

SURVEYING MODULE-II

Asit Kumar Dandapat
GCE KEONJHAR

Semester Questions Topic Wise

Assignment for you?

Module II

LEVELLING: -

1. What is levelling?
2. What is use of levelling?
3. Define level surface and datum surface?
4. What is bench mark? What are the types?
5. What is difference between line of collimation and axis of telescope?
6. What is change point?
7. What is flying levelling?
8. Distinguish between levelling and reciprocal levelling?
9. What is parallax?
10. Compare collimation system and rise fall system?
11. What are the arithmetical checks for the HI method and rise fall method?
12. What are the different corrections applied to levelling?
13. What are the parts of a dumpy level?
14. What are the different sources of error in levelling?
15. Find correction for curvature and for refraction for a distance of 1200m?
16. What is visible horizon distance from a tower 250m high?
17. How do you estimate the distance to the visible horizon?

Reference books: -

- Surveying Volume 1, **S.K. Duggal.**
- Surveying Volume-I, **B.C. Punima,**
- A Text Book of Surveying and Levelling, **R. Agor.**
- Surveying and leveling, **N.N. Basak**

B.Tech(Civil Engineering) Detail Syllabus For Admission Batch 2015-16

(PCI3I102) Survey (3-0-1)

Module I (10 classes)

Linear measurement and chain survey: Use of chains and tapes for measurement of correct length of lines, direct and indirect ranging, chaining along sloping ground. Obstacle in chaining, errors and their elimination.

Compass surveying: Use of prismatic compass, temporary adjustment, bearing of a line, local attractions, correction of bearing

Module II (8 classes)

Levelling: Use of dumpy level and levelling staff. Temporary and Permanent adjustment of dumpy level, Reduction of levels by height of instrument and rise and fall method. Curvature and refraction error, sensitiveness of level tube, reciprocal levelling, levelling difficulties and common errors, Automatic and Electronic or Digital levels

Module III (10 classes)

Contouring: Contour interval and horizontal equivalent, characteristics of contours, methods of contouring- different and indirect method, contour gradient

Theodolite Survey: Use of theodolite, temporary adjustment, measuring horizontal and vertical angles, theodolite traversing

Module IV (8 classes)

Modern Surveying Instruments – Electromagnetic Spectrum, Radar, Electronic Distance Measurement, EDM Equipment, Corrections to measurement, Digital Theodolite, Total Stations, Introduction to Remote Sensing and GIS

Text Books

1. Surveying & Levelling. Vol-I by T.P.Kanethar & S.V.Kulkarni, Pune Vidyarthi Griha Prakashan
2. Surveying and Leveling by R. Subramanian, Oxford University Press
3. Surveying- Vol.I, by B.C. Punmia, Laxmi Publications

Reference Books

1. Surveying Vol-I by R Agor, Khanna Publishers
 2. A Textbook of Surveying, C. Venkatramaiah, Universities Press
 4. Surveying And Levelling, N.N. Basak, McGraw-Hill Education
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***3. LEVELLING

MODULE-II

1. Levelling can be defined as the art of determining the relative positions of the points on the ground surface of the earth to compute earth work. i.e., the volume of soil to be fill or to be excavated.
2. Levelling is applied in vertical plane, where a levelling staff and a dumpy level (telescope) are required.
3. The level line is always a * curved line parallel to earth surface.
4. The top water surface of a pond at rest condition (still) is nothing but level surface.
- **
5. The line of collimation or the line of sight is the line joining the point of intersection of cross hair of diaphragm to the optical center of object glass & its continuation. It plays an important role in determining the staff readings like Back sight, Intermediate sight & Fore sight.

5. The axis of telescope is the line joining the optical center of the eye piece & the optical center of object glass of a telescope.
6. The axis of the bubble tube is the tangential line of the bubble which is always horizontal if the bubble is at the central position, which is the basic principle adopted in dumpy level. i.e., Before starting the levelling work, the bubble must be brought to the central position so that the line of collimation will be truly horizontal, which is obtained by temporary adjustment at every new setup of the telescope instrument.
By turning the foot screws inwards or outwards the bubble is brought to the central position.
7. The known elevation of the ground surface above Mean sea level (Bombay sea level) is known as Bench Mark. Levelling work must start from the Bench mark to fix the RL of line of collimation known as Height of Instrument (HI) / Height of collimation (HC)
- GTS (Great Trigonometrical Survey of India (Central Govt.))
 - Permanent Bench mark (Used at State Govt.)
 - Temporary Bench mark (Used at site)
 - Arbitrary Bench mark (^{if} RL not known, MSL not known)

8. Back sight :- (BS)

It is the first staff reading obtained on the levelling staff if the staff is kept on a bench mark. The back sight is always required to find the reduced level (RL) of the line of collimation.

$$\text{i.e. } * \text{ the RL of line of collimation} = \text{RL of Bench Mark} + \text{Back Sight}$$

Since BS is added to the RL of BM to get RL of line of collimation, BM is also known as Plus sight.

9. Intermediate Sight (IS) :-

It is the staff reading obtained at different positions (stations) to determine the RL of ground stations.

$$\text{i.e. RL of stations} = \text{RL of line of collimation} - \text{Intermediate Sight}$$

10. Fore Sight (FS) :-

It is the last staff reading obtained from one set up of the instrument to determine the RL of change point (turning point).

$$\text{i.e. The RL of change point} = \text{RL of line of collimation} - \text{Fore Sight}$$

$$\text{HC/NI} - \text{FS}$$

The fore sight is also known as **Minus sight**.

The fore sight and back sight are taken at one place called **change point**.

The Back sight at every change point is required to determine new RL of collimation of second set up of instrument. and hence the RL of change point plays an important role.

i.e. RL of new line of collimation = RL of

Change point + BS

= RL of previous line of collimation - Fore sight + Back sight

While shifting the instrument, the staff should be at one place only called change point.

i.e. Fore sight and back sight are obtained at change point. The position of instrument on the ground is NOT the station. i.e. the position of levelling staff is the station.

11. Effect of curvature and refraction:-

1. Since the level line is a curve line but the line of sight / line of collimation is a horizontal line due to which the **staff reading increases** (more than the actual). Therefore an error takes place in the staff reading known as **curvature effect**.

2. Due to curvature effect, the error is positive and hence the correction shall be negative.
3. Since the ray of light passes through different density of atmosphere due to which the line of collimation bend down, which is not truly horizontal called refraction effect.

Due to refraction effect, the staff reading decreases and hence the error is negative due to refraction. It means the correction shall be positive.

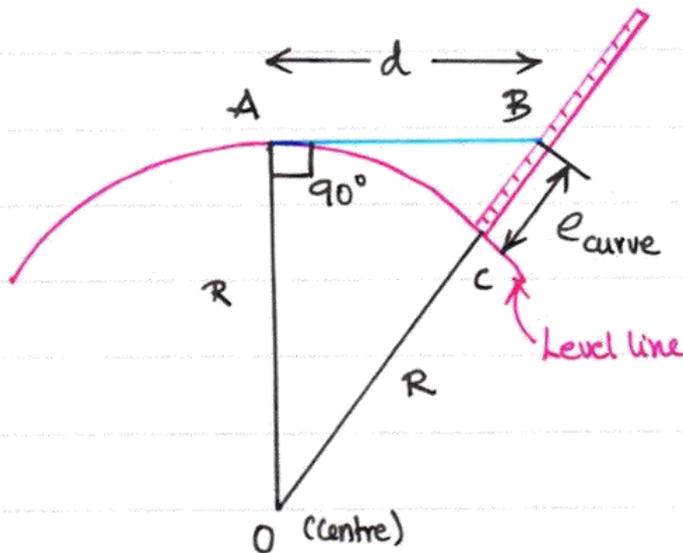
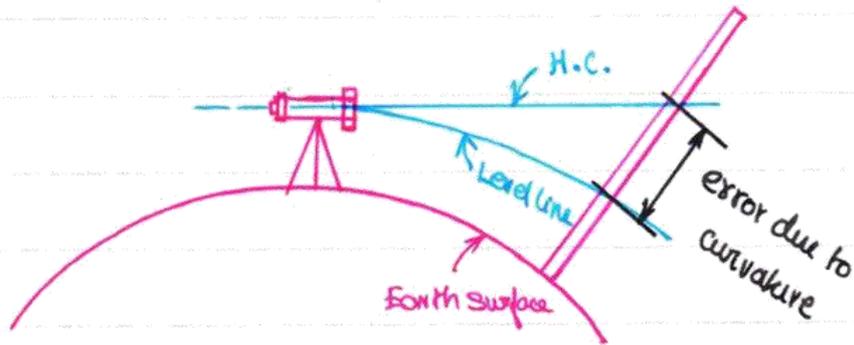
4. The combined error due to curvature and refraction is always positive and hence the combined correction is always negative.

Due to the curvature of the earth, the object sighted appears lower than they really are but due to refraction effect the object sighted appears higher than they really are.

Due to combined effect the objects sighted appears lower than they really are.

- * * 5. To neutralize (to take care of) curvature and refraction effect, 'reciprocal levelling' is adopted.
6. To neutralize curvature and refraction effect while determining the difference of RL of two ground station (staff station), the instrument is kept exactly at Midway between two ground station.

7.



$\triangle OAB$

$$OB^2 = OA^2 + AB^2$$

$$(R + e_c)^2 = R^2 + d^2$$

$$R^2 + e_c^2 + 2Re_c = R^2 + d^2$$

$$e_c^2 + 2Re_c = d^2$$

$$2Re_c = d^2 - e_c^2$$

$$100^2 - 0.1^2$$

$$199.99^2$$

$$2Re_c = d^2$$

$$e_c = \frac{d^2}{2R}$$

$$e_c = \frac{d^2}{2R} \leftarrow \text{km}^2$$

$$e_c = \frac{d^2}{D} \times 1000$$

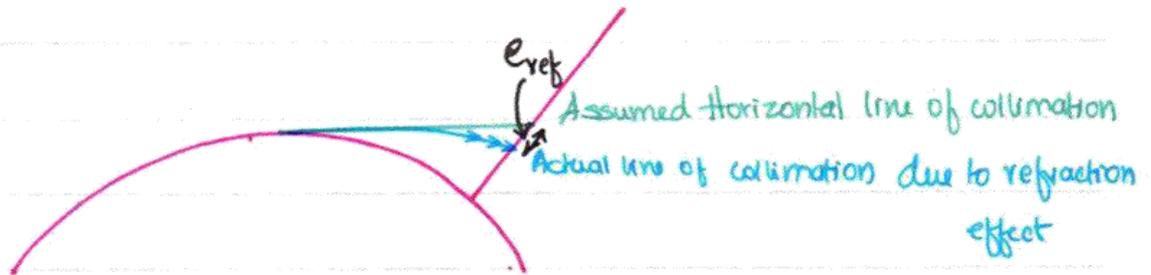
$$e_c = \frac{d^2}{12742} \times 1000$$

$$e_c = 0.0785 d^2$$

$e_c = 0.0785 d^2$

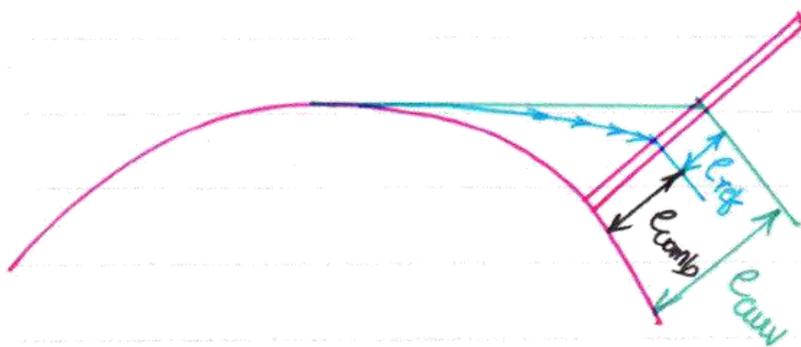
m

* km



$$\begin{aligned}
 \text{Error due to refraction} &= \frac{1}{7} \times e_{\text{curv}} \\
 &= \frac{1}{7} \times e_{\text{curv}} \\
 &= \frac{1}{7} \times 0.0785 d^2 \\
 &= 0.0112 d^2
 \end{aligned}$$

$$e_{\text{ref}} = 0.0112 d^2$$



Combined error

$$= \frac{6}{7} \times 0.0785 d^2$$

$$e_{\text{comb}} = 0.0673 d^2$$

$$\begin{aligned}
 e_{\text{comb}} &= e_{\text{curv}} - e_{\text{ref}} \\
 &= e_{\text{curv}} - \frac{1}{7} e_{\text{curv}} \\
 &= \frac{6}{7} e_{\text{curv}}
 \end{aligned}$$

$$\text{Corrected Staff Reading} = \text{Observed Staff reading} - (0.0673 d^2)$$

Error	Correction
i) Curvature (+ve)	$0.0785 d^2$ (-ve)
ii) Refraction (-ve)	$0.0112 d^2$ (+ve)
iii) Combined (+ve)	$0.0673 d^2$ (-ve)

8. Dip of Horizon :-

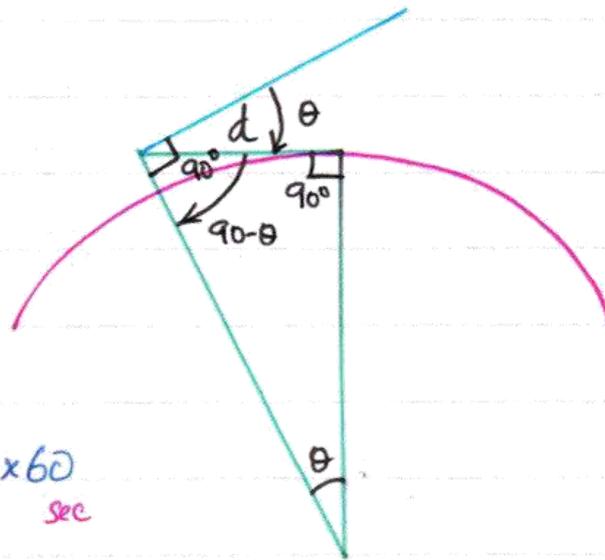
$$\theta = \frac{\text{Arc length}}{\text{radius}}$$

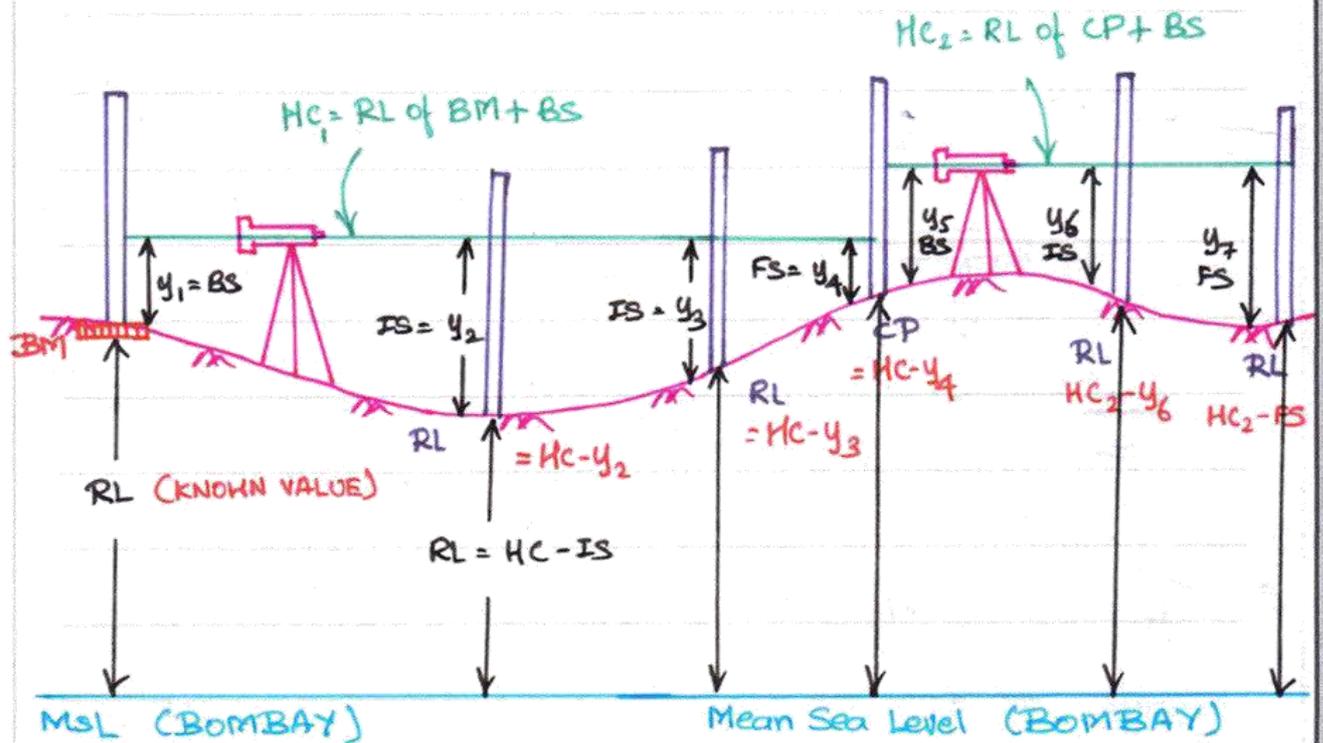
$$\theta = \frac{d}{R}$$

$$\theta = \frac{d}{R} \times \frac{180}{\pi} \times 60 \times 60$$

km
deg
min
sec

$$\frac{12742}{2} \text{ km}$$

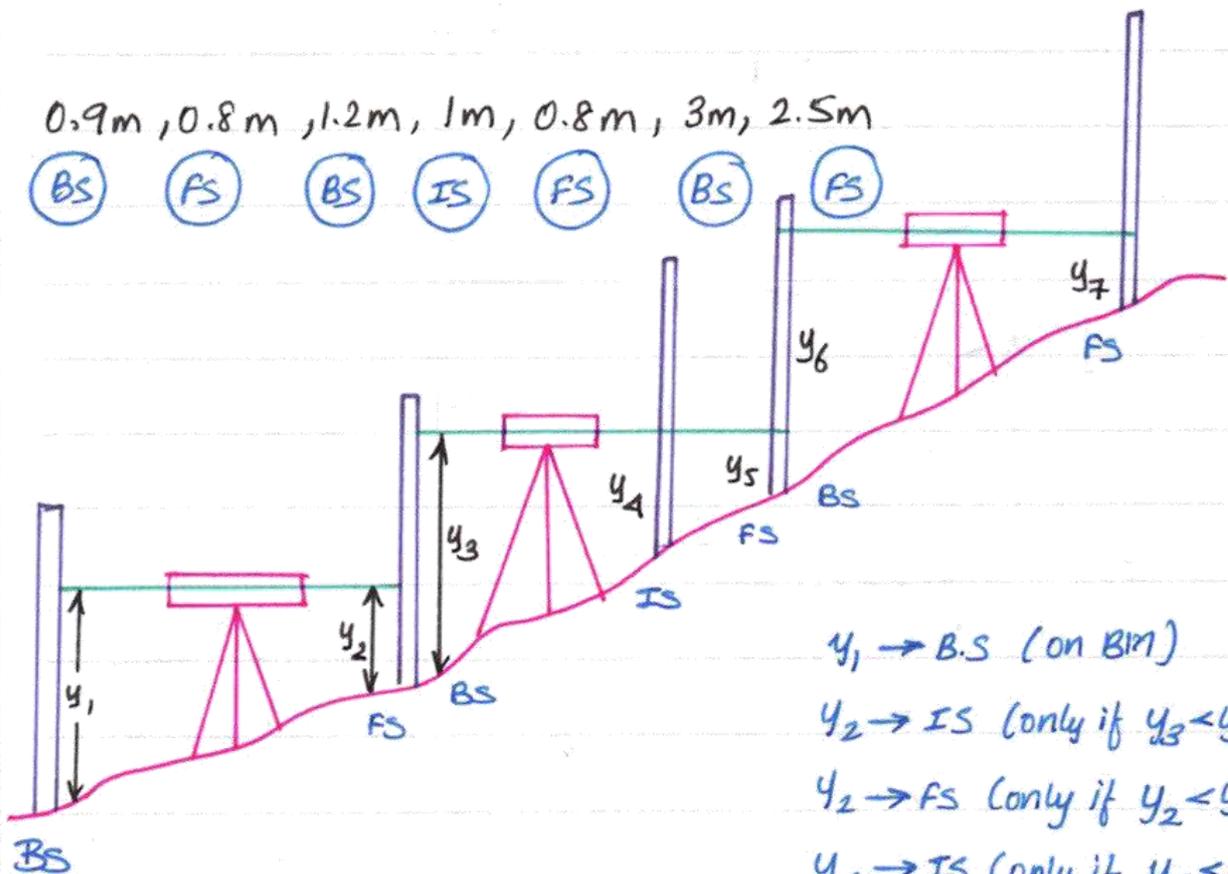




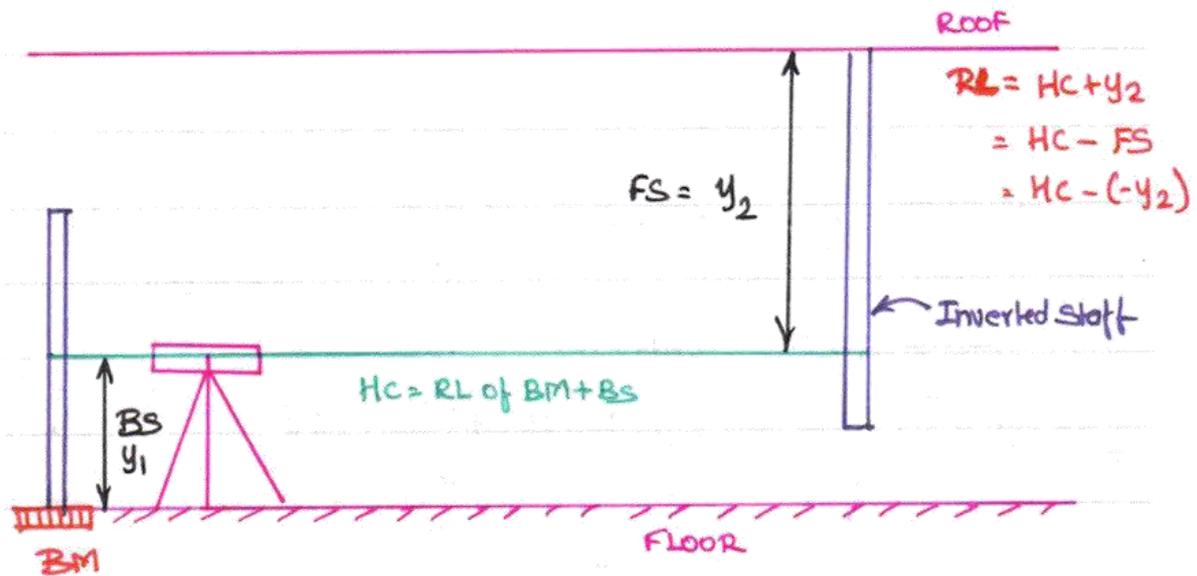
#

0.9m, 0.8m, 1.2m, 1m, 0.8m, 3m, 2.5m

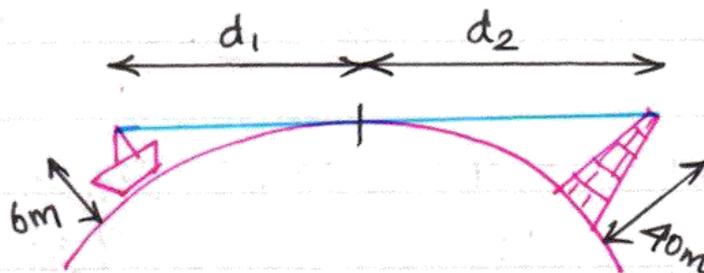
(BS) (FS) (BS) (IS) (FS) (BS) (FS)



$y_1 \rightarrow BS \text{ (on BM)}$
 $y_2 \rightarrow IS \text{ (only if } y_3 < y_2)$
 $y_2 \rightarrow FS \text{ (only if } y_2 < y_3)$
 $y_3 \rightarrow IS \text{ (only if } y_3 < y_2 \text{ and } y_3 > y_4)$



#1. The captain of ship having elevation of his eye 6m above the sea surface just sees (graze) the top of the light house having height 40m above the sea surface. Determine the distance between the ship and the light house.



$$e_{\text{comb}} = 0.0673 d_1^2$$

$$6\text{m} = 0.0673 d_1^2$$

$$d_1 = 9.44 \text{ km}$$

$$40\text{m} = 0.0673 d_2^2$$

$$d_2 = 24.38 \text{ km}$$

$$\text{Distance} = 33.82 \text{ km}$$

- #2. The height of a light house is 40 m. Determine the maximum possible horizontal distance visible from the top of the light house and dip of horizon.



$$e = 0.0673 d^2$$

$$40 = 0.0673 d^2$$

$$d = 24.38 \text{ km}$$

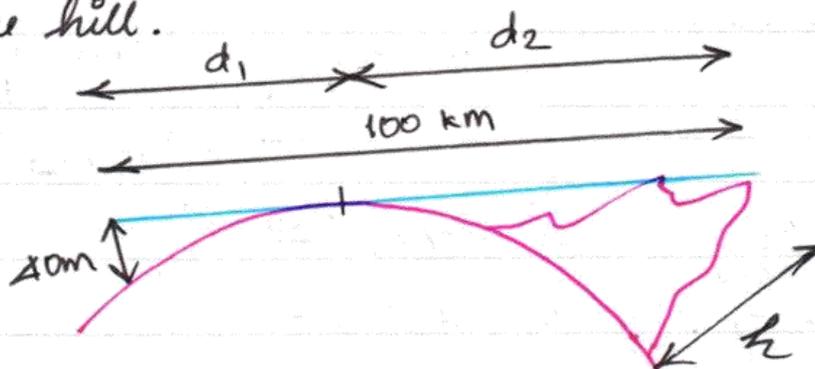
$$\theta = \frac{d}{R} = \frac{24.38}{\frac{12742}{2}}$$

$$= 3.826 \times 10^{-3}$$

$$= 3.826 \times 10^{-3} \times \frac{180}{\pi} \times 60$$

$$\theta = 13.15' \text{ minutes}$$

- #3. A person situated at 40 m above the sea surface just sees the top of a hill. The distance between the hill and that person is 100 km. Determine the height of the hill.



$$40 \text{ m} = 0.0673 d_1^2$$

$$d_1 = 24.38 \text{ km}$$

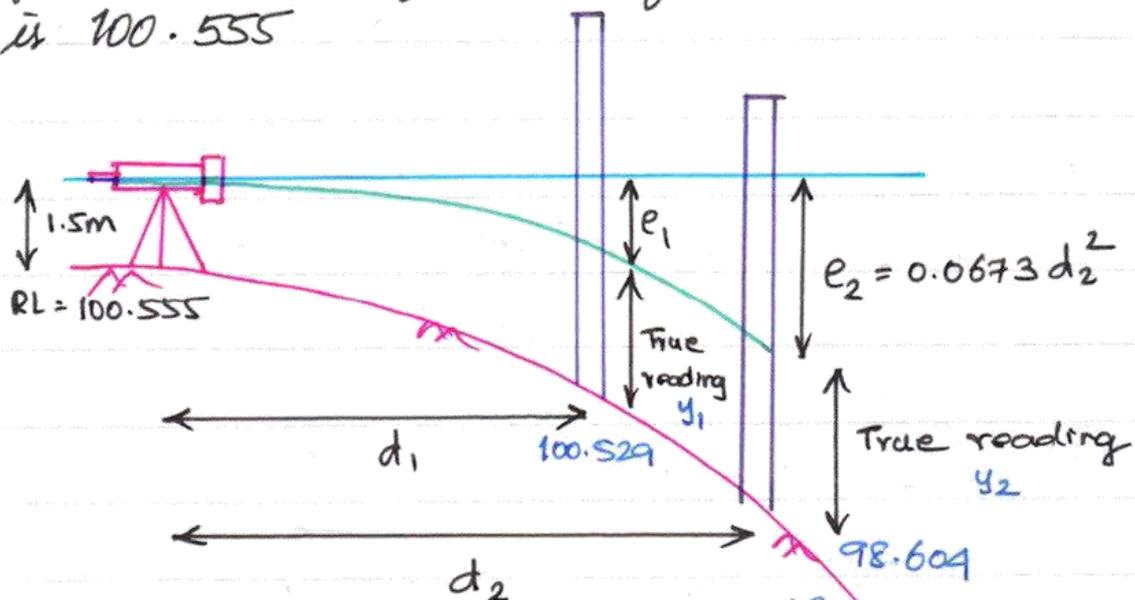
$$d_2 = 100 - d_1$$

$$= 75.62 \text{ km}$$

$$h = 0.0673 d_2^2$$

$$= 384.85 \text{ m}$$

#4. Determine the true difference between two ground stations where staff readings are 1.527 and 3.457. The distance between instrument and staff station are 100 m and 300 m. Also determine the RL of ground stations if the RL of instrument station is 100.555



$$e_1 = 0.0673 \left(\frac{100}{1000} \right)^2$$

$$y_1 = 1.527 - e_1$$

$$= 1.526 \text{ m}$$

$$e_2 = 0.0673 \left(\frac{300}{1000} \right)^2$$

$$y_2 = 3.457 - e_2$$

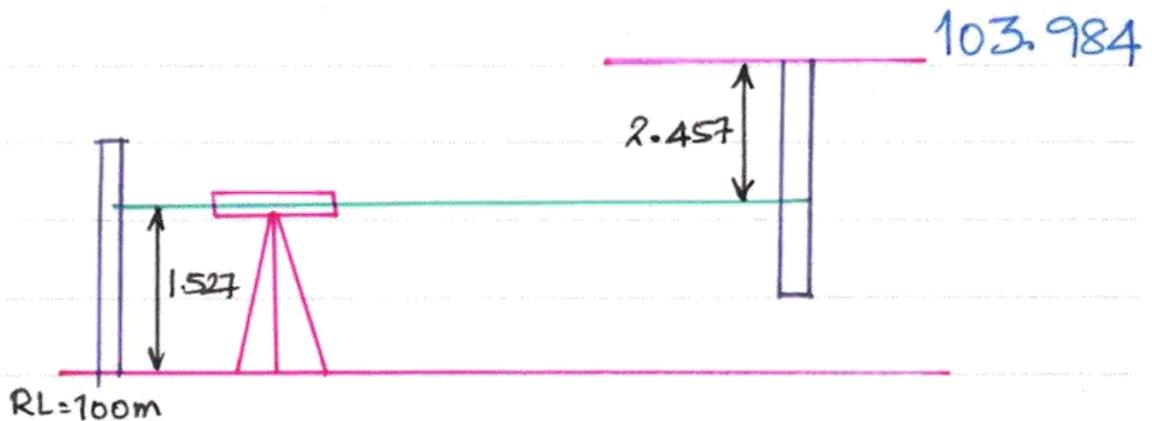
$$= 3.451 \text{ m}$$

Diff in level

$$= 100.529 - 98.604$$

$$= 1.925$$

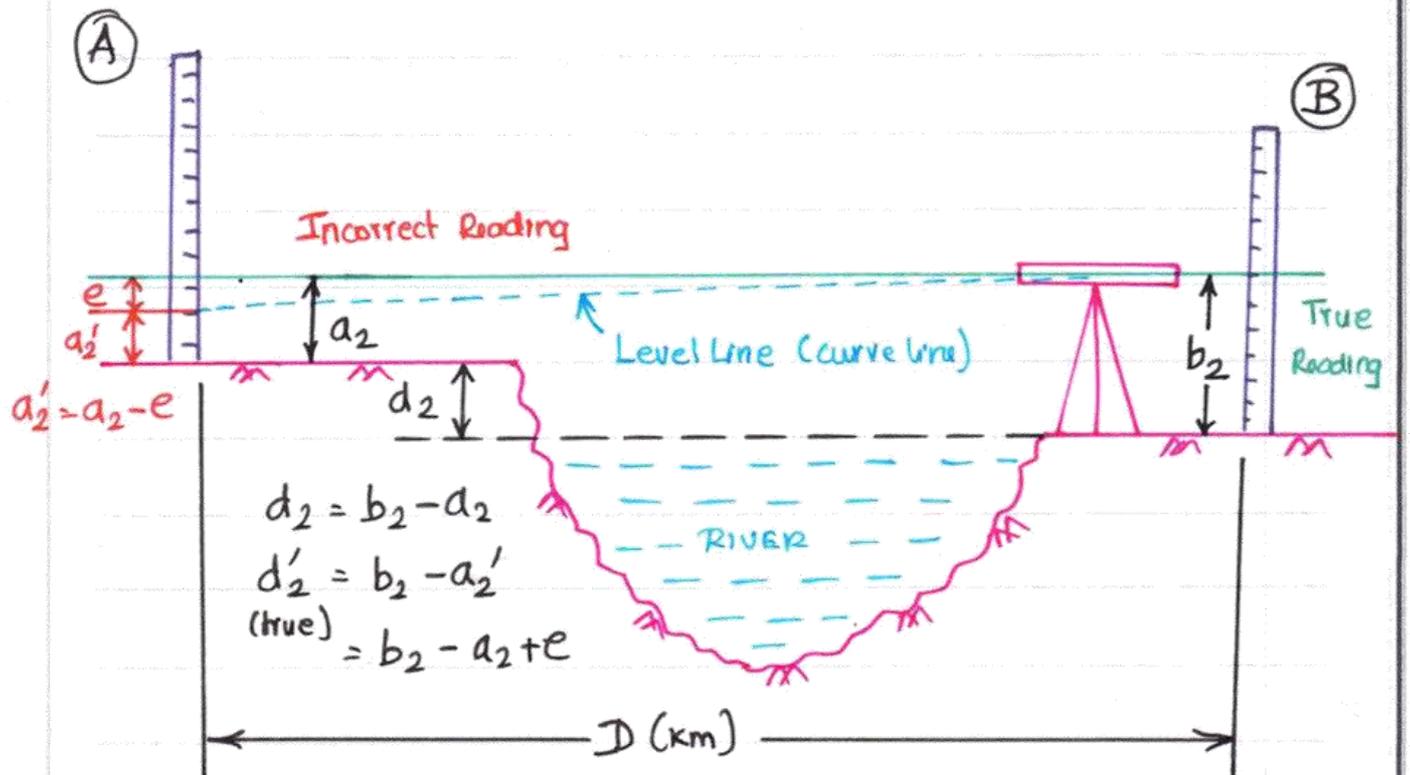
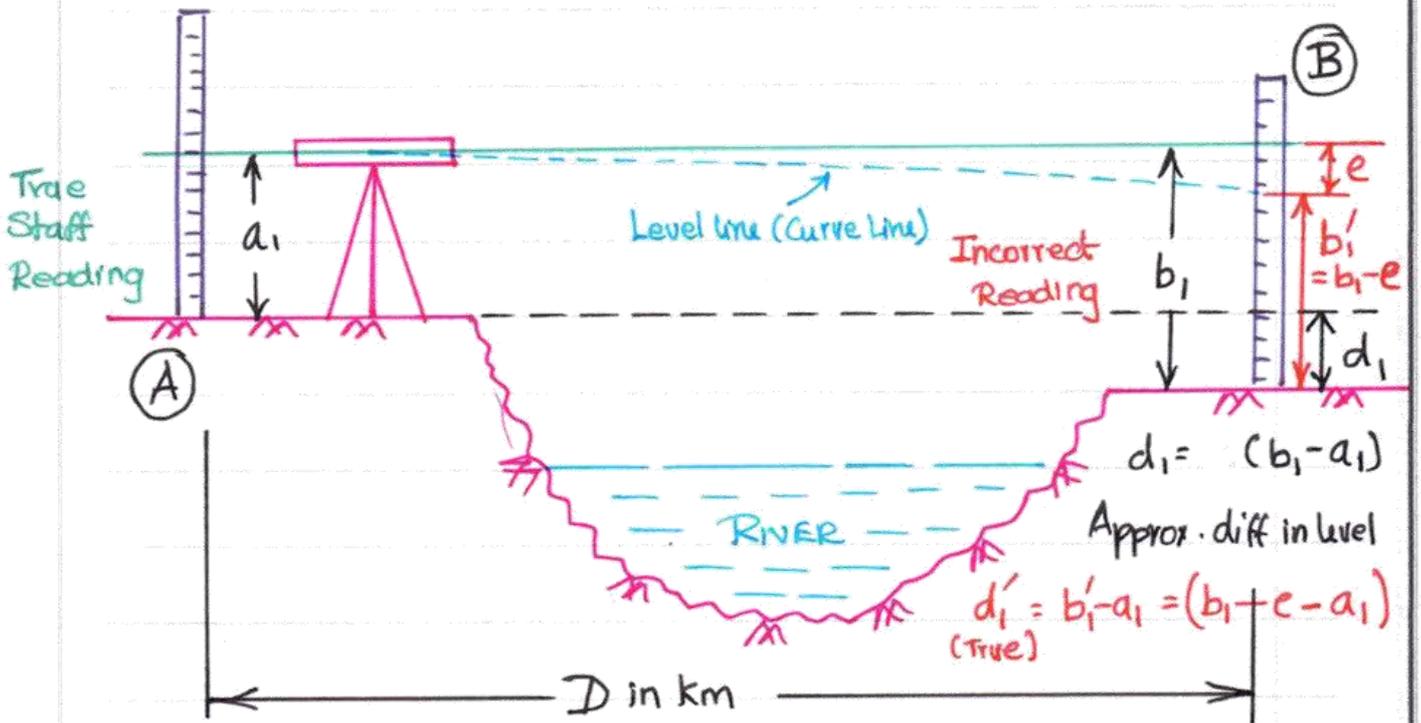
#5. The staff reading on a bench mark having RL = 100m is 1.527. The reading on an inverted staff touching the ceiling of a roof is 2.457 m. Determine the RL of the ceiling.



*** RECIPROCAL LEVELLING

1. Reciprocal levelling is a type of ** spirit levelling* where two sets of staff readings are observed (recorded)
2. Reciprocal levelling is adopted under following conditions / circumstances.
 - a) When it is not possible to keep the instrument at middle of two ground stations.
 - b) If it is not possible to make the distance between instrument and the backsight equal to the distance between instrument and the foresight.
 - c) To determine the difference in banks of a river.
 - d) Levelling work of river across the cross section.
 - e) Levelling work of valley zone.
 - f) Levelling work for a long distance having obstacles.
 - g) Error due to curvature (positive) is to be eliminated.
 - h) Error due to refraction (negative) is to be eliminated.
 - i) Combined error is to be eliminated. (Positive)
 - j) Instrumental error (line of collimation) is to be eliminated.
3. In reciprocal levelling the true difference in level is determined by taking the average of two values of difference in levels.

During taking the average value, the error due to curvature, refraction and instrumental error cancelled, as shown in figure below.



$$d_{true} = d_{av} = \frac{d_1 + d_2}{2}$$

$$d'_{true} = d'_{av} = \frac{d'_1 + d'_2}{2}$$

$$d_{av} = \frac{(b_1 - a_1) + (b_2 - a_2)}{2}$$

$$\begin{aligned} d'_{av} &= \frac{(b_1' - a_1) + (b_2 - a_2')}{2} \\ &= \frac{(b_1 - e - a_1) + (b_2 - a_2 + e)}{2} \\ &= \frac{b_1 - \cancel{e} - a_1 + b_2 - a_2 + \cancel{e}}{2} \\ &= \frac{(b_1 - a_1) + (b_2 - a_2)}{2} \end{aligned}$$

$$d_{true} = d'_{true} //$$

Determination of error in line of collimation :-

1. Find out the approximate difference in level,
 $d_1 = b_1 - a_1$
 (approx)
2. Find out the true difference in level $d = \frac{(b_1 - a_1) + (b_2 - a_2)}{2}$
 (True)
3. Find out the total error, $d_{(approx)} - d_{(True)}$
4. Total error consists of
 - a) Error due to curvature, $e = 0.0785 D^2$ (+) ^{← km}
 - b) Error due to refraction, $e = 0.0112 D^2$ (-) ($\frac{1}{3} \times e_{curv}$)
 - c) Error in line of collimation, (Unknown)

NOTE:- If it is asked to determine, error due to refraction, where error due to line of collimation is given, the $e_{\text{refraction}} = 0.0112D^2$ shall NOT be used

5. Find out the error in line of collimation,

$$e_{\text{total}} = e_{\text{curvature}} - e_{\text{refraction}} + e_{\text{line of collimation}}$$

$$\begin{array}{l} d_1 \quad - \quad d \\ \text{(approx)} \quad \text{(true)} \end{array} = 0.0785D^2 - 0.0112D^2 + e_{\text{line}}$$

$$\left[(b_1 - a_1) - \left[\frac{(b_1 - a_1) + (b_2 - a_2)}{2} \right] \right] = 0.0785D^2 - 0.0112D^2 + e_{\text{line}}$$

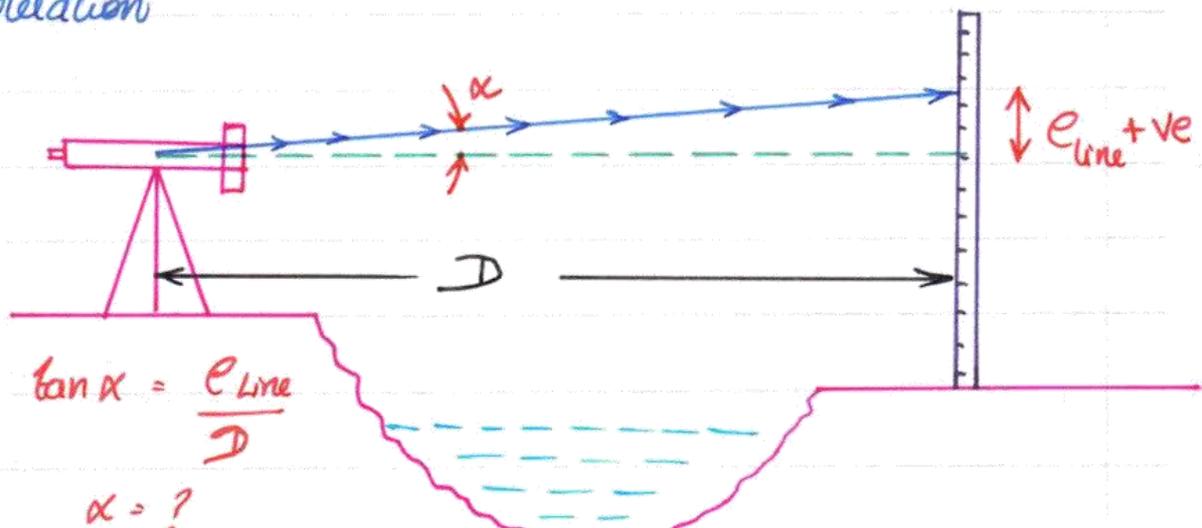
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D in km

$$e_{\text{line}} = ?$$

= +ve Upward line of collimation

= -ve Downward line of collimation

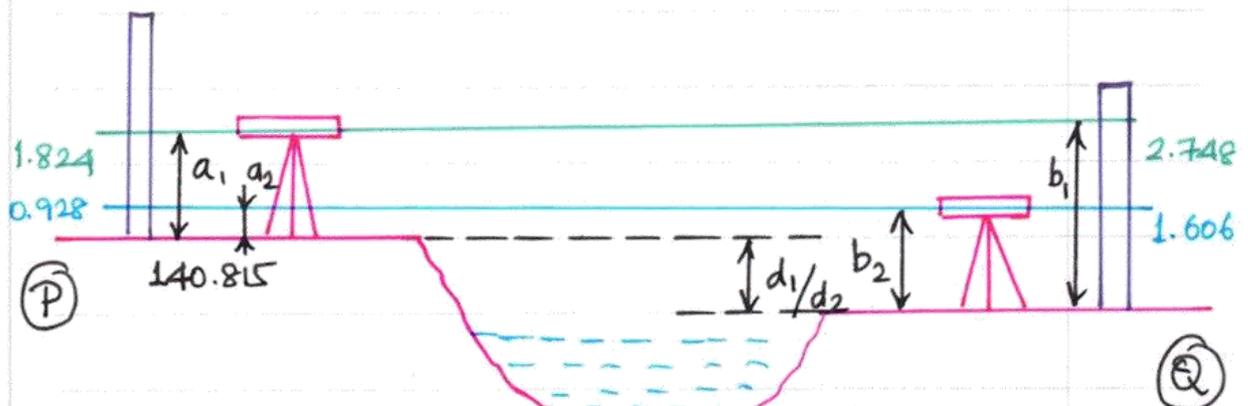
6. The angular error in line of collimation is determined by relation



#1. Following observations were taken during a reciprocal levelling. If RL of P is 140.815. Determine the RL of Q.

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Instrument Position	Staff Position (P)	Staff Position (Q)
Instrument near P	1.824	2.748
" near Q	0.928	1.606



$$d_1 = b_1 - a_1$$

$$= 2.748 - 1.824 = 0.924 \text{ m}$$

$$d_2 = b_2 - a_2$$

$$= 1.606 - 0.928$$

$$= 0.678 \text{ m}$$

$$d_{\text{true}} = \frac{d_1 + d_2}{2} = \frac{0.924 + 0.678}{2} = 0.801 \text{ m}$$

$$\text{RL of Q} = \text{RL of P} - d_{\text{true}}$$

$$= 140.815 - 0.801$$

$$= 140.014 \text{ m}$$

- #2. Two stations P & Q are on the opposite bank of a river. The RL of P is 200.000. Determine the RL of Q. Take staff readings as below.

Instrument	Staff reading at	
	P	Q
near P	1.400	3.500
Q	0.600	2.200

$$d_1 = 3.5 - 1.4 = 2.1$$

$$d_2 = 2.2 - 0.6 = 1.6$$

$$d_{\text{true}} = \frac{d_1 + d_2}{2} = 1.85$$

$$\text{RL of Q} = 200.000 - 1.85 = 198.150$$

- #3. Following observations were taken during a reciprocal levelling. The distance between P & Q is 1100 m. Determine true difference in level.

Error due to curvature effect

Error due to refraction effect

Error in line of collimation

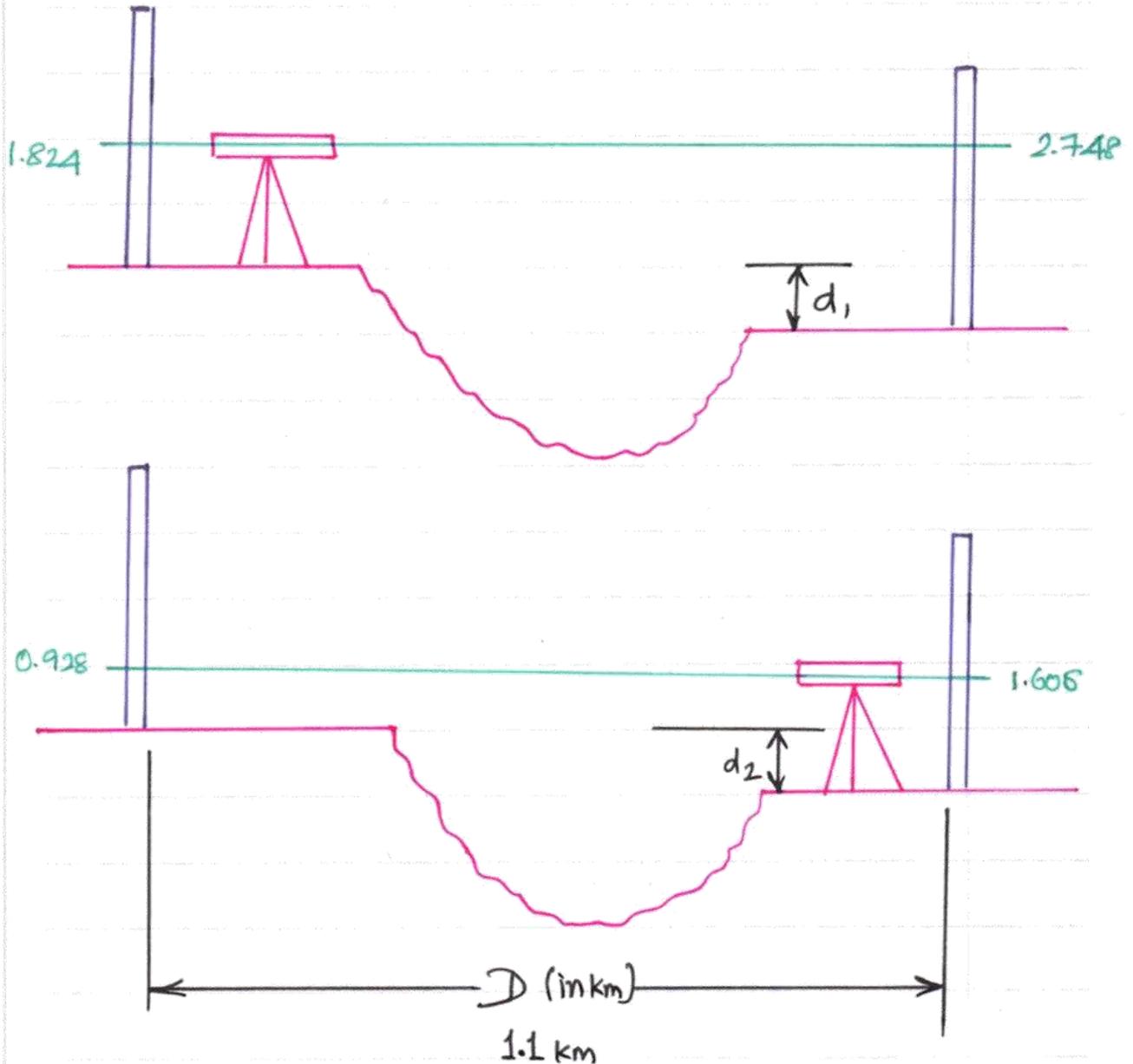
Angular error

Instrument Near	P	Q
Staff @ P	1.824	0.928
Staff @ Q	2.748	1.606

$$d_1 = 1.824 - 2.748 = 0.924 \text{ (with error)}$$

$$d_2 = 1.606 - 0.928 = 0.678 \text{ (with error)}$$

$$d_{\text{True}} = \frac{0.924 + 0.678}{2} = 0.801 //$$



$$\begin{aligned} e_{\text{(curvature)}} &= 0.0785 D^2 \\ &= 0.0785 (1.1)^2 \\ &= 0.09499 \text{ m} \end{aligned}$$

$$\begin{aligned} e_{\text{(refraction)}} &= 0.0112 D^2 \\ &= 0.0112 \times 1.1^2 \\ &= 0.01355 \end{aligned}$$

$$\begin{aligned}
 \text{Total error}_1 &= d_{(\text{approx})} - d_{\text{true}} \\
 &= 0.924 - 0.801 \\
 &= 0.123
 \end{aligned}$$

$$\begin{aligned}
 \text{Total error}_2 &= d_2(\text{approx}) - d_{\text{true}} \\
 &= 0.676 - 0.801 \\
 &= -0.123
 \end{aligned}$$

Angular error:

$$\begin{aligned}
 \text{Total error} &= e_{\text{curv}} - e_{\text{refrac}} + e_{\text{Line}} \\
 0.123 &= 0.09499 - 0.01355 + e_{\text{Line}_1}
 \end{aligned}$$

$$e_{\text{Line}_1} = 0.04156 \text{ m (upward)}$$

$$-0.123 = 0.09499 - 0.01355 + e_{\text{Line}_2}$$

$$e_{\text{Line}_2} = -0.204 \text{ (Downward)}$$

Angular error,

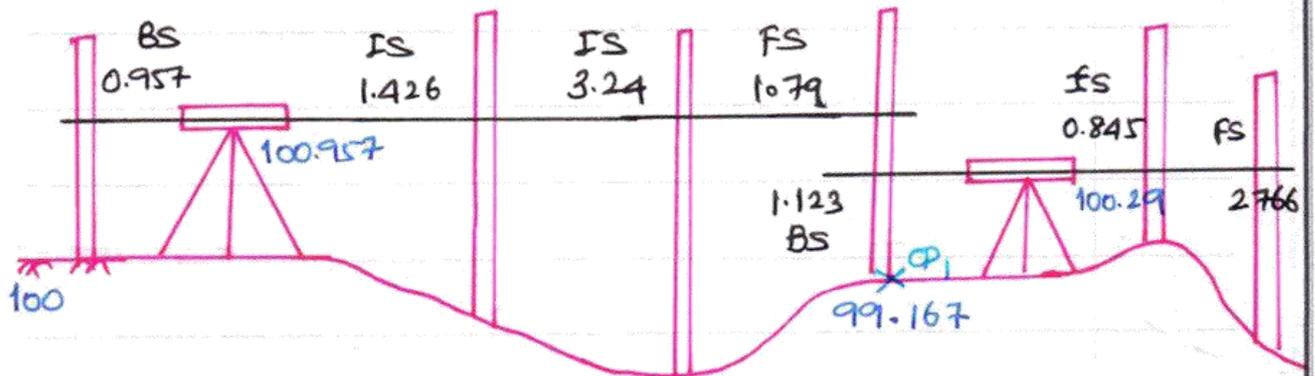
$$\tan \alpha_1 = \frac{e_{\text{Line}_1}}{D} = \frac{0.04156}{1100}$$

$$\alpha_1 = 0^\circ 1' 17.93'' \quad 0^\circ 0' 7.79''$$

$$\tan \alpha_2 = \frac{e_{\text{Line}_2}}{D} = \frac{-0.123}{1100}$$

$$\alpha_2 = -0^\circ 0' 38.25''$$

#1. Determine the RL of change point if the RL of bench mark is 100.000 with the following data.
 The instrument has been shifted after 4th and 7th reading
 0.957, 1.426, 3.24, 1.79, 1.123, 0.845, 2.766, 2.222, 1.111



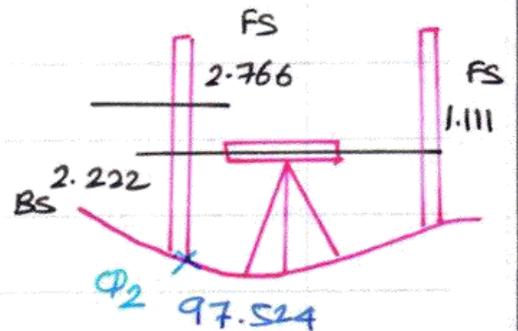
$$HC_1 = 100.000 + 0.957 = 100.957$$

$$RL \text{ of } 1^{st} \text{ C.P} = 100.957 - 1.79$$

$$HC_1 - FS = 99.167$$

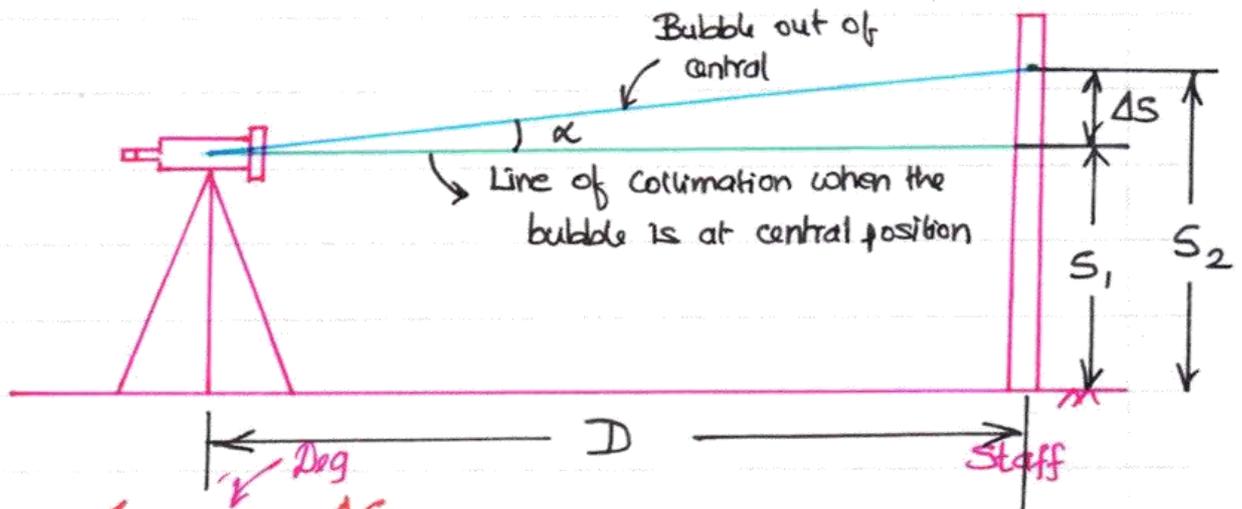
$$HC_2 = 99.167 + 1.123 = 100.29$$

$$RL \text{ of } 2^{nd} \text{ C.P} = 100.29 - 2.766 = 97.524$$



Sensitiveness of bubble tube :-

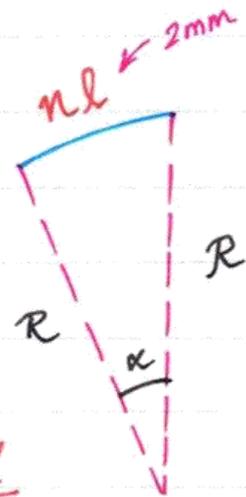
It is defined as the angular value of one division of displacement of bubble having unit rad/mm or sec/mm which is determined by relation



$$\tan \alpha = \frac{\Delta S}{D}$$

rad $\rightarrow \alpha = \frac{\Delta S}{D}$

deg $\rightarrow \alpha = \frac{\Delta S}{D} \times \frac{180}{\pi} \times 60 \times 60$ (seconds)



$$\alpha = \frac{nl}{R}$$

$$\alpha = \frac{S_2 - S_1}{D} = \frac{nl}{R}$$

Sensitiveness = α per division

$$\frac{\alpha}{n} = \frac{l}{R}$$

- #1. Determine the radius of the bubble and angular error if the bubble moves 3 div of out of the centre and the staff readings are 1.456 and 1.500 if the bubble is at central and out of the central respectively. Take distance between staff and instrument as 100 m.

$$\alpha = \frac{S_2 - S_1}{D} = \frac{1.500 - 1.456}{100}$$

$$= 4.4 \times 10^{-4} \times \frac{180}{\pi} \times 60 \times 60$$

$$= 90.756 \text{ seconds}$$

$$\alpha = \frac{n l}{R}$$

$$\frac{1.5 - 1.456}{100} = \frac{3 \times 2 \text{ mm}}{R}$$

$$R = 13.636 \text{ mm}$$

- #2. Determine the true staff reading if the observed staff reading is 1.765 m and the sensitiveness of the bubble is $30''$. Assume the bubble is out of central by 4 div.

$$\alpha = \frac{S_2 - S_1}{D}$$

$$\frac{\alpha}{n} = 30''$$

$$4 \times 1.454 \times 10^{-4} = \frac{1.765 - S_1}{100}$$

$$\alpha = \frac{n \times 30''}{60 \times 60 \times \frac{180}{\pi}}$$

$$S_1 = 1.7068 \text{ m}$$

$$\alpha = n \times 1.454 \times 10^{-4} \text{ deg rad}$$

1. Height of collimation method :- (HC)

i) In this method the RL of one of the points is established with the help of back sight on the bench mark. i.e., $HC = RL \text{ of Bench Mark} + \text{Back sight}$

ii) The levelling staff is placed at known distant point and the staff readings are obtained called intermediate sights. (IS)

iii) The RL of all intermediate sights are determined by subtracting the IS from the height of collimation. i.e., $RL \text{ of Intermediate sight} = HC - IS$

iv) At the change point (CP), there are ^{*} two readings on the same staff (FS & BS). With the help of fore sight, the RL of change point is obtained and with the help of Back sight, the RL of new height of collimation is obtained.

v) This method is less tedious.

vi) This method is rapid.

vii) This method requires less calculations.

viii) This method is mainly adopted for profile levelling. i.e. field work at the site to layout actual (structure) work.

ix) The greatest drawback of this method is that any mistake made at intermediate sight cannot be detected. i.e., it remains as it is.

x) In this method, the arithmetical check applied is

$$\sum BS - \sum FS = \text{Last RL} - \text{First RL}$$

$$\# \quad \text{If } \Sigma BS = 4.756$$

$$\Sigma FS = 3.125$$

$$\text{First RL} = 200.123$$

$$\text{Observed last RL} = 202.985$$

Determine the error in levelling?

$$\Sigma BS - \Sigma FS = \text{True Last RL} - \text{First RL}$$

$$4.756 - 3.125 = \text{TRUE Last RL} - 200.123$$

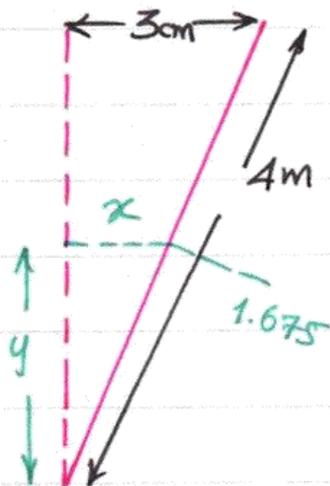
$$\text{True Last RL} = 201.754$$

$$\therefore \text{Error} = \text{True Last RL} - \text{Observed Last RL}$$

$$= 201.754 - 202.985$$

$$= -1.231$$

If a 4m staff is 3cm away from the plumbline then what will be the true staff reading if observed staff reading is 1.675



$$\frac{0.03}{x} = \frac{4}{1.675}$$

$$x = 0.0125 \text{ m}$$

$$y = \sqrt{1.675^2 - 0.0125^2}$$

$$y = 1.6749 \text{ m}$$

2. Rise and Fall Method

i) In this method, the RL of intermediate points are determined with the help of comparison of successive staff readings only. i.e., the height of collimation is not used at all. If the staff reading is higher then the ground station will be at lower level but if the staff reading is lower, then the ground station will be at higher level.

ii) The RL at successive points = $\left[\text{RL at the Bench Mark} + \begin{matrix} \text{Rise} \\ \text{Fall} \end{matrix} \right]$
 or $\left[\text{RL of the Bench mark} - \text{Fall} \right]$

iii) If $y_2 > y_1$, then RL of the second point
 = RL at the first point - $(y_2 - y_1)$ (Fall)
 If $y_2 < y_1$, then RL of second point
 = RL at the first point + $(y_1 - y_2)$ (Rise)

iv) This method is mainly adopted for differential levelling. i.e., to fix the bench mark at a known distance away from the permanent bench mark.

v) In this method, there is a complete check at every ground station and hence the error at intermediate stations can be detected easily.

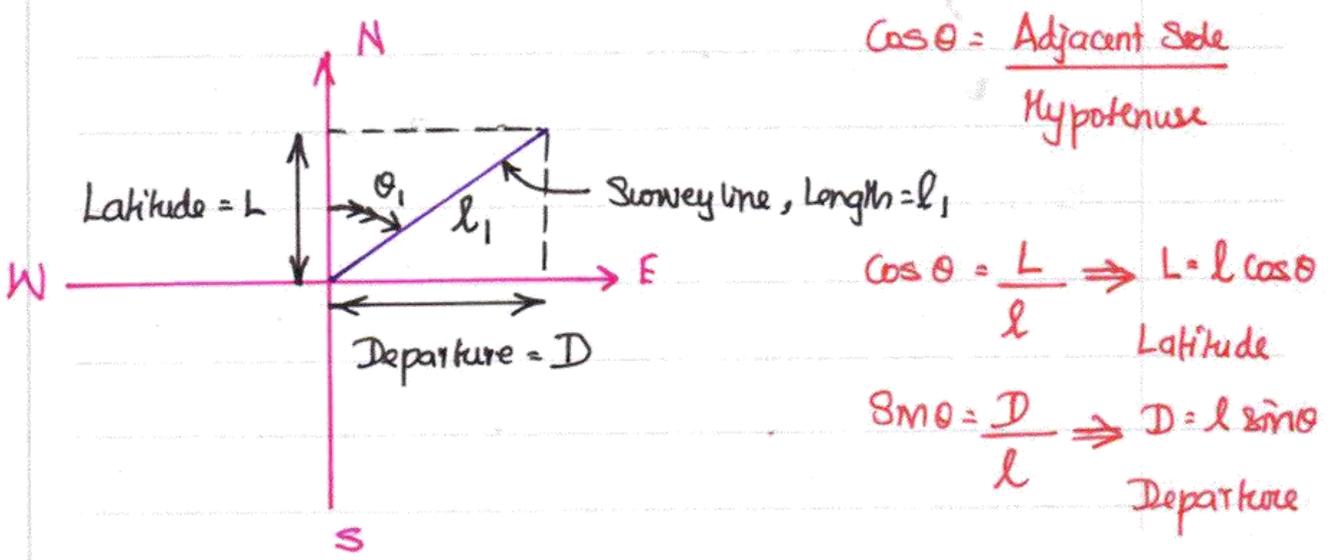
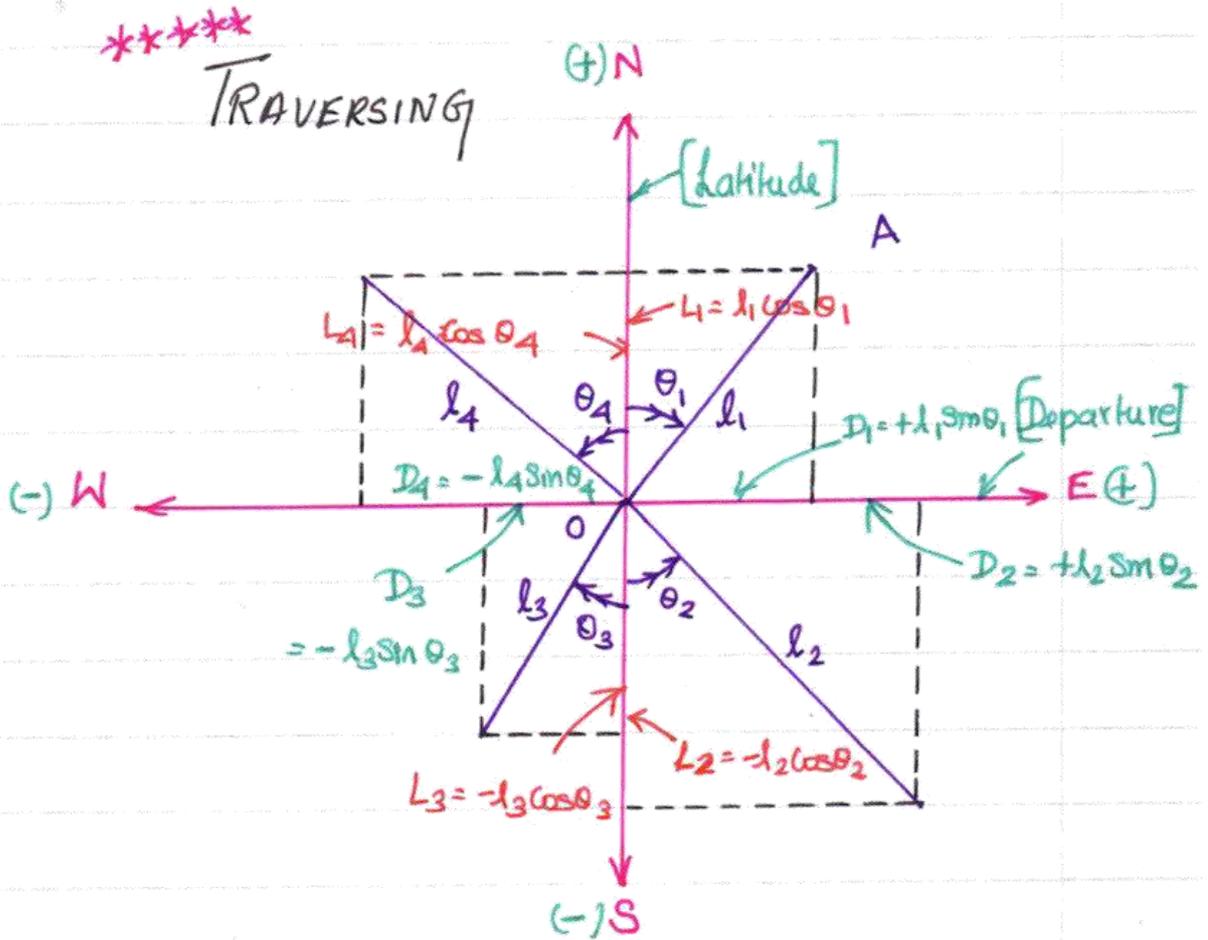
vi) There is an arithmetical check.

$$\sum BS - \sum FS = \sum \text{Rise} - \sum \text{Fall} = \text{Last RL} - \text{First RL}$$

Classifications of Levelling :-

Name of levelling	Description
1. Differential levelling	Mainly used to determine RL at far away points from the permanent
	sly levelling where BS and FS are only observed.
2. Profile levelling	It is the actual levelling adopted at worksight to construct railway, highway or canal where every intermediate sight is considered accurately at fixed interval of distances. In this levelling all types of sights (BS, FS, IS) are observed. It is also known as longitudinal levelling or sectioning.
3. Cross sectioning	In this type of levelling the undulations of ground surface in a transverse direction are obtained. i.e., To find the cross sectional area of earth work it is required.
4. Reciprocal levelling	In this method, two sets of staff reading are required if the instrument cannot be kept at mid way between two points. It is mainly used for survey of rivers, valley and marshy land.

5. **Barometric levelling** → In this method the curvature, refraction and instrumental errors are eliminated. In this method the atmospheric pressure is determined at two different elevations with the help of a barometer. The difference of atmospheric pressure, $\Delta p = \gamma h$, $\therefore h = \frac{\Delta p}{\gamma}$
- It is mainly used to find the approx. height of mountain or hill. $\rho_{air} = 1.24 \text{ kg/m}^3$
 $\gamma = \rho g$
6. **Hypsometry levelling** → In this method the temperature at which water boils (boiling point) is determined at the two different elevations and then the fundamental relations of pressure and temperature (**Charles' Law**) to determine the elevation between two points - It is mainly used to determine the approx. height of the mountain or hill.
7. **Trigonometrical levelling** → In this method the elevation of any point above the ground surface is determined with the help of horizontal measurement of distance and the vertical angle of the top of the object. The relation used is $\tan \theta = \frac{y}{x}$, $\therefore y = x \tan \theta$
- (Not part of spirit levelling)



- Latitude (L) = Co-ordinate length of a survey line measured along meridian direction. i.e. Along N-S.
 $L = +l \cos \theta$, measured along North direction
 $\theta \rightarrow$ NE or NW

$$L = -l \cos \theta,$$

Measured along south direction

$\theta \rightarrow$ SE or SW

2. Departure (D) = Co-ordinate length of survey line measured perpendicular to meridian direction. i.e., E-W. direction.

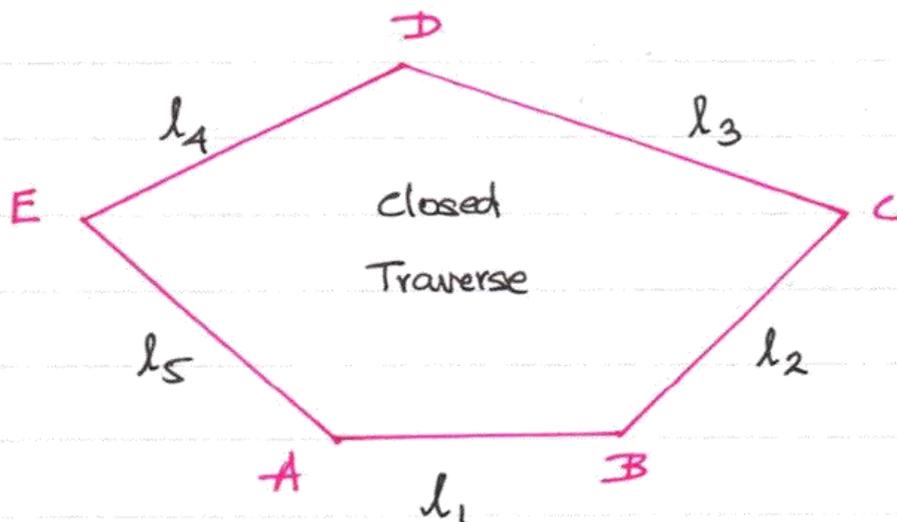
$$D = (+)l \sin \theta, \text{ measured along East}$$

$\theta \rightarrow$ NE, SE

$$D = (-)l \sin \theta, \text{ measured along West}$$

$\theta \rightarrow$ NW, SW

3. For a closed traverse.



$$\pm L_{AB} \pm L_{BC} \pm L_{CD} \pm L_{DE} \pm L_{EA} = \text{Zero}$$

$$\sum L_{(all)} = \text{ZERO}$$

$$l_1 = ?$$

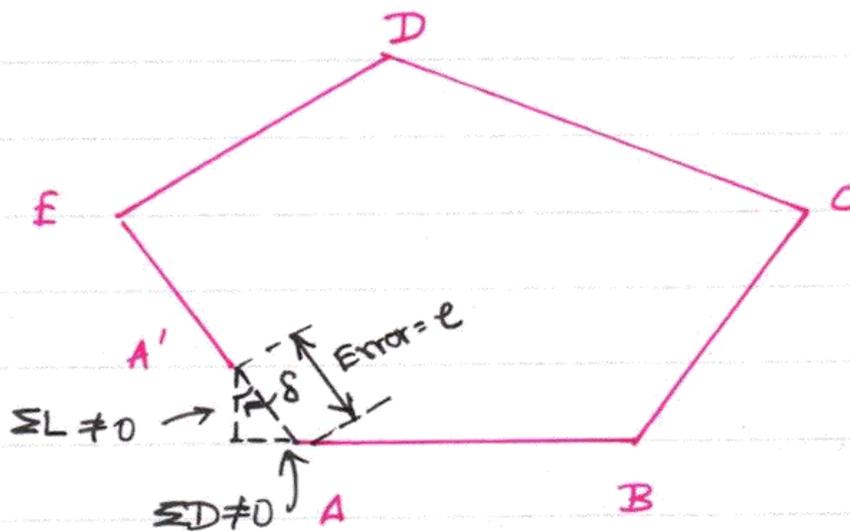
$$\theta_1 = ?$$

$$\pm D_{AB} \pm D_{BC} \pm D_{CD} \pm D_{DE} \pm D_{EA} = \text{Zero}$$

$$\sum D_{(all)} = \text{Zero}$$

But if $\sum L \neq 0$ or $\sum D \neq 0$ or $\sum L \neq 0$ & $\sum D \neq 0$

Error in closed traverse occurs //



$$\text{Error, } e = \sqrt{(\sum L)^2 + (\sum D)^2}$$

AA'

$$\text{Angular error, } \tan s = \frac{\sum D}{\sum L}$$

$$s = \tan^{-1} \left(\frac{\sum D}{\sum L} \right)$$

4. Balancing of Error is obtained by **Bowditch rule**

(Most popular rule)

#1. A closed traverse ABCDA is given below. Survey Determine the missing length and Bearing of line FA.

Survey line	Length	Bearing	Latitude	Departure
AB	204m	$87^{\circ}30'$	-8.898	203.805
BC	226m	$20^{\circ}20'$	211.91	78.530
CD	187m	280°	32.472	-184.159
DE	192m	$220^{\circ}3'$	-166.193	-96.145
FA	?	?	L_{FA}	D_{FA}

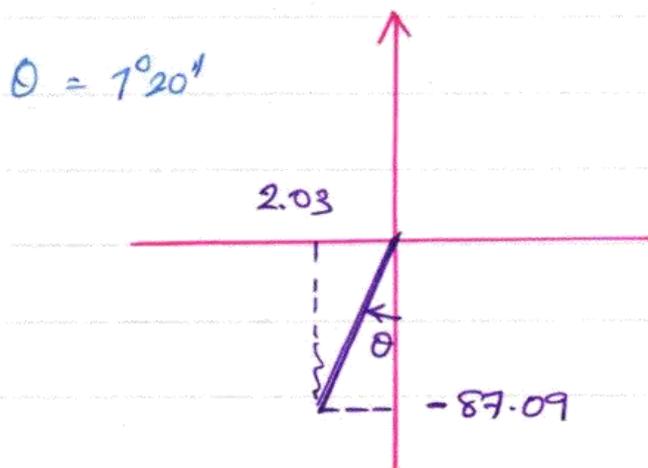
$$\Sigma L = 87.09 \quad \Sigma D = 2.03 + L_{FA} + D_{FA}$$

$$\Sigma L = 0$$

$$87.09 + L_{FA} = 0 \Rightarrow L_{FA} = -87.09 \text{ m}$$

$$\Sigma D = 0$$

$$2.03 + D_{FA} = 0 \Rightarrow D_{FA} = -2.03 \text{ m}$$



$$L_{FA} = \sqrt{L^2 + D^2}$$

$$= \sqrt{(-87.09)^2 + (-2.03)^2}$$

$$= 87.113 \text{ m}$$

$$\approx 87.12 \text{ m}$$

$$WCB = 180^{\circ} + 1^{\circ}20'$$

$$= 181^{\circ}20'$$

#2. A closed traverse ABCDA is given below. Let error in the traverse be. Assume given bearings are magnetic bearing and the declination is $5^{\circ}30'$ West.

Survey line	Length	Bearing MCB	True Bearing	Latitude	Departure
AB	470m	$343^{\circ}52'$	$338^{\circ}22'$	N $21^{\circ}38'W$ 436.89	-173.27
BC	635m	$87^{\circ}50'$	$N82^{\circ}20'E$	84.715	629.324
CD	430m	$172^{\circ}40'$	$167^{\circ}10'$	S $12^{\circ}50'E$ -419.26	95.51
DA	563m	$265^{\circ}12'$	$259^{\circ}42'$	S $7^{\circ}42'W$ -100.665	-553.93
				$\Sigma L = +1.69$	$\Sigma D = -2.37m$

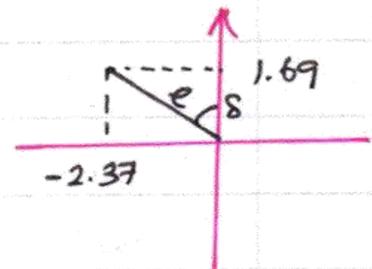
$$T.B = M.B \pm \text{Declination}$$

$$\text{Error } e = \sqrt{(\Sigma L)^2 + (\Sigma D)^2}$$

$$= 2.91$$

$$\delta = \tan^{-1} \left(\frac{2.37}{1.69} \right)$$

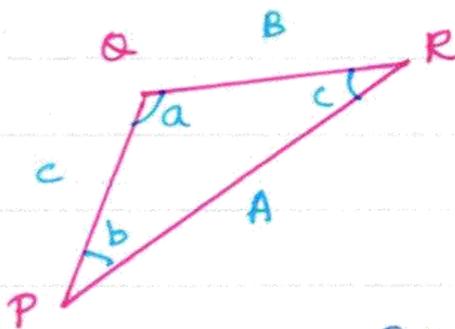
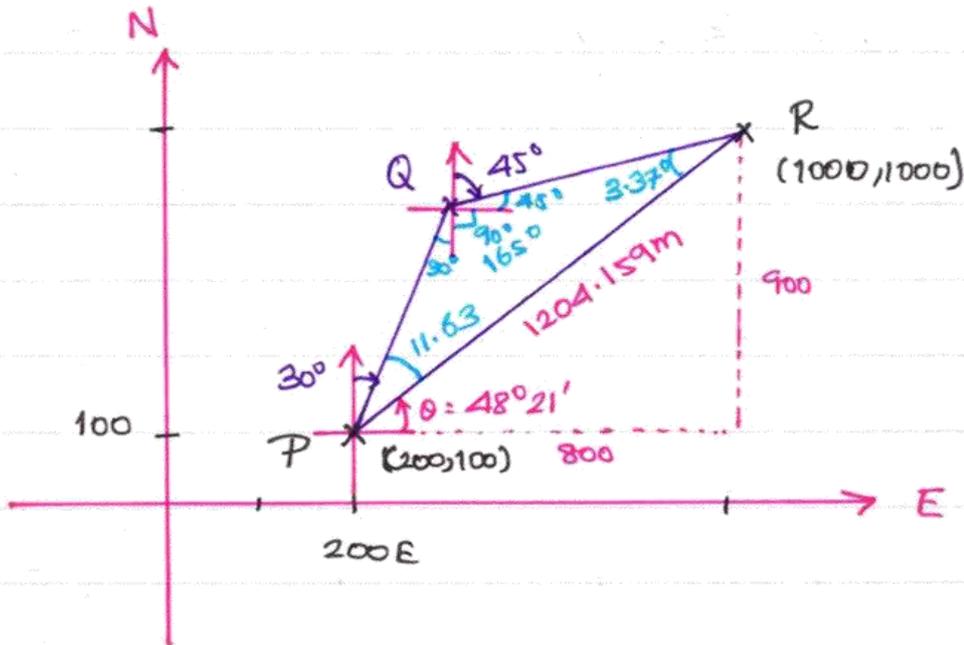
$$= 54^{\circ}30' \text{ (NW)}$$



$$\text{Closing error} = \frac{e}{\text{Total perimeter}} = \frac{e}{\Sigma l_{\text{actual length}}} = \frac{2.91}{2098}$$

$$= 1 \text{ in } 721$$

#3. In a closed traverse PQR has coordinates of P and R as (100, 200) and (1000, 1000) WCB of PQ 30°
 PQ & QR = ? WCB of QR 45°



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin(165^\circ)}{1204.159} = \frac{\sin 11.63}{B} = \frac{\sin 3.37}{C}$$

$$QR = B = 937.9$$

$$PQ = C = 273.49$$

$$PR = A = 1204.159$$

IES
#4.
5m

The following data pertains to a closed traverse ABDC

i) Latitude of A = +542.7m Departure of A = (-) 331.2m

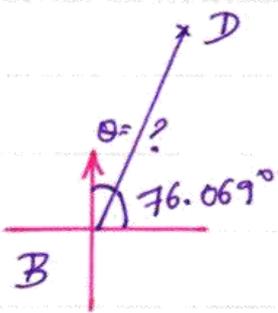
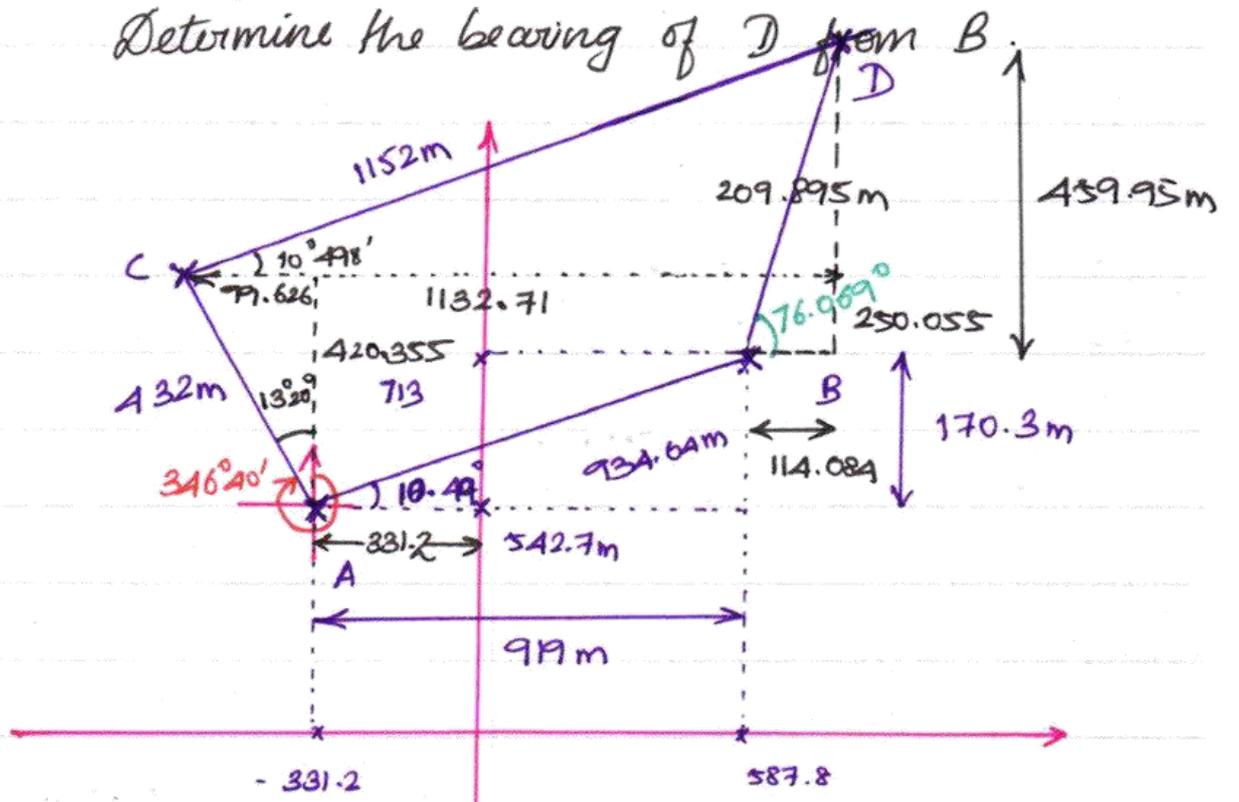
ii) Latitude of B = +713m Departure of B = (+) 587.8m

The distance between Station A and C is 432m

and the azimuth survey line AC = $346^{\circ}40'$

The survey line CD = 1152m which is parallel to survey line AB.

Determine the bearing of D from B.



$$l_{AB} = 568.792 \text{ m}$$

$$l_{AB} = 934.64 \text{ m}$$

$$\alpha = 10.498^{\circ}$$

$$\theta = 13.931 \quad N 13.931 E$$

$$l_{BD} = \sqrt{459.95^2 + 114.084^2}$$

$$= 473.887 \text{ m}$$

#5. A closed traverse ABCDEA has been given in tabular form. Determine the missing lengths of survey lines DE and EA.

Survey line	Length	Azimuth	Latitude $L = +l \cos(\theta(N))$ $-l \cos(\theta(S))$	Departure $D = +l \sin(\theta(E))$ $= -l \sin(\theta(W))$
AB	500m	$98^\circ 30'$	-73.9047	494.51
BC	620m	$30^\circ 20'$	535.123	313.12
CD	468m	$278^\circ 30'$	223.31	-411.286
DE	?	$230^\circ 0'$	$-l_1(0.643)$	$-l_1(0.766)$
EA	?	$150^\circ 10'$	$-l_2(0.867)$	$l_2(0.497)$

$$-l_1(0.643) - l_2(0.867) = -684.528$$

$$-l_1(0.766) + l_2(0.497) = -396.344$$

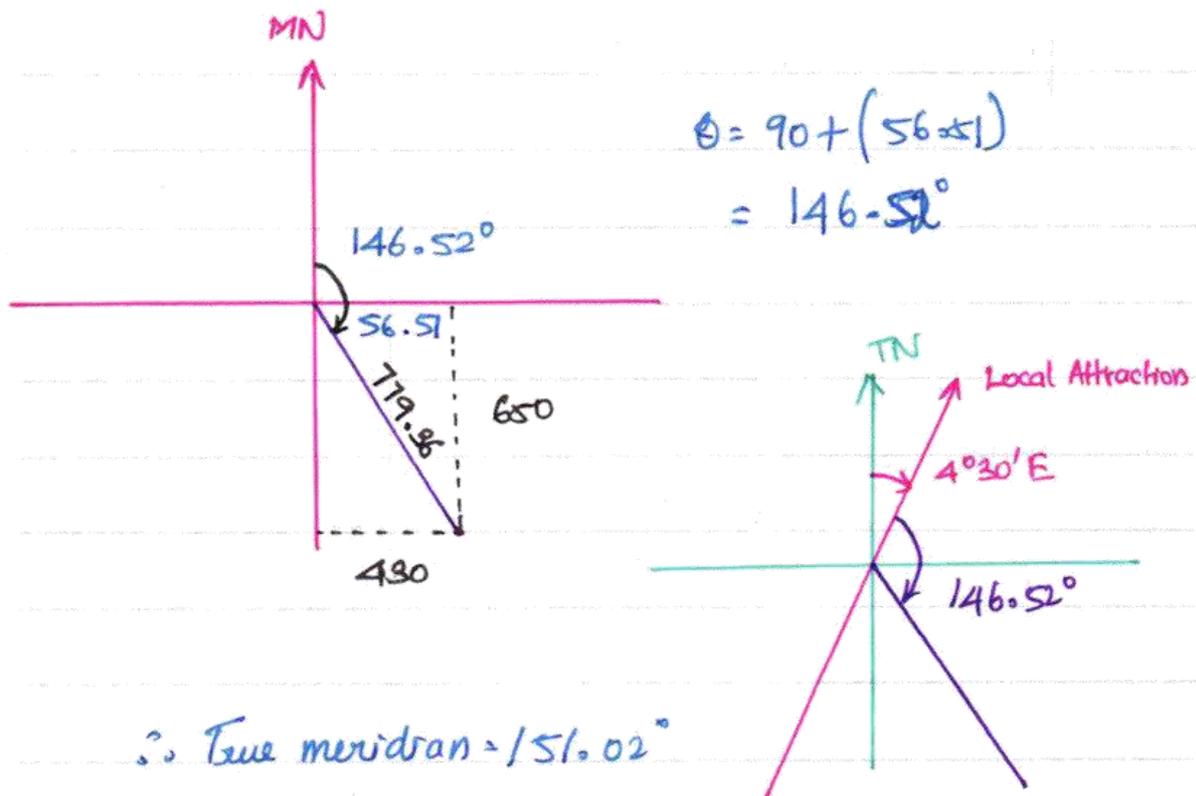
$$l_1 = 695.17$$

$$l_2 = 273.96$$

Second departure

(In survey first always Latitude,

#6. A survey line AB has latitude -650m and departure +430 m. The local attraction at the survey station is $4^\circ 30' E$. Determine true latitude and true departure



$$\text{True Latitude} = 779.36 \cos 151.02^\circ = -681.78$$

$$\text{True Departure} = 779.36 \sin 151.02^\circ = +377.60$$

Balancing the closed traverse :-

1. If the summation of all latitudes or/and summation of all departure of sides of a closed traverse are not equal to zero individually, then an error takes place in measurement of a closed traverse.
2. The error in latitude and departure is adjusted with the given dimensions of traverse, known as Balancing the closed traverse.
3. There are following rules or methods to balance a closed traverse.

- a) BOWDITCH Rule.
- b) Transit Rule.
- c) Graphical method.
- d) Aerial method.

BOWDITCH Rule :-

1. Bowditch rule is mainly adopted where the linear measurement and angular measurement are of equal precision.
- ** 2. The Bowditch rule or method is also known as COMPASS RULE.
3. In this method it is assumed that, the error in linear measurement is directly proportional to \sqrt{l}

$$e \propto \sqrt{l}$$

And error in angular measurement is inversely proportional to $\sqrt{\frac{1}{l}}$

$$e \propto \sqrt{\frac{1}{l}}$$
4. In this method, the correction in latitude and departure is based on $\frac{l}{\sum l}$ proportion of the actual length in a given traverse.

$$C_L = \sum L \times \frac{l}{\sum l}$$

$$C_D = \sum D \times \frac{l}{\sum l}$$