ELECTROMAGNETIC WAVE PROPAGATION

Wave:- In general wave means transporting energy or information

A wave is a function of both space and time.

uniform plane wave: A uniform plane wave is defined a wave whose value remains constant throughout a plane which is transverse to the dir^n of propagation of the wave.

Properties:

① At every point in space, both E and H are perpendicular to each other and to the dir^n of propagation

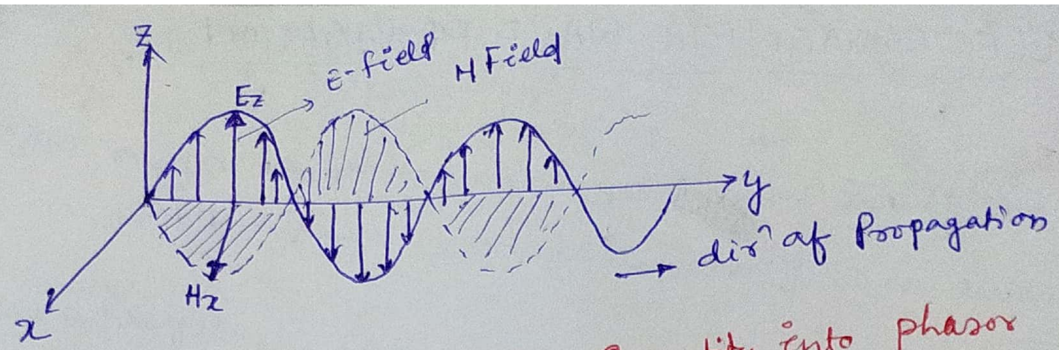
A uniform plane wave is the one in which E and H lie in the same plane at any given instant. E and H are uniform over plane surface.

② velocity of propagation of the wave in free space is $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8$ m/sec.

③ A plane wave is transverse i.e. E and H are transverse (90°) to each other and to the dir^n of propagation. Such waves are also called as transverse electromagnetic (TEM).

Ex: If the wave is travelling in any dir^n, then the plane y = cont will be perpendicular to the dir^n of propagation. Hence (in the plane y = cont), the variables are x and z.

In the plane y = cont, the electric field intensity and the uniform plane wave, the electric field ... are independent of x and z



Transformation of a time Varying Quantity into phasor.

Let E_x be the time varying quantities which is either a function of sine or Cosine. Then the phasor of E_x is written as E_{xs} and it can be obtain by dropping Re and suppressing $e^{j\omega t}$.

If $E_x = E_0 \cos(\omega t + \psi)$

$$= E_0 \, \text{Real}[\cos(\omega t + \psi) + j \sin(\omega t + \psi)]$$

$$= E_0 \, \text{Real}[e^{j(\omega t + \psi)}]$$

$$= E_0 \, \text{Real}[e^{j\omega t} \cdot e^{j\psi}]$$

$$= E_0 \, \text{Real}[$$

Real means the real parts of the following quantity taken.

then $\boxed{E_{xs} = E_0 e^{j\psi}}$

Further it can be proved that the phasor of $\frac{dE_x}{dt}$ is given by multiplying the phasor of E_x by $j\omega$.

phasor of $\boxed{\frac{dE_x}{dt} = j\omega E_{xs}}$

Transformation of phasor into a time Varying Quantity:

To transform a phasor into a time varying quantity. multiplying that phasor by $e^{j\omega t}$ and then consider the real parts of the product.

If $E_{xs} = E_0 e^{j\psi}$ then

$$E_x = \text{Real}[E_{xs} e^{j\omega t}]$$

$$= \text{Real}[E_0 e^{j\psi} e^{j\omega t}]$$

$$= \text{Real}[E_0 e^{j(\omega t + \psi)}]$$

REDMI NOTE 8
AI QUAD CAMERA

Similarly it can be proved that, the ... magnetic field intensity is

$$\boxed{\nabla^2 H_s - \gamma^2 H_s = 0} \quad \text{——— (2)}$$

$\nabla \times H_s = (\sigma + j\omega\epsilon) E_s$
$\nabla \times \nabla \times H_s = (\sigma + j\omega\epsilon) \nabla \times E_s$
$\nabla(\nabla \cdot H_s) - \nabla^2 H_s = (\sigma + j\omega\epsilon) \times (-j\omega\mu H_s)$
$-\nabla^2 H_s = -(\sigma + j\omega\epsilon) j\omega\mu H_s$
$\nabla^2 H_s = \gamma^2 H_s.$
$\nabla \cdot H_s = \nabla \cdot \frac{B_s}{\mu} = \frac{1}{\mu}(\nabla \cdot B_s) = 0$

The eqn (1) and (2) are known as homogeneous vector Helmholtz's eqn or simply vector wave equation.

Since γ in eqn (1) to (2) is a complex quantity.

$$\gamma = \alpha + j\beta \quad \gamma^2 = \alpha^2 + \beta^2$$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

When we solve its, we get

$$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{\left\{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2\right\}} - 1\right]}$$

$$\beta = \omega \sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{\left\{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2\right\}} + 1\right]}$$

Solutation of wave Equations:

The wave eqn for electric field intensity is

$$\nabla^2 E_s = \gamma^2 E_s.$$

It is vector differential eqn. Let us consider x component of electric field intensity. Then

$$\nabla^2 E_{xs} = \gamma^2 E_{xs}.$$

$$\frac{\partial^2 E_{xs}}{\partial x^2} + \frac{\partial^2 E_{xs}}{\partial y^2} + \frac{\partial^2 E_{xs}}{\partial z^2} = \gamma^2 E_{xs}.$$

$$\boxed{\nabla^2 E_s - \gamma^2 E_s = 0} \quad \text{——} \quad (1)$$

This is the wave equation for electric field intensity.

Similarly it can be proved that, the wave equation for Magnetic field intensity is

$\nabla \times H_s = (\sigma + j\omega\epsilon)E_s$

$\nabla \times \nabla \times H_s = (\sigma + j\omega\epsilon)\nabla \times E_s$

$\nabla(\nabla \cdot H_s) - \nabla^2 H_s = (\sigma + j\omega\epsilon)(-j\omega\mu H_s)$

$-\nabla^2 H_s = -(\sigma + j\omega\epsilon)j\omega\mu H_s$

$\nabla^2 H_s = \gamma^2 H_s$

$\nabla \cdot H_s = \nabla \cdot \frac{B_s}{\mu} = \frac{1}{\mu}(\nabla \cdot B_s) = 0$

$$\boxed{\nabla^2 H_s - \gamma^2 H_s = 0} \quad \text{——} \quad (2)$$

The eqn (1) and (2) are known as homogenous vector Helmholtz's eqn or simply vector wave equation.

Since γ in eqn (1) to (2) is a complex quantity.

$$\gamma = \alpha + j\beta \qquad \gamma^2 = \alpha^2 + \beta^2$$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

when we solve it, we get

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{\left\{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2\right\}} - 1\right]}$$

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{\left\{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2\right\}} + 1\right]}$$

Sol- ... wave Equations:

... electric field intensity is

wave eqn for

...etic Scalar lossy di...

Assuming that E_{xs} does not vary with both x and y but varies only with z-variable. We can reduce the equation as

$$\frac{\partial^2 E_{xs}}{\partial z^2} = \gamma^2 E_{xs}$$

* wave is travelling in a_z dirⁿ, E_{xs} will not change with x and y co-ordinate

The solⁿ of the above differential equⁿ may be written as

$$E_{xs} = A e^{-\gamma z}$$

$\left[E_{xs} = \underbrace{A e^{-\gamma z}}_{\text{along } a_z \text{ dir}} + \underbrace{B e^{\gamma z}}_{\text{along } -a_z \text{ dir}} \right]$

But we can consider a_z dirⁿ only. So $B = 0$.

Multiplying both sides by $e^{j\omega t}$ and consider the real part. Then we get

$$E_x = \text{Real}\left[A e^{-\gamma z} e^{j\omega t}\right]$$

$$= \text{Real}\left[A e^{-(\alpha + j\beta)z} e^{j\omega t}\right]$$

$$= \text{Real}\left[A e^{-\alpha z} e^{j(\omega t - \beta z)}\right]$$

$$= A e^{-\alpha z} \cos(\omega t - \beta z)$$

Let E_{xo} be the value of E_x at $t = 0$ and $z = 0$. Then

$$E_{xo} = A.$$

$$\boxed{E_x = E_{xo} e^{-\alpha z} \cos(\omega t - \beta z)}$$

$$\boxed{E(z,t) = E_o e^{-\alpha z} \cos(\omega t - \beta z)\, a_x}$$

So

$$\boxed{H(z,t) = H_o e^{-\alpha z} \cos(\omega t - \beta z)\, a_y}$$

Frequency, wavelength and Intrinsic impedance.

Frequency: The number of oscillations carried out by a ... frequency of the wave.

Maxwell's Equation in phasor form:

The four Maxwell's equation in point form or differential form are

$$\nabla \times E = -\frac{\partial B}{\partial t} = -\mu \frac{\partial H}{\partial t}$$

$$\nabla \times H = J + \frac{\partial D}{\partial t} = \sigma E + \epsilon \frac{\partial E}{\partial t}$$

$$\nabla \cdot D = \rho_v$$

$$\nabla \cdot B = 0$$

These eq^ns in phasor form may be written as

$$\nabla \times \vec{E_s} = -j\omega\mu \vec{H_s}$$

$$\nabla \times \vec{H_s} = (\sigma + j\omega\epsilon)\vec{E_s}$$

$$\nabla \cdot D_s = \rho_v$$

$$\nabla \cdot B_s = 0$$

Wave Equation for Electric field intensity.

using the vector identity.

$$\nabla \times \nabla \times A = \nabla(\nabla \cdot A) - \nabla^2 A$$

$$\nabla \times \nabla \times E_s = \nabla(\nabla \cdot E_s) - \nabla^2 E_s. \quad \text{——— ①}$$

$$\text{LHS} = \nabla \times \nabla \times E_s = \nabla \times (-j\omega\mu \vec{H_s}) = -j\omega\mu \nabla \times \vec{H_s}$$

$$= -j\omega\mu(\sigma + j\omega\epsilon) E_s.$$

$$\text{RHS.} \quad \nabla(\nabla \cdot E_s) - \nabla^2 E_s$$

$$= \nabla\left(\nabla \cdot \frac{D_s}{\epsilon}\right) - \nabla^2 E_s$$

$$= \frac{1}{\epsilon}\nabla(\rho_v) - \nabla^2 E_s$$

$$= -\nabla^2 E_s.$$

Use this in eqn ①, $\nabla^2 E_s = j\omega\mu(\sigma + j\omega\epsilon) E_s$.

$$\nabla^2 E_s = \gamma^2 E_s \text{ where } \gamma^2 = j\omega\mu(\sigma + j\omega\epsilon)$$

...e Motion in lossy Dielectric :

A lossy dielectric is a medium in which an EM wave los[es] Power as it Propagates due to Poor Conduction.

A lossy dielectric is characterized by σ (very sm[all])

$\mu = \mu_0 \mu_r, \quad \epsilon = \epsilon_0 \epsilon_r.$

The maxwell's equation for time varying fields in phasor is

$$\nabla \times H = J + \frac{\partial D}{\partial t} = J_c + J_d = \sigma E + \epsilon \frac{\partial E}{\partial t}$$

When displacement current ($J_d = \omega\epsilon \gg \sigma$) is very much greater than the conduction current, the medium behave like a dielectric. If $\sigma = 0$, the medium is a perfect loss less dielectric. If $\sigma \neq 0$, the medium is a lossy or imperfect dielectric.

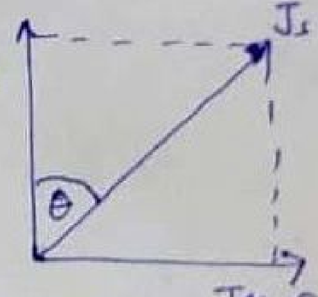
When displacement current density is much greater than conduction current density then the medium is good conductor

* If σ is not zero
  1. $\frac{\sigma}{\omega\epsilon} \ll 1$, good dielectric
  2. $\frac{\sigma}{\omega\epsilon} \gg 1$, good conductor
  3. $\frac{\sigma}{\omega\epsilon} = 1$ quasi conductor

we have

$$J_c = \nabla \times H_s = J_{cs} + J_{ds} = \sigma E_s + j\omega\epsilon E_s$$

The time phase relationship J_{ds}, J_{cs}, J_s and E_s as shown



The angle θ may be identified as the angle by which the displacement current density leads the total current density, we have,

$$\frac{|J_{cs}|}{|J_{ds}|} = \frac{|\sigma E_s|}{|j\omega\epsilon E_s|} = \frac{\sigma}{\omega\epsilon} = \tan\theta$$

$$\boxed{\tan\theta = \frac{\sigma}{\omega\epsilon}}$$ where $\tan\theta$ is known as loss tangent.

The common Power factor angle is the angle by which J_c leads E_s is equal to $90°$

Wave Motion in Good Conductors

A good conductor is characterised by $\sigma(\text{high}) \simeq \infty$.

$\mu = \mu_0$ (non magnetic conductor)

$\epsilon = \epsilon_0$.

Since the propagation const γ is given by

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

$$= \sqrt{j\omega\mu \times j\omega\epsilon\left(1 + \frac{\sigma}{j\omega\epsilon}\right)} = j\omega\sqrt{\mu\epsilon}\sqrt{1 - j\frac{\sigma}{\omega\epsilon}}$$

As $\frac{\sigma}{\omega\epsilon} \ggg 1$, $\quad \gamma = j\omega\sqrt{\mu\epsilon}\sqrt{-j\frac{\sigma}{\omega\epsilon}}$

$$\gamma = j\omega\sqrt{\frac{-j\sigma\mu\epsilon}{\omega\epsilon}} = j\omega\sqrt{\frac{-j\sigma\mu}{\omega}}$$

$$\gamma = j\sqrt{-j\sigma\mu\omega} \qquad \sqrt{-j} = \sqrt{1\angle -90} = 1\angle -45 = \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$$

$$\gamma = j\left(\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right)\sqrt{\omega\mu\sigma} \qquad \omega = 2\pi f$$

$$\gamma = \left(\frac{j}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\sqrt{2\pi f\mu\sigma} \qquad \gamma = (1+j)\sqrt{\pi f\mu\sigma}$$

$$\gamma = \sqrt{\pi f\mu\sigma} + j\sqrt{\pi f\mu\sigma} = \alpha + j\beta$$

$$\boxed{\alpha = \beta = \sqrt{\pi f\mu\sigma}} \qquad v = \frac{\omega}{\beta} = \frac{2\pi f}{\beta} = \frac{2\pi f}{\sqrt{\pi f\mu\sigma}}$$

$$= \frac{2\sqrt{\pi f}}{\sqrt{\mu\sigma}} = \frac{\sqrt{2\omega}}{\sqrt{\mu\sigma}}$$

$$\boxed{v = \frac{\sqrt{2\omega}}{\sqrt{\mu\sigma}}} \qquad \lambda = \frac{2\pi}{\beta}$$

$$\boxed{\lambda = \frac{2\pi}{\sqrt{\pi f\mu\sigma}}}$$

$$\boxed{\eta = \sqrt{\frac{\mu\omega}{\epsilon}}\angle 45^\circ} \qquad \eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{j\omega\mu}{j\omega\epsilon\left[1 + \frac{\sigma}{j\omega\epsilon}\right]}}$$

$$= \sqrt{\frac{\mu}{\epsilon\left[1 - j\frac{\sigma}{\omega\epsilon}\right]}}$$

$$= \sqrt{\frac{\mu}{\epsilon \times -j\frac{\sigma}{\omega\epsilon}}} = \sqrt{\frac{\mu\omega}{\sigma}j}$$

$$\boxed{\eta = \sqrt{\frac{\mu\omega}{\sigma}}\angle 45^\circ} \qquad \eta = \sqrt{\frac{\mu\omega}{\sigma}}\angle 45^\circ$$

Scanned with CamScanner

Wave Motion in Free Space:

The free space is characterized by

$$\boxed{\sigma = 0, \ \mu = \mu_0, \ \epsilon = \epsilon_0}$$

$$\alpha = 0, \quad \beta = \omega\sqrt{\mu_0\epsilon_0}$$

$$\lambda = \frac{2\pi}{\beta}$$

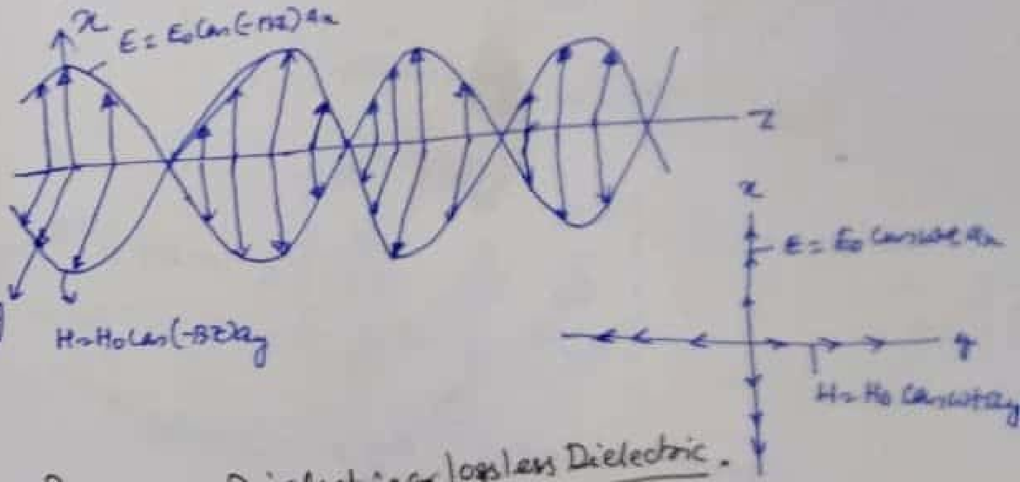
$$\eta = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377 = 120\pi$$

$$v = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu_0\epsilon_0}} = \frac{1}{\sqrt{\mu_0\epsilon_0}} = 3\times10^8 \text{ m/sec}$$

Hence electromagnetic wave in free space travels at speed of light (3×10^8 m/sec).

$$E = E_0 \cos(\omega t - \beta z)\, a_x$$

then $H = H_0 \cos(\omega t - \beta z)\, a_y = \frac{E_0}{\eta_0}\cos(\omega t - \beta z)\, a_y$.



Wave Motion in Perfect Dielectric or lossless Dielectric.

A Perfect dielectric is characterized by

$$\sigma = 0, \ \mu = \mu_0\mu_r, \ \epsilon = \epsilon_0\epsilon_r.$$

$$\alpha = 0, \quad \beta = \omega\sqrt{\mu_0\mu_r\epsilon_0\epsilon_r}$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0\mu_r}{\epsilon_0\epsilon_r}}$$

$$v = \frac{1}{\sqrt{\mu_0\mu_r\epsilon_0\epsilon_r}}$$

Intrinsic Impedance:

The ratio of amplitude of electric field intensity to the magnetic field intensity has the dimensions of impedance. This ratio is known as intrinsic impedance of the medium in which the wave is travelling.
It is denoted by (η).

$$\eta = \frac{E}{H}.$$

Since the wave equation for electric field intensity is

$$\nabla^2 E_s = \gamma^2 E_s.$$

The solⁿ which is assumed for electric field intensity is:

$$E_{xs} = A e^{-\gamma z}$$

$$\bar{E}_s = A e^{-\gamma z} a_x$$

$$\nabla \times \bar{E}_s = -j\omega\mu H_s.$$

$$\text{So } \nabla \times \vec{E}_s = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A e^{-\gamma z} & 0 & 0 \end{vmatrix} = -A\gamma e^{-\gamma z} a_y.$$

Hence $\quad -A\gamma e^{-\gamma z} a_y = -j\omega\mu H_s.$

$$H_s = \frac{A\gamma e^{-\gamma z}}{j\omega\mu} a_y.$$

$$\eta = \frac{A e^{-\gamma z}}{A\gamma e^{-\gamma z}/j\omega\mu} = \frac{j\omega\mu}{\gamma} = \frac{j\omega\mu}{\sqrt{j\omega\mu(\sigma + j\omega\epsilon)}}$$

$$= \sqrt{\frac{j^2\omega^2\mu^2}{j\omega\mu(\sigma + j\omega\epsilon)}} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

$$\boxed{\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}}$$

Wave length :

Wave length means the distance travelled by the wave in which its phase angle changes by 2π radians. It is denoted by λ. and unit meter.

As per Def^n $\beta\lambda = 2\pi$

$$\boxed{\lambda = \frac{2\pi}{\beta}}$$

β = phase Const rad/m

Velocity —

As $v = f\lambda$ where v is the phase velocity of the wave.

$$v = \frac{\omega}{2\pi} \times \frac{2\pi}{\beta} = \frac{\omega}{\beta}$$

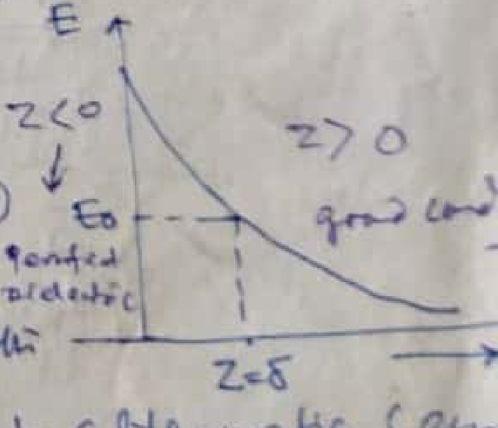
$$\boxed{v = \frac{\omega}{\beta} \text{ m/sec}}$$

Skin effect: (Depth of Penetration).

In a medium of high conductivity the wave is attenuated as it progresses due to losses which occur in the medium. In a good conductor the rate of attenuation is very great and the wave may penetrate only a very small distance.

The depth of penetration is defined as the depth in which the wave has been attenuated to $1/e \approx 37\%$ of its initial values.

The amplitude of the wave decreases by a factor $e^{-\alpha z}$ where α is attenuation const. It is apparent that a distance z which makes $\alpha z = 1$, the amplitude is only $\frac{1}{e}$ times its values at $z = 0$. This distance is equal to δ, the depth of penetration



we have $\alpha z = 1$, ~~$\alpha z \cdot \delta$~~

$\alpha \delta = 1$ $\boxed{\delta = \frac{1}{\alpha}}$ —— (1)

The eqn (1) represent as skin depth which is inversely proportional to attenuation const

for a good conductors, $\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$

$$\text{Skin depth } (\delta) = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}} = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

$$\boxed{\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}}$$

The normal component ... the boundary

Boundary Conditions to Determine the Refraction of the Electric field across the interface

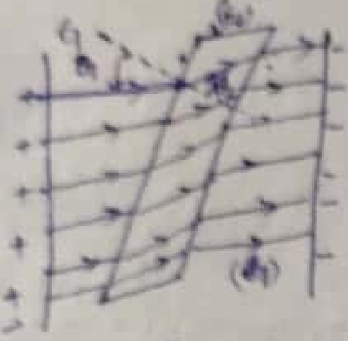
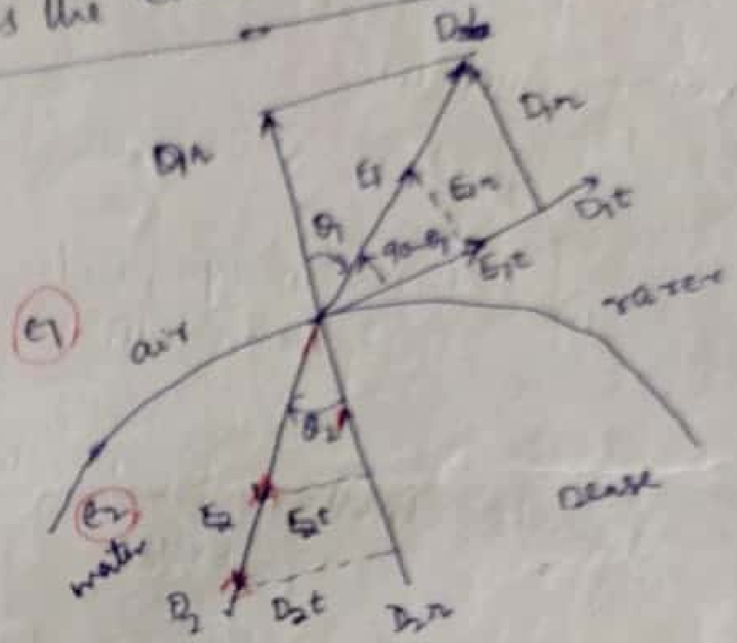


fig ①



②

we can use the boundary conditions to determine "Refraction" of the electric field across the interface as shown in fig ②

(1) Dielectric- Dielectric

Let D_1 & E_1 makes an angle θ_1 with a normal D_{n_1} & E_{n_1}
Let D_2 & E_2 makes an angle θ_2 with the normal D_{n_2} & E_{n_2}
to the interface as shown in fig ②

using normal component of $\vec{D}$ is continuous across the surface.

$$D_{n_1} = D_1 \cos\theta_1 , \quad D_{n_2} = D_2 \cos\theta_2$$

$$\boxed{D_{n_1} = D_{n_2}}$$

$$D_1 \cos\theta_1 = D_2 \cos\theta_2 \quad (1)$$

from $\epsilon_1 E_1 \cos\theta_1 = \epsilon_2 E_2 \cos\theta_2$

(Space for Writting)

The tangential component of $\vec{E}$ are the same on the two sides of the boundary.

$$\boxed{E_{t1} = E_{t2}}$$

$E_{1t} = E_1 \cos(90-\theta_1) = E_1 \sin\theta_1$

$E_{2t} = E_2 \sin\theta_2$

$\therefore \; E_1 \sin\theta_1 = E_2 \sin\theta_2$ — (2)

~~$\epsilon_1 E_1 \cos\theta_1 = \epsilon_2 E_2 \cos\theta_2$~~ —

~~from eqn (1)~~

$$\cancel{\frac{\epsilon_1}{\epsilon_2} = \frac{E_2 \cos\theta_2}{E_1 \cos\theta_1}}$$ — (3)

~~from eqn (1)~~

Dividing eqn (2/1) $\dfrac{E_1 \sin\theta_1}{\epsilon_1 E_1 \cos\theta_1} = \dfrac{E_2 \sin\theta_2}{\epsilon_2 E_2 \cos\theta_2}$

$\Rightarrow \dfrac{\tan\theta_1}{\epsilon_1} = \dfrac{\tan\theta_2}{\epsilon_2}$

$\dfrac{\tan\theta_1}{\tan\theta_2} = \dfrac{\epsilon_1}{\epsilon_2}$ since $\epsilon_1 = \epsilon_0 \epsilon_{r1}$, $\epsilon_2 = \epsilon_0 \epsilon_{r2}$

$\dfrac{\tan\theta_1}{\tan\theta_2} = \dfrac{\epsilon_0 \epsilon_{r1}}{\epsilon_0 \epsilon_{r2}} \Rightarrow \boxed{\dfrac{\tan\theta_1}{\tan\theta_2} = \dfrac{\epsilon_{r1}}{\epsilon_{r2}}}$

From eqn (5) $D_1 \cos\theta_1 = D_2 \cos\theta_2$

from eqn (4) $D_1 \sin\theta_1 = \dfrac{D_2 \epsilon_1}{\epsilon_2} \sin\theta_2$

$D_1^2 = D_2 \left[\cos^2\theta_2 + \left(\dfrac{\epsilon_1}{\epsilon_2}\right)^2 \sin^2\theta_2\right]$

$D_1 = D_2 \sqrt{\cos^2\theta_2 + \left(\dfrac{\epsilon_1}{\epsilon_2}\right)^2 \sin^2\theta_2}$

From eqn (3)

$D_2 \sin\theta_2 = \dfrac{\epsilon_2}{\epsilon_1} D_1 \sin\theta_1$ — from eqn

$D_2 \cos\theta_2 = D_1 \cos\theta_1$ — (5)

$\Rightarrow$ $\dfrac{D_1}{\epsilon_1} \sin\theta_1 = \dfrac{D_2}{\epsilon_2} \sin\theta_2$

$\Rightarrow D_2 \sin\theta_2 = \ldots$ — (4)

$(4)^2 + (5)^2 =$

$D_2^2 \sin^2\theta_2 + D_2^2 \cos^2\theta_2$

$\boxed{D_2 = D_1 \sqrt{\cos^2\theta_1 + \ldots}}$

This is the law of refraction of the electric field at a boundary free of charge. (since $\rho_s = 0$ is assumed at the interface

Conductor - Dielectric Boundary Conditions.

Δs

(1) A uniform plane wave in a medium having $\sigma = 10^{-3}$ S/m, $\epsilon = 80\epsilon_0$ and $\mu = \mu_0$ is having a frequency of 10kHz. Calculate the different parameters of wave.

Sol^n:

$\sigma = 10^{-3}$ S/m, $\epsilon = 80\epsilon_0$, $\mu = \mu_0$

$$\frac{\sigma}{\omega\epsilon} = \frac{10^{-3}}{2\pi \times 10^4 \times 80 \times 8.85 \times 10^{-12}} = 22.47771.$$

The medium is a good conductor. so attenuation const

$$\alpha = \sqrt{\pi f \mu \sigma} = \sqrt{\pi \times 10^4 \times 4\pi \times 10^{-7} \times 10^{-3}} = 2\pi \times 10^{-3} \text{ rad/m}$$

$$\text{Phase Const } \beta = \alpha = 2\pi \times 10^{-3} \text{ rad/m.}$$

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45 = 2\pi(1+j)\,\Omega$$

$$\lambda = \frac{2\pi}{\beta} = 1000\text{m.}$$

$$v = \frac{\omega}{\beta} = \frac{2\pi \times 10^4}{2\pi \times 10^{-3}} = 10^7 \text{ m/sec.}$$

(2) The magnetic field intensity of a uniform plane wave in air is 30A/m in the a_y dir^n. The wave is propagating in the a_z dir^n at a frequency of 3×10^9 rad/sec. Find (a) the wavelength (b) the frequency (c) the time period (d) the amplitude of E.

sn^n - (a)

$$\beta = \omega\sqrt{\mu_0\epsilon_0} = 3\times10^9\sqrt{4\pi\times10^{-7}\times8.85\times10^{-12}} = 10 \text{ rad/m.}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{10} = 0.628 \text{ m}$$

$$f = \frac{\omega}{2\pi} = \frac{3\times10^9}{2\pi} = 477.464 \text{ mHz}$$

$$T = \frac{1}{f} = 2.094 \text{ nsec}$$

$$\eta = \frac{E}{H} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 376.734$$

$E = 376.73 \times 30$

$E = 11302.03$ V/m

(3) Determine the propagation constant at 400KHz for a medium in which $\mu_r = 2$, $\epsilon_r = 17$, and ~~σ~~ $\sigma = 0$. At what velocity will an electromagnetic wave travel in this medium?

Sol: $f = 400\,kHz$, $\mu_r = 2$, $\epsilon_r = 17$, $\sigma = 0$. Since $\sigma = 0$, ϵ_r is very...

The medium is Perfect ~~good~~ dielectric material,

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = j\omega\sqrt{\mu\epsilon}$$

$$\alpha = 0, \quad \beta = \omega\sqrt{\mu\epsilon} = 2\pi f\sqrt{\mu_0\mu_r\epsilon_0\epsilon_r}$$

$$= 2\pi \times 400 \times 10^3 \sqrt{4\pi \times 10^{-7} \times 2 \times 8.854 \times 10^{-12} \times 17} = 0.0484.$$

$$\gamma = \alpha + j\beta, \quad \gamma = 0.0484 \angle 90^\circ.$$

Velocity of propagation $v = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_0\mu_r\epsilon_0\epsilon_r}}$

$$= \frac{1}{\sqrt{\mu_0\epsilon_0}} \times \frac{1}{\sqrt{\mu_r\epsilon_r}} = 3 \times 10^8 \times \frac{1}{\sqrt{2 \times 17}}$$

$$= 0.51 \times 10^8\ m/sec$$

(4) Determine the skin depth for copper with conductivity of 66 mmho/meter at 15 MHz frequency.

Sol: $\sigma = 6.6$ mmho/m, $f = 15\,MHz$.

$\mu = \mu_0\mu_r = \mu_0$ since $\mu_r = 1$ for copper which is a non magnetic medium.

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

$$= \frac{1}{\sqrt{\pi \times 15 \times 10^6 \times 4\pi \times 10^{-7} \times 6.6 \times 10^6}} = 5.06 \times 10^{-5}\ m.$$

(5) An E field in free space is given by $E = 900\cos(10^9 t - \beta z)$ V/m. find (a) β (b) λ (c) H at $P(0.1, 0.5, 0.4)$ at $t = 10$ nsec.

Sol:

(a) $\omega = 10^9$ $\quad \beta = \omega\sqrt{\mu_0\epsilon_0} = 3.334$ rad/m.

(b) $\lambda = 2\pi/\beta = 1.883$.

$$\eta = \frac{E}{H} = 376.7343$$

$$E = 900, \quad H = \frac{900}{376.73} = 2.388\ A/m.$$

$$H = 2.388\cos(10^9 t - \beta z)\ a_x\ A/m.$$

Since ~~E×H~~ = ~~space~~ since dir of propagation along z dir.

E is along z mean H along x d

Q. Two extensive homogeneous dielectrics meet on plane $z=0$. For $z>0$, $\epsilon_{r_1}=4$ and for $z<0$ $\epsilon_{r_2}=3$.
A uniform electric field $E_1 = 5a_x - 2a_y + 3a_z$ kV/m exists for $z \geq 0$. Find

(a) E_2 for $z \leq 0$

(b) The angles E_1 and E_2 make the interface.

(c) The energy densities (in J/m³) in both dielectric.

Sol (a) Since a_z is normal to the boundary plane.

$$E_{1n} = E_1 \cdot a_n = E_1 \cdot a_z = 3.$$

$$\vec{E}_{1n} = 3a_z$$

$$E_{2n} = E_2 \cdot a_z$$

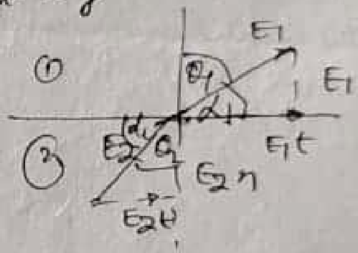
$$\vec{E}_{2n} = (E_2 \cdot a_z)a_z \qquad E = E_n + E_t$$

$$E_{1t} = E_1 - E_{1n} = (5a_x - 2a_y + 3a_z) - 3a_z$$

$$E_{1t} = 5a_x - 2a_y$$

$E_{1t} = E_{2t}$

$$\Rightarrow E_{2t} = 5a_x - 2a_y$$



$D_{2n} = D_{1n} \qquad \epsilon_{o2} E_{2n} = \epsilon_{o1} E_{1n}$

$$E_{2n} = \frac{\epsilon_{r1}}{\epsilon_{r2}} E_{1n} = \frac{4}{3} \times 3a_z = 4a_z$$

$$E_2 = E_{2t} + E_{2n} = (5a_x - 2a_y) + 4a_z \text{ kV/m.}$$

(b) $\alpha_1 = 90 - \theta_1$, $\alpha_2 = 90 - \theta_2$, $E_{1n} = 3$, $E_{1t} = \sqrt{25+4} = \sqrt{29}$

$$\tan\theta_1 = \frac{E_{1t}}{E_{1n}} = \frac{\sqrt{29}}{3} = 1.795, \quad \theta_1 = 60.9$$

$$\alpha_1 = 29.1^\circ$$

$E_{2n} = 4$, $\quad E_{2t} = E_{1t} = \sqrt{29}$, $\quad \tan\theta_2 = \frac{E_{2t}}{E_{2n}} = \frac{\sqrt{29}}{4} = 1.346$

$$\theta_2 = 53.4^\circ, \; \alpha_2 = 36.6^\circ$$

d * $\left\{ \frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_{r1}}{\epsilon_{r2}} \text{ is satisfied} \right)$

(c) $w_{E_1} = \frac{1}{2}\epsilon_1 |E_1|^2 = \frac{1}{2} \times 4 \times \frac{10^{-9}}{36\pi} \times [25+4+9) \times 10^{6}$

$w_{E_2} = \frac{1}{2}\epsilon_2 |E_2|^2 =$

Polarization:

The polarization of a uniform plane wave refers to the time varying behaviour of the electric field entensity at some fixed point in space.

There are three types of polarization

1. linear polarization
2. Elliptical polarization
3. circular polarization.

Elliptical polarization:

Let E_x = instantaneous electric field of the horizontal polarized wave (component)

E_y = instantaneous electric field of the vertically polarized wave (component).

Then the electric field component as a function of time and distance in x and y dirn are

$$\left.\begin{aligned} E_x &= E_1 \sin(\omega t - \beta z) \\ E_y &= E_2 \sin(\omega t - \beta z + \delta) \end{aligned}\right\} \quad \text{—— ①}$$

Where E_1, E_2 are amplitudes of plane polarized wave in x and y dir respectively and δ the time phase angle by which E_y leads E_x.

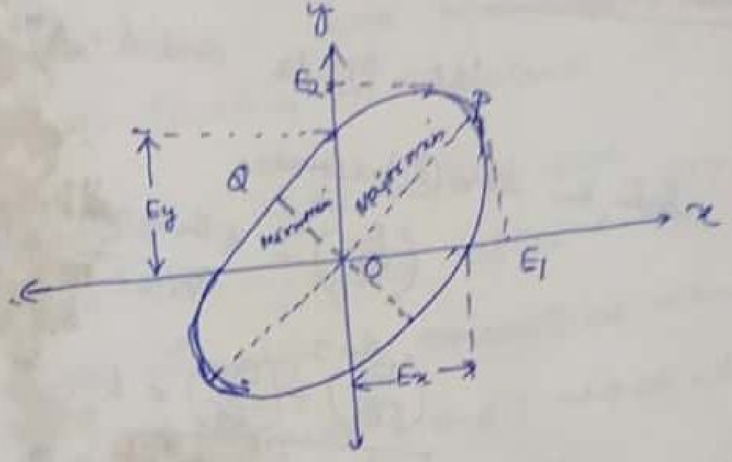
Now $E = E_x a_x + E_y a_y$

At $z=0$ the eqn ① becomes

$$E_x = E_1 \sin \omega t$$
$$E_y = E_2 \sin(\omega t + \delta)$$

The eqn which represent as the general form of ellipse.

OP = semi major axis.
OQ = semi minor axis.



Linear polarization.

Let E_y be in phase or $180°$ out of phase with E_x then $\delta = 0$,

The eqn (2-A) becomes

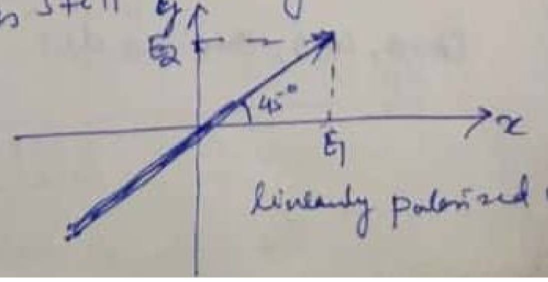
$$\left(\frac{E_y}{E_2}\right)^2 \pm \frac{2 E_x E_y}{E_1 E_2} + \left(\frac{E_x}{E_1}\right)^2 = 0$$

$$\left(\frac{E_y}{E_2} \pm \frac{E_x}{E_1}\right)^2 = 0$$

$$\frac{E_y}{E_2} = \pm \frac{E_x}{E_1} \qquad E_y = \pm \frac{E_2}{E_1} E_x$$

$$E_y = \frac{E_2}{E_1} E_x \qquad \boxed{E_y = mE_x} \quad \text{—— (4)}$$

This eqn (4) represent an straight line where $m = \frac{E_2}{E_1}$ = slope of the line. Hence when two linearly polarized component waves are in phase or out of phase, the resultant wave is a plane (or linearly) polarized with E. If $E_1 = 0$, the wave is polarized in y-dirn and when $E_2 = 0$, the wave is polarized in x dirn and when $E_1 = E_2$, and $\delta = 0$, the wave is still linearly polarized but at $45°$



linearly polarized wave.

Circular polarization:

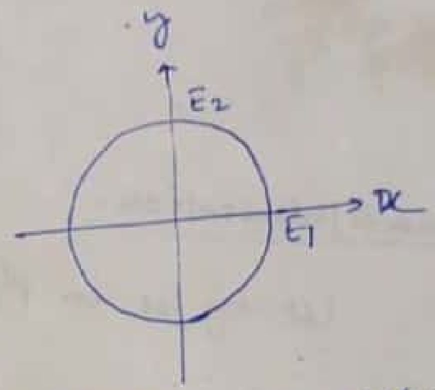
let $E_1 = E_2$ and $\delta = \pm 90^\circ$.

then the eqn (3) becomes

$$\left(\frac{E_x}{E_1}\right)^2 - \frac{2 E_x E_y \cos 90}{E_1 E_2 \sin^2 90} + \left(\frac{E_y}{E_1}\right)^2 = 1.$$

$$\left(\frac{E_x}{E_1}\right)^2 + \left(\frac{E_y}{E_1}\right)^2 = 1$$

$$\left.\begin{array}{l} \boxed{E_x^2 + E_y^2 = E_1^2} \\ \text{or } \boxed{E_x^2 + E_y^2 = E_2^2} \end{array}\right\} \text{— (4)}$$



The eqn (4) are the eqn of circle.

Therefore, when two linearly polarized component waves are in time phase quadrature (ie 90°) and also are equal in magnitude, the resultant wave is circularly polarized.

Direction Cosine:

The uniform plane wave travelling in x-dirn is given by

$$E(x) = E_0 e^{-j\beta x}$$

The plane of constant phase is given by $x =$ Constant

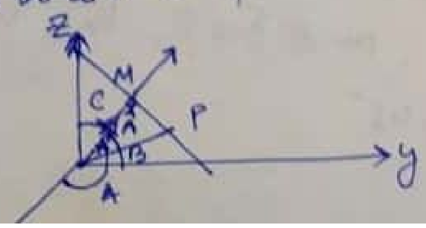
For a plane wave travelling in arbitrary dirn then the equation of the plane is $\hat{n} \cdot \bar{r} =$ Const where $\bar{r} \rightarrow$ radius vector from the origin to the point 'P'.

$$\hat{n} \cdot \bar{r} = x \cos A + y \cos B + z \cos C$$

where $\bar{r} = x a_x + y a_y + z a_z$.

$A, B, C \rightarrow$ angle that the unit vector makes with x, y and z axes

$\cos A, \cos B, \cos C \rightarrow$ dirn Cosines or dirn Components of the vector.



Reflection by Perfect Dielectric (Normal incidence)

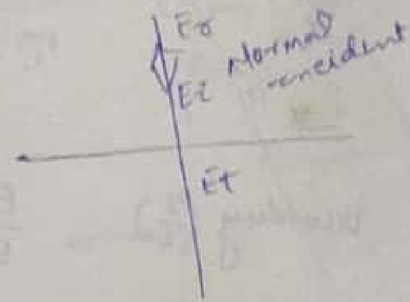
Let us consider the two media with parameters ϵ_1, μ_1 and ϵ_2, μ_2. The boundary is parallel to $x = 0$ plane.

$E_i \rightarrow$ electric field intensity of the incident wave.

$E_r \rightarrow$ electric field intensity of the reflected wave

$E_t \rightarrow$ electric field intensity of transmitted wave.

η_1, η_2 are the impedances of the media 1 and 2 respectively and are given by $\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}}$ and $\eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}}$.



$$E_i = \eta_1 H_i \quad \text{——— (1)}$$

$$E_r = -\eta_1 H_r \quad \text{——— (2)}$$

and $$E_t = \eta_2 H_t \quad \text{——— (3)}$$

Since the total field in medium (1) comprises both incident and reflected fields, whereas medium 2 has only transmitted field.

So $$E_i + E_r = E_t$$

and $$H_i + H_r = H_t.$$

From eqn (3), $H_t = \frac{E_t}{\eta_2} = \frac{1}{\eta_2}[E_i + E_r]$

$$H_i + H_r = \frac{1}{\eta_2}[E_i + E_r]$$

$$\Rightarrow \frac{E_i}{\eta_1} - \frac{E_r}{\eta_1} = \frac{1}{\eta_2}[E_i + E_r]$$

$$\left[\frac{1}{\eta_1} - \frac{1}{\eta_2}\right] E_i = \left[\frac{1}{\eta_2} + \frac{1}{\eta_1}\right] E_r$$

$$\frac{\eta_2 - \eta_1}{\eta_1 \eta_2} E_i = \frac{\eta_1 + \eta_2}{\eta_1 \eta_2} E_r$$

$$\boxed{\frac{E_r}{E_i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}} \quad \text{——— (5)}$$

$\Gamma = \frac{\eta_2 - \eta_1}{}$ — reflection coefficient.

~~Put~~ ~~we~~ Know that $E_i + E_r = E_t$

$$1 + \frac{E_r}{E_i} = \frac{E_t}{E_i}$$

$$\Rightarrow \frac{E_t}{E_i} = 1 + \frac{E_r}{E_i} = 1 + \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\boxed{\frac{E_t}{E_i} = \frac{2\eta_2}{\eta_1 + \eta_2}} \quad \text{——— (6)}$$

$$\tau = \frac{E_t}{E_i} = \frac{2\eta_2}{\eta_1 + \eta_2} = \text{Transmission co-efficient.}$$

Dividing $\left(\frac{2}{1}\right)$ $\quad \frac{E_r}{E_i} = -\frac{H_r}{H_i}$

$$\frac{H_r}{H_i} = -\frac{E_r}{E_i} = -\left[\frac{\eta_2 - \eta_1}{\eta_1 + \eta_2}\right] = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}$$

$$\boxed{\frac{H_r}{H_i} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}}$$

Dividing (3/4). $\quad \frac{E_t}{E_i} = \frac{\eta_2 H_t}{\eta_1 H_i}$

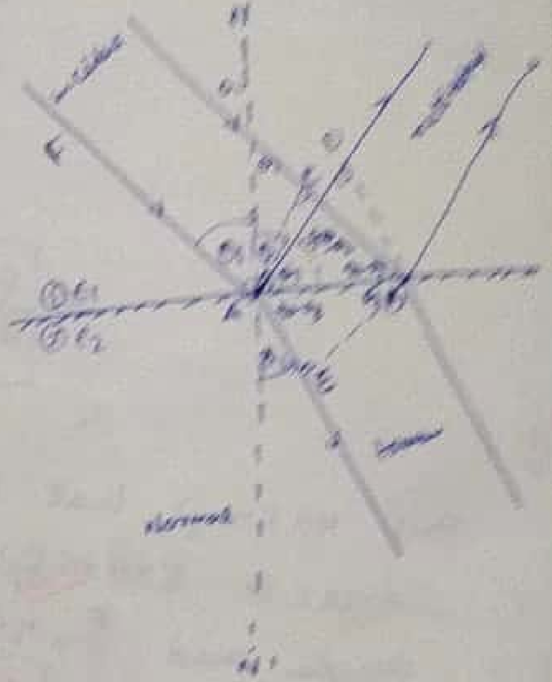
$$\frac{H_t}{H_i} = \frac{\eta_1}{\eta_2}\frac{E_t}{E_i} = \frac{\eta_1}{\eta_2} \times \frac{2\eta_2}{\eta_1 + \eta_2}$$

$$\boxed{\frac{H_t}{H_i} = \frac{2\eta_1}{\eta_1 + \eta_2}}$$

Wave incident obliquely on boundary bet^n Perfect Dielectric

Let us consider an incident waves meets on the boundary over the region AB and gets partly reflected and partly transmitted. The incident ray FA, the reflected ray AD and transmitted ray AE all lies in one plane, θ_1, θ_2 and θ_3 angle of incident, reflected and transmitted wave respectively.

The normal NAN' to the boundary at the point A, then we can draw AC perpendicular to the incident wave, AD perpendicular to reflected wave and AE perpendicular to the transmitted wave.



Let ϵ_1 and ϵ_2 be the permittivities of medium 1 and 2 and μ_0 are permeability of both media. Since the permeability of all known dielectric is same as for free space.

Then the velocities of propagation of wave in the two media will be

$$V_1 = \frac{1}{\sqrt{\mu_0 \epsilon_1}}, \quad V_2 = \frac{1}{\sqrt{\mu_0 \epsilon_2}}$$

Let us assume that the incident beam reaches the point A at time t_1 and the point B at time t_2, then

$$\left.\begin{aligned} CB &= v_1 (t_2 - t_1) \\ AD &= v_1 (t_2 - t_1) \\ AE &= v_2 (t_2 - t_1) \end{aligned}\right] \quad \text{--- (1)}$$

But from the above fig, $\angle NAF = \theta_1 = \angle BAC$
$\angle NAD = \theta_2 = \angle ABD$
$\angle N'AE = \theta_3 = \angle ABE$.

From (1) $\quad BC = AD$.

$\Rightarrow \quad AB \sin\theta_1 = AB \sin\theta_2$

$$\boxed{\theta_1 = \theta_2}$$

This is really law of optics showing that the angle of incident is

From eqn ① $\frac{CB}{AE} = \frac{V_1}{V_2}$

$$\frac{AB \sin\theta_1}{AB \sin\theta_3} = \frac{V_1}{V_2}$$

$$\frac{\sin\theta_1}{\sin\theta_3} = \frac{V_1}{V_2}$$

Put $V_1 = \frac{1}{\sqrt{\mu_0 \epsilon_1}}$

$V_2 = \frac{1}{\sqrt{\mu_0 \epsilon_2}}$

$$\Rightarrow \boxed{\frac{\sin\theta_1}{\sin\theta_3} = \frac{\sqrt{\epsilon_2}}{\sqrt{\epsilon_1}}} \longrightarrow ②$$

$\rightarrow$ Snell's law. $\frac{V_1}{V_2} = \frac{\sqrt{\epsilon_2}}{\sqrt{\epsilon_1}}$

But we know that the power flow in the dirⁿ of P which is equal to E×H. Since in the wave E and H are at right angles and $\frac{E}{H} = \eta$, Therefore this power is equal to EH sin 90

$= EH = \frac{E^2}{\eta}$.

Let us consider the component of power flowing in the dirⁿ of the normal NAN', we get

$$\frac{E_i^2}{\eta_1}\cos\theta_1 - \frac{E_r^2}{\eta_1}\cos\theta_2 = \frac{E_t^2}{\eta_2}\cos\theta_3.$$

Now dividing throughout $\frac{E_i^2}{\eta_1}\cos\theta_1$. and $(\theta_1 = \theta_2)$

$$1 = \frac{E_r^2}{E_i^2} + \frac{\eta_1}{\eta_2}\frac{\cos\theta_3}{\cos\theta_1}\left(\frac{E_t}{E_i}\right)^2$$

$$1 - \frac{E_r^2}{E_i^2} = \frac{\eta_1 \cos\theta_3}{\eta_2 \cos\theta_1}\left(\frac{E_t}{E_i}\right)^2$$

$$\boxed{\frac{E_r^2}{E_i^2} = 1 - \frac{\eta_1 \cos\theta_3}{\eta_2 \cos\theta_1}\left(\frac{E_t}{E_i}\right)^2} \longrightarrow ⑤$$

REDMI NOTE 8
AI QUAD CAMERA

Perpendicular polarization:

$$T_\perp = \frac{\sin(\theta_3-\theta_1)}{\sin(\theta_3+\theta_1)}$$

$$= \frac{\sin\theta_3 . \cos\theta_1 - \cos\theta_3 . \sin\theta_1}{\sin\theta_3 . \cos\theta_1 + \cos\theta_3 . \sin\theta_1}$$

$$= \frac{\cos\theta_1 - \cos\theta_3 . \frac{\sin\theta_1}{\sin\theta_3}}{\cos\theta_1 + \cos\theta_3 . \frac{\sin\theta_1}{\sin\theta_3}} = \frac{\cos\theta_1 - \cos\theta_3 . \frac{\eta_1}{\eta_2}}{\cos\theta_1 + \cos\theta_3 \frac{\eta_1}{\eta_2}}$$

$$= \frac{\eta_2 \cos\theta_1 - \eta_1 \cos\theta_3}{\eta_2 \cos\theta_1 + \eta_1 \cos\theta_3} = \frac{E}{E}$$

But $\frac{\sin\theta_1}{\sin\theta_3} = \frac{\sqrt{\epsilon_2}}{\sqrt{\epsilon_1}}$

$\eta_1 = \frac{1}{\sqrt{\epsilon_1}}$, $\eta_2 = \frac{1}{\sqrt{\epsilon_2}}$ $\quad \frac{\eta_1}{\eta_2} = \frac{\sqrt{\epsilon_2}}{\sqrt{\epsilon_1}}$

Parallel polarization:

$$T_{\parallel} = \frac{\tan(\theta_3-\theta_1)}{\tan(\theta_3+\theta_1)} = \frac{\sin(\theta_3-\theta_1)}{\cos(\theta_3-\theta_1)} \times \frac{\cos(\theta_3+\theta_1)}{\sin(\theta_3+\theta_1)}$$

$$= \frac{2\sin(\theta_3-\theta_1).\cos(\theta_3+\theta_1)}{2\cos(\theta_3-\theta_1).\sin(\theta_3+\theta_1)}$$

$$= \frac{\sin(2\theta_3) + \sin(-2\theta_1)}{\sin(2\theta_3) - \sin(-2\theta_1)}$$

$$= \frac{2\sin\theta_3 . \cos\theta_3 - 2\sin\theta_1 \cos\theta_1}{2\sin\theta_3 . \cos\theta_3 + 2\sin\theta_1 \cos\theta_1}$$

$$= \frac{\cos\theta_3 - \cos\theta_1 . \frac{\sin\theta_1}{\sin\theta_3}}{\cos\theta_3 + \cos\theta_1 . \frac{\sin\theta_1}{\sin\theta_3}}$$

$$= \frac{\cos\theta_3 - \cos\theta_1 . \frac{\eta_1}{\eta_2}}{\cos\theta_3 + \cos\theta_1 \frac{\eta_1}{\eta_2}}$$

$$= \frac{\eta_2 \cos\theta_3 - \eta_1 \cos\theta_1}{\eta_2 \cos\theta_3 + \eta_1 \cos\theta_1} =$$

(1) Perpendicular palarization. (5)

As we know that $E_i + E_r = E_t$

$$\frac{E_t}{E_i} = 1 + \frac{E_r}{E_i}$$

Now substituting $\frac{E_t}{E_i}$ in Eqn (3).

$$\frac{E_r^2}{E_i^2} = 1 - \frac{\eta_1 \cos\theta_3}{\eta_2 \cos\theta_1}\left(\frac{E_t}{E_i}\right)^2$$

$$= 1 - \frac{\eta_1 \cos\theta_3}{\eta_2 \cos\theta_1}\left(1 + \frac{E_r}{E_i}\right)^2$$

$$\Rightarrow \quad 1 - \frac{E_r^2}{E_i^2} = \frac{\eta_1 \cos\theta_3}{\eta_2 \cos\theta_1}\left(1 + \frac{E_r}{E_i}\right)^2$$

$$\Rightarrow \left(1 + \frac{E_r}{E_i}\right)\left(1 - \frac{E_r}{E_i}\right) = \frac{\eta_1 \cos\theta_3}{\eta_2 \cos\theta_1}\left(1 + \frac{E_r}{E_i}\right)^2$$

$$1 - \frac{E_r}{E_i} = \frac{\eta_1 \cos\theta_3}{\eta_2 \cos\theta_1}\left(1 + \frac{E_r}{E_i}\right)$$

$$1 - \frac{\eta_1 \cos\theta_3}{\eta_2 \cos\theta_1} = \frac{E_r}{E_i}\left(1 + \frac{\eta_1 \cos\theta_3}{\eta_2 \cos\theta_1}\right)$$

$$\frac{\eta_2 \cos\theta_1 - \eta_1 \cos\theta_3}{\eta_2 \cos\theta_1} = \frac{E_r}{E_i}\left(\frac{\eta_2 \cos\theta_1 + \eta_1 \cos\theta_3}{\eta_2 \cos\theta_1}\right)$$

$$\boxed{\frac{E_r}{E_i} = \frac{\eta_2 \cos\theta_1 - \eta_1 \cos\theta_3}{\eta_2 \cos\theta_1 + \eta_1 \cos\theta_3}}$$

$\Gamma_\perp = \frac{E_r}{E_i}$ = Reflection co-efficient for perpendicular palarization.

But $\frac{E_t}{E_i} = 1 + \frac{E_r}{E_i} = 1 + \frac{\eta_2 \cos\theta_1 - \eta_1 \cos\theta_3}{\eta_2 \cos\theta_1 + \eta_1 \cos\theta_3}$

Apply the boundary cond that the tangential component of E must be the same value on both sides at the boundary. let "medium ① and medium ②. Since $\theta_1 = \theta_2$,

$$E_i \cos\theta_1 - E_r \cos\theta_2 = E_t \cos\theta_3$$

$$\Rightarrow E_i \cos\theta_1 = E_r \cos\theta_2 + E_t \cos\theta_3$$

Dividing throughout by $E_i \cos\theta_1$

$$1 = \frac{E_r}{E_i} + \frac{E_t \cos\theta_3}{E_i \cos\theta_1}$$

$$1 - \frac{E_r}{E_i} = \frac{E_t \cos\theta_3}{E_i \cos\theta_1}$$

$$\Rightarrow \frac{E_t}{E_i} = \left(1 - \frac{E_r}{E_i}\right) \frac{\cos\theta_1}{\cos\theta_3} \quad \text{—— (4)}$$

But from Eqn (3), $\frac{E_r^2}{E_i^2} = 1 - \frac{\eta_1 \cos\theta_3}{\eta_2 \cos\theta_1} \left(\frac{E_t}{E_i}\right)^2$

$$\Rightarrow \frac{\eta_1 \cos\theta_3}{\eta_2 \cos\theta_1} \left(\frac{E_t}{E_i}\right)^2 = 1 - \frac{E_r^2}{E_i^2}$$

$$\Rightarrow \frac{\eta_1 \cos\theta_3}{\eta_2 \cos\theta_1} \left(1 - \frac{E_r}{E_i}\right)^2 \frac{\cos^2\theta_1}{\cos^2\theta_3} = \left(1 + \frac{E_r}{E_i}\right)\left(1 - \frac{E_r}{E_i}\right)$$

$$\frac{\eta_1 \cos\theta_1}{\eta_2 \cos\theta_3} \left(1 - \frac{E_r}{E_i}\right) = 1 + \frac{E_r}{E_i}$$

$$\frac{\eta_1 \cos\theta_1}{\eta_2 \cos\theta_3} - 1 = \frac{E_r}{E_i}\left[1 + \frac{\eta_1 \cos\theta_1}{\eta_2 \cos\theta_3}\right]$$

$$\frac{\eta_1 \cos\theta_1 - \eta_2 \cos\theta_3}{\eta_2 \cos\theta_3} = \frac{E_r}{E_i}\left[\frac{\eta_2 \cos\theta_3 + \eta_1 \cos\theta_1}{\eta_2 \cos\theta_3}\right]$$

$$\boxed{\frac{E_r}{E_i} = \frac{\eta_1 \cos\theta_1 - \eta_2 \cos\theta_3}{\eta_1 \cos\theta_1 + \eta_2 \cos\theta_3}} = T_{11} \quad \text{—— (5)}$$

Brewster angle :

(6)

This is the angle of incidence for which angle of reflection is zero. ie $\frac{E_r}{E_i} = 0$.

From eqn (5), $\eta_1 \cos\theta_1 = \eta_2 \cos\theta_3$

$$\eta_1^2 \cos^2\theta_1 = \eta_2^2 \cos^2\theta_3 = \eta_2^2(1-\sin^2\theta_3)$$

But from Snell's eqn $\frac{\sin\theta_1}{\sin\theta_3} = \frac{\sqrt{\epsilon_2}}{\sqrt{\epsilon_1}}$

$$\Rightarrow \sin\theta_3 = \frac{\sqrt{\epsilon_1}}{\sqrt{\epsilon_2}} \sin\theta_1$$

$$\therefore \quad \eta_1^2 \cos^2\theta_1 = \eta_2^2\left[1 - \frac{\epsilon_1}{\epsilon_2}\sin^2\theta_1\right]$$

$$\eta_1^2 = \eta_2^2\left[\frac{1}{\cos^2\theta_1} - \frac{\epsilon_1}{\epsilon_2}\frac{\sin^2\theta_1}{\cos^2\theta_1}\right]$$

$$\eta_1^2 = \eta_2^2\left[\sec^2\theta_1 - \frac{\epsilon_1}{\epsilon_2}\tan^2\theta_1\right]$$

$$\eta_1^2 = \eta_2^2(1+\tan^2\theta_1) - \eta_2^2\frac{\epsilon_1}{\epsilon_2}\tan^2\theta_1$$

$\eta_1 \sqrt{\frac{\mu_0}{\epsilon_1}}$

$$\eta_1^2 = \eta_2^2 + \eta_2^2\tan^2\theta_1 - \eta_2^2\frac{\epsilon_1}{\epsilon_2}\tan^2\theta_1$$

$$\eta_1^2 - \eta_2^2 = \eta_2^2\tan^2\theta_1\left[1 - \frac{\epsilon_1}{\epsilon_2}\right]$$

$\eta_1 = \frac{\sqrt{\mu_0}}{\sqrt{\epsilon_1}}$

$\eta_2 = \frac{\sqrt{\mu_0}}{\sqrt{\epsilon_2}}$ $\quad \eta_2 \sqrt{\frac{\mu_0}{\epsilon_2}}$

$$\frac{\cancel{\mu_0}}{\epsilon_1} - \frac{\cancel{\mu_0}}{\epsilon_2} = \frac{\cancel{\mu_0}}{\epsilon_2}\tan^2\theta_1\left[\frac{\epsilon_2-\epsilon_1}{\epsilon_2}\right]$$

$$\frac{\epsilon_2-\epsilon_1}{\epsilon_1 \cancel{\epsilon_2}} = \frac{1}{\cancel{\epsilon_2}}\tan^2\theta_1\left\{\frac{\epsilon_2-\epsilon_1}{\epsilon_2}\right\}$$

$$\tan^2\theta_1 = \frac{\epsilon_2}{\epsilon_1}$$

$$\tan\theta_1 = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \qquad \boxed{\theta_1 = \tan^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}}}$$

This is called Brewster angle. For this angle of incidence in this case there is no reflected ray, and the entire