Structural Analysis
Civil Engg.

Module - IV

Influence line Diagram and Rolling Loads

Introduction :-

Most common load acting on structure are dead Load and live load.

→ Dead Loads are those loads which are stationary and don't change their position through out the working life of structure.

eg:- self weight of the structure, flooring etc.

→ live Load can change their position during the working life of structure

eg:- wheel Loads and weight of passanger etc.

→ For dead Load the Loads are fixed, the variation of B.M, S.F and deflection is computed at different sections. But the variation of B.M, S.F and deflection for liveload is computed with the help of Influence line diagram (ILD).

Influence line diagram (ILD).

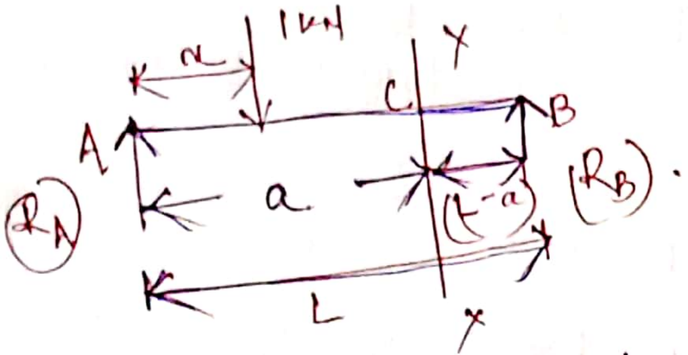
Influence line diagram represents variation of stress function such as Bending Moment, reaction, shearforce slope or deflection at a fixed section when a unit concentration load moves from one end to other end.

Or ILD represents the changing value of reaction, shearforce or bending moment due to change in position of loads.

→ Especially the vehicle loads on roads, bridges are Moving Load.

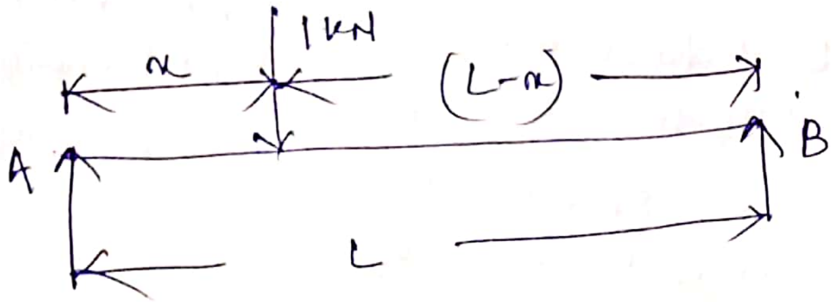
Influence line diagram for determinate structure

(i) Simply supported beam :-



Consider a simply supported beam and a unit load of 1kN is at a distance of xm from support A.

or



anticlock = −ve
Clock = +ve

$$\Sigma M_A = 0$$

$$\Rightarrow -R_B \times L + 1 \times x = 0$$

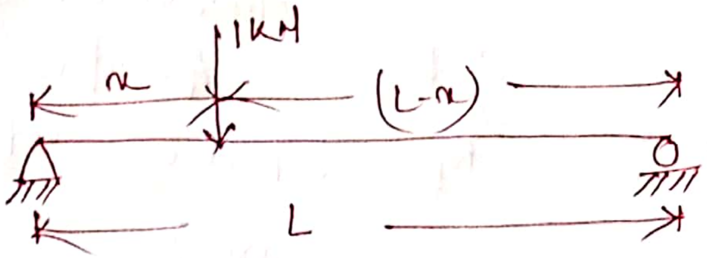
$$\Rightarrow \boxed{R_B = \frac{x}{L}}$$

$$\Sigma M_B = 0$$

$$\Rightarrow R_A \times L - 1 \times (L - x) = 0$$

$$\Rightarrow \boxed{R_A = \frac{L-x}{L}}$$

Influence line diagram for support Reaction :-



Boundary condition (R_B)

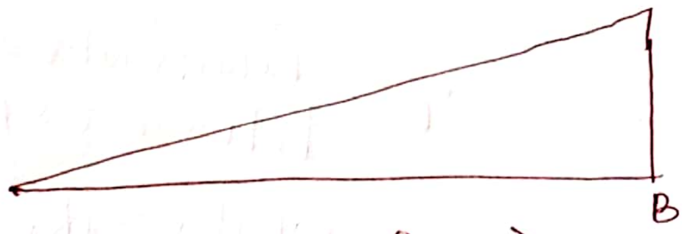
When unit Load is at A

$x = 0$

$R_B = 0$ (as $\frac{x}{L}$ ^{0})

(ii) When unit load is at B then $x = L$

$R_B = \frac{x}{L} = \frac{L}{L} = 1$



A B

(Influence line for R_B)

Boundary condition for R_A

A B

(Influence line for R_A)

When unit load is at A then $x = 0$

$R_A = \frac{L-x}{L} = \frac{L-0}{L} = \frac{L}{L} = 1$

$\boxed{R_A = 1}$

When unit load is at B then $x = L$

$R_A = \frac{L-x}{L} = \frac{L-L}{L} = \frac{0}{L} = 0$

$\boxed{R_A = 0}$

At $x = 0$ $R_A = 1$

$R_B = 0$

at $x = L$ $R_A = 0$

$R_B = 1$.

* Influence line diagram for shear force at a given point

ILD for Shear force

Case-1

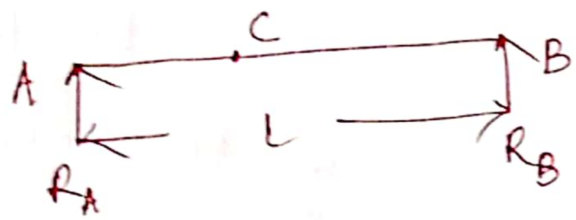
When unit load is between A and C

S.F at C = $-R_B$

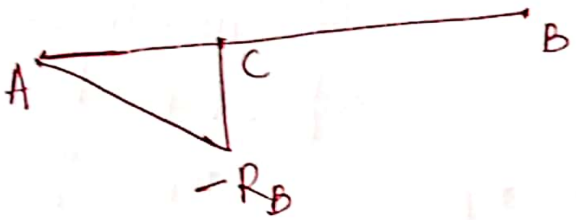
Case-2

when unit load is between C & B

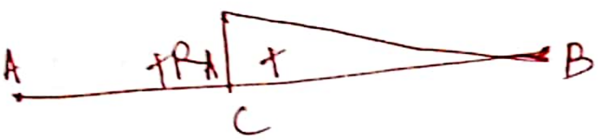
S.F at C = $+R_A$

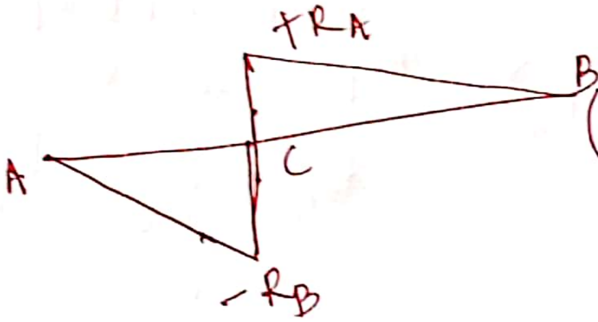


Case-1



Case-2





(Final diagram)

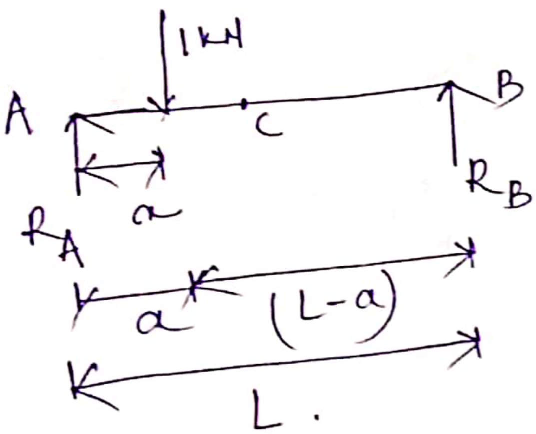
* ILD for Bending moment

Consider unit load 1 kN at distance x.

Case-1

When unit load is between A and C

we know value of R_A and R_B



$$\boxed{R_A = \frac{l-x}{L}} \qquad \boxed{R_B = \frac{x}{L}}$$

(3)

Case-1 :- When Bending Moment is calculate when load is in between A and C.

$B.M @ C = +R_B \times (l-a)$

$= \frac{x}{l} \times (l-a) = \left(\frac{l-a}{l}\right) x$

Applying boundary condition at $x=0$ when unit load is at A then $x=0$

$B.M \text{ at } C = \left(\frac{l-a}{l}\right) \times \cancel{x}^{0} = 0.$

when $x = a$ (unit load is at C)

$B.M \text{ at } C = (x=a)$

$= \left(\frac{l-a}{l}\right) \times x$

$= \left(\frac{l-a}{l}\right) \times a.$

Case-2 :- When unit load is in between C and B

$B.M @ C = + R_A \times a$

$= \left(\frac{l-x}{l}\right) \times a$

Applying boundary condition when unit load is at B then $x=l$

$B.M @ C = \left(\frac{l-x}{l}\right) \times a$

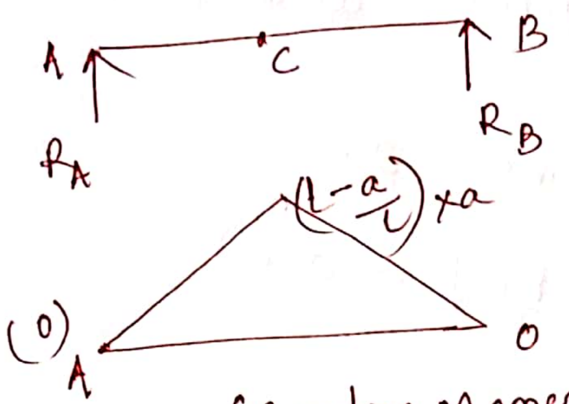
$= \left(\frac{l-l}{l}\right) \times a = \frac{0}{l} \times a = 0$

when unit load is at C, then $x = a$

$B.M @ C = \left(\frac{l-x}{l}\right) \times a$

$$B.M @ C = \left(\frac{l-a}{L}\right) \times a$$
$$= \left(\frac{l-a}{L}\right) a$$

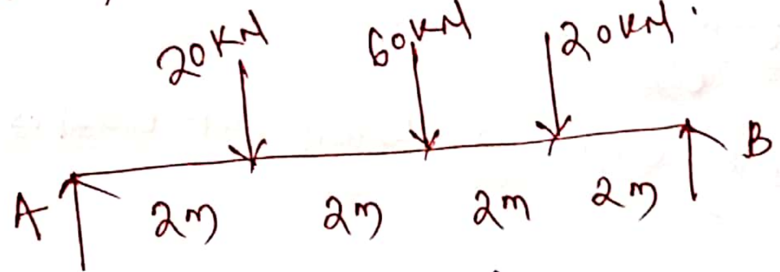
B.M.D :-



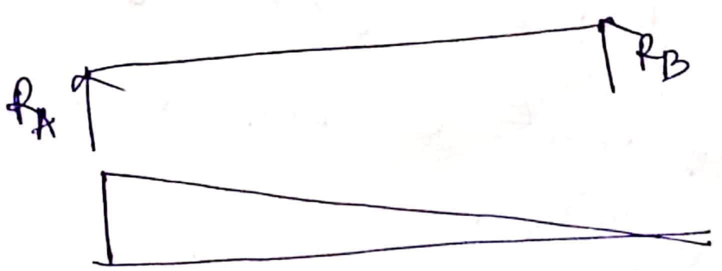
(Bending Moment @ C)

Example of
Simply supported Beam

Q A Simply supported beam of span 8m loaded as shown infigure. find the shear force and Bending Moment at section 4m from leftend. Draw ILD for support reaction, shearforce and Bending moment.

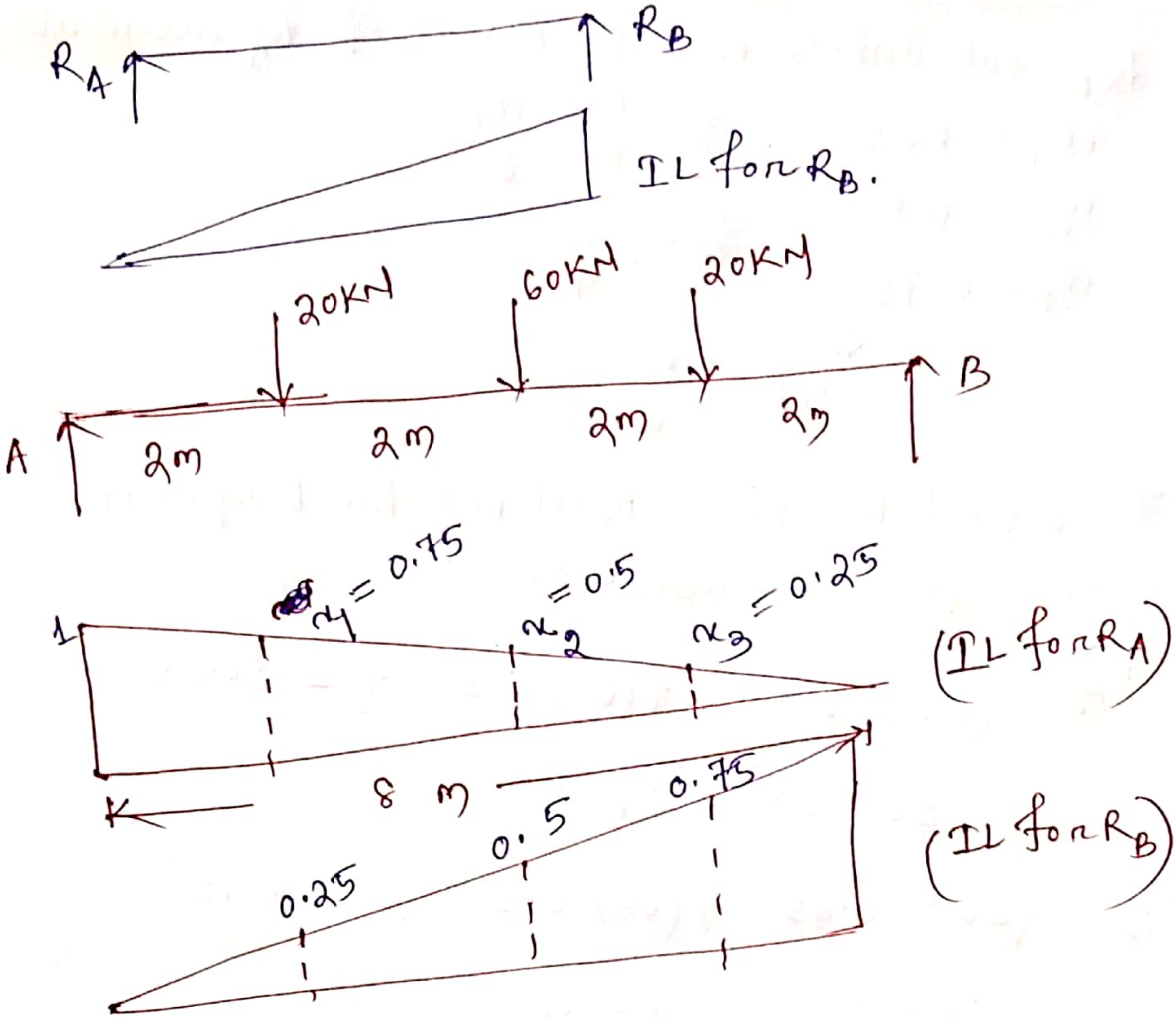


Ans
Step-1 :- Support reaction.



[IL for R_A]

(2)



for R_A

For R_A calculation Take first triangle

$$\frac{1}{8} = \frac{x'}{6}$$

$$\Rightarrow \boxed{x_1' = 0.75} \quad (x_1 = 0.75)$$

Take 2nd triangle

$$\frac{1}{8} = \frac{x_2}{4}$$

$$\Rightarrow x_2 = 0.5$$

Take 3rd triangle

$$\frac{1}{8} = \frac{x_3}{2}$$

$$\Rightarrow x_2 = 0.25.$$

For R_B calculation.

R_B calculation is just reverse of R_A calculation.

$x_1 = 0.25 \Rightarrow \frac{1}{8} = \frac{x_1}{6}$

$x_2 = 0.5 \Rightarrow \frac{1}{8} = \frac{x_2}{4}$

$x_3 = 0.75$

$\Downarrow$

$1/8 = \frac{x_3}{2}$.

* To find reaction multiply the load and corresponding co-ordinates.

$R_A = (20 \times 0.75) + (60 \times 0.5) + (20 \times 0.25)$

$= 50$ (+ve so upward)

$R_B = (20 \times 0.25) + (60 \times 0.5) + (20 \times 0.75)$

$= 50 \uparrow$ (upward +ve)

As distance are same the value for R_A and R_B are same.

formula = Load $\times$ corresponding co-ordinate of R_A and R_B

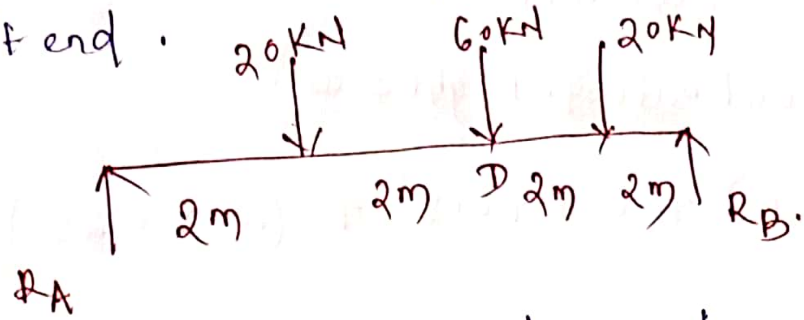
2nd part.

Shear force and Bending Moment calculation

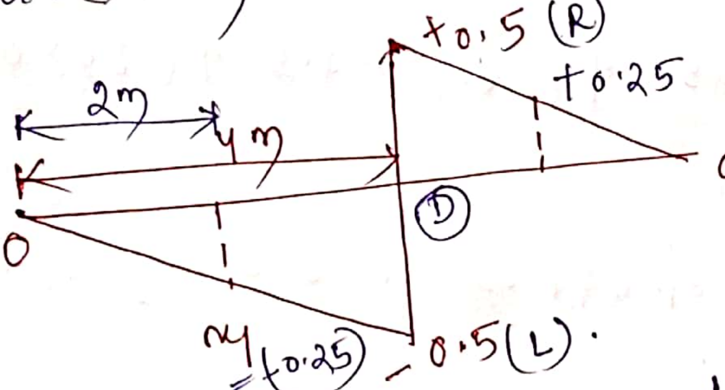
In 1st part we have studied ILD for support reaction.

2nd part Shear force calculation

In question given that Shear force and Bending moment is calculated at distance 4m from the left end.



Note: S.F diagram and B.M diagram for simply supported beam are studied before.



⇒ IL for shearforce at D

$$\frac{0.5}{4} = \frac{x_1}{2}$$

$\Rightarrow x_1 = -0.25$ } Leftpart (downward so −ve)

In this case triangle is similar so both left and right portion are similar. In different cases extra calculation needed.

$$\frac{0.5}{4} = \frac{x_1}{2} \Rightarrow \text{right part}$$

$x_1 = +0.25$ (upward)

* There will be two values (right side of D and left side of D).

(Note)
For left side shear take right side ordinates
For right side shear take left side ordinates.

* Take shearforce at left

Load × (ordinate of right side)

$= (20 \times -0.25) + (60 \times +0.5) + (20 \times 0.25)$

$= 30\,KN.$

Shearforce at right :- Load × left ordinate

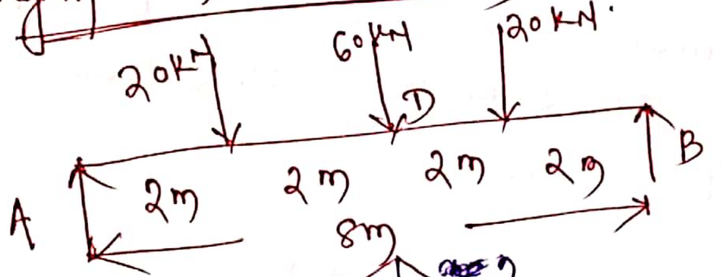
$= (20 \times -0.25) + (60 \times -0.5) + (20 \times 0.25)$

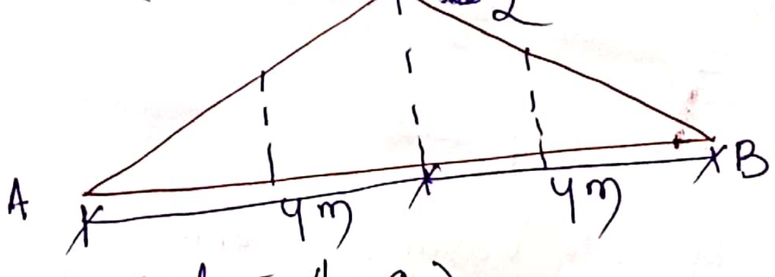
$= -30\,KN.$

Note: sign of left is -ve and sign of right part is +ve.

* D point is taken because it is mid point.

Bending moment calculation





$$\text{Bending Moment} = \left(\frac{L-a}{L}\right)a$$

$$= \left(\frac{8-4}{8}\right) \times 4 = 2.$$

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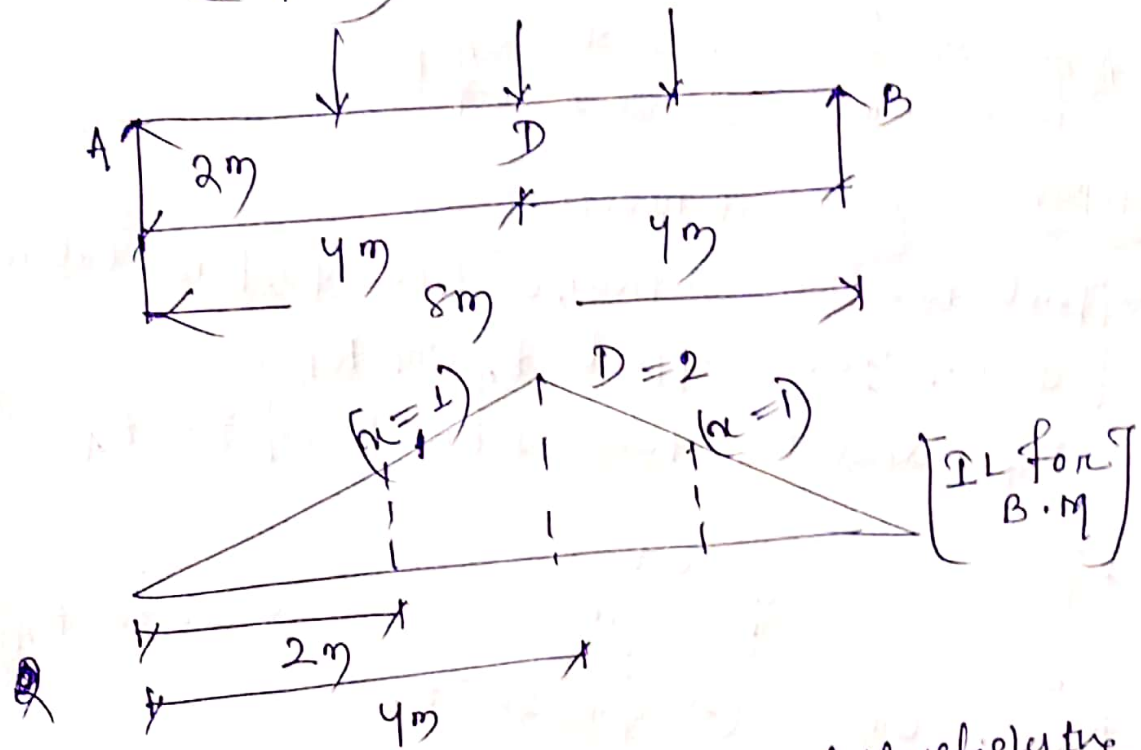
(6)

Take D (Midpoint) = 2



$$\frac{2}{4} = \frac{x}{2}$$

$\Rightarrow x = 1$

For Bending Moment Multiply the Load with corresponding co-ordinates.

Bending moment at D $= (20 \times 1) + (60 \times 2) + (20 \times 1)$

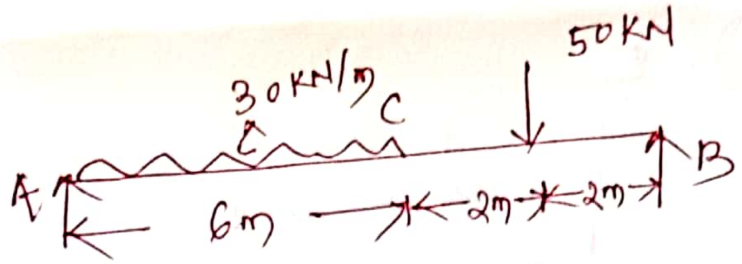
$= 160$ KNm.

* The sign of Bending Moment for simply supported beam is +ve.

Example-2

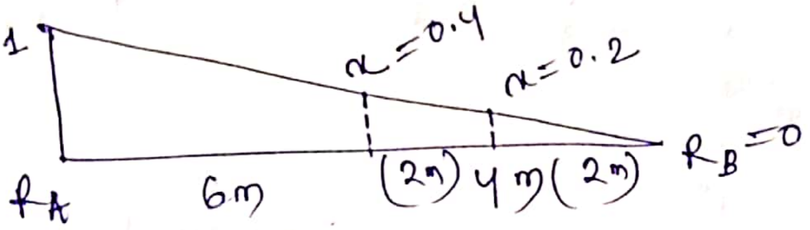
Simply supported beam with uniformly distributed load and point load.

Q. Simply supported beam as shown in figure determine support reaction, shearforce and Bending moment at point C using ILD.

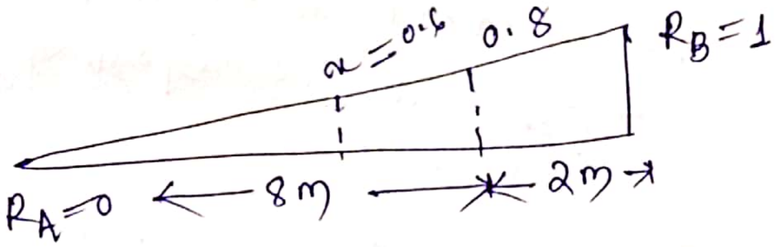


Step-1 :- Support reaction :-

Support reaction Influence line must be calculated and drawn for support R_A and R_B.
for R_A calculate take 1 KN Load for R_A



[IL for R_A]



[IL for R_B]

for R_A

$\frac{1}{10} = \frac{x}{4} \Rightarrow x = 0.4$

$\frac{1}{10} = \frac{x}{2} \Rightarrow x = 0.2$

for R_B

$\frac{1}{10} = \frac{x}{8} \Rightarrow x = 0.8$

$\frac{1}{10} = \frac{x}{6} \Rightarrow x = 0.6$

* This isn't simillar case as previous problem so value for R_A and R_B are different.

Calculation of R_A

for udL = (udL intensity × Area of ILD under Load)

(7)

$R_A = (30^{w} \times \text{trapezoidal area})\left(\frac{a+b}{2}\right) \times h$

$= \left[30 \times \left(\frac{1+0.4}{2}\right) \times 6 + 50 \times 0.2\right] \Rightarrow (h = 6)$

$= 136 \text{ KNm}.$

Calculation for R_B

$R_B =$ for udl calculation formula

$= (\text{udl intensity} \times \text{area under udl})$

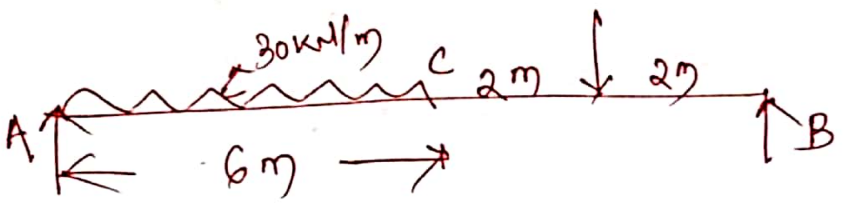
$= 30 \times \left(\frac{1}{2} \times 6 \times 0.6\right) + 50 \times 0.8$

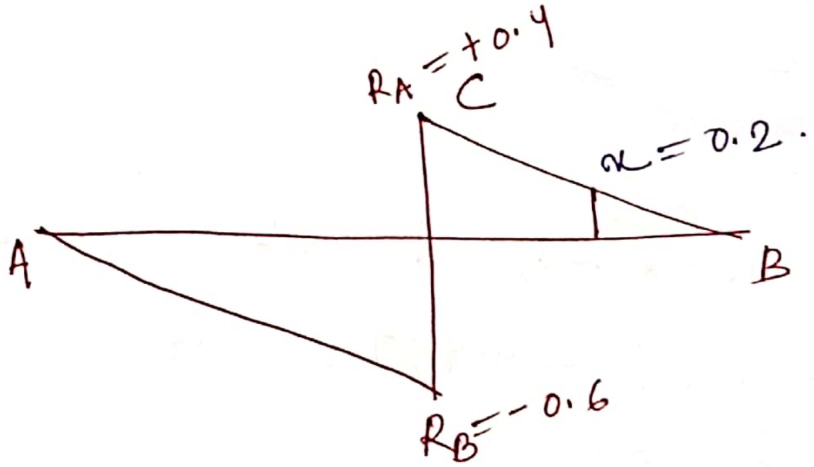
$\Downarrow$

triangle area

$= 94 \text{ KNm}.$

Step-2 :- calculation of shearforce (S.F@C)





$\Rightarrow \frac{0.4}{4} = \frac{x}{2}$

$\Rightarrow x = 0.2$

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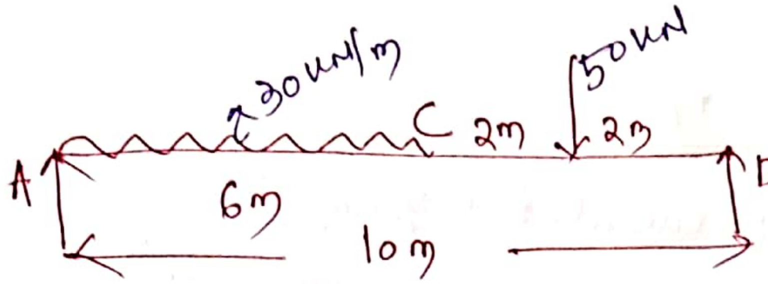
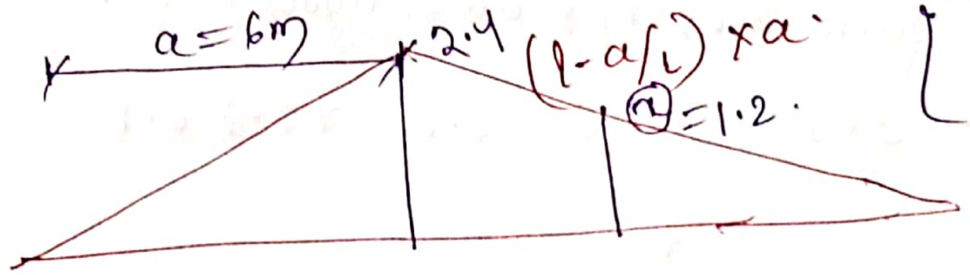
S.F at C = udl × Area of ILD

$= 30 \times \left(\frac{1}{2} \times 6 \times -0.6\right) + 50 \times 0.2 = -44$ KN.

Bending moment calculation





[always a is in leftside]

Height formula for B.M = $\left(\frac{l-a}{l}\right) \times a$

$= \left(\frac{10-6}{10}\right) \times 6$

$= 2.4$ m

$\Rightarrow \frac{2.4}{4} = \frac{x}{2}$

$\Rightarrow x = 1.2$

B.M @ C = udl intensity × Area of ILD under load

$= 30 \times \left(\frac{1}{2} \times 6 \times 2.4\right) + 50 \times 1.2$

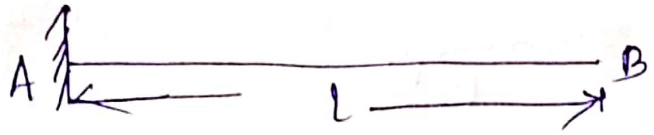
$= 276$ KNm.

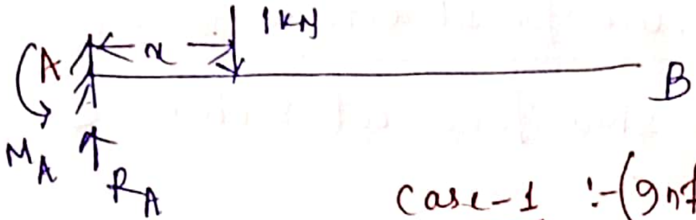
(Ans).

⑧

Influence line diagram for cantilever Beam :-

Take a cantilever beam AB at a distance L m.





(Free body diagram)

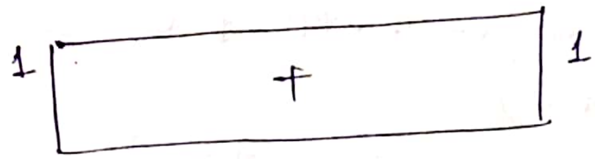
Case-1 :- (Influence Line for support Rea?)

Consider a unit load at A

as $\Sigma f_y = 0$ upward force = downward force

$$\boxed{R_A = 1}$$

When unit load is at B, $\boxed{R_A = 1}$, No change.



(IL for support reaction R_A)

Case-2

* Influence line for Moment about A

$M_A = 1 \times x$

$= -1 \times x$ (as Load clockwise)

anticlock = +ve
clockwise = -ve
In case of cantilever beam

When unit load is at A, then $x = 0$.

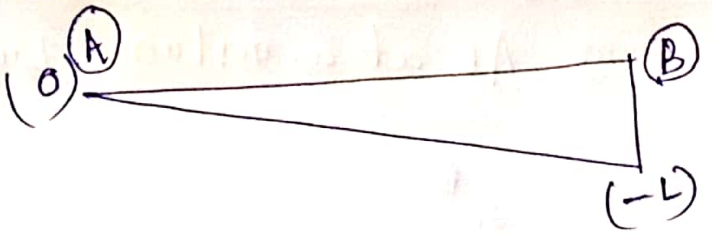
$M_A = -1 \times 0 = 0$

when unit load is at B, then $x = L$

Moment about A $M_A = -1 \times x$

$= -1 \times L = -L$.

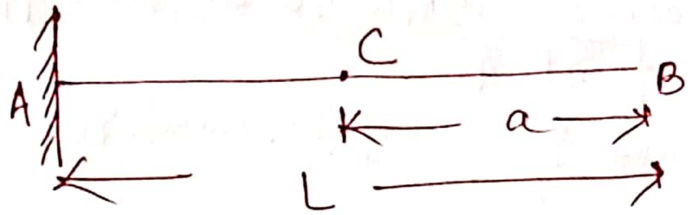
$$\boxed{\text{Moment about A} = -L}$$



(IL for Moment about A)

Case-3 :- Influence line for shear force

Influence line for shear force at section C



Consider a cantilever beam AB in which point C is there at a distance 'a' from support B.

Case-1 :- When unit load is on left of C (between A and C)

Shear force = 0 (any where between A & C)

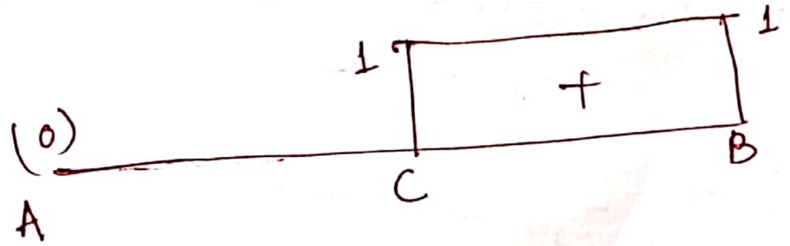
Because at A there is $R_A = 1$ and then between A and C unit load 1 kN comes downward so

$R_A - 1 \Rightarrow 1 - 1 = 0$.

Case-2 :- When unit load is on right of C (between C and B)

Shear force at C = 1 (when unit load betn C and B)

because Right downward is +ve.



(IL for shear force)

(9)

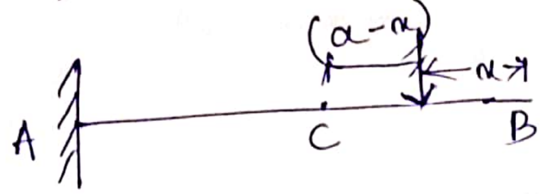
Bending Moment at point C

Case-1:- When unit load is on left (between A and C)

$(B.M)_C = 0$

Case-2:- When unit load is on right (between C and B)

$(B.M)_C =$



(unit Load betn C and B)

$(B.M)_C = -1 \times (a-x)$

When unit load is on B then $x = 0$

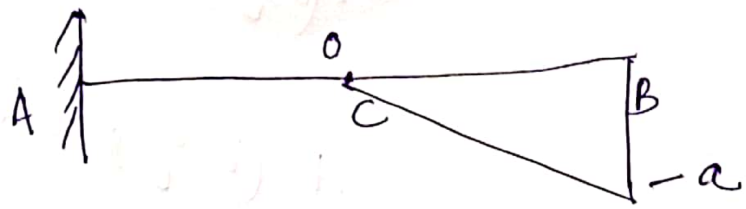
$(B.M)_C = -1 \times (a-0)$

$= -1 \times a = -a$

When unit load is on C $(x = a)$

$(B.M)_C = -1 \times (a-a)$

$= -1 \times 0 = 0$



[Influence line for B.M]

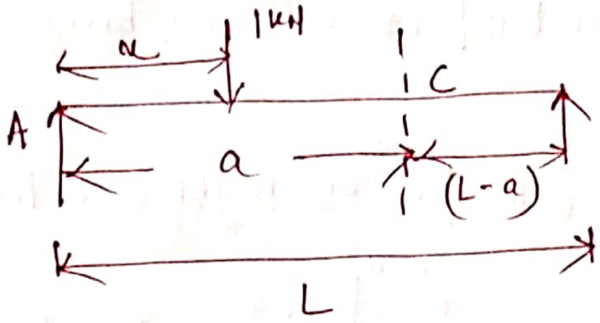
Part-1:- IL for support Reaction

Part-2:- Influence line for shear force.

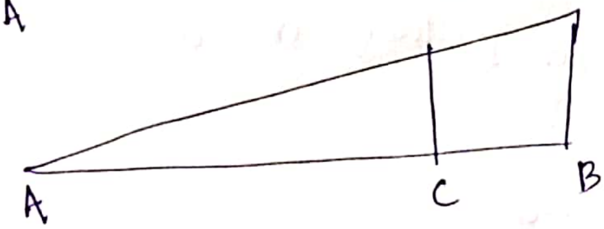
Part-3:- Influence line for Bending Moment @ C.

Note

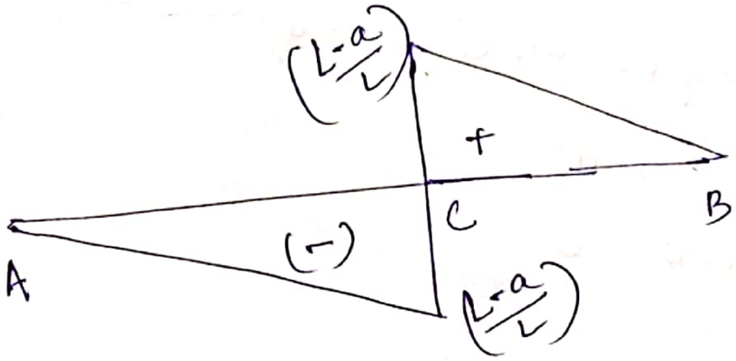
Influence line diagram for Simply supported beam



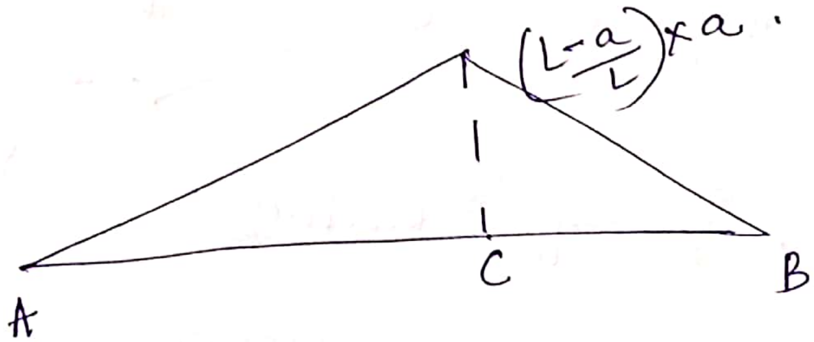
[ILD for R_A]



[ILD for R_B]



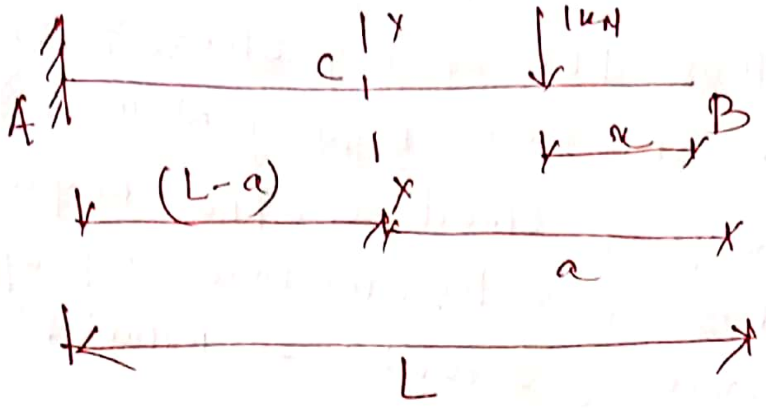
[ILD for $(S.F)_C$]

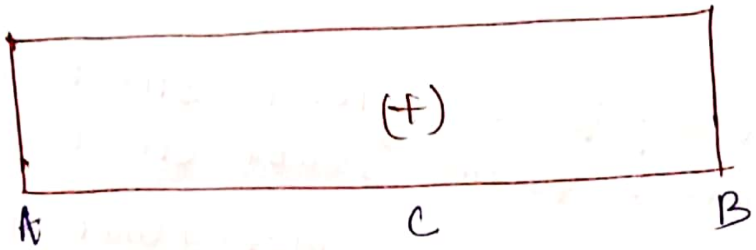


(ILD)

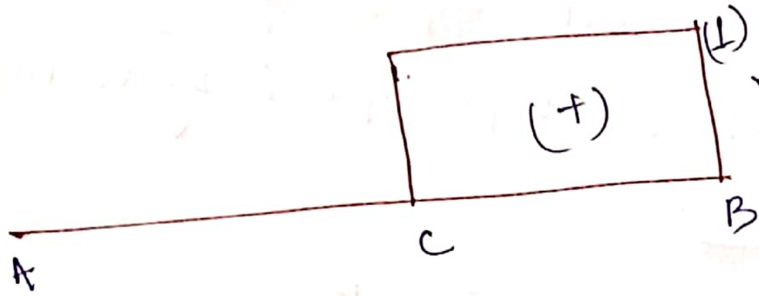
[Influence line Diagram for B.M @ C]

Influence line diagram for cantilever beam with unit load

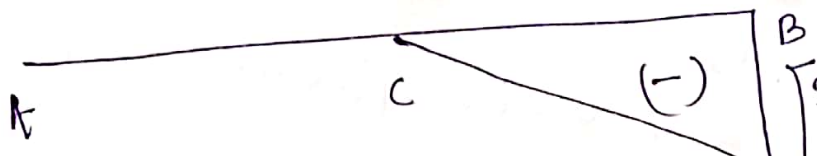




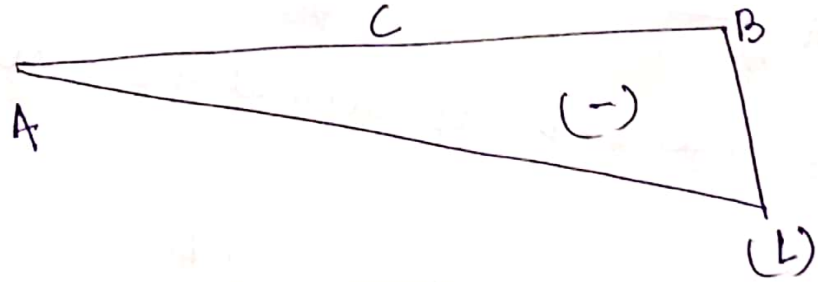
[ILD for R_A]



[ILD for $(SF)_C$]



[ILD for $(M)_C$]



[Influence line diagram for $(M)_A$]

Muller Breslau's principle:

It states that ILD of any stress function (Reaction, shear force, Bending Moment) is the deflected shape of structure after removing stress function from the structure and applying unit displacement (deflection or rotation) in the positive direction of stress function

Notes:-

(1) It is valid for all type of structure such as arch, truss, frame, cable structure etc.

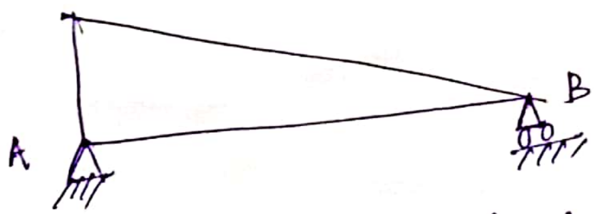
(2) It is not valid for moving unit moment.

(3) proof of this principle is by virtual work method.

Examples:

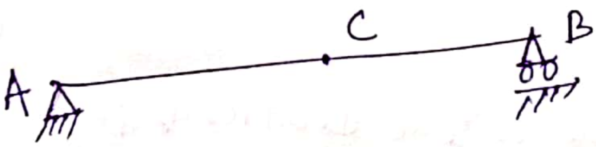
Q

In R_A there is unit displacement

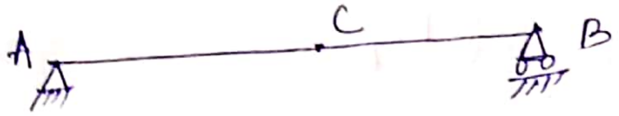


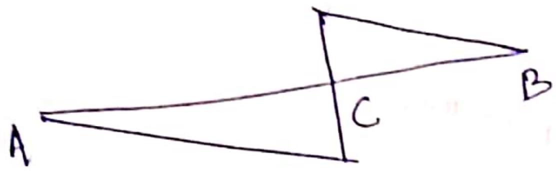
(ILD for R_A)

Shear force ILD is calculated at middle point 'c'

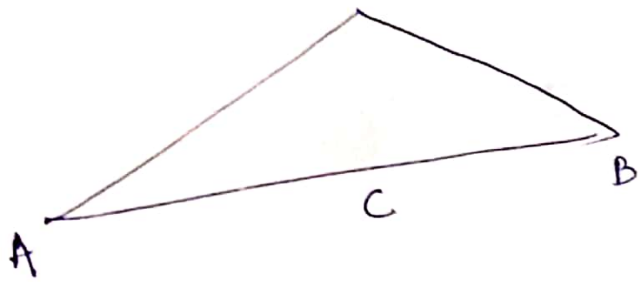


Some Examples of ILD





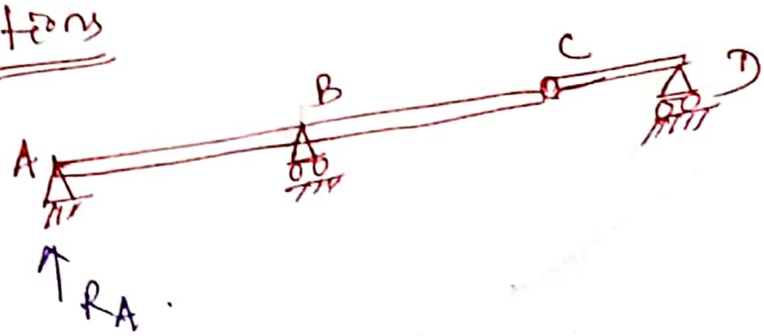
(ILD for shear force)
(Both segment will parallel)



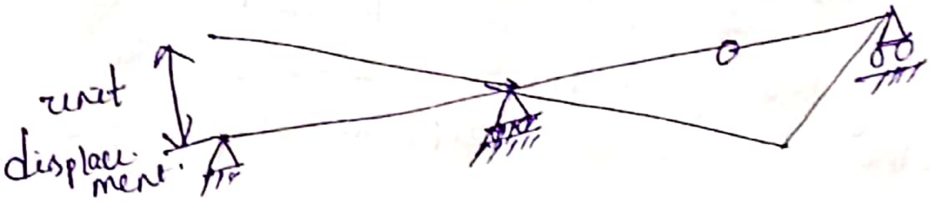
(ILD for Bending Moment)

Questions

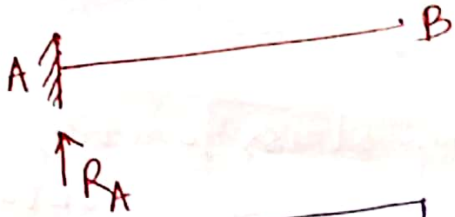
(1)



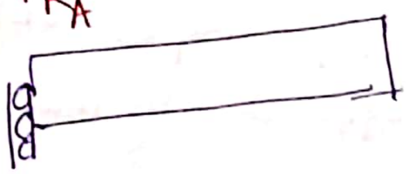
ILD of R_A = ?



(2)

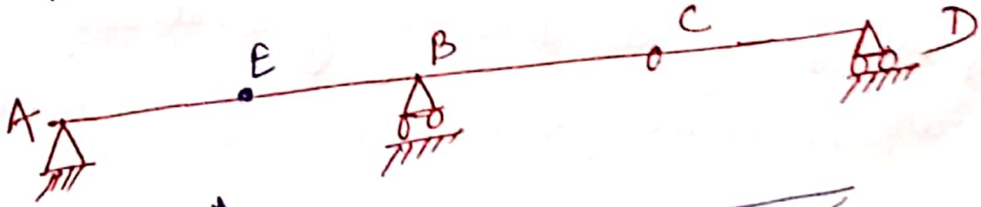


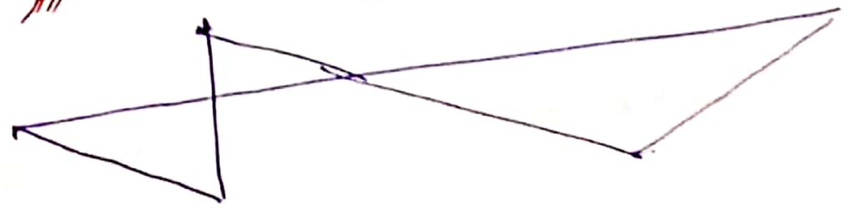
ILD of R_A?



(Same)

Q(3)



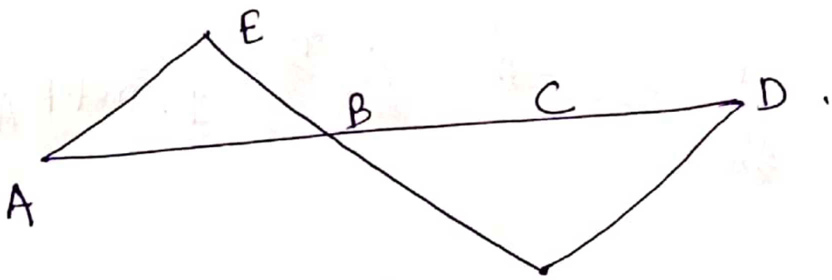


Q

$(S.F)_B = ?$

Q

$(B.M)_E = ?$



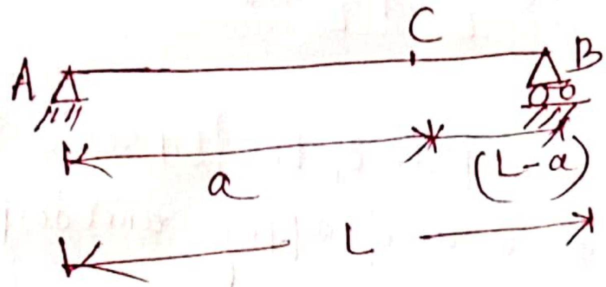
* **Description of Muller Breslau's principle**

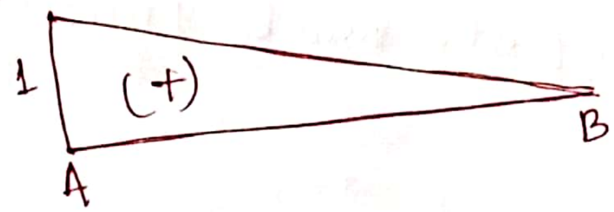
This principle gives qualitative ILD for any structure either statically determinate or indeterminate structure.

→ According to this principle

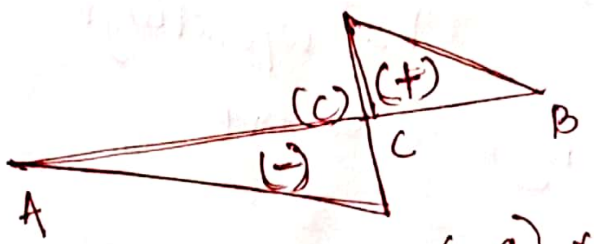
"The ILD for any stress function in a structure is represented by its deflected shape obtained by removing the restraint offered by that stress function and introducing a directly generalized unit displacement in the positive direction of that stress function".

Description:-

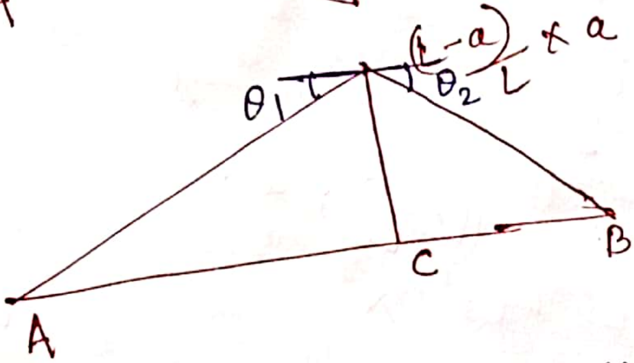




[ILD for R_A]



[ILD for $(S.F)_C$]



$(\theta_1 + \theta_2 = 1)$

[ILD for BM_C]

* To draw ILD for R_A, remove restraint offered by R_A. i,e remove R_A and apply unit displacement in the positive direction of R_A

→ The resulted deflected shape of beam will represent ILD for R_A.

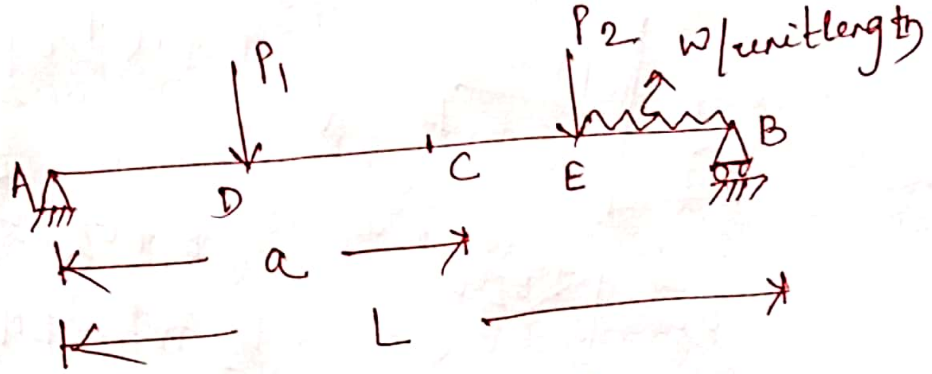
* To draw ILD for M_C, relese the B.M at C i.e Provide hinge at C and apply unit displacement in the positive direction of Moment.

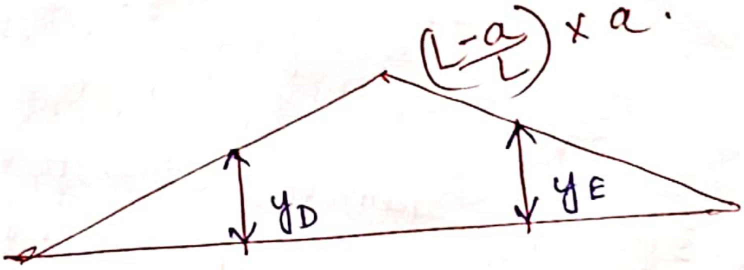
$$\theta_1 = \frac{(L-a)}{L} \qquad \theta_1 + \theta_2 = \frac{(L-a)}{L} + \frac{a}{L} = 1$$

$$\theta_2 = \frac{a}{L}$$

* To draw ILD for S.F at C relese the S.F at C i,e provide internal roller at C and apply unit displacement in the direction of shearforce.
Hence total displacement at C will be 1 unit.

Application of ILD :-





(ILD for Mc)

(i) Application of ILD :- ILD can be used to study the effect of moving Load on the structure.

(ii) ILD can be used to find position of live load which will produce maximum value of a perticular stress function.

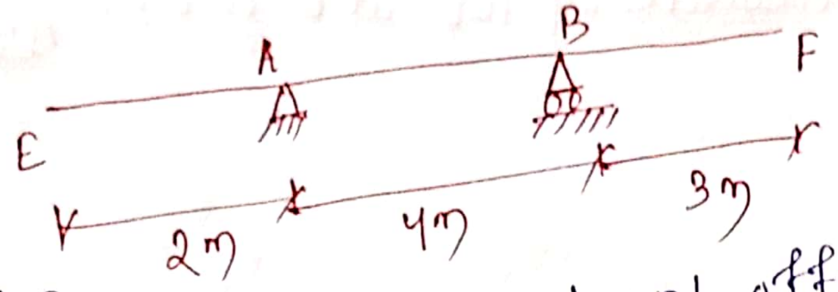
(iii) ILD can be used to calculate total value of a perticular stress function due to a multiple Load System.

Net B.M at C due to load system will be given by

$$\boxed{M_C = P_1 y_D + P_2 y_E + w \times \text{area of ILD below udl.}}$$

Question:-

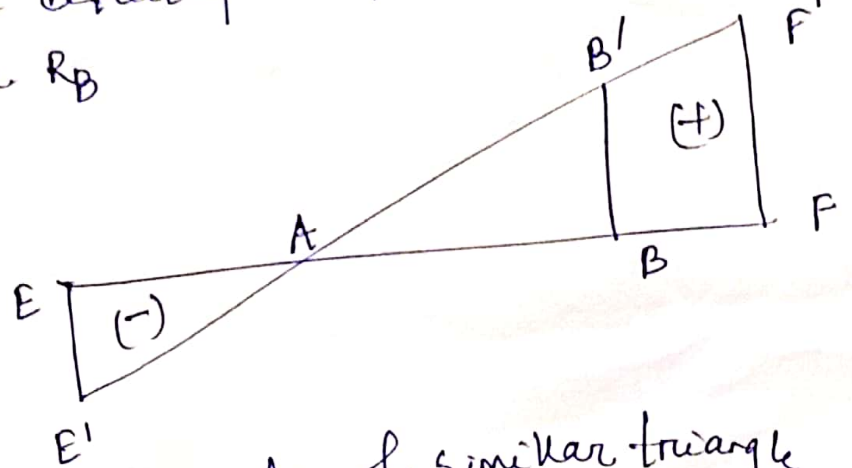
using Muller Breslau's principle draw ILD for R_B and find the ordinate of ILD at E and F



Solution:-

For ILD of R_B remove restraint offered by reaction and provide unit deflection in the direction of R_B. i,e in upward direction and observed the deflected shape of beam.

The deflected shape of beam will represent ILD for R_B



By the property of similar triangle

$$\frac{BB'}{FF'} = \frac{AB}{AF}$$

$$\Rightarrow \frac{1}{FF'} = \frac{4}{7}$$

Thus the ordinate of ILD at F is $\frac{7}{4}$

$$\frac{EE'}{BB'} = \frac{AE}{AB}$$

$$\Rightarrow EE' = BB' \times \frac{AE}{AB}$$

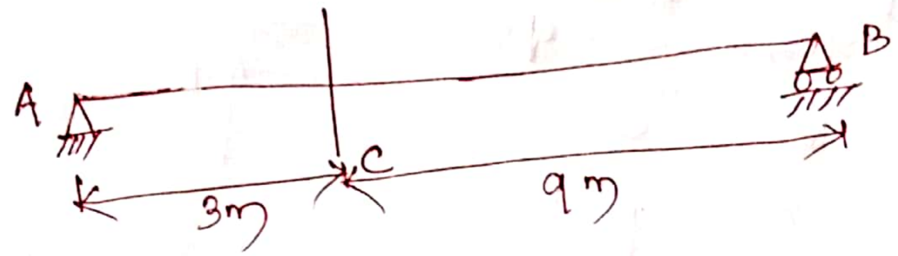
$$= 1 \times \frac{2}{4} = 0.5.$$

Thus the ordinate of ILD at E is 0.5. Ans.

(14)

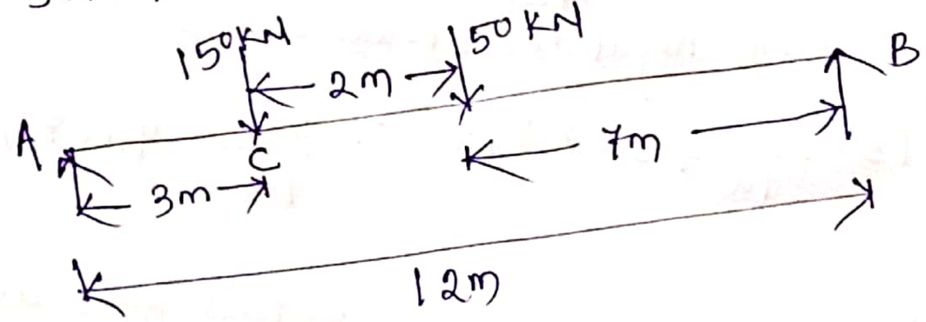
Examples :- [Wheel Load problem]

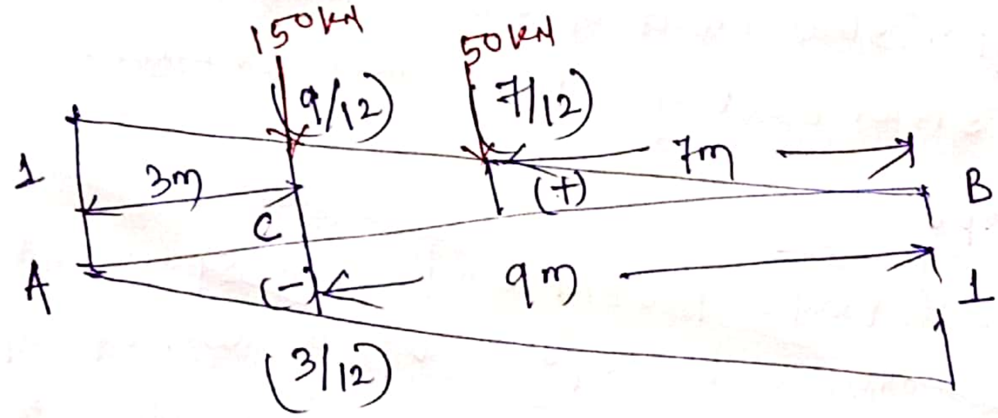
Two wheel loads 50 KN and 150 KN, spaced at 2m move on a girder of 12m long. find the values of Maximum positive and negative shear force at a section 3m from the left end. Any wheel load can lead the other.



Solution

Maximum positive shear force at section will occur when 150 KN Load is just to the right of section and 50 KN is ahead of 150 KN Load.





Hence Maximum Positive shear at C

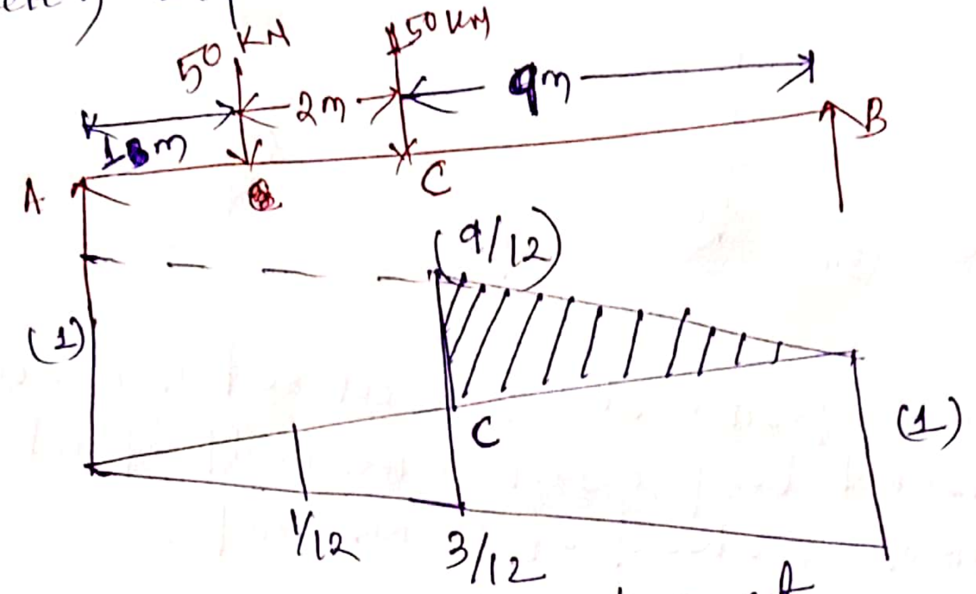
$$(S.F)_c = 150 \times \frac{9}{12} + 50 \times \frac{7}{12}$$

$$= 1700/12 = 141.67 \text{KN}.$$

Maximum positive Shear force at C = 141.67 kN.

Maximum negative shear force at C :-

Maximum negative shear force and section will occur when 150 kN load is just to the left of section and 50 kN is behind the 150 kN Load.



Hence Maximum negative shear at

$$C = 150 \times \frac{3}{12} + 50 \times 1/12 = \frac{500}{12} = 41.67\,kN.$$

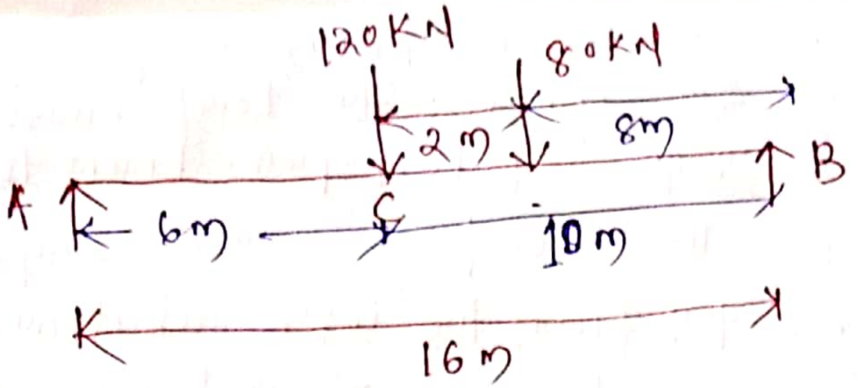
Note

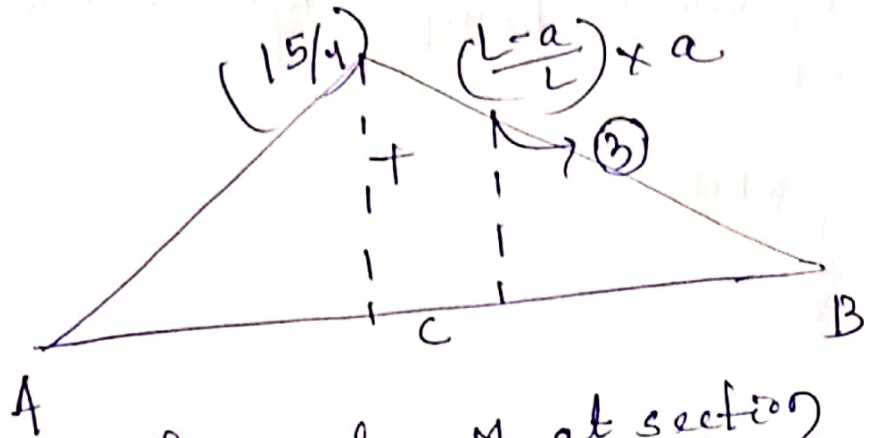
(1) positive shear created above neutral axis and -ve shear created below neutral axis.

Example-2

Two wheel loads 120 kN and 80 kN spaced at 2m apart moves on the span of girder of span 16m long. Find the maximum bending moment at section 6m from the left end. Any wheel load can lead the other.

(15)





Ordinate of ILD for M_c at section

$= \left(\frac{L-a}{L}\right) \times a$

$= \left(\frac{16-6}{16}\right) \times 6$

$= \frac{10}{16} \times 6 = \frac{15}{4}$ unit.

For 120 KN $y = \frac{15}{4}$

$\Rightarrow$ For 80 KN

$\frac{3.75}{10} = \frac{x}{8}$

$\Rightarrow x = 3m$

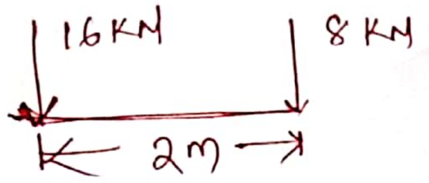
The Maximum Bending Moment at C will occur when lighter wheel Load dead and heavier Load just at the section.

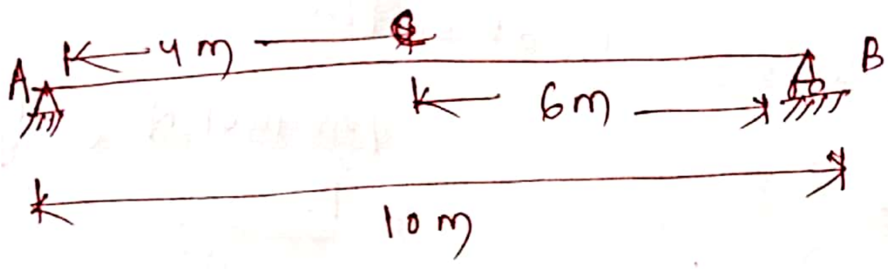
Hence Bending Moment (BM) at C

$= 120 \times \frac{15}{4} + 80 \times 3$

$= \cancel{450 + 480}$ $450 + 240 = 690$ KNm

Q Two wheel loads of 16 KN and 8 KN are placed at fixed distance of 2m apart. The Load cross a simply supported beam AB of 10 m span. Draw the influence line for the Bending moment and shear force at section 'C' 4 m from the left abutement and find the Maximum bending moment and S.F at that point.





Ans

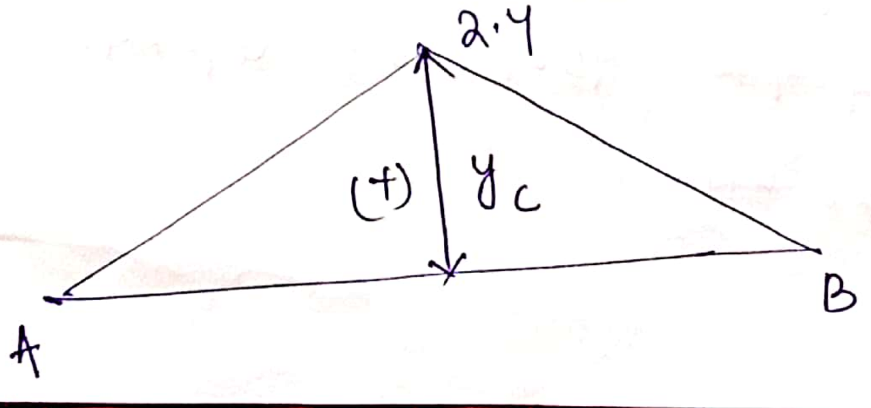
(i) ILD for BM at C

Maximum ordinate of ILD of BM at C

$$y_c = \frac{a(L-a)}{L}$$

$a = 4m$, $(L-a) = 6m$ $L = 10m$

$$y_c = \frac{4 \times (6)}{10} = 2.4$$



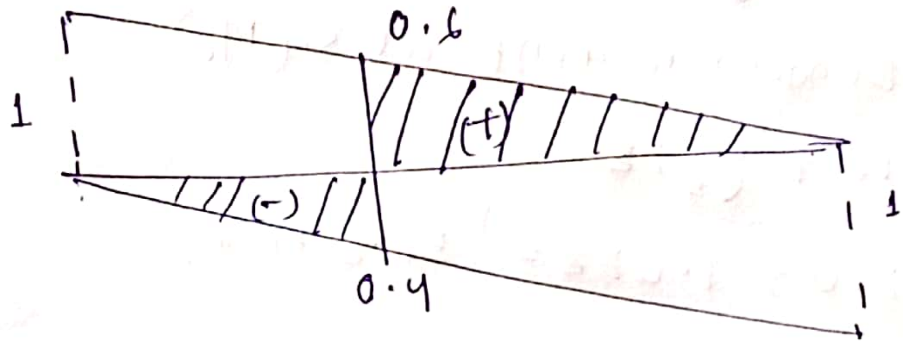
(16)

(II) ILD for S.F at C

Maximum ordinate of ILD just to the left of section

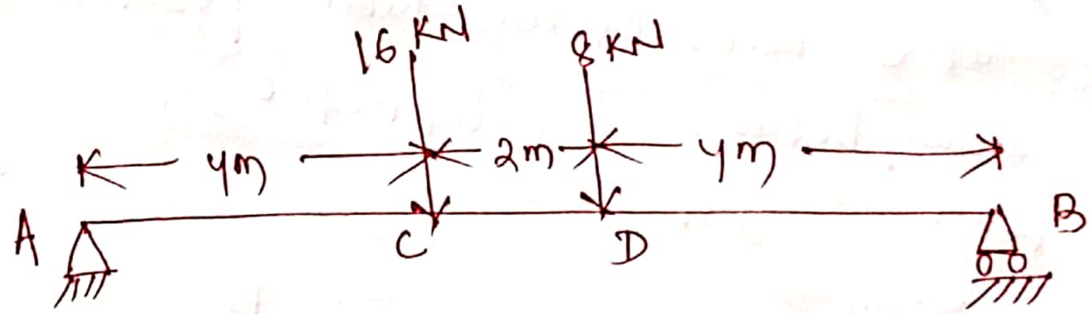
$= \frac{a}{L} = \frac{4}{10} = 0.4$

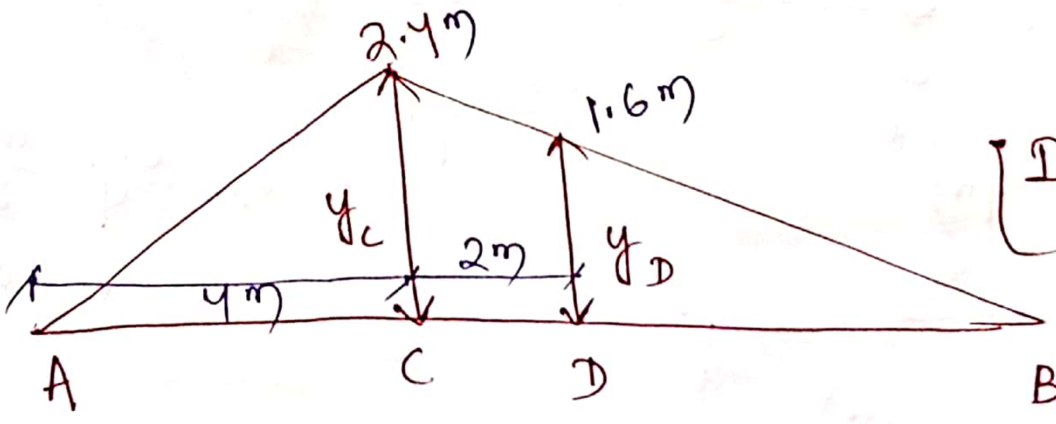
Maximum ordinate of ILD just to the right of the section $= \frac{L-a}{L} = \frac{6}{10} = 0.6.$



(ILD for shearforce)

(III) Maximum Bending moment :-





[ILD for Max.m B.M]

We know $y_D = x$

$y_C = \dfrac{a(L-a)}{L} = \dfrac{4\times 6}{10} = 2.4$

for y_D calculation

$$\frac{2.4}{6} = \frac{y_D}{4}$$

$$\Rightarrow y_D = \frac{2.4 \times 4}{6} = 1.6\,m$$

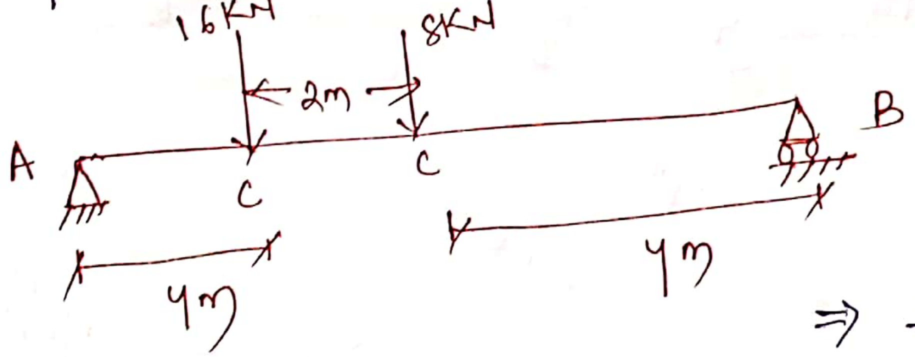
Maximum Bending moment at C (M_C)

$= y_C \times 16 + y_D \times 8$

$= 2.4 \times 16 + 1.6 \times 8 = 51.2$ KNm.

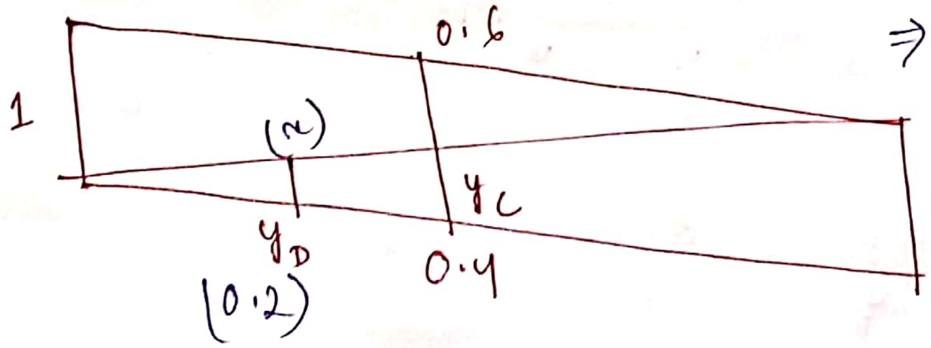
(iv) Maximum Shear force :-

(a) Maximum -ve shear force at C :- Maximum -ve s.f at C will occur when position of Load is shown below. (Left of C).



$\Rightarrow \dfrac{0.4}{4} = \dfrac{x}{2}$

$\Rightarrow x = 0.2$



(17)

Maximum negative shear at C

$= y_D \times 16 + y_C \times 8$

$= 0.2 \times 16 + 0.4 \times 8 = 6.4$ KN.

Note:- negative shear force always occur left of C and posite shear force always right of C.

* calculation of y_D and y_C for left (negative shear)

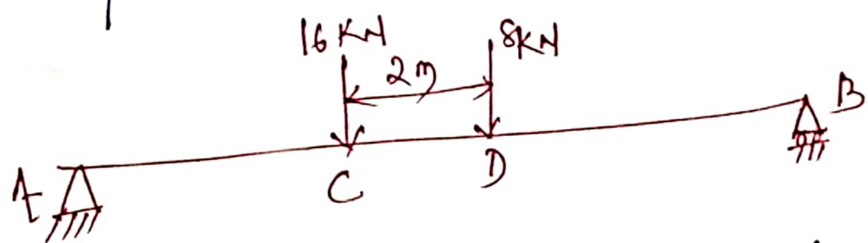
$y_D = x$ $\quad\quad\quad$ $y_C = \frac{a}{L} = \frac{4}{10} = 0.4$

$\Rightarrow \frac{0.4}{4} = \frac{x}{2}$

$\Rightarrow x = y_D = 0.2$

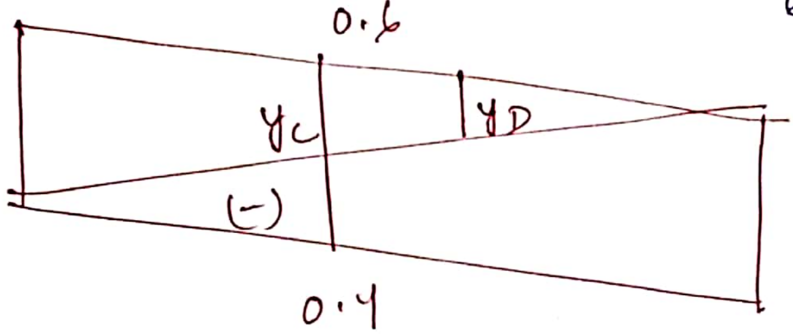
(b) Maximum positive shear force at C

Maximum positive S.F at C will occur when position of Load as shown below.



$\frac{0.6}{6} = \frac{y_D}{4}$

$y_D = 0.4$



Maximum positive shear at C

$= y_C \times 16 + y_D \times 8$

$= 0.6 \times 16 + 0.4 \times 8 = 12.8 KN.$

Max^m S.F at C = 12.8KN.

* Effect of Rolling Load:

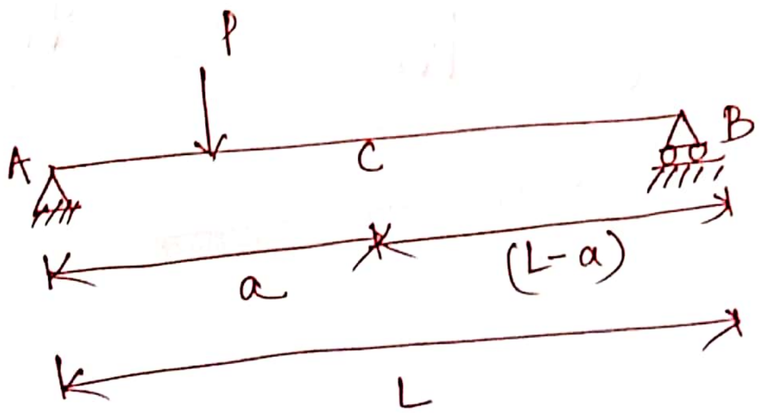
The work of structural Engineer is to find the position of Moving Loads for which shearforce and Bending Moment values are maximum

→ To find the maximum value of shearforce and bending Moment influence line diagrams are used.

Generally simply supported girders are subjected to following types of Rolling Loads.

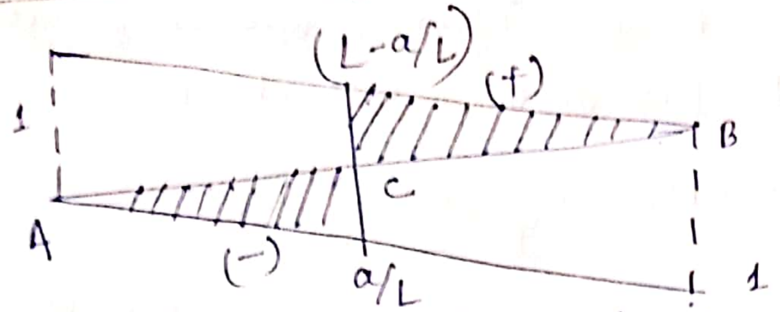
1) Single point Load
2) uniformly distributed Load Longer than the span
3) uniformly distributed Load shorter than span
4) series of point Loads.

1) Single point Load:-



(18)

For Point Load — Maximum shearforce at given section



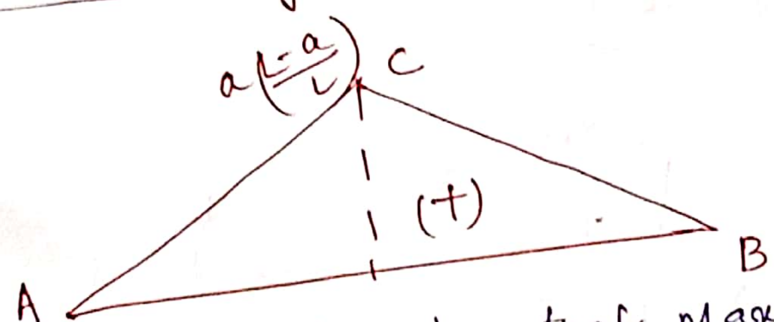
From the ILD, it is clear that Maximum positive Shearforce will occur when the Load is just to the right of section C.

$$\therefore \text{Maximum positive Shearforce} = \frac{(L-a)}{L} \times \text{per unit}$$

From ILD it is also clear that Maximum negative shear force will occur when the Load is just to the left of section C.

$$\therefore \text{Maximum negative shearforce} = \frac{a}{L} \times \text{per unit}$$

Maximum Bending Moment at given section for point Load



From the ILD it is clear that Maximum bending Moment at C will occur when the Load is at the section itself.

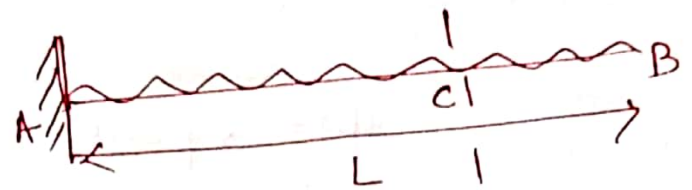
$$\therefore \text{Maximum Bending Moment} = a\left(\frac{L-a}{L}\right) \times \text{per unit}$$

Absolute Maximum Bending Moment anywhere in Beam

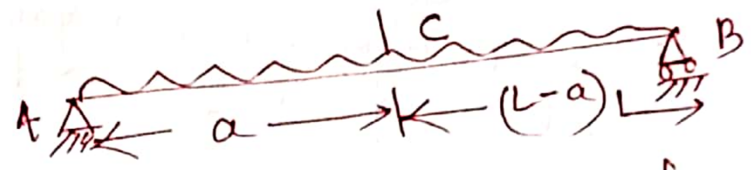
Absolute Maximum Bending Moment will occur is at $a = L/2$ and load is on the section itself

$\therefore$ $\boxed{\text{Absolute Max}^m \text{ BM} = \frac{PL}{4} \text{ unit}}$

(2) uniformly distributed Load Longer than the span



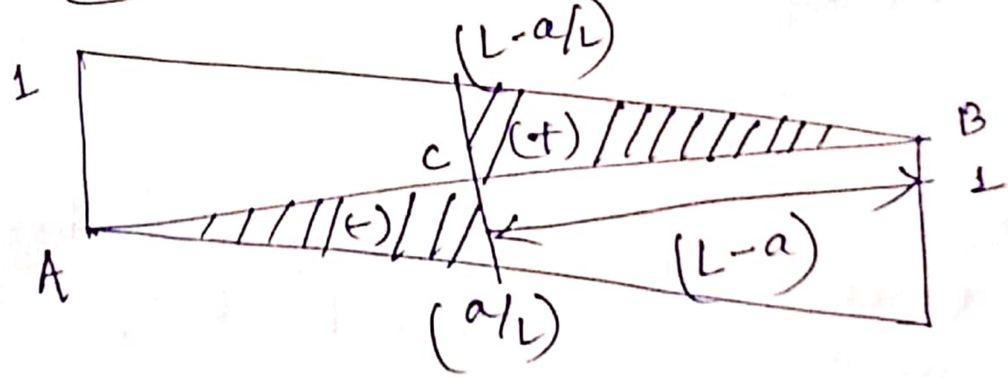
and



Maximum positive s.f at given section (C)

s.f at C = Area of ILD below length CB × Intensity of Loading.

$$= \left[\frac{1}{2}(L-a) \times \left(\frac{L-a}{L}\right)\right] \times w$$



$$= \frac{w(L-a)^2}{2L}.$$

19

Maximum Negative shear force at given section (C)

Note

+ve S.F occur bet.ⁿ right part may C & B

-ve shear force occurs between left part (A & C)

Negative S.F at C = Area of ILD below length AC × intensity of load

$$\boxed{= \left(\frac{1}{2} \times a \times \frac{a}{L}\right) \times w}$$

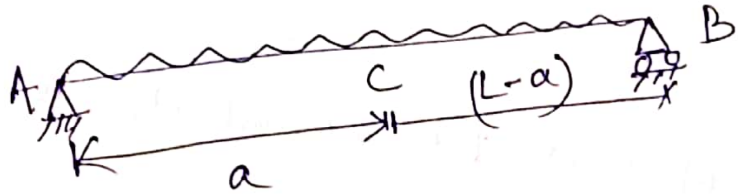
Note

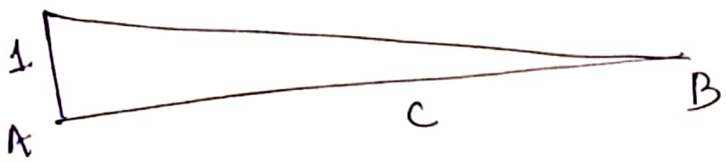
1) always +ve S.F is above to the datum/base line and must be right of section

2) -ve shear force below the base line and left of the section.

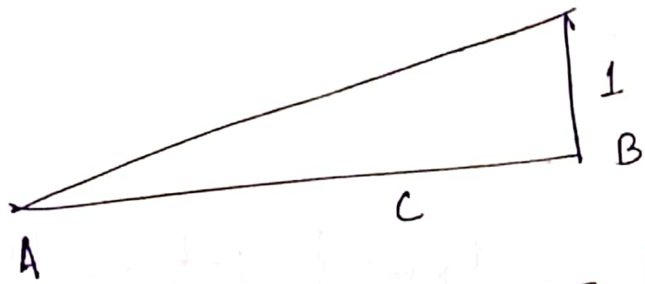
3) At support Absolute Maximum +ve shear force

$$\boxed{= w \times \left(\frac{1}{2} \times L \times 1\right) = \frac{wL}{2}}$$





(IL for +ve S.F)

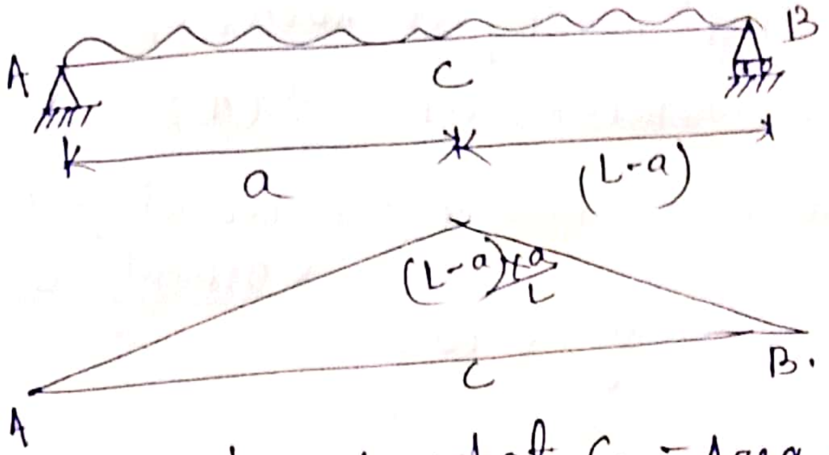


(IL for Maximum -ve shear force)

Formula for absolute -ve S.F at support

$$\boxed{= W \times \left(\frac{1}{2} \times a \times L\right) = \frac{WL}{2}}$$

Maximum bending Moment at given section:-



Maximum Bending Moment at C = Area of ILD for M_C × w

$$= \left[\frac{1}{2} \times L \times a\left(\frac{L-a}{L}\right)\right] \times w$$

⇓
area of triangle

$$= \frac{wa(L-a)}{2} \text{ unit.}$$

Absolute Maximum Bending Moment anywhere in Beam :-

$$M_{max} = \frac{wa(L-a)}{2}$$

for Max.m Bending Moment to be absolute Maximum

$$\frac{dM_{max}}{da} = 0$$

$$\frac{w}{2}[L-2a] = 0$$

$$\Rightarrow a = \frac{L}{2}$$

Hence Absolute Max.m B.M = $\frac{1}{2} \times w \times \frac{L}{2}\left[L - \frac{L}{2}\right] = \frac{wL^2}{8}$

(at mid span)

(20)

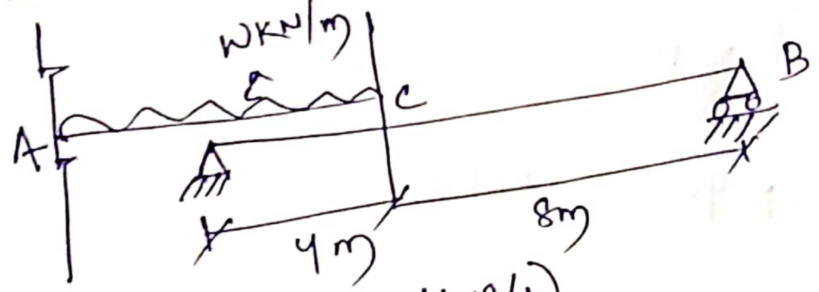
A uniformly distributed load of 90 KN/m of length Longer span moves on a simply supported girder of span 12 meters. find

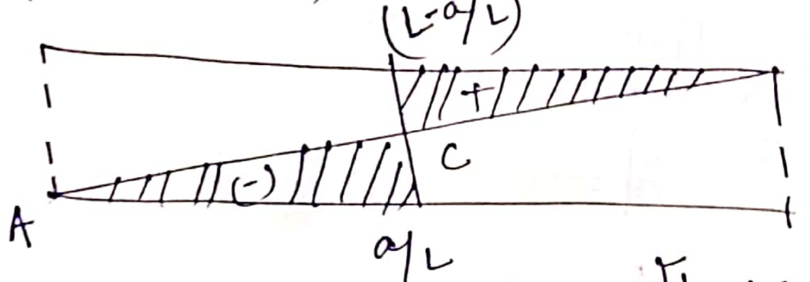
(a) Maximum +ve and -ve shear force

(b) Maximum Bending Moment.

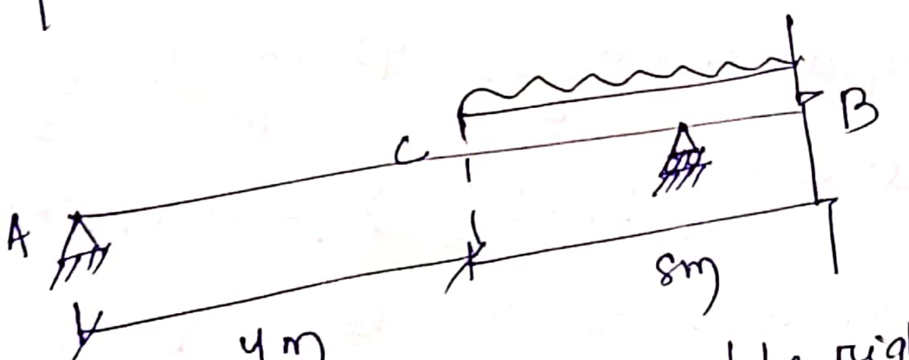
at a section 4 meters from the left end. Also find the absolute Maximum values of Maximum +ve and -ve shear force.

Solution





Maximum positive S.F at C $= \left[\frac{1}{2} \times 8 \times \left(\frac{L-a}{L}\right) \times w\right]$



for +ve shear force udL must be right of the section 'c'

$$\text{Max}^m \text{ height} = \frac{L-a}{L} = \frac{12-4}{12} = \frac{2}{3}$$

and $w = 90$ KN/m

Maximum positive S.F at C $= \left(\frac{1}{2} \times 8 \times \frac{2}{3}\right) \times 90$

$= 240 KN$.

Maximum -ve S.F at C

for this we have to shift udL Left of the section (C)

$= \left[\frac{1}{2} \times 4 \times \cancel{\left(\frac{L-a}{L}\right)}\right] \times w$ (a/L)

$\frac{L-a}{L}$ = Here a = 4m whereas $\frac{a}{L} = \frac{4}{12} = \frac{1}{3}$

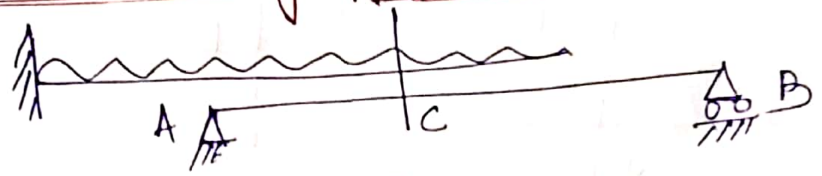
⇓ for +ve shear force

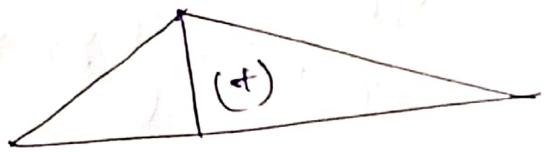
⇓ for -ve shear force

$= \left[\frac{1}{2} \times 4 \times \left(\frac{a}{L}\right)\right] \times w$

$= \left[\frac{1}{2} \times 4 \times \frac{1}{3}\right] \times 90 = 60 KN$.

Maximum Bending Moment at section





Ordinate of B.M at C $= \frac{a(L-a)}{L} = \frac{4 \times (12-4)}{12} = \frac{8}{3}$

Maximum B.M at C $= \left(\frac{1}{2} \times 12 \times \frac{8}{3}\right) \times 90 = 1440 KNm$.

Absolute Max^m +ve and -ve S.F $= \frac{wL}{2} = \frac{90 \times 12}{2} = 540 KN$

Absolute Max^m Bending Moment $= wL^2/8$

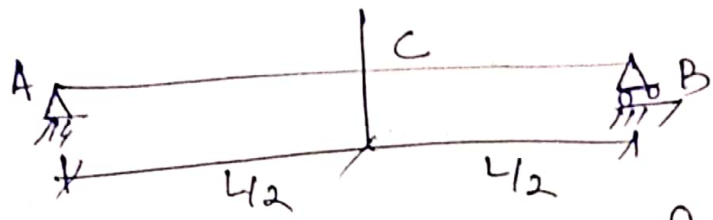
$= \frac{90 \times 12^2}{8} = 1620 KNm$.

(21)

Series of point Load :-

When a series of wheel load crosses the girder then absolute Max^m Bending Moment occurs below one of the wheel Load

→ For example



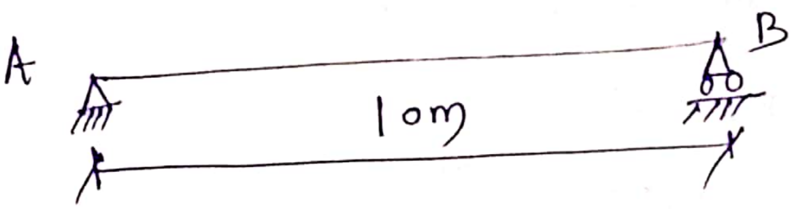
If W_3 is heavier then W_2 and W_3 is closed to C.G. then absolute Max^m B.M will occur below W_3 when Load system occupies such a position that the centre of span is Mid way bet^n C.G of Load system and Load under consideration W_3.

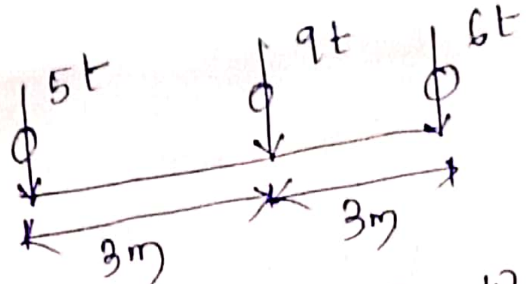
→ If W_2 is greater than W_3 but W_3 is nearest to C.G then absolute Maximum B.M will occur either below W_2 or W_3.

Example:-

The series of three wheel loads 5t, 9t and 6t spaced 3m from centre to centre cross over simply supported girder of span 10m . If Loads Moves from left to right and 6t Load leading, then find position and Max^m B.M which may occur anywhere on the girder.

Ans :-





C.G of Load system $= \bar{x} = \dfrac{W_1\bar{x}_1 + W_2\bar{x}_2 + W_3\bar{x}_3}{W_1+W_2+W_3}$

from right to left $= \dfrac{6\times 6 + 9\times 3 + 5\times 0}{6+9+5}$

$W_1 = 6t$

$W_2 = 9t$

$W_3 = 5t$

$= 3.15m$ from 5t Load.

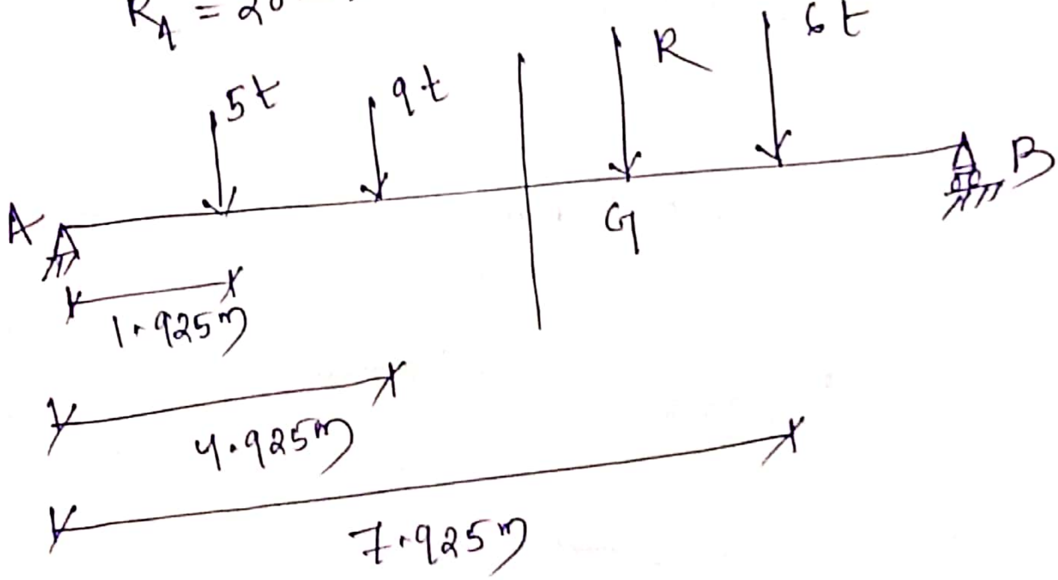
Since heavier load 9t is nearest to C.G of Load group hence Maxm B.M occur below

$\Sigma M_A = 0$

$\Rightarrow R_B \times 10 - 6\times 7.925 - 9\times 4.925 - 5\times 1.925 = 0$

$\Rightarrow R_B = 10.15t$

$R_A = 20 - 10.15 = 9.85t$



Absolute Maxm B.M = B.M under Load 9t

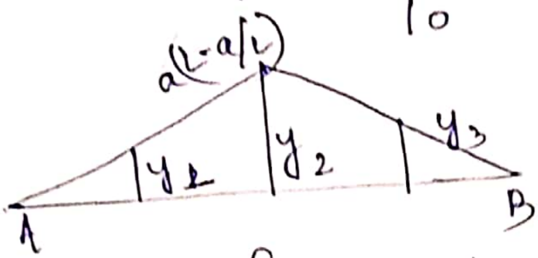
$= R_A \times 4.925 - 5\times 3$

$= 33.511$ t-M

(22)

Absolute Maxm B.M can also be determined by using ILD ordinate of ILD for M at 4.925 m from left support.

$$y_2 = \frac{4.925\,(10-4.295)}{10} = 2.499.$$



From property of similar triangle

$$\frac{y_1}{y_2} = \frac{1.925}{4.925}$$

$$y_1 = \frac{1.925}{4.925} \times 2.499 = 0.976.$$

Also $\frac{y_3}{y_2} = \frac{2.075}{5.075}$

$$\Rightarrow y_3 = \frac{2.075}{5.075} \times 2.499 = 1.029$$

Absolute Maxm Bending Moment

$= $ Bending Moment under 9t Load

$= W_1 \times y_3 + W_2 \times y_2 + W_3 \times y_1$

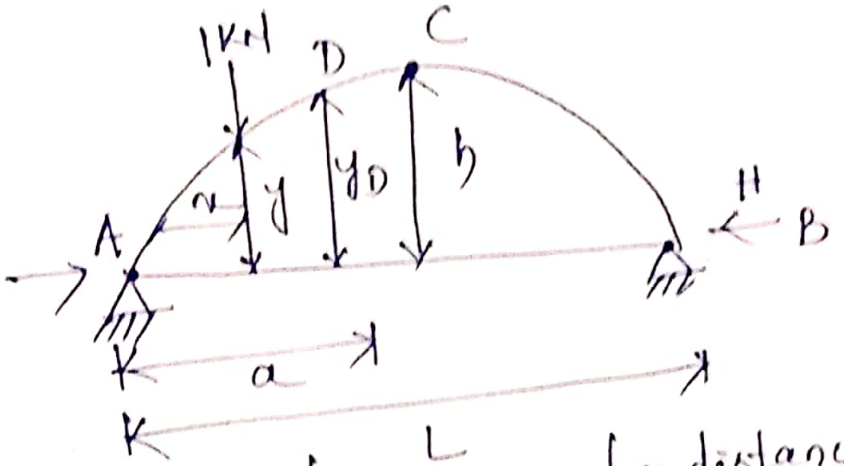
$= 6 \times 1.029 + 9 \times 2.499 + 5 \times 0.976$

$= 33.54$ tm

(22) In 5th module

Influence line Diagrams for 3 Hinged Arches

(1) ILD for Horizontal Thrust (H)



Consider a section x-x at a distance 'a' from left support. at point D, the rise of point D is y_D.

case-1

When unit load in portion AC

$$\Sigma M_B = 0$$

$$\Rightarrow R_A \times L - 1 \times (L-a) = 0$$

$$\Rightarrow R_A = \frac{(L-a)}{L}$$

Also $\Sigma M_A = 0$

$$\Rightarrow -R_B \times L + 1 \times a = 0$$

$$\Rightarrow R_B = a/L$$

Taking $\Sigma M_C = 0$ (from right)

$$\Rightarrow -R_B \times \frac{L}{2} + H \times h = 0.$$

$$\Rightarrow H \times h = \frac{a}{\cancel{L}} \times \frac{\cancel{L}}{2}$$

$$\therefore \quad H = \frac{a}{2h} \text{ Linear}$$

Scanned with CamScanner

When $x = 0,\ H = 0$
$x = \frac{L}{2},\ H = \frac{L}{4h}$ } Boundary Conditions.

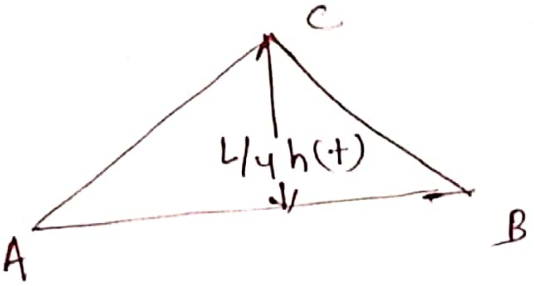
<u>Case-2</u> :- When unit Load is in portion CB

Taking $\Sigma M_C = 0$ (From left)

$\Rightarrow R_A \times \frac{L}{2} - H \times h = 0$

$\Rightarrow H \times h = \frac{L-x}{L} \times \frac{L}{2}$

$\Rightarrow H = \frac{(L-x)}{2h}$ (linear)



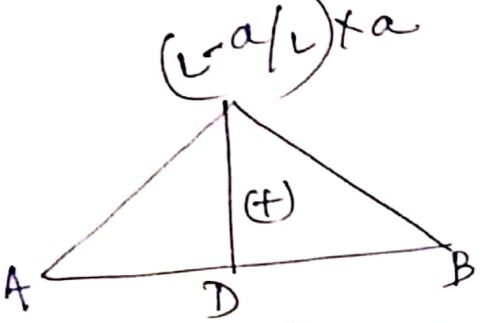
When $x = \frac{L}{2},\ H = \frac{L}{4h}$

$x = L,\ H = 0.$

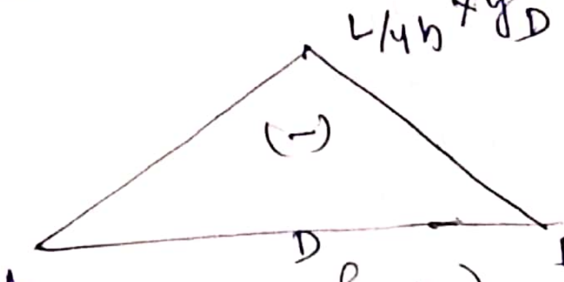
② <u>ILD for Bending Moment D (M_D)</u>

Bending moment at any section = B.M − Hy

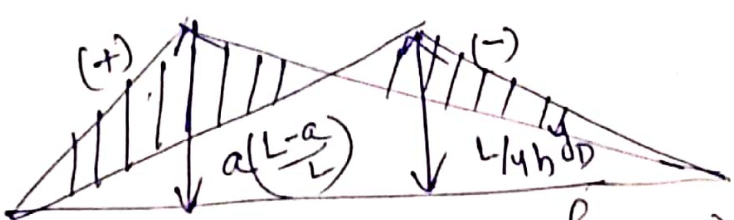
$$\boxed{M_x = B.M - Hy}$$



(ILD for M_D)



(ILD for Hy)



(ILD for M_D)

(23) [3-Hinged Arch ILD]

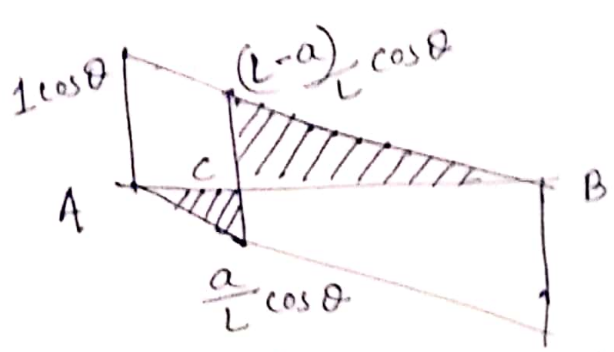
(3) ILD for Radial shear (S_D)

Radial shear at any section is given by

$$S_\alpha = V_\alpha \cos\theta - H_\alpha \sin\theta$$

$$S_\alpha = (\text{Beam shear}) \cos\theta - H_\alpha \sin\theta$$

$$\text{ILD for } S_\alpha = (\text{Beam ILD for shear}) \times (H\text{-ILD}) \times \sin\theta$$



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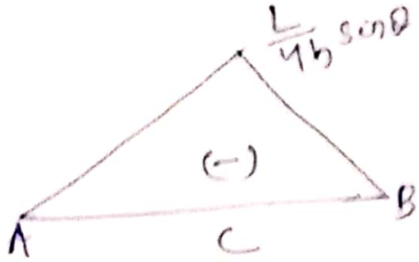
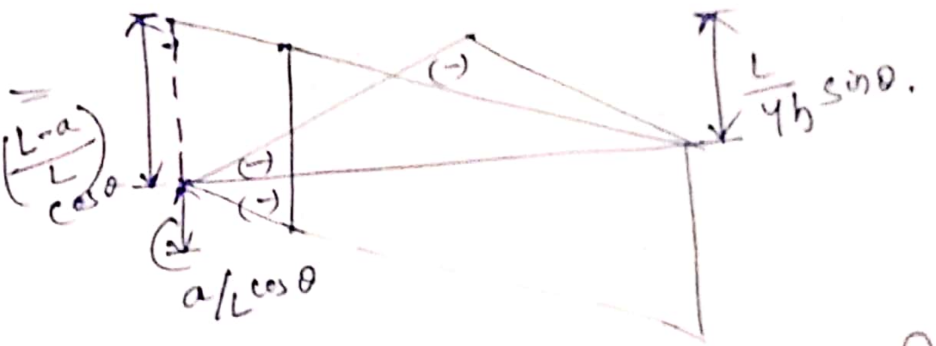


(i) ILD for (Beam shear) × cos θ

(ii) ILD for H × sin θ



(iii) ILD for S_D [(combined diagram of (i) and (ii))]

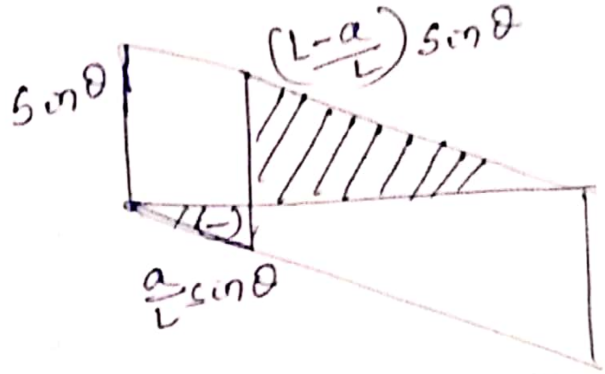
(4) ILD for Normal thrust at D (N_D)

The Normal thrust at any section is given by

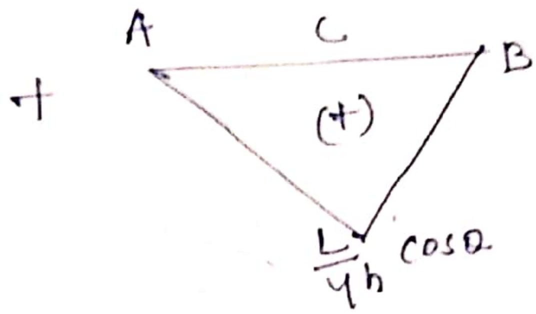
$$\boxed{N_\alpha = V_\alpha \sin\theta + H\cos\theta}$$

$N_\alpha = (\text{Beam shear})\sin\theta - H\cos\theta$

ILD for N_D = (ILD for Beam shear at D) $\times \sin\theta$ (ILD for H_D) $\times \cos\theta$.

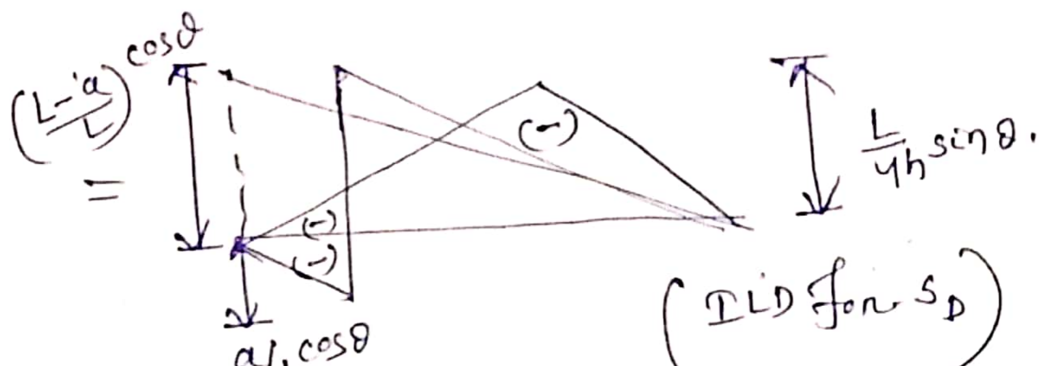


(i) (Beam shear) $\times \sin\theta$



(ii) ILD for $H_D \times \cos\theta$

final diagram of S_D



(ILD for S_D)

* ILD for Normal thrust N_D is the combined diagram of (Beam shear) $\times \sin\theta$ and ILD for $H_D \times \cos\theta$

final diagram of N_D