Structural Analysis
Civil Engg.

Dt:- 13/4/2020

Module - III

Analysis of Redundant plane trusses

Truss :- Trusses are most common type of structure used in constructing building roofs, bridges and towers etc.

→ A truss can be constructed by straight slender members joined together at their end by bolting, riveting or welding.

classification of trusses :

The trusses can be classified into

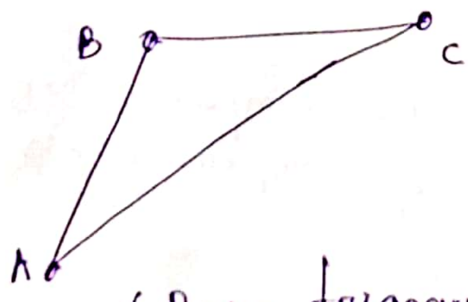
(a) plane truss (2D truss)

(b) space truss (3D truss)

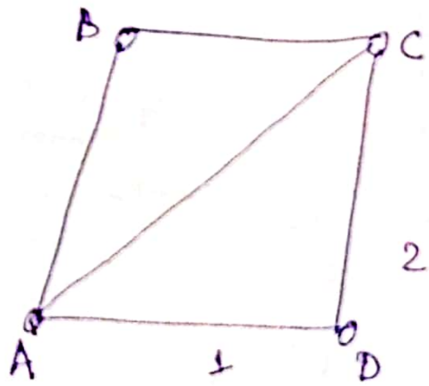
Again plane trusses can also be classified into Simple truss, compound truss and complex truss.

(i) **Simple truss** :- A simple truss is constructed from the basic triangular element as shown in figure. by connecting two members to form an additional element.

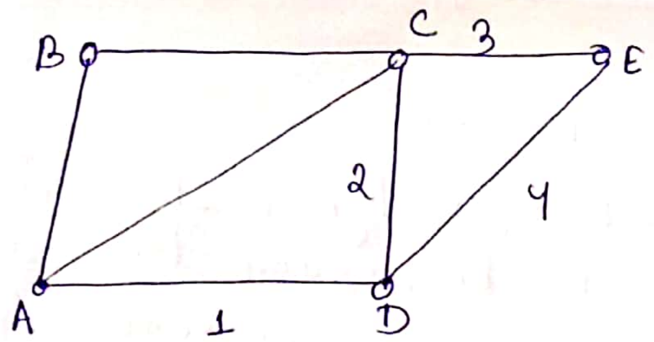
→ The other element can be form by connecting two more members to resulting simple truss.



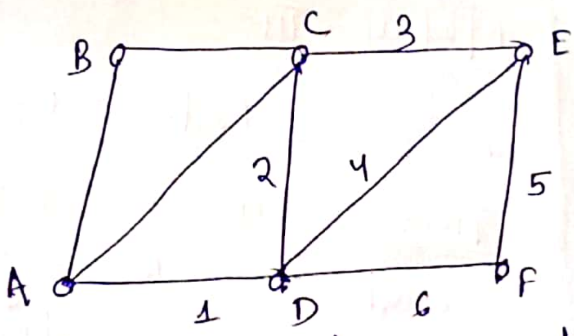
a: (Basic triangular element)



b. (Simple truss)



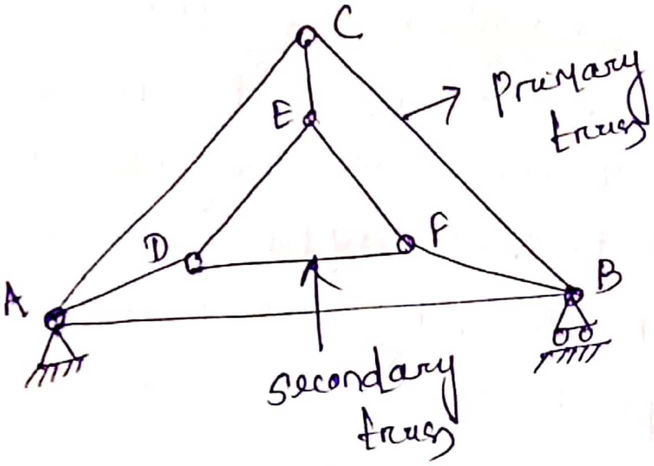
(C) Addition of element



(Resulting simple truss)

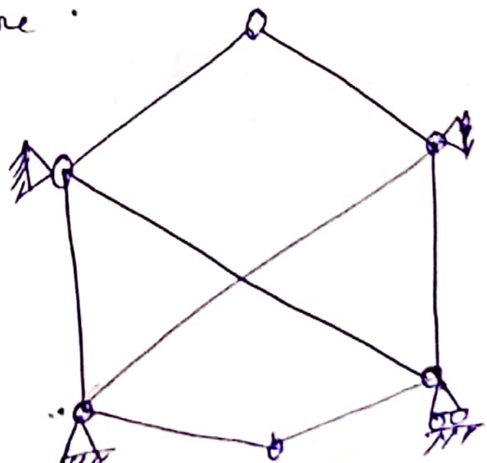
(ii) Compound truss:-

A compound truss is formed by connecting two or more simple trusses together either by hinge or by additional members.



(iii) Complex truss:-

If a truss is neither simple nor compound then its called as complex truss. A complex truss has polygonal structure.



(2)

Stability of trusses :-

(a) External stability :-

For stability of 2D trusses following three condition of equilibrium must be satisfied.

(i) $\Sigma F_x = 0$

(ii) $\Sigma F_y = 0$.

(iii) $\Sigma M_z = 0$.

Total reaction − Equilibrium equation

$r - 3$

(2) Internal stability :-

For plane truss the minimum no. of members needed for geometric stability are

$$M = 2J - 3$$

j = No. of joint

m = No. of members.

Determinate and Indeterminate trusses :-

A truss is said to be determinate when it can be analysed by equation of static equilibrium alone.

→ For determinate truss, the degree of static indeterminacy is zero.

→ If the degree of static indeterminacy is greater than zero then the truss is called indeterminate truss.

→ To analyse the indeterminate truss additional compatibility conditions are required.

The degree of static indeterminacy is given by

$$D_s = M + R - 2j \quad (2D\text{-truss})$$

$$D_s = M + R - 3j \quad (3D \text{ truss})$$

Condition

(1) If $D_s = 0$, then truss is stable and determinate. Such trusses are called as perfect trusses

(2) If $D_s > 0$ then the truss is stable but indeterminate or overstiff

(3) If $D_s < 0$, then truss is ~~not~~ unstable or deficient.

Methods of Analysis of determinate truss

The determinate and stable truss can be analyzed by using equilibrium methods such as

(i) Method of joints

(ii) Method of section

(iii) Graphical method (Willot-Mohr diagram)

(iv) Bar chain method.

Method of Joints:

This method is suitable when forces in all members are to be calculated.

→ At each joint two equation of equilibrium are available for a truss.

③

At each joint of a truss there are two equation of equilibrium such as $\Sigma F_x = 0$

$\Sigma F_y = 0$.

→ As the equation of equilibrium for each joint is 2. Hence this method can not be applicable when no. of unknowns at each joint are more than 2.

→ So proceeding one joint to another joint when no. of unknown forces at a joint are not more than two.

Sign convention:-

(i) For tension arrows are pointing away from each other

← → (Tension)

(Tensile force = +ve).

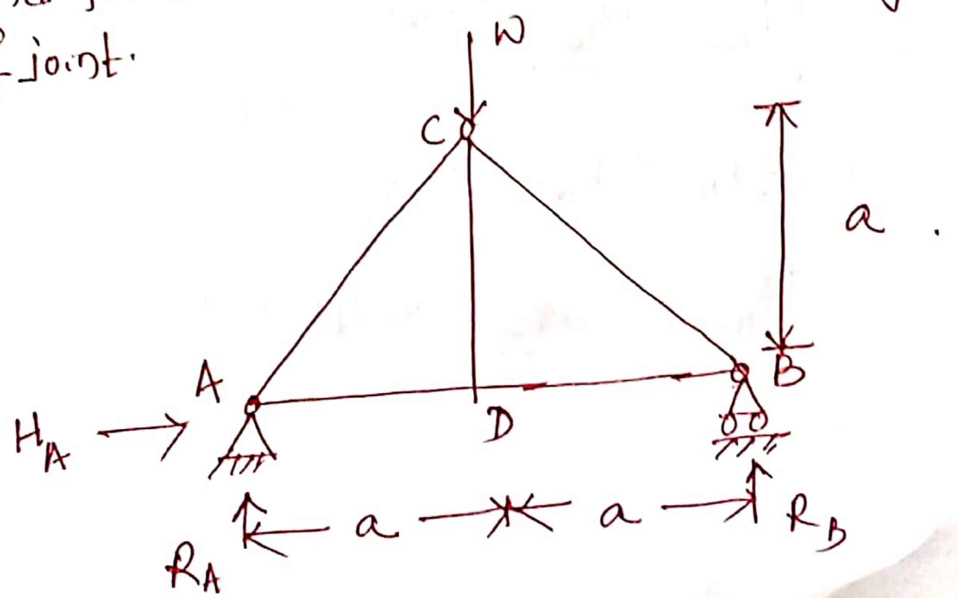
(ii) For compression arrows are pointing towards each other

→ ← (compression)

(Compressive force = -ve)

Example ①

Find forces in all members of truss by using method of joint.



Ans

Step-1

Calculation of support Reaction

$\Sigma F_x = 0 \; (H_A = 0)$

$\Sigma F_y = 0 \; , R_A + R_B = W$

$\Rightarrow$ Taking $\Sigma M_A = 0$.

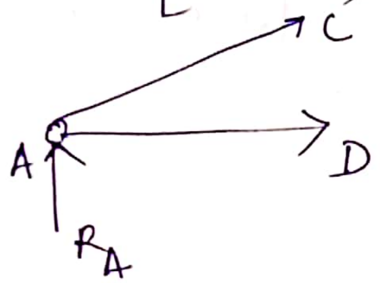
$\Rightarrow R_B \times 2a - W \times a = 0$

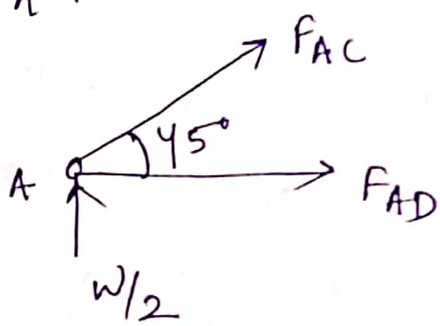
$\Rightarrow \boxed{R_B = \frac{W}{2}} \; , \boxed{R_A = W/2}$

Step-2 : Method of joint

Consider joint A

Consider equilibrium at joint A.





$\Sigma F_x = 0 \; , \quad F_{AD} + F_{AC} \cos 45° = 0$

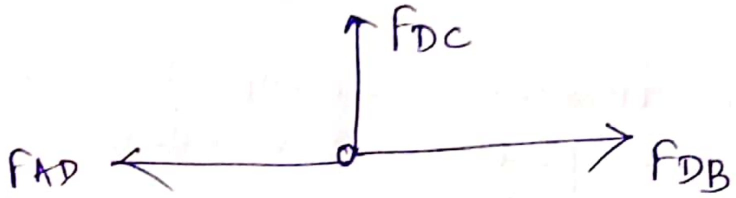
$\boxed{F_{AD} = W/2}$ (Tension)

$\Sigma F_y = 0 \quad \frac{W}{2} + F_{AC} \sin 45° = 0$

$\boxed{F_{AC} = -W/\sqrt{2}}$ compression.

Joint D :-

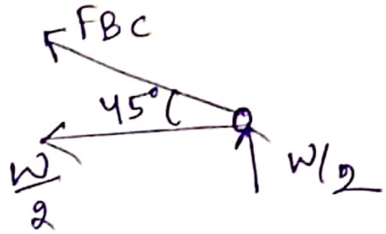
Consider equilibrium of joint D.



$\Sigma F_x = 0$, $F_{DB} = \frac{W}{2}$

$\Sigma F_y = 0$, $F_{DC} = 0$.

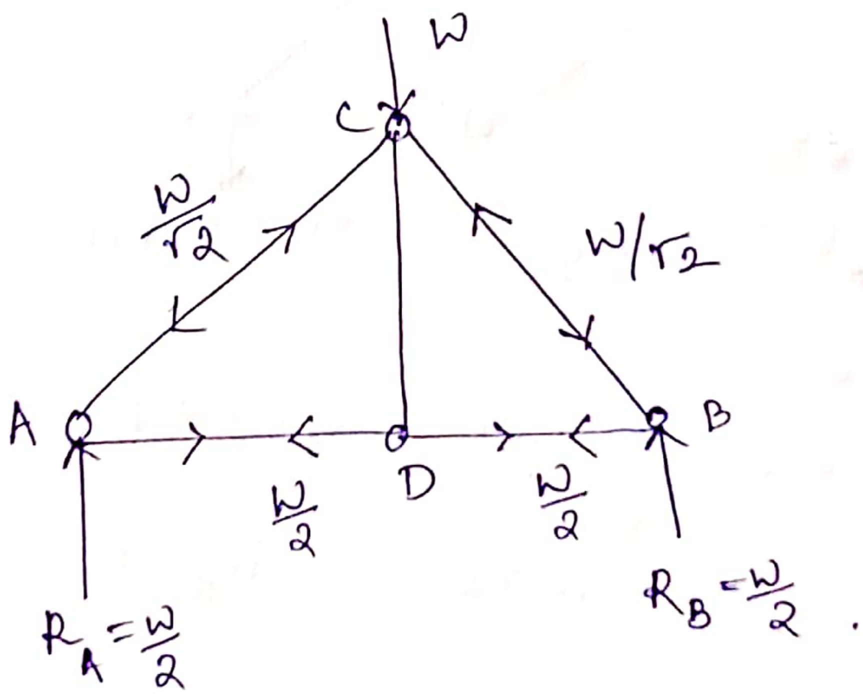
Joint B



Consider equilibrium of joint B

$\Sigma F_y = 0$, $\frac{W}{2} + F_{BC} \sin 45° = 0$

$\left(\Sigma F_y = 0 \right)$ $\boxed{F_{BC} = \frac{-W}{\sqrt{2}}}$ compression.



(2) Method of section

This method is suitable when forces are required only in few members

→ In this method whole truss is cut into two portions and equilibrium of each portion is considered.

→ For two portion following equation of equilibrium should be satisfied.

(i) $\Sigma F_x = 0$

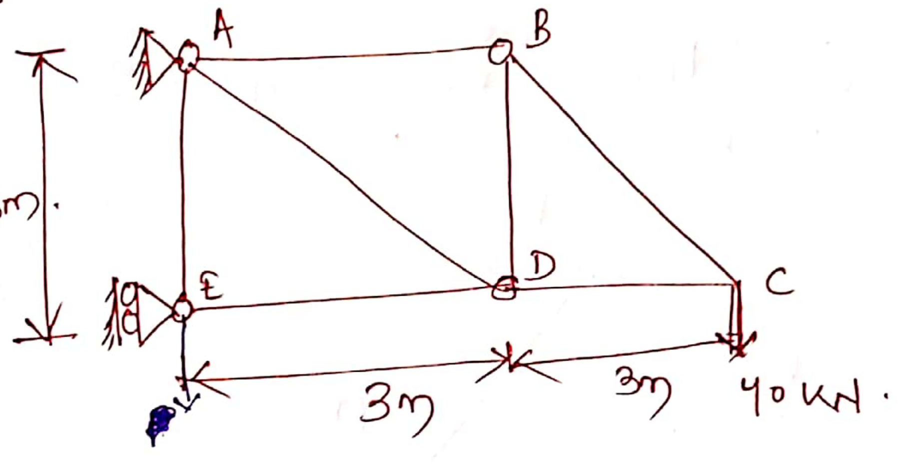
(ii) $\Sigma F_y = 0$

(iii) $\Sigma M_z = 0$.

Example-2:

The pin jointed cantilever truss is Loaded as shown in figure. The forces in member ED is

(ans)



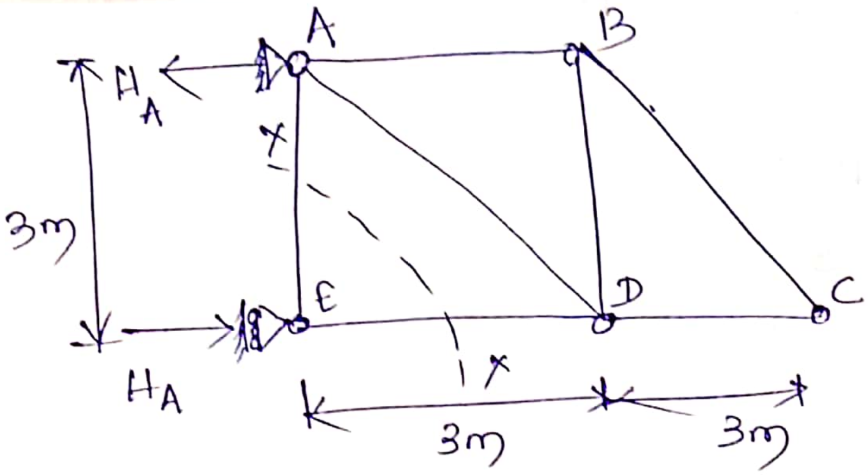
$\Sigma F_y = 0$

$R_A = 40\,kN$ [As we know upward force = downward force]

$\Sigma M_A = 0 \quad 40 \times 6 - H_F \times 3 = 0 \quad (H_A = H_F)$

$H_F = 80\,kN$

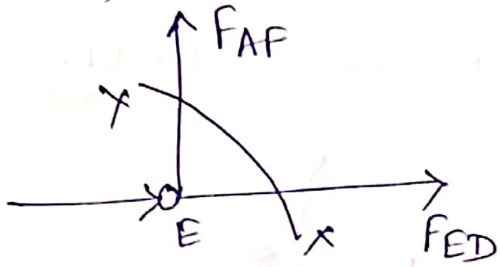
$H_A = 80\,kN$.



$H_A = 80kN$

and $H_F = 80kN$.

using method of section



$\Sigma F_x = 0$

$\Rightarrow F_{ED} + 80 = 0$

$\boxed{F_{ED} = -80kN}$ (Compressive)

Deflection of pin jointed Truss :-

A truss consists of the no. of members connected at the joint called as pin jointed truss.

→ It is assumed that pin joint truss carries only axial force.

→ A truss which satisfies the equation $(m = 2j - 3)$ is known as perfect truss.

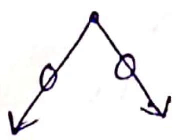
* Truss can be classified as various types such as (1) Plane truss

(2) Space truss

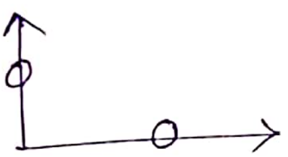
(3) Statically Determinate truss
(4) Statically Indeterminate truss
(5) perfect truss
(6) Imperfect truss
(7) Simple truss
(8) compound truss
(9) complex truss.

* zero force member :-

(i)



This is the pinjointed truss and is going in the same direction and opposite to the both value of zero. So known as zero force member.



[This can't be balanced so zero force member]

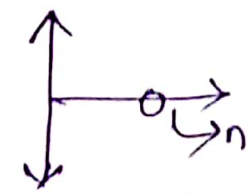
(ii)

⇒ In this figure the horizontal line are balanced with same magnitude but vertical axis can't be balanced as no force at lower portion. So vertical axis will be only zero force member.

(iii)

This both value are same but 3rd one is zero.

(iv)



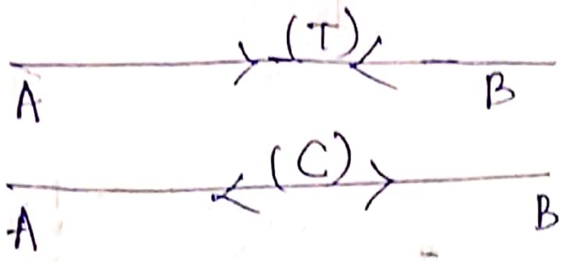
↳ not same so zero.

(6)

Sign convention of Truss member.

Tension is taken as +ve.

and compression " ~~+ve~~ (-ve).



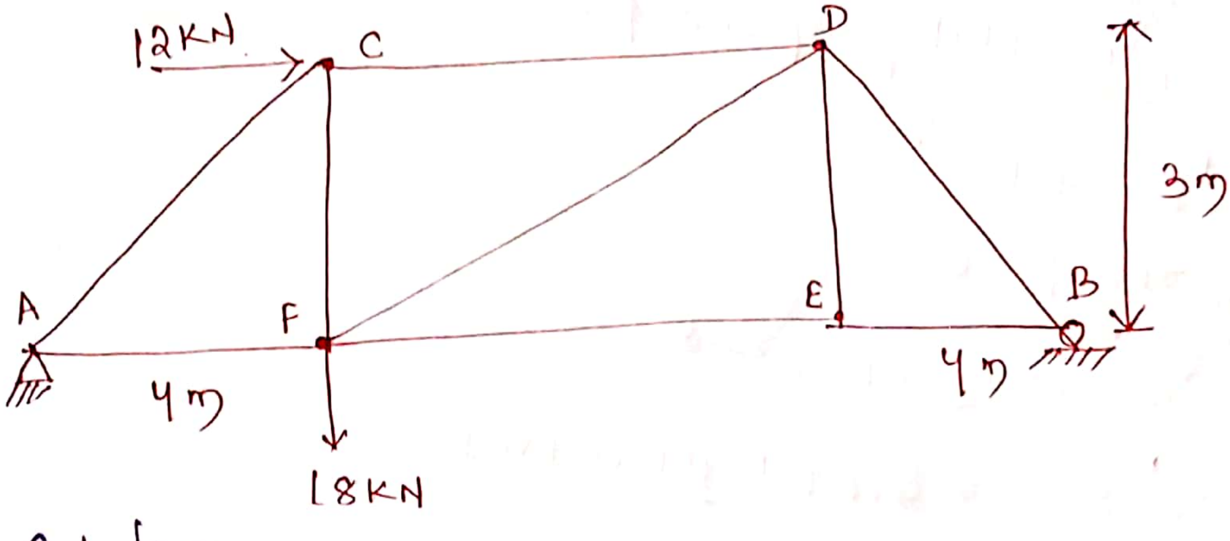
(facing eachother)

(away from eachother)

Example - 3

using unit load method or any other method, find the vertical deflection of joint 'E' of a pinjointed truss loaded and supported as shown in figure.

Take AE = constant in all member.



Solution

Step-1 :- find Reaction at support

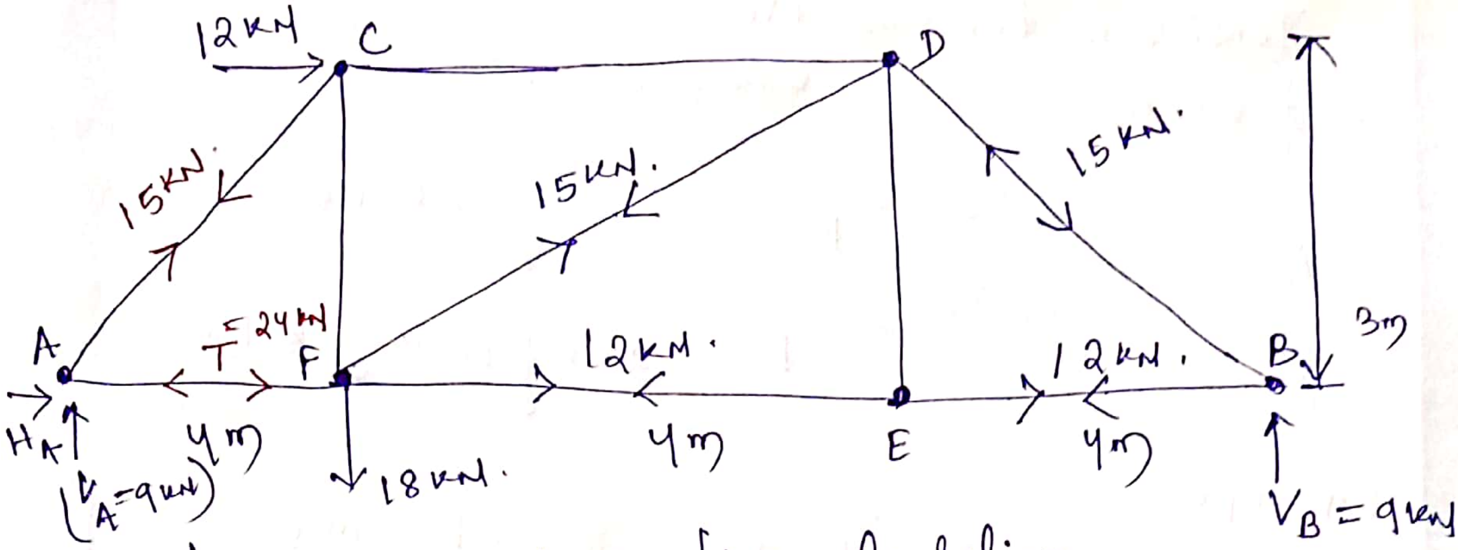
Step-2 :- P-Analysis.

Step-3 :- K-analysis

Step-4 :- Table

Step-5 :- Final Answer.

Step-1: Support Reaction calculation



Step-1: Support Reaction calculation

$\Sigma M_A = 0$ (Clockwise +ve)

$\Rightarrow 12 \times 3 + 18 \times 4 - V_B \times 12 = 0$

$\Rightarrow \boxed{V_B = 9 \text{ kN}}$ (upward ↑)

$\Sigma F_y = 0$ (+ve ↑)

$\Rightarrow V_A - 18 + 9 = 0$

$\Rightarrow \boxed{V_A = 9 \text{ kN}}$ (↑ upward)

$\Sigma F_x = 0$ (→ left to right is +ve)

$H_A + 12 = 0$

$\Rightarrow H_A = -12 \text{ KN}$.

$\Rightarrow \boxed{H_A = 12 \text{ KN}}$ (← from right to left compressive)

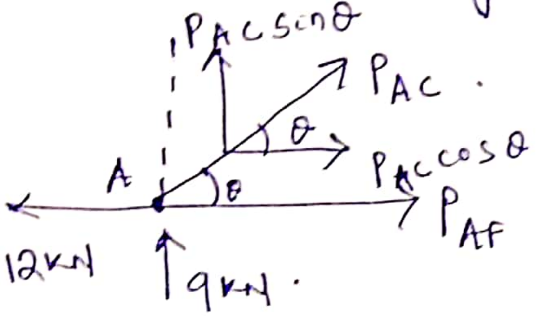
Finally we got $V_A = 9 \text{ KN}$ and $H_A = 12 \text{ KN}$.
$V_B = 9 \text{ KN}$

(7)

Step-2:- P-analysis

In this case you have to take either A or B Joint because the joint which having only two equation of equilibrium is taken not other joint.

Joint A :- Free Body diagram of Joint A



$\tan\theta = \frac{3}{4}$

$\theta = \tan^{-1}\left(\frac{3}{4}\right)$

$(\theta = 36.86^\circ)$

(*) $\Sigma F_y = 0$ ($\uparrow$ +ve)

$\Rightarrow 9 + P_{AC}\sin\theta = 0$

$\Rightarrow 9 + P_{AC}\sin(36.86) = 0$

$\Rightarrow \boxed{P_{AC} = -15\text{ kN}}$ (C) (-ve means compression)

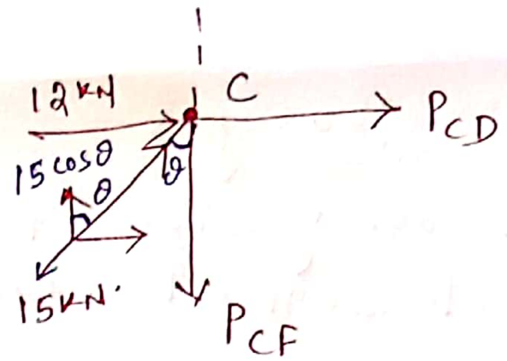
(*) $\Sigma F_x = 0$ ($\rightarrow$ +ve)

$P_{AF} + P_{AC}\cos\theta - 12 = 0$

$\Rightarrow P_{AF} + (-15\cos 36.86) - 12 = 0$

$\Rightarrow \boxed{P_{AF} = 24\text{ KN}}$ (T).

Then next Joint 'C' is taken.



Joint C:-

$\tan\theta = 4/3$

$\theta = \tan^{-1}(4/3)$

$\boxed{\theta = 53.13^\circ}$

$\Sigma f_y = 0 \quad (+ve \uparrow)$

$\Rightarrow 15\cos(53.13) - P_{CF} = 0$

$\Rightarrow \boxed{P_{CF} = 9\,kN} \quad (T)$

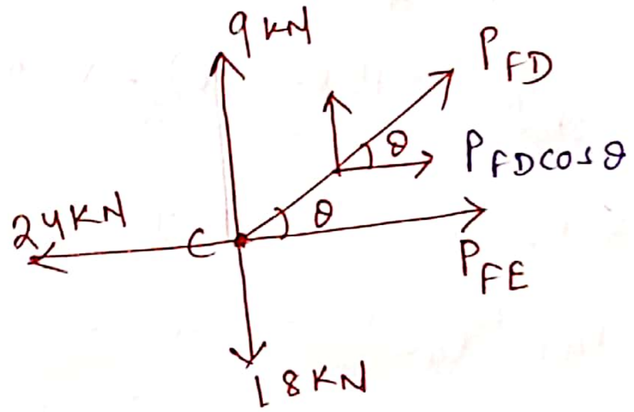
$\Sigma f_x = 0 \quad (\rightarrow +ve)$

$\Rightarrow P_{CD} + 15\sin(53.13) + 12 = 0$

$\Rightarrow P_{CD} = -24\,KN$

$\boxed{P_{CD} = 24\,KN} \quad (C)$

Joint F :-



$\tan\theta = \frac{3}{4}$

$\theta = \tan^{-1}(3/4)$

$\boxed{\theta = 36.86^\circ}$

$\Sigma f_y = 0 \quad (+ve \uparrow)$

$\Rightarrow 9 - 18 + P_{FD}\sin\theta = 0$

$\Rightarrow 9 - 18 + P_{FD}\sin(36.86) = 0$

$\Rightarrow \boxed{P_{FD} = 15\,kN} \quad$ (Tension T)

(8)

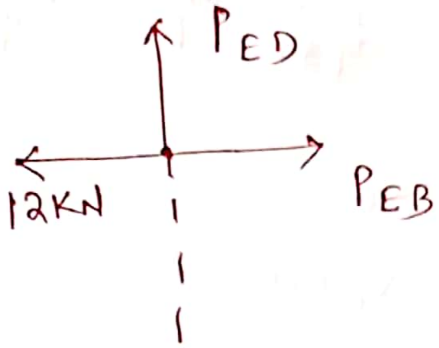
$\Sigma F_x = 0 \quad (\rightarrow +ve)$

$\Rightarrow P_{FE} + P_{ED}\cos\theta - 24 = 0$

$\Rightarrow P_{FE} + 15\cos(36.86) - 24 = 0$

$\Rightarrow \boxed{P_{FE} = 12KN}$ (T).

Joint E :-



$\Sigma F_y = 0 \quad (\uparrow +ve)$

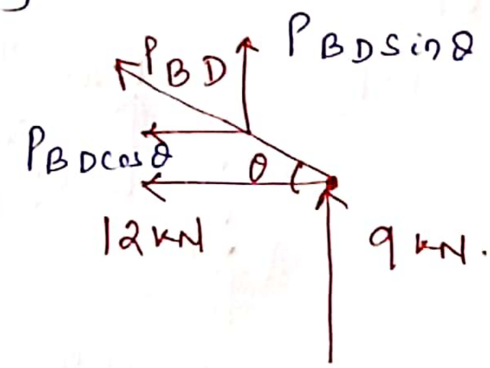
$\boxed{P_{ED} = 0}$

$\Sigma F_x = 0 \quad (\rightarrow +ve)$

$-12 + P_{EB} = 0$

$\boxed{P_{EB} = 12KN}$ [T].

Joint B :-



$\tan\theta = 3/4$

$\theta = \tan^{-1}(3/4)$

$\Rightarrow 36.86^\circ$.

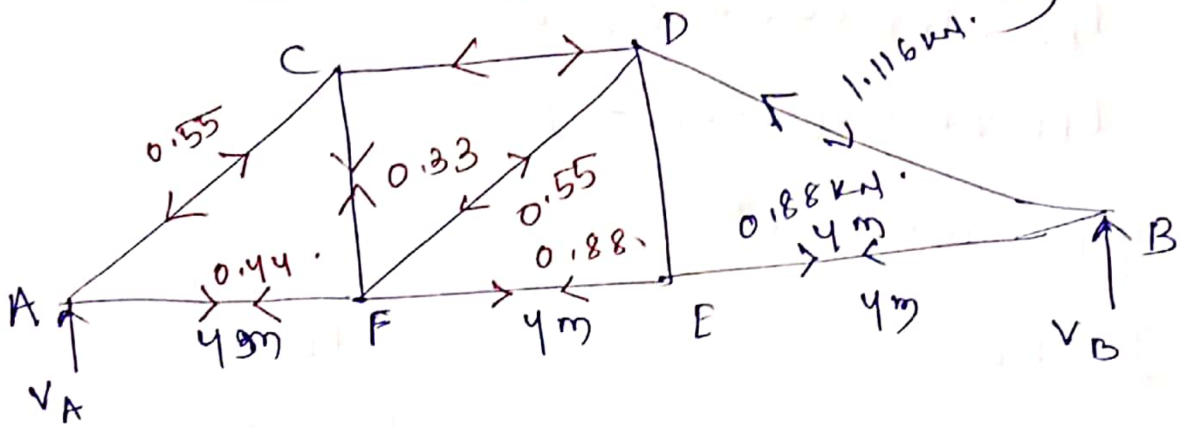
$\Sigma F_y = 0 \quad (\uparrow +ve)$

$P_{BD}\sin(36.86) + 9 = 0$

$P_{BD} = -15KN$

$\boxed{P_{BD} = 15KN}$ [C].

Step-3 (K-analysis) (Remove all the external Loads)



$\Sigma M_A = 0$

$1 \times 8 - V_B \times 12 = 0$

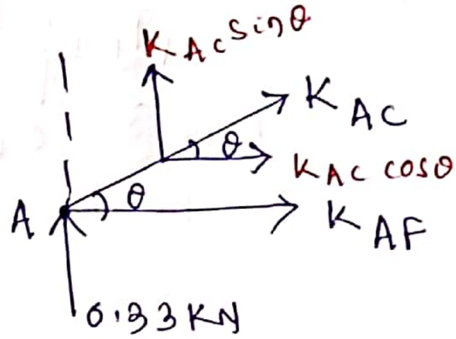
$\boxed{V_B = 0.67\,KN}$

$\Sigma f_y = 0$ ($\uparrow$+ve)

$V_A - 1 + 0.67 = 0$

$\boxed{\Rightarrow V_A = 0.33\,KN}$

Joint A



$\tan\theta = \frac{3}{4}$

$\theta = \tan^{-1}(3/4)$

$\Rightarrow \boxed{\theta = 36.86^\circ}$

$\Sigma f_y = 0$ ($\uparrow$+ve)

$0.33 + K_{AC}\sin\theta = 0$

$\Rightarrow 0.33 + K_{AC}(36.86) = 0$

$\boxed{K_{AC} = 0.55\,KN}$ [C].

Scanned with CamScanner

⑨

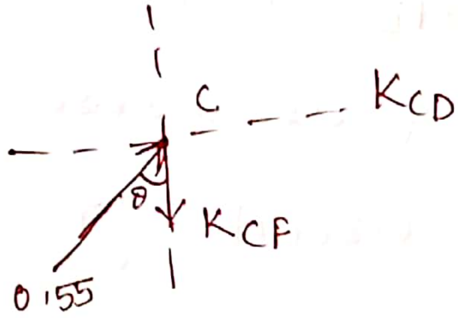
$\Sigma F_x = 0 \; (\rightarrow +ve)$

$K_{AF} + K_{AC}\cos\theta = 0$

$\Rightarrow K_{AF} + (-0.55\cos\cancel{\theta}) = 0$ 36.86

$\Rightarrow \boxed{K_{AF} = 0.44\,KN}$ [T].

Consider Joint C :→



$\tan\theta = 4/3$

$\Rightarrow \theta = \tan^{-1}(4/3)$

$\boxed{\theta = 53.13^\circ}$

$\Sigma F_y = 0 \; (\uparrow +ve)$

$0.55\cos(53.13) - K_{CF} = 0$

$\Rightarrow \boxed{K_{CF} = 0.33\,KN}$ [T]

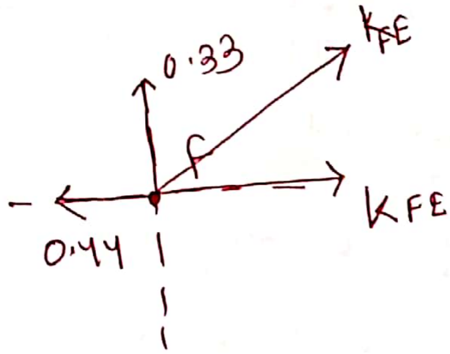
$\Sigma F_x = 0 \; (\rightarrow +ve)$

$0.55\sin\theta + K_{CD} = 0$

$\Rightarrow \boxed{K_{CD} = -0.44\,KN}$

$\boxed{K_{CD} = 0.44\,KN}$

Joint F



$\tan\theta = 3/4$

$\boxed{\theta = 36.86^\circ}$

$\Sigma F_y = 0$ (+ve ↑)

$\Rightarrow 0.33 + K_{FD} \sin(36.86) = 0$

$\boxed{K_{FD} = -0.55\,KN}$ (-ve sign indicated compression)

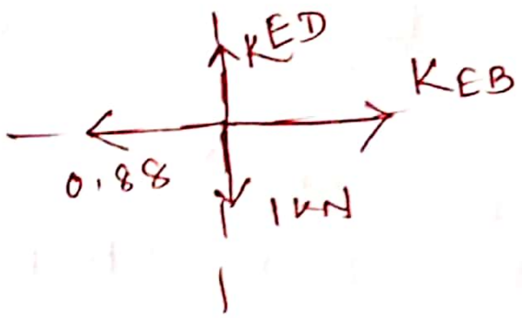
$\Sigma F_x = 0$ (→ +ve)

$K_{FE} + K_{FD} \cos\theta = 0$

$\Rightarrow K_{FE} + (-0.55 \cos 36.86) = 0$

$\Rightarrow \boxed{K_{FE} = 0.88\,KN}$ [T].

Joint E :-



$\Sigma F_y = 0$. (↑+ve)

$K_{ED} = +1\,KN$ [T]

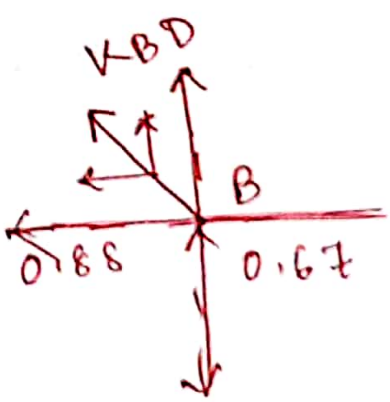
as $\boxed{K_{ED} - 1 = 0}$

$\Sigma F_x = 0$ (→ +ve)

$-0.88 + K_{EB} = 0$

$\Rightarrow \boxed{K_{EB} = 0.88\,KN}$ [T]

Joint B



$\tan\theta = 3/4$

$\boxed{\theta = 36.86}$

(10)

$\Sigma f_y = 0$ (+ve ↑)

$0.67 + K_{BD} \sin(36.86) = 0$

$\Rightarrow \boxed{K_{BD} = -1.116\ KN}$ [C]

Step-4 Table

Member	P(KN)	K(KN)	L(m)	PKL
AF	24	0.44	4	42.24
AC	(-15KN)	(-0.55)	5	41.25
CF	9	0.33	3	8.91
CD	(-24)	(-0.44)	4	42.24
FD	15	(-0.55)	5	41.25
FE	12	0.88	4	42.24
DE	0	1	3	0
BD	(-15)	(-1.116)	5	83.745
BE	12	0.88	4	42.24
				$\Sigma = 261.615$

To find length

$AC = \sqrt{3^2 + 4^2} = 5m$

for inclined $AC = 5m$

Now Deflection at E at vertical direction

$$\boxed{y_{E(V)} = \frac{261.615}{AE}}$$

AE = constant.

Ans.

Willot Mohr's diagram :-

The Willot diagram which is drawn first gives the movements of all joints of the truss, with respect to one of the joints and a member that enters that joint.

* Deflection of truss joints :-

The deflection of truss joint can be found by following methods.

(1) Force method :- Maxwell's method / unit load method and strain energy method. These methods can be used to find deflection of beam rigid frame and trusses.

(2) Stiffness / Displacement method :- These method can also be used to find deflection of beam, rigid frame or truss.

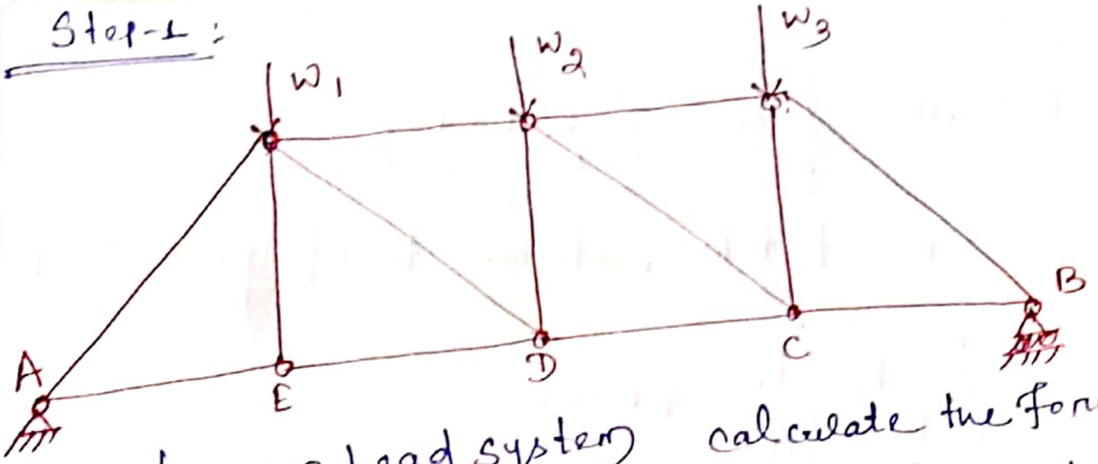
(3) Graphical Method :- Bar chain method and Willot Mohr's diagram. These methods used only for truss.

Maxwell's Method / unit Load Method :-

Case-1 :- Due to external Loading :

To find the deflection of joint 'c' following steps may be used.

Step-1 :



Due to given Load system calculate the forces in all members say $P_1, P_2, P_3 \ldots\ldots P_n$ (P-system of forces)

→ The P-system of forces can be calculated by using method of joint.

Step-2 :-

Remove all given external loading and apply unit load at C in the direction of desired deflection.

Due to unit load alone find forces in members says $K_1, K_2 \ldots\ldots K_n$ respectively.

Step-3 :- The deflection of joint C in the direction of applied unit load is given by

$$\Delta_C = \frac{\Sigma PKL}{AE}$$

Case-2 Due to variation of temperature

If temperature changes occours then deflection of each member will be

$$\boxed{\Delta = L\alpha T}$$

T = change in temperature.

T is taken as +ve if rise in temperature and T is -ve if there is fall in temperature.

Deflection at C $\boxed{\Delta_C = \sum_{i=1}^{n} K_i (L\alpha T)}$

Case-3:- Due to both external Loading and Temperature

$$\Delta_C = \sum_{i=1}^{n} K_i \left[\frac{PL}{AE} + L\alpha T\right]$$

P = +ve (Tension) P = -ve (compression)

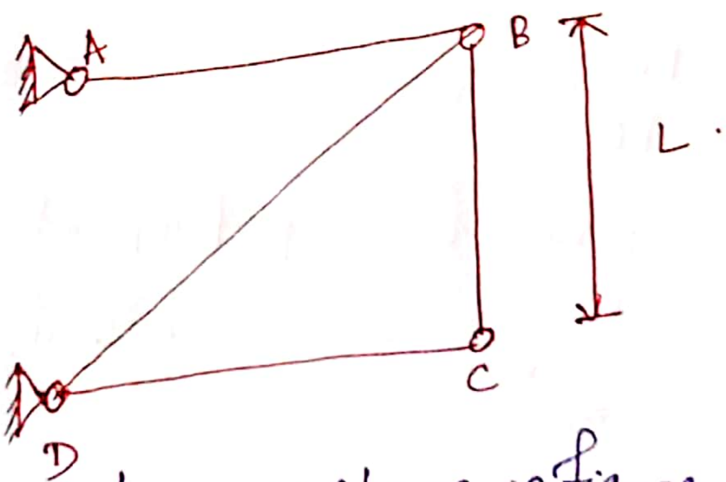
T = +ve (Rise) T = -ve (fall).

Case-4:- Due to Lack of fit of some members:

2 Marks Lack of fit :-

It is an occurrence in trusses in which there is a difference between the length of a member and the distance between the nodes it is supposed to fit.

→ This prestress occurring due to lack of fit may be used advantageously in design of truss.



Consider a truss as shown in figure.

If member BC is too long then the deflection of point 'C' may be given as

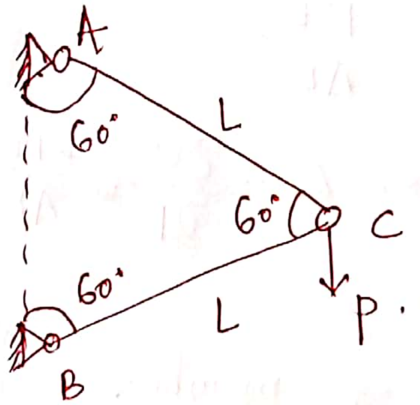
$$\Delta_c = \sum_{i=1}^{n} K_i \Delta_i$$

K_i = Force induced in member due to unit load.
Δ_i = Discrepancy in length (Δ)
= -ve for too short
= +ve for too long.

Hence deflection of joint C $\boxed{\Delta_c = \sum_{i=1}^{n} K_i \Delta_i}$

Δ_i = Axial deflection of member due to external Loading.
= $L\alpha T$, due to temperature effect.
= Δ, due to some other effect such as lack of fit.

Example:- The vertical deflection of node C for the truss as shown in figure.



Solution P-system of forces

Consider joint C

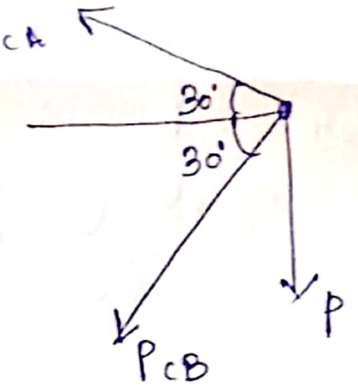
$\Sigma F_x = 0 \quad -P_{CA}\cos 30° - P_{CB}\cos 30° = 0. \Rightarrow \boxed{P_{CA} = -P_{CB}}$

$\Sigma F_y = 0$

$P_{CA} \sin 30° - P - P_{CB} \sin 30° = 0$

$P_{CA} = +P$

$P_{CB} = -P$



K-system of forces:-

Remove load P and apply unit load at P in downward direction.

$K_{CA} = +1\,KN.$

$K_{CB} = -1\,KN.$

The deflection at C = $\Delta_C = \dfrac{\Sigma KPL}{AE} = \dfrac{(1 \times P \times L) + (-1 \times -P \times L)}{AE}$

$= \dfrac{2PL}{AE}$

Alternatively

$P_{CA} = P$, $P_{CB} = P$

The strain energy stored in truss

$U = U_{AC} + U_{BC}$

$= \dfrac{P^2L}{2AE} + \dfrac{P^2L}{2AE} = \dfrac{P^2L}{AE}$

The deflection at C will be $\Delta_C = \dfrac{\partial U}{\partial P} = \dfrac{2PL}{AE}$

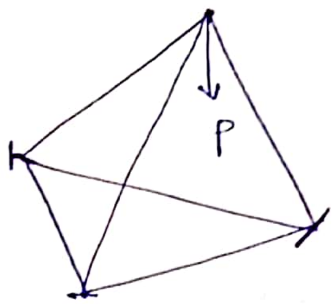
(Ans).-

* Space Trusses :-

A space truss consists of members joined together at the ends to form a stable 3-D structure.

→ Simple Space Truss

Simple Space Truss



Assumptions of truss analysis

(i) Members are weightless

(ii) All members are pin jointed.

Assumptions:

Members may be treated as a two force member provided the external loading is applied at the joints. and the joints are ball-socket connection.

→ Valid for welded or bolted connections when the weight of a member is neglected.

Procedure:-

It is same for 3-D trusses.

Method of Joints: When forces in all members need to be determined which utilize $\Sigma\vec{F}=0$ to analyze method of sections.

Method of section

When the few forces are to be determined cut truss into two section then apply

$$\Sigma\vec{F}=0$$

$$\Sigma\vec{M}=0$$

S.Q perfect Truss

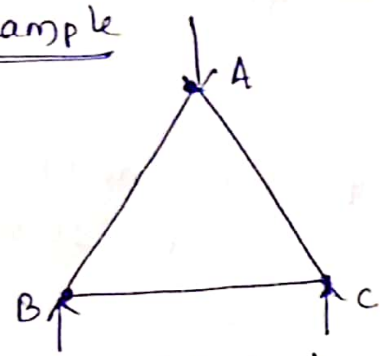
Perfect truss is that which is made up of members just sufficient to keep it in equilibrium.

The No. of members in perfect frame may be expressed by relation $(M = 2j - 3)$

M = no. of members

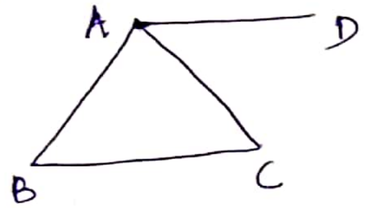
j = no. of joints.

Example



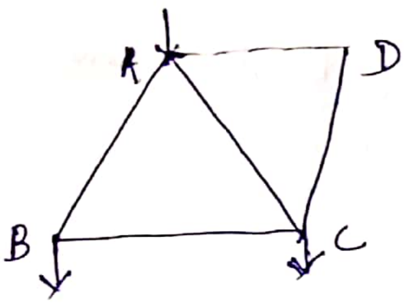
$$M = 2J - 3$$
$$\Rightarrow 3 = 2 \times 3 - 3$$
$$\boxed{3 = 3}$$

Now we added two members to again make it perfect by adding 1 member it will not perfect truss



$M = 4$
$j = 4$
$M = 2J - 3$
$\Rightarrow 4 = 2 \times 4 - 3$
$\Rightarrow 4 \neq 5$ (Not perfect)

If we add 2 members



$M = 5$
$J = 4$
$M = 2J - 3$
$\Rightarrow 5 = 2 \times 4 - 3$
$\Rightarrow 5 = 5$ (perfect)

So for each joint two member will add then that will be perfect.

If $M \neq 2J - 3$ (Imperfect truss)

(14)

Keypoints of truss

(i) A simple truss is composed from basic triangular units.

(ii) A compound truss is formed when two or more simple trusses are joined either by hinge or additional members.

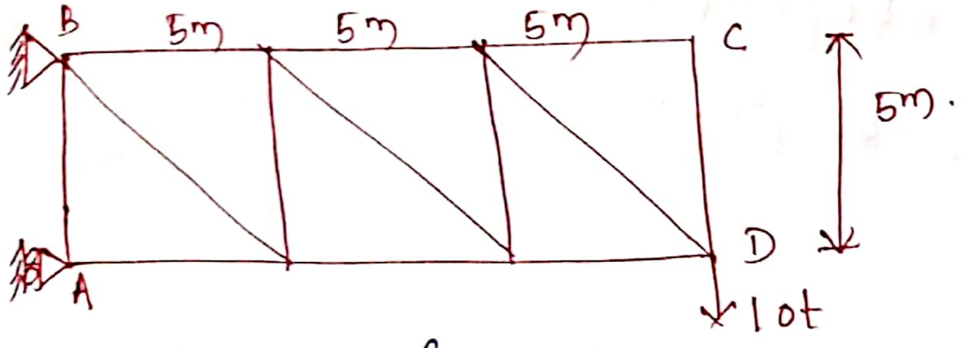
(iii) A plane truss is externally stable if minimum 3 independent support reactions are available.

(iv) A truss is internally stable if $(m = 2j - 3)$

(v) There are two methods to analyse statically determinate truss such as method of section and method of joint.

(vi) If at a joint two members meet and no external load is there then both member will carry zero force.

Q In the cantilever truss shown in the figure the reaction at A is

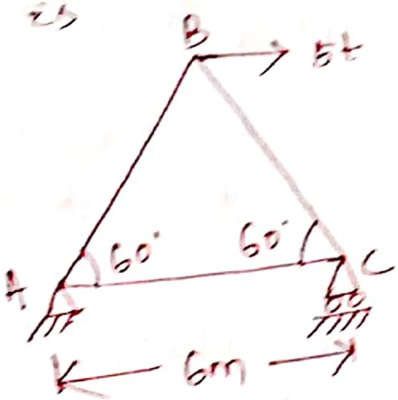


A:- Only a horizontal reaction can exist at A

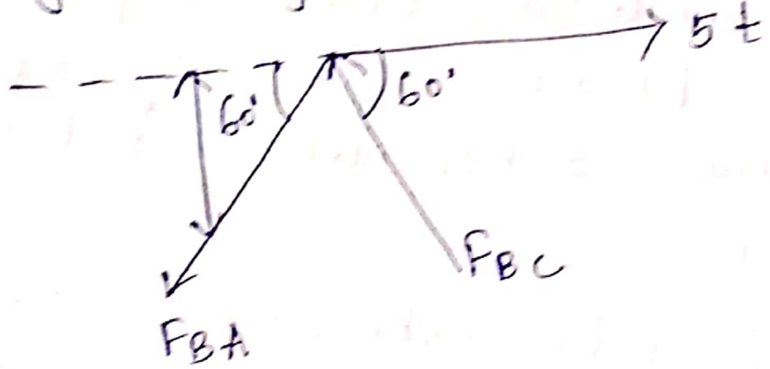
Taking moments about hinge.

$H_A \times 5 = 10 \times 15$ $\boxed{\Rightarrow H_A = 30t}$

Q Force in the member BC of the truss shown in figure is



Ans:- By considering joint equilibrium at B



$F_{BC} = F_{BA}$

$F_{BC} \cos 60° + F_{BA} \cos 60° = 5$

$F_{BC} = 5t$ (Compressive)

$F_{BA} = 5t$ (Tensile)

— 0 —