4.4.1. Short-Run Production Function

Short-run refers to a time period where some factors are variable, while some other factors are fixed. The variable factors are ordinary labour, raw materials etc. The fixed factors are land, capital, and superior labour etc. In a simple production function 'labour' is the variable factor and 'capital' is the fixed factor. In the short run, there are following important concepts related to production, viz.,

1. Total Production(TP)
2. Average Production(AP)
3. Marginal Production(MP)
4. Output Elasticity

1.   Total Production (TP)

Total production refers to total amount of goods and services produced in a given period. TP is the amount of total output produced by a given amount of variable factor (labour), keeping capital fixed. As the amount of variable factor, i.e., labour increases, the total production at first, increases rapidly. Thereafter, it increases at a slow rate and a stage comes when total production instead of increasing begins to fall. In other words, TP, first, increases at an increasing rate and then at a diminishing rate. It is because due to initial use of limited number of units of variable factors, the fixed factor is not fully utilised. As more and more units of variable factors are employed the fixed factor is utilised more efficiently; but after a point, when fixed factor is fully utilised, then any more employment of variable factor is followed by decrease in production. The concept of total production has been made clear in Table - 4.1.

Table - 4.1 (Total Production)

Units of Capital(K)	Units of Labour(L)	Total Production
5	1	3
5	2	6
5	3	10
5	4	13
5	5	15
5	6	16
5	7	16
5	8	14

The nature of total production gives rise to a curve, called total product curve, as shown below. This curve has different slopes. At first, TP increases at an increasing rate; then at a diminishing rate; at a point it also reaches maximum and thereafter falls.
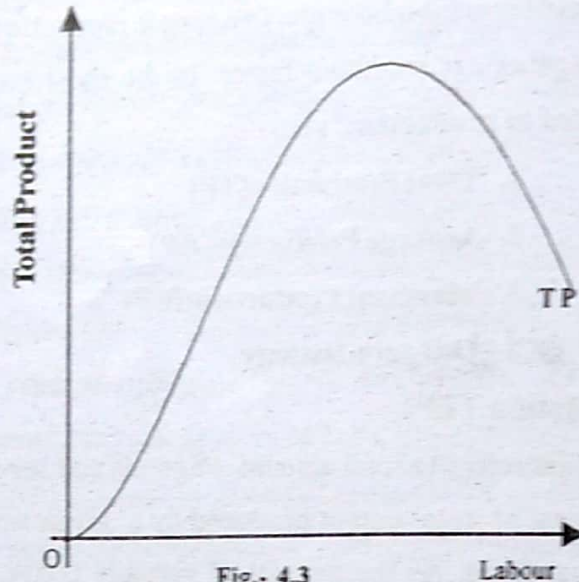

Fig.- 4.3

2. **Average production (AP) :**

Average production means per unit production of the variable factor. It is total production per unit of labour. AP is calculated by dividing the total production by the total units of variable factor of production,

i.e., $AP = \dfrac{\text{Total production}}{\text{Labour}}$

The concept of average production has been made clear in **Table -4.2.**

Table - 4.2

Units of Capital	Units of Labour	Total Production	Average Production
5	1	3	$\frac{3}{1} = 3$
5	2	6	$\frac{6}{2} = 3$
5	3	10	$\frac{10}{3} = 3.3$
5	4	13	$\frac{13}{4} = 3.4$
5	5	15	$\frac{15}{5} = 3$
5	6	16	$\frac{16}{6} = 2.7$
5	7	16	$\frac{16}{7} = 2.3$
5	8	14	$\frac{14}{8} = 1.75$

It is clear from the above table that in the beginning AP is increasing. After the fifth unit of labour, it begins to diminish, but remains positive always. On the basis of this nature, the average production curve can be derived as shown here:
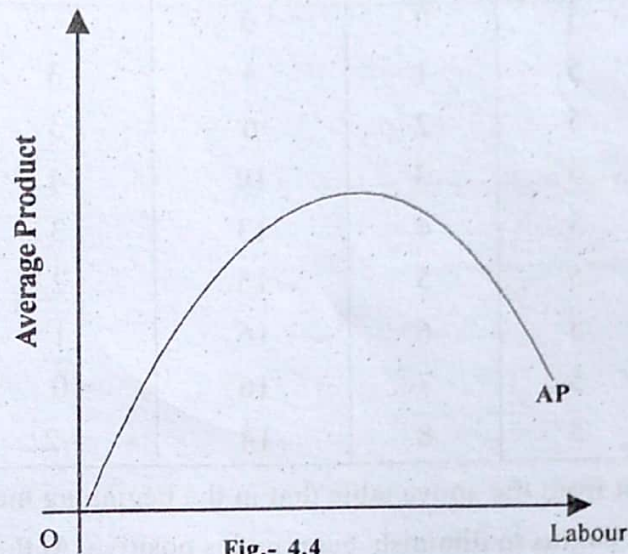

Fig.- 4.4

3. **Marginal production(MP) :**

Marginal production refers to change in total production due to the application of one more or one less unit of variable factor. MP can be calculated with the help of the following formula :

$$MP_n = TP_n - TP_{n-1}$$

or, $$MP_n = \frac{\Delta Q}{\Delta L}$$

where, MP_n is the marginal production of 'n' - units;

TP_n is the total production of 'n' - units;

TP_{n-1} is the total production of 'n -1' units.

ΔQ is the change in total production (Q);

ΔL is the change in the quantity of variable factor labour.

The concept of MP is explained with the help of the **Table - 4.3**

Table - 4.3

Units of Capital	Units of Labour	Total Production	Marginal Production
5	0	0	--
5	1	3	3
5	2	6	3
5	3	10	4
5	4	13	3
5	5	15	2
5	6	16	1
5	7	16	0
5	8	14	-2

It is evident from the above table that in the beginning marginal production tends to rise; thereafter it begins to diminish but remains positive. At the seventh unit, MP is zero. After that, it turns negative. This feature of MP gives rise to the Mp curve as shown here.
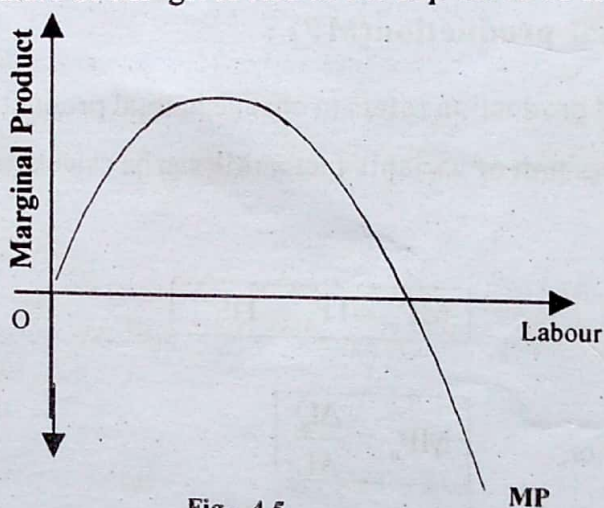

Fig.- 4.5

4. **Output Elasticity**

It is defined as the degree of responsiveness of output of a firm due to small change in the volume of labour, capital being the fixed factor. Thus, it is calculated as interpreted as follows:

$$\text{Output Elasticity } (\varepsilon) = \frac{\text{Proportionate Change in Output}}{\text{Proportionate Change in Labour}}$$

$$= \frac{d(\ln Q)}{d(\ln L)} = \frac{dQ}{dL} \cdot \frac{L}{Q} = \frac{MP_L}{AP_L}$$

4.6 LAW OF VARIABLE PROPORTIONS

From the above discussion we come to know that there were three different laws of returns, as classical economists put forward. But according to the modern economists these are not three separate laws of returns. There is only one law of returns, i.e., the law of variable proportions. And these three laws are nothing but three different phases of the law of variable proportion.

Law of variable proportions occupies an important place in economic theory. The law studies the output behaviour of a firm in the short-run. The law is so named because under this law we are studying the effects of variation in factor proportion on total output. Factor proportion means the ratio of capital to labour i.e., $K : L$.

In the short-run capital is the fixed factor and labour is the variable factor. The factor proportion will change, when the amount of labour changes. For examples, the volume of capital is fixed at 10 units. The production may be carried on by employing various units of labour. Then factor proportions may be 10 : 1, 10 : 2, 10 : 3 and so on. Due to this change in factor proportions, there will emerge a change in total output at different rates. This tendency is the subject matter of the law of variable proportions.

"The *law of variable proportion* states that if the inputs of one resources is increased by equal increment per unit of time while the inputs of other resources are held constant, total output will increase, but beyond some point the resulting output increases will become smaller and smaller".

According to *Benham*, "as the proportion of the factor in a combination of factors is increased after a point, first the marginal and then the average product of that factor will diminish".

Assumptions :

1. There should be only one factor which is variable i.e., labour.
2. All units of labour are homogeneous.
3. It is possible to change the factor proportion.
4. The state of technology is assumed to be given and constant.
5. The law is valid only in the short run.
6. The output is measured in physical units.
7. The variable factor, labour, must increase in equal increments. There should not be any sudden jump.
8. There must be atleast one factor whose quantity can be held constant.

Explanation of the Law :

The behaviour of output of a firm when one factor is varied and the quantities of other factors are kept constant, can be divided into three stages. These stages are :

Stage - I   : The law of Increasing Returns (IR)

Stage - II   : The law of Diminishing Returns (DR)

Stage - III   : The law of Negative Returns (NR)

We may illustrate these three stages of production in the following table.

Table - 4.8

Volume of Capital	Volume of Labour	Total Product (TP)	Average Product (AP)	Marginal Product (MP)	Stages of Production
10	1	8	8	8	Stage -I (IR)
10	2	20	10	12	
10	3	36	12	16	
10	4	48	12	12	Stage -II (DR)
10	5	55	11	7	
10	6	60	10	5	
10	7	60	8.6	0	Stage -III (NR)
10	8	56	7	-4	

The above table illustrates the short run production function of a firm with one fixed factor and another variable factor. The factor capital is fixed at 10 units and labour is the variable factor. The table shows the nature of Total Product (TP), Average Product (AP) and Marginal Product (MP) as the quantity of labour is changed. This can be studied in three stages.

In the first stage, TP, AP and MP increases but MP is greater than AP. Both AP and MP increases upto a point i.e., upto the employment of 3rd unit of labour with 10 units of capital. Then both start decreasing because proportion of labour to capital was sufficient and capital is not properly used. This is the end of the first stage. The first stage ends where AP is equal to MP, or where AP is maximum. This stage is known as the stage of Increasing Returns.

In the second stage, AP and MP start falling. MP falls at a faster rate than the AP. So AP is greater than MP. Here, TP increases at a diminishing rate. TP is also maximum at 6th unit of labour where MP become zero. This stage is known as the stage of diminishing returns.

The third stage begins where second stage ends or where MP is zero. In this stage, MP is negative and TP falls but AP is still positive. This stage is known as the stage of Negative Returns. The law of variable proportions can be explained with the help of this diagram.
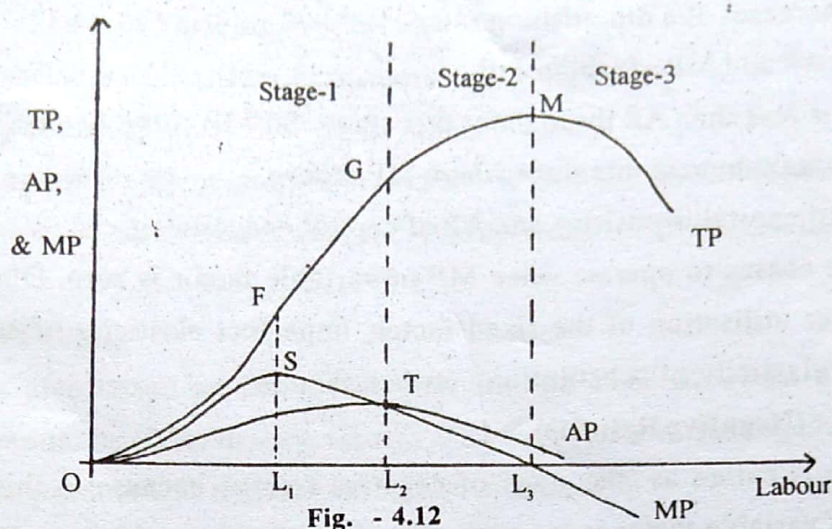
Fig. - 4.12

In the diagram, horizontal axis represents the units of the variable factor labour and vertical axis measures TP, AP and MP of the variable factor labour. TP is the total product curve. AP is the average product curve. MP is the marginal product curve.

The above diagram shows three stages of the law of variable proportions.

Stage - I : (Increasing Returns)

This stage is known as the stage of increasing returns because the AP of the variable factor increases throughout this stage and reaches its maximum.

The characteristics of Stage - I are :

(i)   TP increases at an increasing rate.

(ii) MP rises in part and then it falls.

(iii) Although in this stage MP falls after a point, MP is greater than AP. So MP curve lies above the AP curve throughout this stage.

(iv) There is partial utilisation of fixed factor, capital.

(v) The MP of capital is negative and AP of capital is positive and increasing.

The stage of Increasing Returns comes to an end when MP is equal to maximum of AP. Increasing returns are due to indivisibility of factors, division of labour and specialisation and internal economies like labour economies, technical economies, managerial economies, financial economies, marketing economies and external economies like economy of concentration, economy of information and economy of disintegration.

Stage - II : (Diminishing Returns)

The second stage begins from where AP is equal to MP. This stage is called as the stage of diminishing returns, since the AP and MP both decline throughout this stage. The characteristics of Stage - II are :

(i)   TP increases at a diminishing rate.

(ii) Both AP and MP of labour fall.

(iii) MP is less than AP throughout this stage. So MP curve lies below the AP curve.

(iv) TP is maximum in this stage where MP is zero.

(v)   MP of capital is positive and AP of capital is declining.

This stage ceases to operate when MP of variable factor is zero. Diminishing returns are due to over utilisation of the fixed factor, imperfect elasticity of supply of factors and imperfect elasticity of substitution between factors.

Stage - III : (Negative Returns)

This stage is called as the stage of negative returns because in this stage the marginal product of variable factor is negative. The characteristics of Stage - III are :

(i)   TP falls

(ii) MP of labour is negative

(iii) AP falls throughout but positive

(iv) There is too much of labour in relation to capital

(v)   MP of capital is positive.

The Stage of Operation :

Already we have discussed about the three stages of production. Now an important question arises in which stage rational producer will like to operate.

In the first stage, the AP and TP are continuously rising. The MP is also rising. A rational producer will not stop his production at this stage. Because, if he stops production during this stage, he will not be able to make the best use of the fixed factor capital. Similarly, he will also fail to fully utilise the benefit of the increasing quantity of labour, when AP is continuously rising in this stage. So he will expand his production further.

Similarly, a rational producer can't choose to operate in Stage - III, because MP of labour is negative.

Therefore, the second stage is the only theoretical possibility. In the Stage - II, there is full and efficient utilisation of the fixed factor of production. This stage is called as the Rational Producer's Operation Stage.

Mrs. Joan Robinson said that increasing returns is an empirical fact but diminishing returns is a logical necessity. The operation of the law of diminishing returns is a logical necessity because of the fact that different factors of production are imperfect substitutes for one another. The operation of this law can be postponed for some time by changing the factor proportions, but it can not be permanently put off. The law of diminishing returns is universal.

Limitations of Law of Variable Proportions

The law of Variable Proportions has following limitations:

1. State of Technology

This law is based on the assumption that state or technology remains constant. So if there is improvement in technique of production, the marginal product will increase instead of diminishing.

2. Different Factors

This law is also based on the assumption that the ratio in which different factors are combined should be liable to vary. Hence, if all factors are varied together in equal quantity, the marginal product will increase instead of diminishing.

3. New Land

If the land is just brought under cultivation, the application of more and more labour will lead to increase in marginal product in the beginning.

4. Unrealistic Assumptions

The assumptions which are considered in law of variable proportions are highly unrealistic. For example, the assumptions such as homogeneity of labour, perfect competition, and constancy of capital are unrealistic

Despite these weaknesses, the law of variable proportion is universal in application. Prof. Marshall and his followers believed that the law of increasing returns is applicable in manufacturing industries while the law of diminishing returns is applicable in agriculture. But this is not true. When a factor is increased keeping constant the other factors, the level of technology being held constant, the product definitely decrease both in agriculture and industry. This law has been proved by empirical evidence.

4.7 RETURNS TO SCALE

Returns to scale relates to the behaviour of total output as all inputs are carried and is a long run concept. The term returns to scale refers to the changes in output as all factors change by the same proportion. When all factors change in the same proportion, the scale of production will change. The behaviour of production due to a change in the scale of production forms the subject-matter of the law of returns to scale. In case all inputs are increased in the same proportion and the scale of production is expanded, the effect on output may take three forms or stages. Such as increasing, constant and diminishing returns to scale.

When output increases more than proportionately to the increase in inputs, returns to scale are said to be increasing. Thus, if the quantity of inputs is increased by 20% and the output