.
 $\frac{v^2}{2g}$: velocity head.
 z : Elevation head.

Where, h = Total head.

u = pressure.

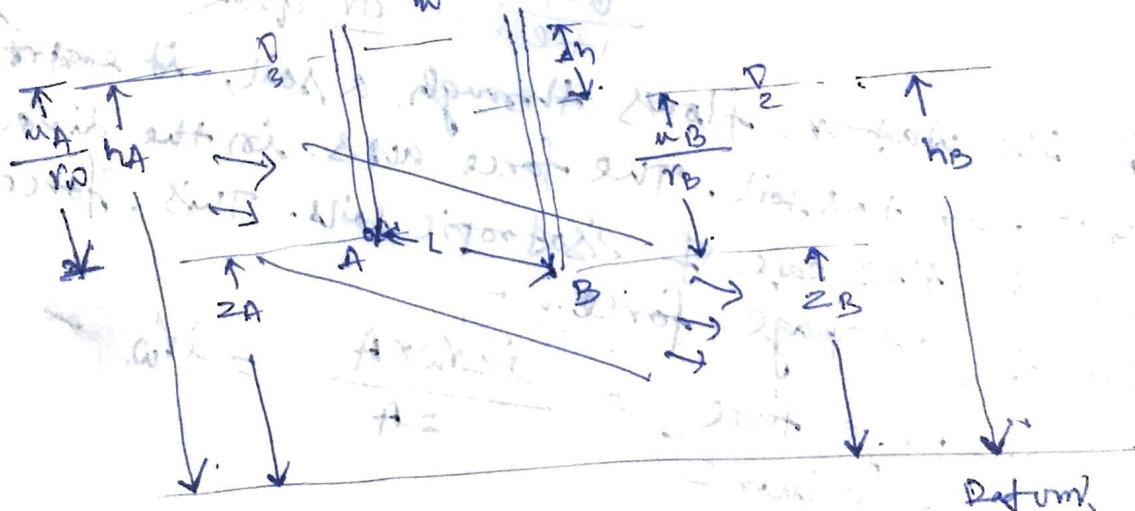
v = velocity.

g = Accⁿ due to gravity.

γ_w = unit wt of water.

* Since the flow of water through porous soil-medium, and the seepage velocity is small, the total head at any point, can be represented by,

$$h = \frac{u}{\gamma_w} + z$$



* The level upto which, water rises and known as -

piezometric levels.

The loss of head between two points A and B can be given -

by,
$$\Delta h = h_A - h_B = \left(\frac{u_A}{\gamma_B} + z_A \right) - \left(\frac{u_B}{\gamma_B} + z_B \right)$$

The head loss can be expressed as $i = \frac{\Delta h}{L}$

Darcy's Law:

The discharge velocity of water through saturated soils, which may be expressed as,

$$v = Ki$$

(valid for laminar flow & for wide range)

where, v = discharge velocity, which is the quantity of water, flowing in unit time, through a unit gross cross-sectional area, of soil at right angles to the direction of flow.

K = hydraulic conductivity, (otherwise known as coefficient of permeability).

$$Q = vA = A_v \times v_s$$

where, v_s = seepage velocity,

A_v = Area of void in the c/s of the specimen.

The actual velocity of water i.e., v_s (seepage velocity) through the void space is greater than the

discharge velocity i.e., v .

$$Q = \frac{A_v}{A_s + A_v} = \frac{A_v \times v_s}{A_s + A_v}$$

$$\Rightarrow v_s = \frac{v(A_s + A_v)}{A_v} = \frac{v(1 + e)}{e}$$

$$\Rightarrow v_s = \left(\frac{1+e}{e} \right) v = \frac{v}{\eta}$$

Hydraulic Conductivity:

Hydraulic conductivity of soils depends on several factors: fluid viscosity, pore size distribution, grain size distribution, void ratio, roughness of mineral particles, and degree of saturation.

H. conductivity is also related to, properties of fluid

flow, i.e., $K = \frac{\gamma_w}{\eta} \bar{K}$

where, γ_w = unit wt of water.

η = viscosity of water.

K = absolute permeability of soil (L^2).

* Hydraulic conductivity is a function of the unit wt and the viscosity of water, which in turn, function of the temp^r, at which the test is conducted.

$$\therefore, \frac{K_{T_1}}{K_{T_2}} = \left(\frac{\eta_{T_2}}{\eta_{T_1}} \right) \left[\frac{\gamma_w(T_1)}{\gamma_w(T_2)} \right]$$

where, K_{T_1}, K_{T_2} = hydraulic conductivity at

Temp^r T_1 and T_2 , respectively.

η_{T_1}, η_{T_2} = viscosity of water at temp^r T_1 and T_2 respectively.

① $\gamma_w(T_1), \gamma_w(T_2)$ = unit wt of water, at temp^r T_1 and T_2 respectively.

* It is conventional to express the value of K at 20°C. within the range of, test temp^r, we can assume

$$\gamma_w(T_1) \approx \gamma_w(T_2) \quad \therefore K_{20^\circ C} = \left(\frac{\eta_{T^\circ C}}{\eta_{20^\circ C}} \right) \times K_{T^\circ C}$$

Laboratory Determination of Hydraulic conductivity

Constant head Test

The water supply at the inlet is, adjusted in such a way that, the difference of head between the inlet and the outlet remains constant during the test period.

The total volume of water collected, may be expressed as,

$$Q = Avt = A(k_i)t$$

where, Q = volume of water collected.

A = Area of c/s of the soil specimen.

t = duration of water collection.

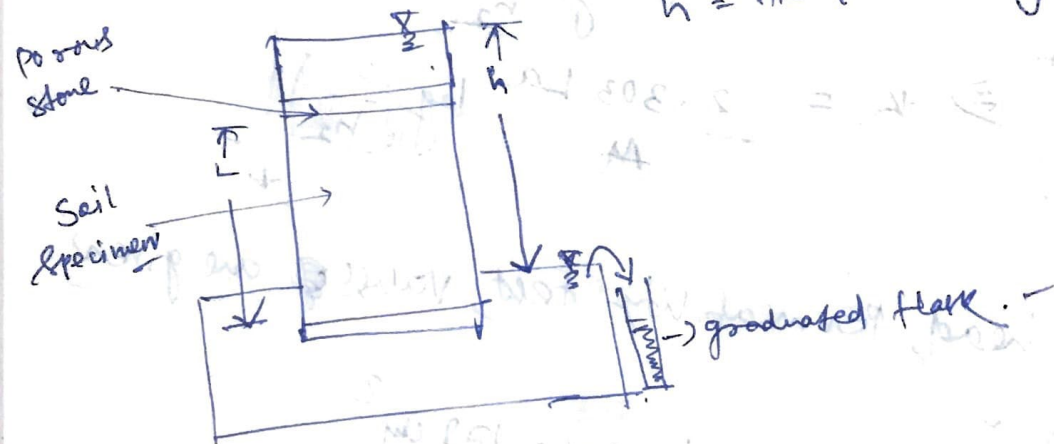
$$= A\left(k \times \frac{h}{L}\right)t$$

$$\Rightarrow k = \frac{QL}{Aht}$$

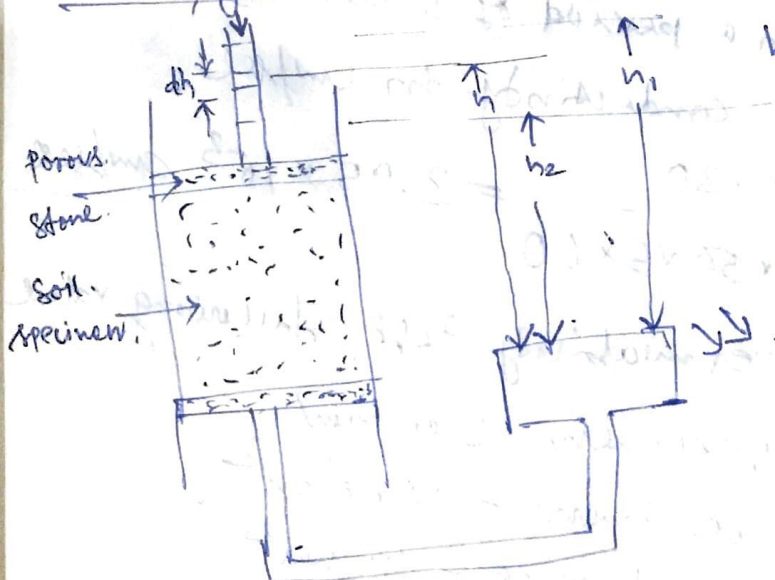
where k = co-efficient of permeability.

L = Length of the specimen.

h = Head causing flow.



Falling Head Test :-



h_1 = Head causing flow at time $t = t_1$

h_2 = Head causing flow at time $t = t_2$

The rate of flow of water, through the specimen, at any time t , can be given by,

$$q = k \frac{h}{L} \times A = -a \frac{dh}{dt}$$

where, q = flow rate.

a = c/s area of stand pipe.

A = c/s area of soil specimen.

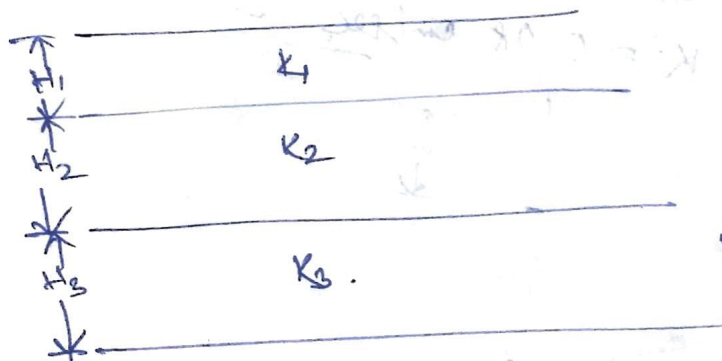
Equivalent hydraulic conductivity in stratified soil :-

The average permeability of the whole deposit will depend upon the direction of flow, with relation to the direction of bedding planes.

Two cases may arise :-

1. Average permeability parallel to the bedding planes
2. Average permeability per to the bedding planes.

Ans A layered soil is shown in fig:-



$$\begin{aligned} H_1 &= 1.5 \text{ m}, K_1 = 10^{-4} \text{ cm/sec} \\ H_2 &= 3 \text{ m}, K_2 = 3.2 \times 10^{-2} \text{ cm/sec} \\ H_3 &= 2 \text{ m}, K_3 = 4.1 \times 10^{-5} \text{ cm/sec} \end{aligned}$$

Estimate the ratio of hydraulic conductivity.

$$\frac{K_H (\text{eq})}{K_V (\text{eq})}$$

$$\begin{aligned} \text{Ans:} \rightarrow K_H (\text{eq}) &= \frac{1}{H} (K_1 H_1 + K_2 H_2 + K_3 H_3) \\ &= \frac{1}{(1.5 + 3 + 2)} [10^{-4} \times 1.5 + (3.2 \times 10^{-2}) \times 3 + (4.1 \times 10^{-5}) \times 2] \\ &= 148.05 \times 10^{-4} \text{ cm/sec.} \end{aligned}$$

$$\begin{aligned} K_V (\text{eq}) &= \frac{H}{\left(\frac{H_1}{K_1}\right) + \left(\frac{H_2}{K_2}\right) + \left(\frac{H_3}{K_3}\right)} \\ &= \frac{1.5 + 3 + 2}{\left(\frac{1.5}{10^{-4}}\right) + \left(\frac{3}{3.2 \times 10^{-2}}\right) + \left(\frac{2}{4.1 \times 10^{-5}}\right)} \\ &= 1.018 \times 10^{-4} \text{ cm/sec.} \end{aligned}$$

Hence,

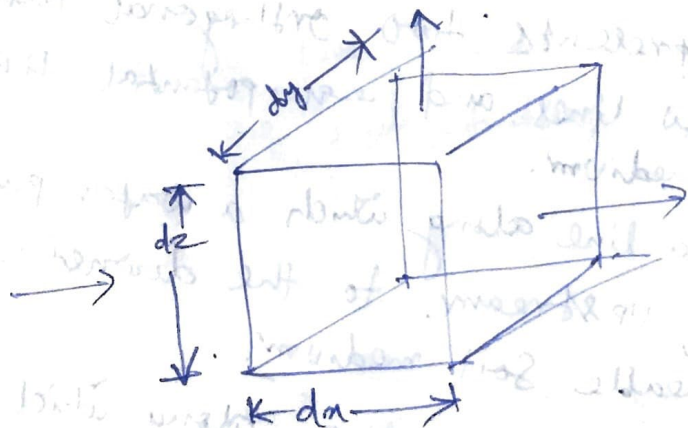
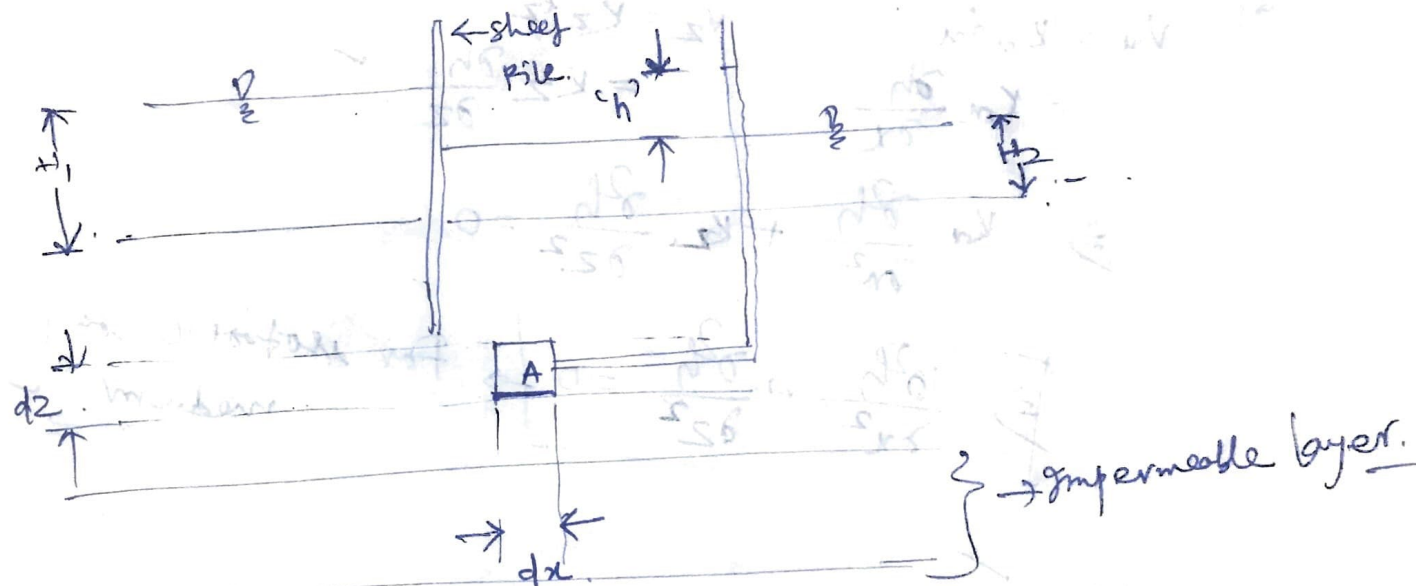
$$\frac{K_H (\text{eq})}{K_V (\text{eq})} = \frac{148.05 \times 10^{-4}}{1.018 \times 10^{-4}} = 145.4$$

Seepage $\rightarrow$

Seepage is the flow of water, under gravitational forces in a permeable medium.

Laplace eqⁿ of continuity $\rightarrow$

Consider a single row of sheet piles driven into a permeable soil layer.



X-direction or Horizontal direction

$$\text{Inflow} = v_x dz dy$$

$$\text{out flow} = \left(v_x + \frac{\partial v_x}{\partial x} dx \right) dz dy$$

Z-direction or Vertical direction

$$\text{Inflow} = v_z dx dy \quad \text{out flow} = \left(v_z + \frac{\partial v_z}{\partial z} dz \right) dx dy$$

Rate of inflow = Rate of outflow

$$\left(v_x + \frac{\partial v_x}{\partial x} dx \right) dy dz + \left(v_z + \frac{\partial v_z}{\partial z} dz \right) dx dy - \left[v_x dz dy + v_z dx dy \right] = 0.$$

$$\Rightarrow \frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} = 0. \quad \checkmark$$

$$\text{If } v_x = K_x i_x \frac{\partial h}{\partial x} \quad v_z = K_z i_z \frac{\partial h}{\partial z} \quad \checkmark$$

$$\Rightarrow K_x \frac{\partial h}{\partial x^2} + K_z \frac{\partial h}{\partial z^2} = 0.$$

$$\Rightarrow \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0 \rightarrow \text{For isotropic soil medium.} \quad \checkmark$$

Flow nets $\rightarrow$

Flow nets represents two orthogonal families of curves, i.e., flow lines and equipotential lines in an isotropic soil medium.

* A flow line is a line along which a water particle will travel from upstream to the downstream side for the permeable soil medium.

* An equipotential line is a line, along which the potential head, at all points is equal.

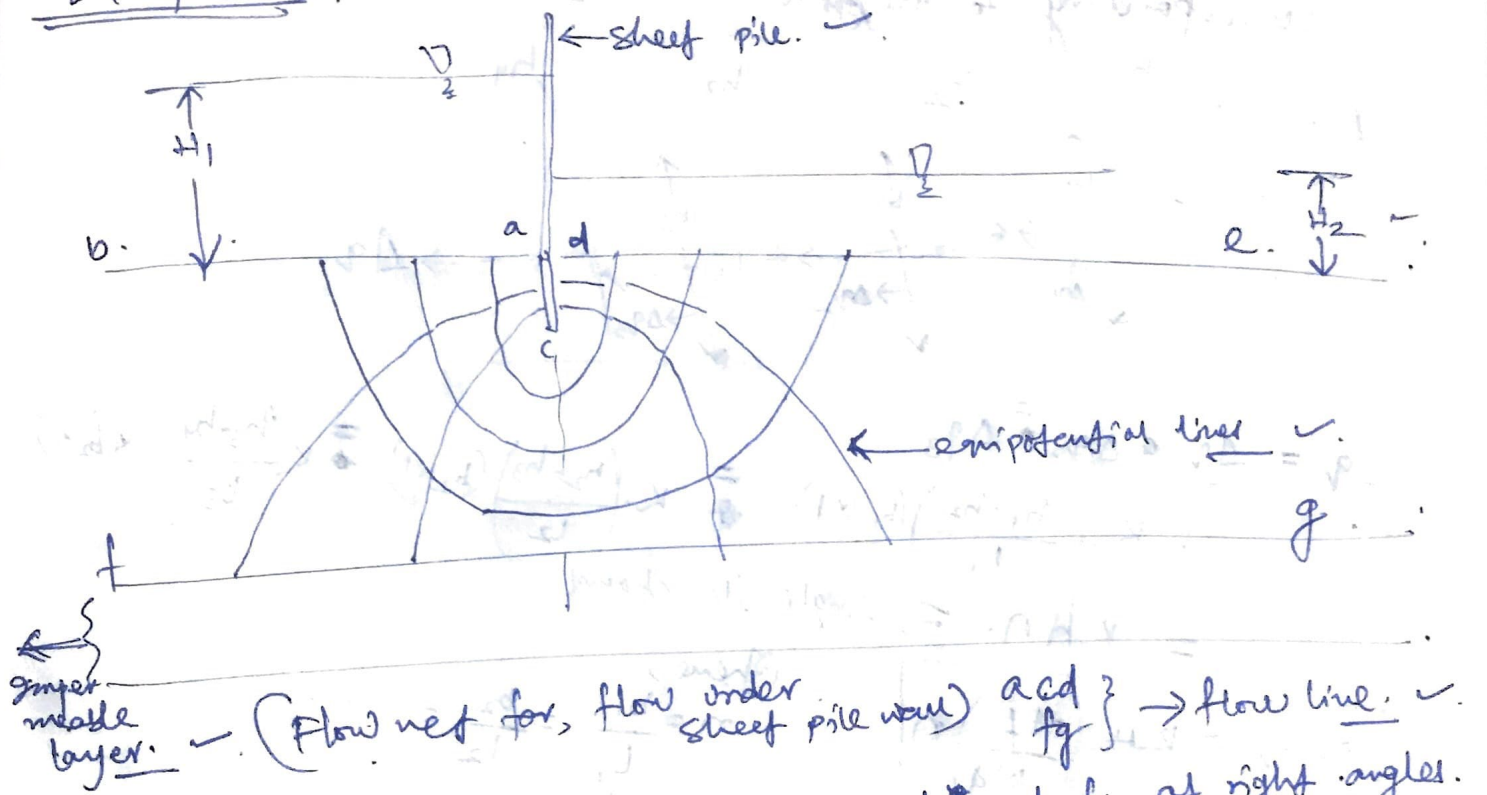
characteristics of flow net $\rightarrow$

1. The equipotential lines intersect the flow lines at right angles.
2. The flow elements formed are approximately same.
3. Quantity of seepage in each flow channel is the same.

4. Drop in head between, adjacent equipotential lines - is the same.

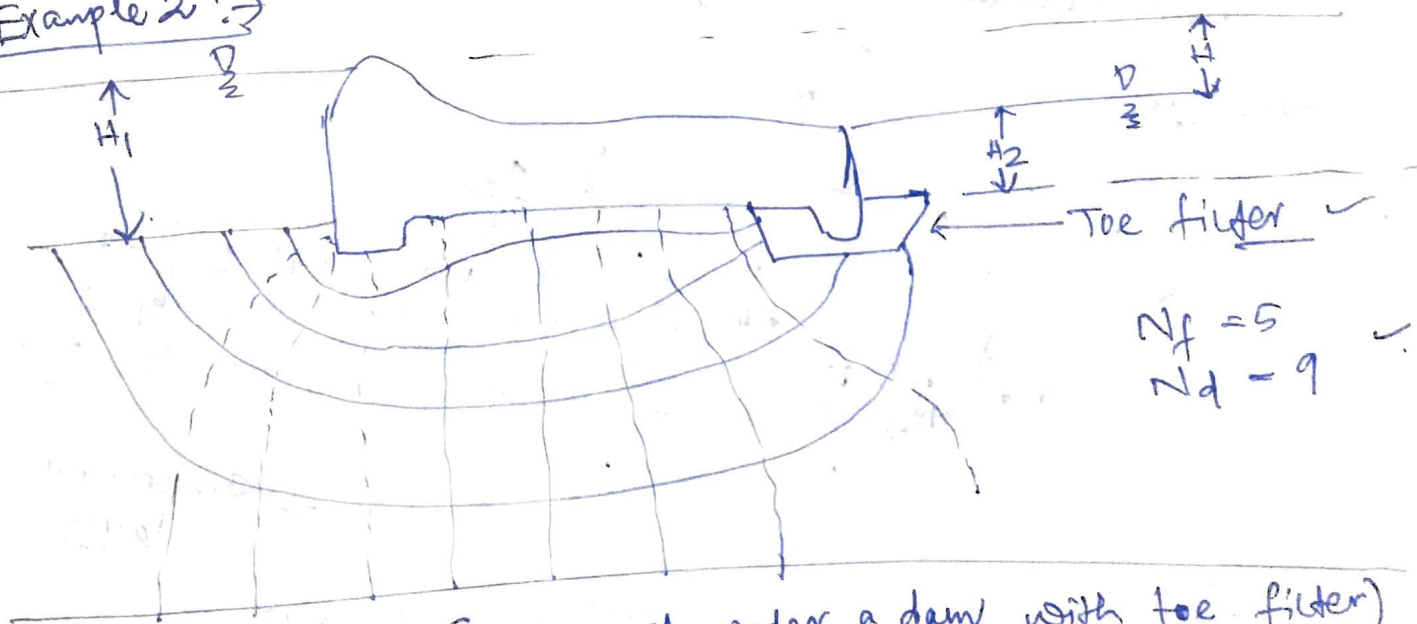
5. Two flow lines or two equipotential lines, can never meet or cross each other.

Example 1: →



* Equipotential lines meet, acd and fg at right angles.
 * ab & de are equipotential lines. $N_f = 4$, $N_d = 6$

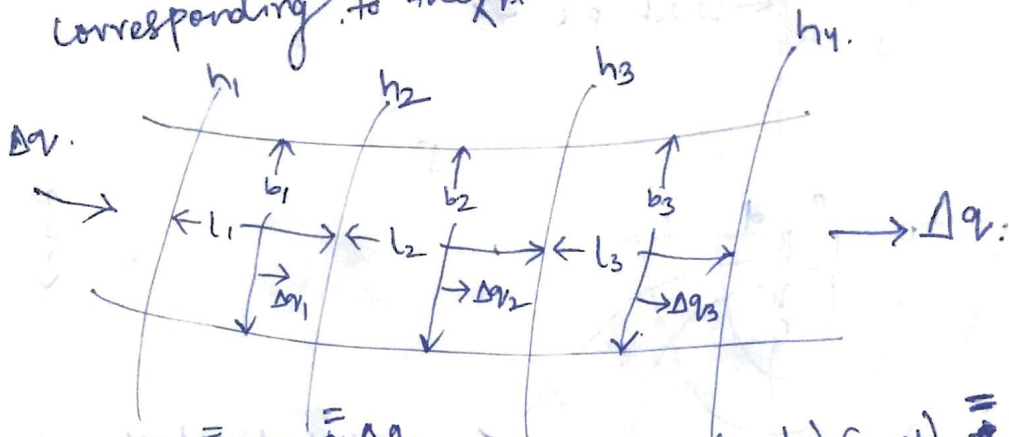
Example 2: →



(Flow net under a dam, with toe filter)

Seepage calculation from a flow net $\rightarrow$

* Strip between any two adjacent flow lines is called a flow channel. Let h_1, h_2 be the piezometric levels corresponding to the potential lines.



$$\Delta q = \Delta q_1 = \Delta q_2 = \Delta q_3$$

$$= K \left(\frac{h_1 - h_2}{l_1} \right) (b_1 \times 1) = K \left(\frac{h_2 - h_3}{l_2} \right) (b_2 \times 1) = K \left(\frac{h_3 - h_4}{l_3} \right) (b_3 \times 1)$$

$$= K H n \Rightarrow \text{Single flow channel.}$$

$$\Rightarrow q = K H \left(\frac{N_f}{N_d} \right) n$$

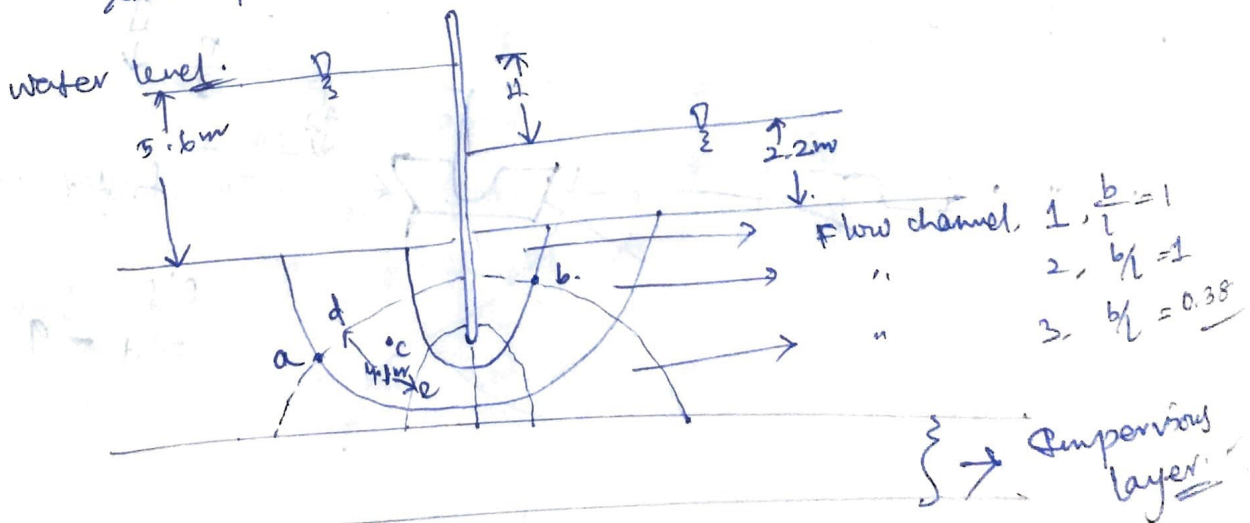
Total flow.

where,

$$n = \frac{b_1}{l_1} = \frac{b_2}{l_2} = \dots$$

$$\& h = H / N_d$$

Ex 1.9 A flow net for flow around a single row of sheet piles in a permeable soil layer is shown in fig.



Given that, $K_1 = K_2 = K = 5 \times 10^{-3}$ cm/sec. determine,
 a. How high (above the ground surface) the water will rise if piezometers are placed at points 'a' and 'b'.

b. The total rate of seepage, through the permeable layer, per unit length. -

c. Approximate average hydraulic gradient at 'C'. -

Ans: → a $N_d = 6$, $H_1 = 5.6 \text{ m}$, $H_2 = 2.2 \text{ m}$. ✓

Head loss in each potential drop, $\Delta H = \frac{H_1 - H_2}{N_d}$

$$= \frac{5.6 - 2.2}{6}$$

$$= 0.567 \text{ m} \quad \checkmark$$

At point a $5.6 - 0.567 = 5.033 \text{ m}$ above the ground surface. ✓

b $5.6 - (5 \times 0.567) = 2.765 \text{ m}$ above ground surface.

b $q = K H \cdot \frac{(2.38)}{N_d} = (5 \times 10^{-5} \text{ m/sec}) (5.6 - 2.2) \times \frac{2.38}{6}$

$$= 6.74 \times 10^{-5} \text{ m}^3/\text{sec}/\text{m} \quad \checkmark$$

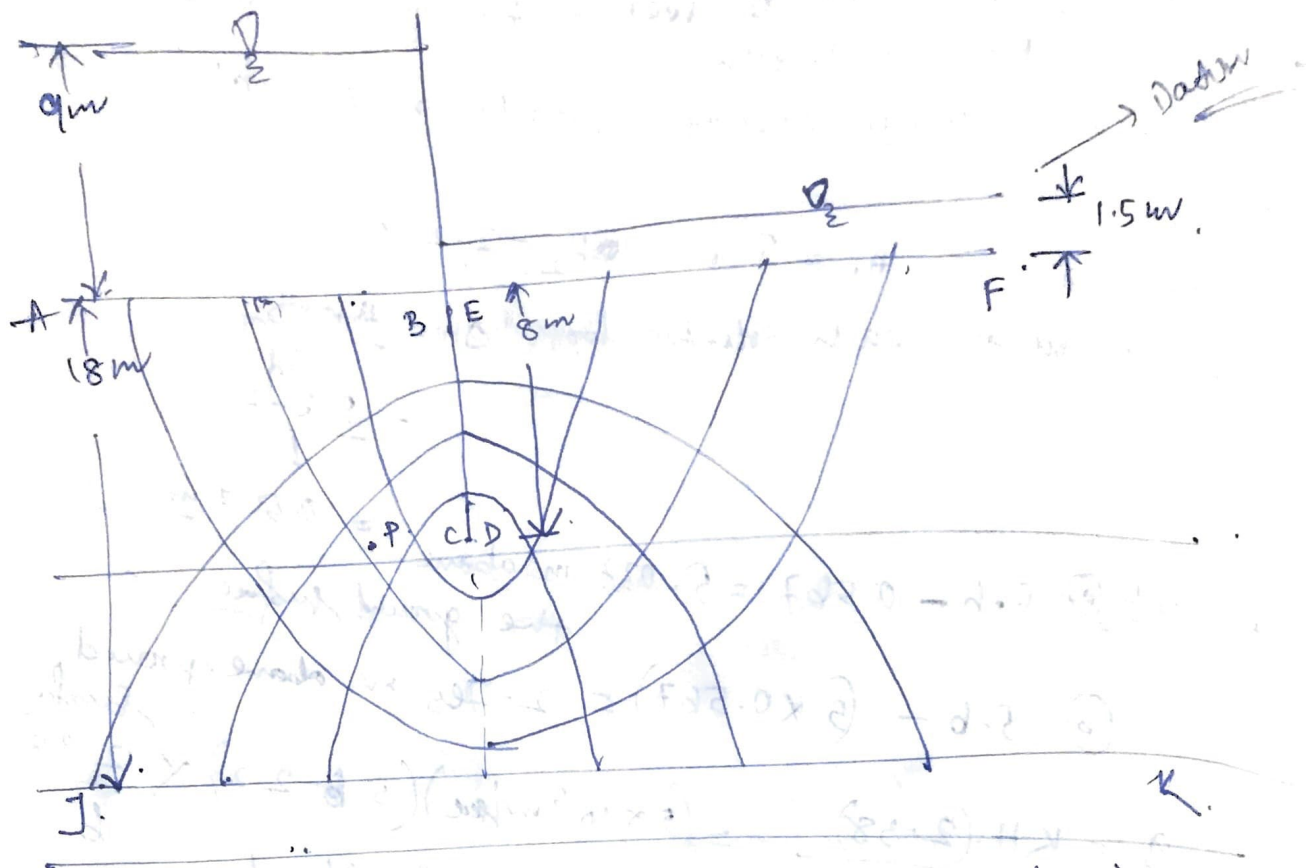
c Average hydraulic gradient, at 'C'

$$= i = \frac{\text{Head loss}}{\text{Avg length of flow between d and e.}}$$

$$= \frac{\Delta H}{\Delta L} = \frac{0.567 \text{ m}}{4.1 \text{ m}} = 0.138.$$

N/10 A soil stratum, with permeability, $K = 5 \times 10^{-7} \text{ cm/sec}$, overlies an impermeable stratum. The impermeable stratum lies at a depth of 18 m below the ground surface (surface of soil stratum). A sheet pile wall penetrates 8 m into the permeable soil stratum. Water stands to a height of 9 m on upstream side and 1.5 m on downstream side, above the surface of soil stratum. sketch the flow net and determine.

- (i) quantity of seepage.
- (ii) seepage pressure, at a point 'p' located 8 m, below surface of soil stratum, and 4 m away from the sheet pile wall on its upstream side.
- (iii) pore pressure, at point 'p'.
- (iv) max^m exit gradient. ✓



Ans: $\rightarrow N_f = 4, N_d = 8$ ✓ (No. of potential drop)

(i) Head causing flow, $H = 9 - 1.5 = 7.5 \text{ m}$.
 quantity of seepage, $q = K H \frac{N_f}{N_d}$

$$= (5 \times 10^9) (7.5) \left(\frac{4}{8}\right) = 18.75 \times 10^9 \text{ m}^3/\text{sec}/\text{m}$$

(ii) potential drop per field, $\Delta h = \frac{H}{N_d} = \frac{7.5}{8} = 0.9375 \text{ m}$

No. of potential drops upto point P,

$$n = 2.5$$

$$\begin{aligned} \text{Total head, at point, 'P', } h &= H - n \cdot \Delta h \\ &= 7.5 - (2.5 \times 0.9375) \\ &= 5.16 \text{ m} \end{aligned}$$

Seepage pressure at 'P',

$$\begin{aligned} p_s &= h \gamma_w = 5.16 \times 9.81 \\ &= 50.62 \text{ kN/m}^2 \end{aligned}$$

(iii) Hydrostatic head, $P = h_w = h - Z$
 $= 5.16 - (-9.5)$
 $= 14.66 \text{ m}$

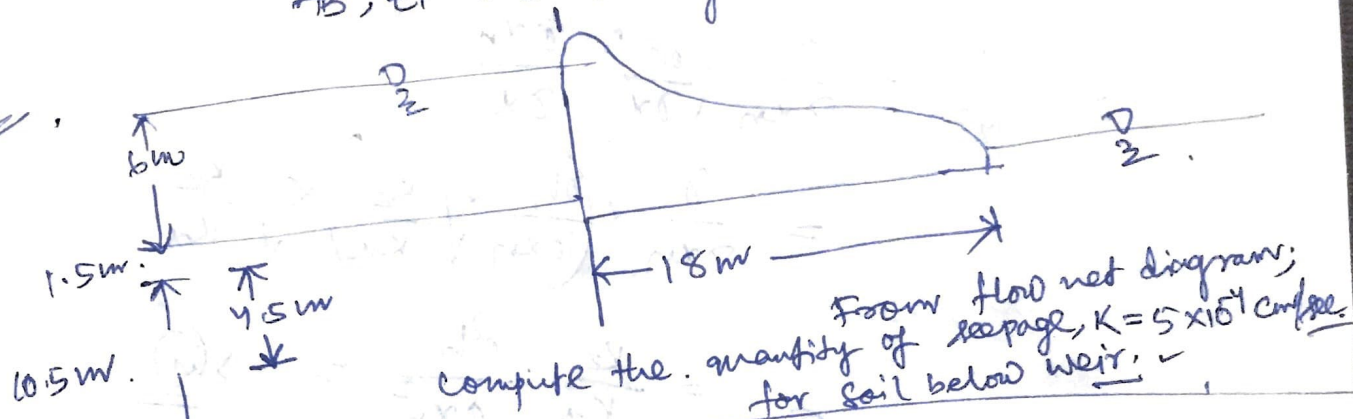
pore pressure, $p_u = h_w \gamma_w$

$$= 14.66 \times 9.81 = 143.81 \text{ kN/m}^2$$

(iv) Average length of exit field, adjacent to sheet pile wall, $l_e = 2.8 b_w$.
 Max^m exit gradient = $i_e = \frac{\Delta h}{l_e} = \frac{0.9375}{2.8} = 0.33$.

AB, BC, DE & JK are boundary flow lines.

AB, EF are boundary equipotential lines.



Flow Nets in Anisotropic soils:

The differential eqⁿ of continuity for 2D flow is.

$$K_x \frac{\partial^2 h}{\partial x^2} + K_z \frac{\partial^2 h}{\partial z^2} = 0. \quad \checkmark \quad \text{This is not in the Laplacian form.}$$

To make it Laplace form, change the scale of x-axis i.e., $x_n = x \sqrt{\frac{K_z}{K_x}}$. & Hold in y-direct plot without change.

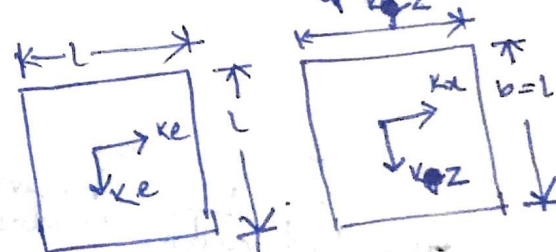
Hence for transformed section,

$$\Delta q = K_e \left(\frac{\Delta h}{l} \right) \times l = K_e \cdot \Delta h. \quad \text{--- (i)}$$

And for true section,

$$\Delta q = K_x \left[\frac{\Delta h}{l \sqrt{\frac{K_x}{K_z}}} \right] \times l$$

$$= K_x \cdot \Delta h \cdot \sqrt{\frac{K_z}{K_x}}$$



$$= \Delta h \cdot \sqrt{K_x K_z} \quad \text{--- (ii)}$$

$$\therefore K_e \cdot \Delta h = \Delta h \sqrt{K_x K_z} \Rightarrow K_e = \sqrt{K_x K_z}$$

$$\Rightarrow q = K_e \cdot H \cdot \frac{N_f}{N_d} = \sqrt{K_x K_z} \times H \times \frac{N_f}{N_d}$$

$$K_x \frac{\partial^2 h}{\partial x^2} + K_z \frac{\partial^2 h}{\partial z^2} = 0$$

$$x_n = n \sqrt{\frac{K_z}{K_x}}$$

$$\Rightarrow \frac{K_x}{K_z} \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0 \rightarrow \textcircled{i}$$

$$\frac{\partial h}{\partial x} = \frac{\partial h}{\partial x_n} \frac{\partial x_n}{\partial x} = \frac{\partial h}{\partial x_n} \times \sqrt{\frac{K_z}{K_x}}$$

$$\Rightarrow \frac{\partial^2 h}{\partial x^2} = \frac{\partial}{\partial x_n} \left(\frac{\partial h}{\partial x} \right) \frac{\partial x_n}{\partial x}$$

$$= \frac{\partial}{\partial x_n} \left(\frac{\partial h}{\partial x_n} \times \sqrt{\frac{K_z}{K_x}} \right) \times \sqrt{\frac{K_z}{K_x}}$$

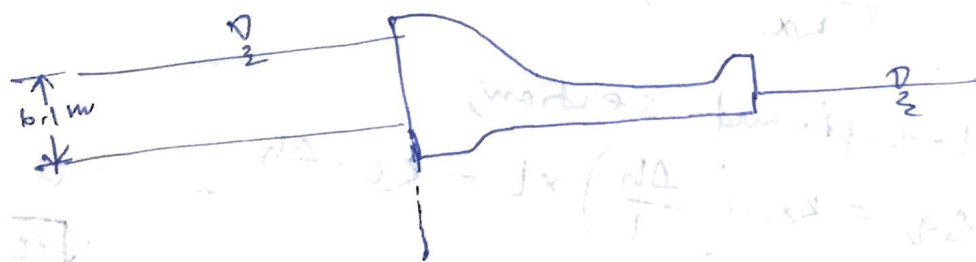
$$= \frac{K_z}{K_x} \frac{\partial^2 h}{\partial x_n^2} \rightarrow \textcircled{ii}$$

Substituting $\textcircled{ii}$ in $\textcircled{i}$.

$$\frac{K_x}{K_z} \cdot \frac{K_z}{K_x} \frac{\partial^2 h}{\partial x_n^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

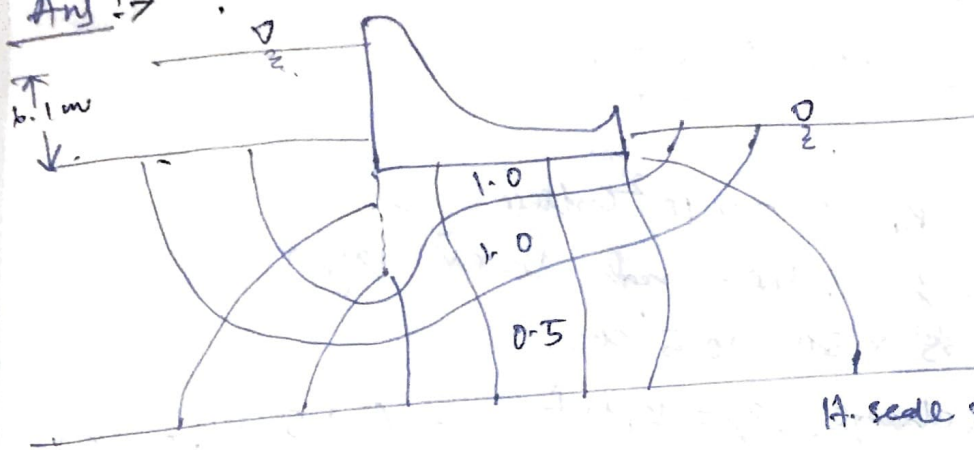
$$\Rightarrow \frac{\partial^2 h}{\partial x_n^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

Q.12 A dam section is shown in fig.



The hydraulic conductivity of the permeable layer in the vertical and horizontal directions are 2×10^{-2} mm/sec. and 4×10^{-2} mm/sec, respectively. Draw a flow net, and calculate the seepage loss of the dam in $\text{m}^3/\text{day}/\text{m}$.

Ans: →



scale in feet, 1 unit = 7.6 m ✓

$$H. \text{ scale} = 7.6 \times \sqrt{2} = 10.75 \text{ m} \checkmark$$

$$V. \text{ scale} = 7.6 \text{ m} \checkmark$$

$$H. \text{ scale} = \sqrt{\frac{2 \times 10^2}{4 \times 10^2}} (V. \text{ scale})$$

$$= \frac{1}{\sqrt{2}} (V. \text{ scale})$$

$$N_d = 8$$

$$N_f = 2.5$$

lower most flow channel has width to length ratio, 0.5. ✓

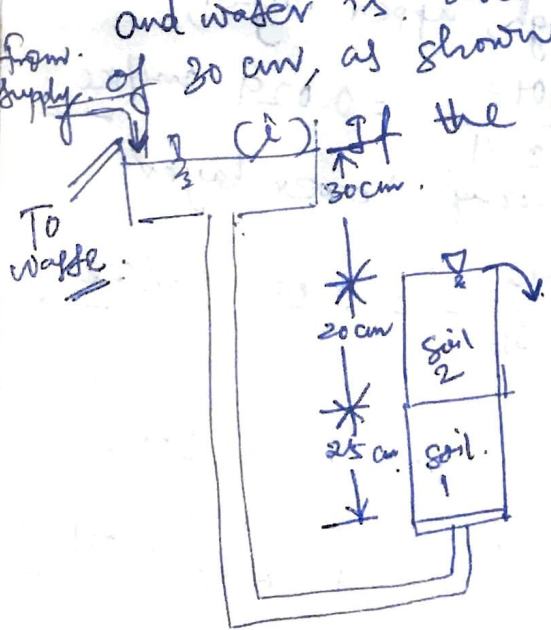
$$Q = \sqrt{1.728 \times 3.456} \times (6.1) \times \left(\frac{2.5}{8}\right) = 4.66 \text{ m}^3/\text{day/m}$$

Q.13 Calculate the critical hydraulic gradient for a coarse-grained soil deposit, with void ratio 0.7. Take $G_s = 2.67$.

Ans: → critical hydraulic gradient, $i_c = \frac{s'}{\gamma_w} = \frac{G_s - 1}{1 + e}$

$$= \frac{2.67 - 1}{1 + 0.7} = 0.98$$

Q.14 Soils of two types 1 and 2 are taken in a permeameter, 8 cm. and water is allowed to flow through them under a head of 20 cm, as shown in fig.



If the permeability of soil 1 is, $2.8 \times 10^{-2} \text{ cm/sec}$ and 35% of total head, causing flow is lost during flow through this layer, calculate the rate of discharge and permeability for soil 2.

- (ii) If the void ratio is 0.55 for soil 1 and 0.70 for soil 2, compute the seepage velocity, for flow through, each layer.
- (iii) If the total head is increased, determine its value, at which either of the soil, will become quick. Assume G_s as 2.65 for soil 1 and 2.7 for soil 2.

Ans : $\rightarrow A = \frac{\pi (8)^2}{4} = 50.26 \text{ cm}^2$ —

(i) For soil 1, $K_1 = 2.8 \times 10^{-2} \text{ cm/sec}$. ✓
 Head lost during flow through lower layer.
 $= 0.35 \times 30 = 10.5 \text{ cm}$.

$\therefore$ Rate of discharge, $q = K_1 i_1 A$
 $= (2.8 \times 10^{-2}) \left(\frac{10.5}{25} \right) (50.26)$
 $= 0.59 \text{ cm}^3/\text{sec}$ —

Head lost, during flow through upper layer $= 0.65 \times 30 = 19.5 \text{ cm}$

Hydraulic gradient, $i_2 = \frac{19.5}{20} = 0.975$.

$\therefore$ coefficient of permeability of soil 2, $K_2 = \frac{q}{i_2 A}$
 $= \frac{0.59}{0.975 \times 50.26}$
 $= 1.2 \times 10^{-2} \text{ cm/sec}$ —

(ii) Discharge velocity, $v = \frac{q}{A} = \frac{0.59}{50.26} = 0.012 \text{ cm/sec}$ —

porosity, $n = \frac{e}{1+e} = \frac{0.7}{1+0.7} = 0.41$ for upper layer

and, $n = \frac{0.55}{1+0.55} = 0.36$ for lower layer.

Seepage velocity for flow through upper layer,
 $v_s = \frac{v}{n} = \frac{0.012}{0.41} = 0.029 \text{ cm/sec}$ —

Seepage velocity for flow through lower layer,

$v_s = \frac{0.012}{0.36} = 0.033 \text{ cm/sec}$ —

(iii) critical hydraulic gradient,

$i_c = \frac{G-1}{1+e}$

For upper soil, $i_c = \frac{2.7-1}{1+0.7} = 1$

for lower soil,

$i_c = \frac{2.65-1}{1+0.55} = 1.07$

$$\text{Head lost in upper soil} = i_c z = 1(20) \\ = 20 = 0.65h$$

$$\therefore h = \frac{20}{0.65} = 30.8 \text{ cm} \checkmark$$

$$\text{Head lost, in lower soil} = i_c z = 1.06 \times 25 = 26.5 \\ = 0.35h$$

$$\Rightarrow h = \frac{26.5}{0.35} = 75.7 \text{ cm} \checkmark$$

quick condition occurs in the ^{0.35} upper soil, when hydraulic head reaches 30.8 cm.

seepage through earth dam: $\rightarrow$

NB: $\rightarrow$ Flow lines and Equipotential lines get deflected, at the interface between, two dissimilar soils, when they pass from one soil to the other. $\checkmark$

$$\therefore \frac{\tan \theta_1}{\tan \theta_2} = \frac{k_1}{k_2}$$

