Converters

Converters:

Converter is basically any circuit that converts electrical power from one form to another

Majorly there five types of power electronics converters, each having different purpose.

Rectifier : Fixed Ac to variable Dc

Inverter : Dc to Ac having variable amplitude & frequency.

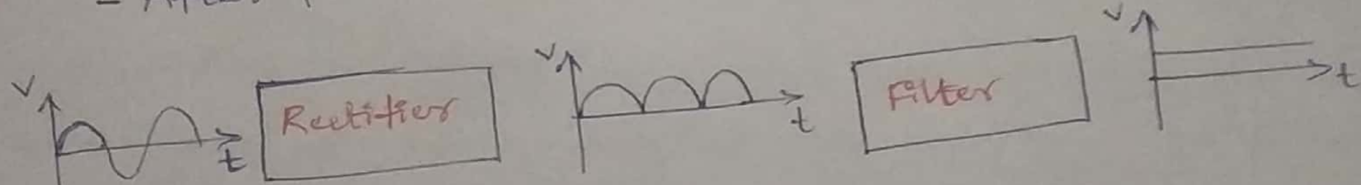
Ac voltage controller : Converts fixed Ac to variable Ac

Dc chopper : Fixed Dc to variable Dc

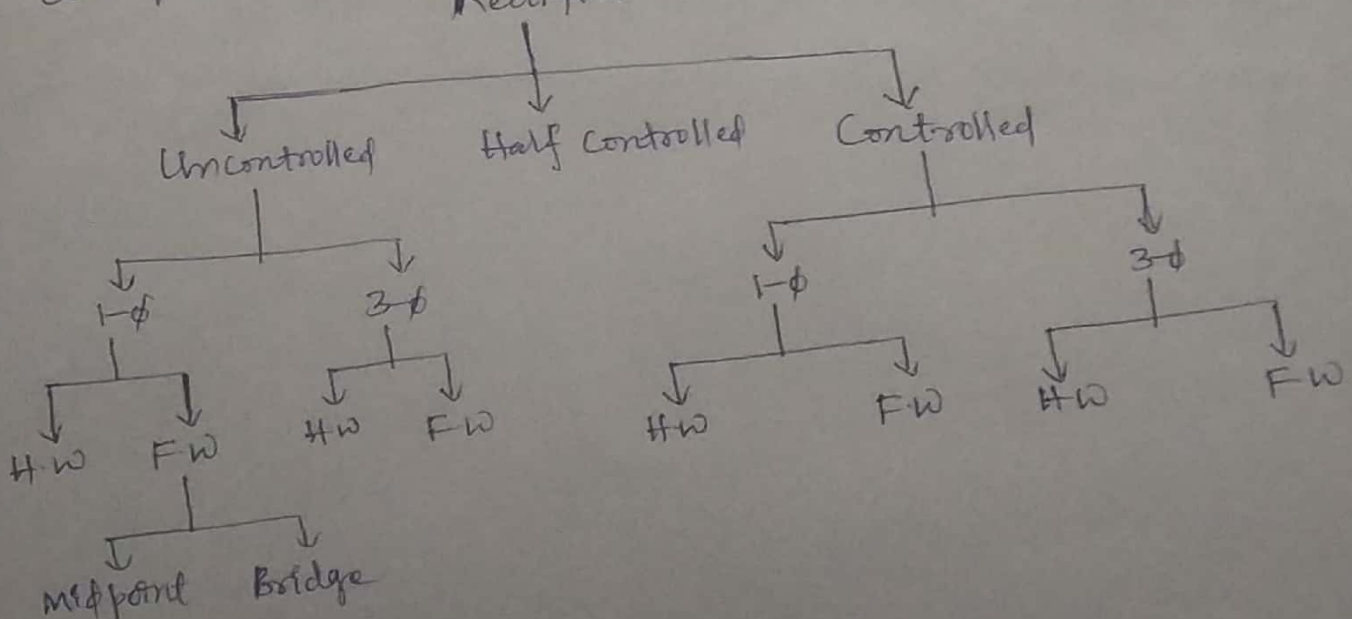
Cyclo converter : fixed Ac to Ac with variable frequency

Rectifier :

- The output is doesn't pure DC (pulsating DC).
- After filter is used to translate pulsating Dc into smooth Dc



classification :



Based on pulse number,

1 pulse converter
2 pulse converter
3 pulse converter
6 pulse converter

Loads connected to the Rectifier.

→ R – load
→ L
→ C
→ RL
→ RL load with free wheeling diode.
→ RLE
→ RE

1. Uncontrolled:

where diodes are used only and provides a fixed dc output voltage for a given ac supply.

2. Controlled:

where thyristors and diodes used, provides an adjustable dc output voltage by controlling the phase at which the device is turned ON.

(a) Half Controlled: Allows electrical power from AC to DC

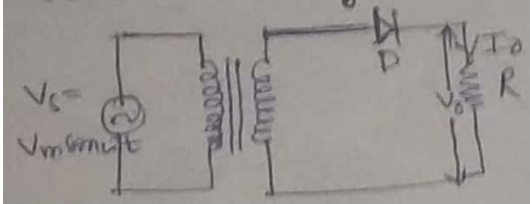
(b) Fully controlled: Allows power from AC to DC as well as DC to AC

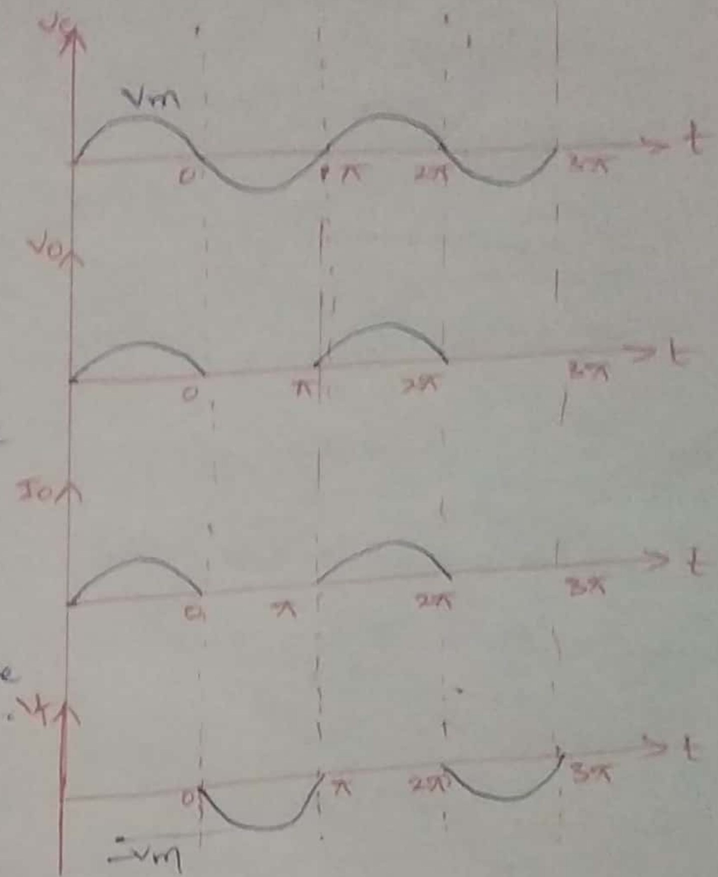
Uncontrolled Rectifiers

Uncontrolled Rectifiers:

1-ϕ Halfwave Uncontrolled Rectifier with R-Load

(a) Circuit Diagram:





(b) Circuit discription:

- The Circuit Consists of a Diode in series with source Voltage(V_s) and Load (R).
- The Source voltage (V_s), that is represented by a Sinusoidal wave given by,

$$V_s = V_m \sin\omega t$$

where $V_m = \sqrt{2}\, V_{rms}$

- V_m is peak to peak value of the supply voltage, ω is the angular frequency and t is the time.

(c) operation:

- **During positive half cycle**

During +ve half Cycle, Anode terminal of diode is +ve w.r.t Cathode. Here, the diode is said to be in the forward bias, so the load voltage follows the supply voltage, from 0 to π.

- **During Negative half cycle:**

During -ve half Cycle, Cathode terminal of diode is +ve w.r.t to Anode so the diode is in reverse biased and blocks the -ve Cycle of the supply voltage. so the voltage appears across the load becomes Zero

1. Average output voltage:

$V_{dc} = V_{avg} = V_0 = \frac{1}{2\pi}\int_0^{\pi} V_s(\omega t)\, d\omega t$

$= \frac{1}{2\pi}\int_0^{\pi} V_m \sin\omega t \cdot d\omega t$

$= \frac{V_m}{2\pi}\left[-\cos\omega t\right]_0^{\pi}$

$= \frac{V_m}{2\pi}\left[\cos\pi - (-\cos 0)\right]$

$= \frac{V_m}{2\pi}\{1+1\} = \frac{V_m}{\pi}$

$$V_{dc} = \frac{V_m}{\pi}$$

2. Average output load current

$$I_{dc} = \frac{V_{dc}}{R} = \frac{V_m}{\pi R}$$

proof:

$I_{dc} = \frac{1}{2\pi}\int_0^{\pi} i(\omega t)\, d\omega t$

$= \frac{1}{2\pi}\int_0^{\pi} I_m \sin\omega t\, d\omega t$

$= \frac{I_m}{2\pi}\int_0^{\pi} \sin\omega t\, d\omega t$

$= \frac{I_m}{2\pi}\left[-\cos\omega t\right]_0^{\pi}$

$= \frac{I_m}{\pi}$

$$I_m = \frac{V_m}{R} \text{ so } I_{dc} = \frac{V_m}{\pi R}$$

3. RMS load voltage.

$V_{or} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} V_s^2(\omega t)\, d\omega t}$

$= \sqrt{\frac{1}{2\pi}\int_0^{\pi} (V_m \sin\omega t)^2\, d\omega t}$

$= \left[\frac{1}{2\pi}\int_0^{\pi} V_m^2 \sin^2\omega t\, d\omega t\right]^{1/2}$

$= \left[\frac{V_m^2}{2\pi}\int_0^{\pi} \sin^2\omega t\, d\omega t\right]^{1/2}$

$= \left[\frac{V_m^2}{2\pi}\int_0^{\pi} \frac{1}{2}(1-\cos 2\omega t)\, d\omega t\right]^{1/2}$

$= \left(\frac{V_m^2}{4\pi}\right)^{1/2} \cdot \left[\int_0^{\pi} 1-\cos 2\omega t \cdot d\omega t\right]^{1/2}$

$= \left(\frac{V_m^2}{4\pi}\right)^{1/2} \cdot \left[\int_0^{\pi} d\omega t - \int_0^{\pi} \cos 2\omega t\, d\omega t\right]^{1/2}$

$= \frac{V_m}{2\sqrt{\pi}}\left[(\omega t)_0^{\pi} - \left[\frac{\sin 2\omega t}{2}\right]_0^{\pi}\right]^{1/2}$

$= \frac{V_m}{2\sqrt{\pi}}\left[(\pi-0) - \left(\frac{\sin 2\pi}{2} - \frac{\sin 2(0)}{2}\right)\right]^{1/2}$

$= \frac{V_m}{2\sqrt{\pi}}(\pi)^{1/2}$

$= \frac{V_m}{2}$

4. rms value of load current

$I_{or} = \frac{V_{rms}}{R} = \frac{V_m}{2R} = \frac{I_m}{2}$

5. output ac power

$(P_{ac})_{out} = V_{rms} I_{rms} = \frac{V_{rms}^2}{R}$ or $\frac{V_m}{2} \times \frac{V_m}{2R}$

$= \frac{V_m^2}{4R}$

6. Efficiency

$$\eta = \frac{P_{dc}}{P_{ac}}$$

$$P_{dc} = V_{dc} I_{dc} \qquad P_{ac} = V_{or} \cdot I_{or}$$

7 output ac component.

$V_{ac} = \sqrt{V_{or}^2 - V_{dc}^2}$

8. form factor

$FF = \frac{V_{or}}{V_{dc}} = \frac{V_m/2}{V_m/\pi} = \frac{\pi}{2}$

9. Ripple factor

$= \frac{\text{rms value of the ac component}}{\text{avg value}}$

$= \frac{V_{ac}}{V_{dc}} = \frac{\sqrt{V_{orm}^2 - V_{dc}^2}}{V_{dc}}$

10. TUF (Transformer utilization factor)

$TUF = \frac{P_{dc}}{V_s I_s}$

[illegible]

11. Input power factor

$$PF = \frac{P_{ac}}{P_{in}} = \frac{V_{or}}{V_s}$$

12. peak Inverse voltage

$$PIV = V_m$$

13. peak factor

$$K_p = \frac{\text{Peak value}}{\text{rms value}}$$

14. Harmonic factor or Total Harmonic distortion (THD)

$$HF = THD = \frac{I_h}{I_{s1}} = \sqrt{\left(\frac{I_s}{I_{s1}}\right)^2 - 1}$$

15. Displacement factor

$$DF = \cos\phi$$

Example 1:

An ideal 1-ϕ source 240 V, 50 Hz, supplies power to a load resistance $R = 100\,\Omega$ via a single ideal diode

(a) Calculate the average and rms values of current and the power dissipation.

(b) Calculate the circuit power factor and ripple factor

(c) what must be the rating of circuit.

Solution:

(a) $$I_{dc} = \frac{V_m}{\pi R} = \frac{240\sqrt{2}}{\pi \times 100} = 1.08\,A$$

$$I_{or} = \frac{V_m}{2R} = \frac{240\sqrt{2}}{2 \times 100} = 1.7\,A$$

power dissipation

$$P_{ac} = I_{or}^2 R = \left(\frac{V_m}{2R}\right)^2 = \frac{V_m^2}{4R^2} = (1.7)^2 \times 100 = 289\,W$$

$$P_{in} = V_s \times I_{or} = 240 \times 1.7 = 408\,W.$$

(b) $$P.F = \frac{P_{ac}}{P_{in}} = \frac{289}{408} = 0.708$$

$$R.F = \frac{\sqrt{V_{or}^2 - V_{dc}^2}}{V_{dc}} = \frac{\sqrt{I_{or}^2 - I_{dc}^2}}{I_{dc}} = \sqrt{\left(\frac{1.7}{1.08}\right)^2 - 1} = 1.21$$

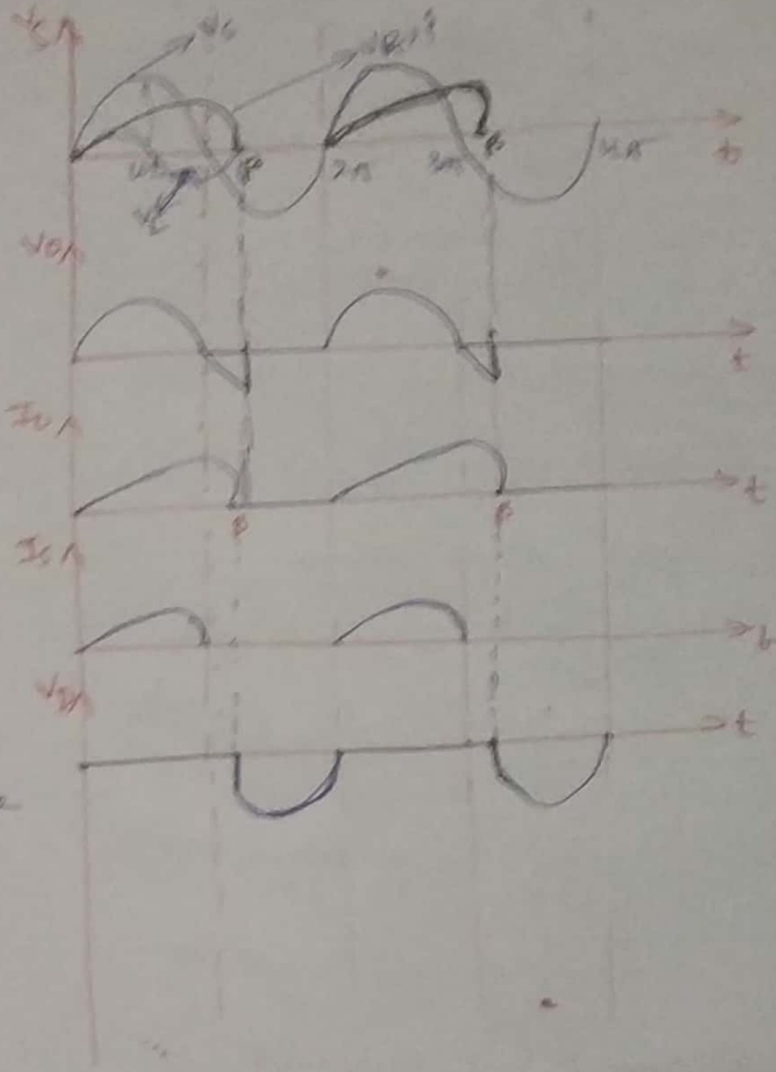
(c) PIV = ~~Vm = √2 × 240~~

$$V_m = \sqrt{2} V_s = \sqrt{2} \times 240 = 339.4\,V$$

diode = 400 V

1-ϕ Half wave un controlled Rectifier with RL load:

(a) Circuit diagram



(b) Circuit discription:

- the circuit consists of a diode in series with source voltage and load (RL)
- If the load consists of a series resistor and inductor, the current will flow through the negative cycle as well

(c) Operation:

Each supply period (cycle) can be divided into 4-distinct regions.

- From 0-ωt: The current rises from 0 to peak which lags the voltage peak due to circuit inductance; V_L is +ve inductor stores energy.
- From ωt-π: the current decays and hence V_L is -ve. both source & inductance supply energy to R.
- From π-β: the current continuous to decay until it reaches to zero V_L remains negative and energy supplied by inductance to both source and resistor
- From β-2π: at β current reaches zero and the diode cut-out. Current remains zero.

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(a) Average output voltage

$$V_{dc} = \frac{1}{2\pi}\int_0^{\beta} V_s(\omega t)\, d\omega t$$

$$= \frac{1}{2\pi}\int_0^{\beta} V_m \sin\omega t\, d\omega t$$

$$\boxed{V_{dc} = \frac{V_m}{2\pi}(1-\cos\beta)}$$

(b) Average value of load current

$$I_{dc} = \frac{V_{dc}}{R} = \frac{V_m}{2\pi R}(1-\cos\beta)$$

(c) rms output voltage

$$V_{or} = \sqrt{\frac{1}{2\pi}\int_0^{\beta} V_m^2 \sin^2\omega t\, d\omega t}$$

$$= \left[\frac{V_m^2}{2\pi}\int_0^{\beta} \sin^2\omega t\, d\omega t\right]^{1/2}$$

$$= \left[\frac{V_m^2}{2\pi}\int_0^{\beta}\frac{1}{2}(1-\cos 2\omega t)\, d\omega t\right]^{1/2}$$

$$= \frac{V_m}{2\sqrt{\pi}}\left(\beta - \frac{\sin 2\beta}{2}\right)^{1/2}$$

(d) rms output current

$$I_{or} = \frac{V_{orms}}{R}$$

⇒ when diode conducts, KVL for ckt

$$V_m \sin\omega t = i_o R + L\frac{di_o}{dt}$$

⇒ Equation of the current

above equation is 1st order differential equation. The solution of this equation has two parts steady state and transient

(1) steady state solution

$$i_{ss}(\omega t) = \frac{V_m}{Z}\sin(\omega t - \theta)$$

(2) transient solution

$$i_{tr}(\omega t) = Ae^{t/\tau}$$

where $Z = \sqrt{R^2 + \omega^2 L^2}$

$\tan\theta = \frac{\omega L}{R}$

$\tau = \frac{L}{R}$ time constant

A = constant

$$i(\omega t) = i_{ss}(\omega t) + i_{tr}(\omega t)$$

$$= \frac{V_m}{Z}\sin(\omega t - \theta) + Ae^{t/\tau}$$

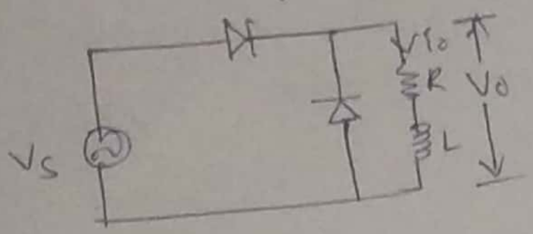
$$\boxed{A = \frac{V_m}{Z}\sin\theta}$$

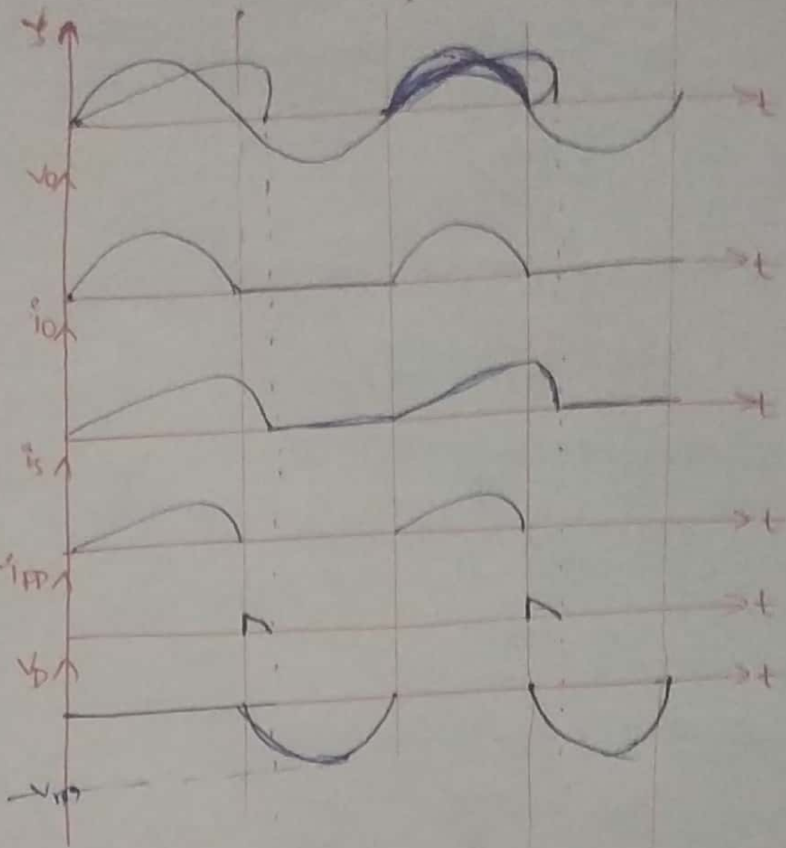
$$i(\omega t) = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}}\left[\sin(\omega t - \theta) + \sin\theta\, e^{-(R/\omega L)\omega t}\right]$$

to find β, put β in ωt.

1-ϕ Halfwave uncontrolled Rectifier with RL and free wheeling diode

(a) Circuit diagram:





(b) Circuit discription:

(c) operation

(a) Average output voltage

$$V_{dc} = \frac{1}{2\pi} \int_0^{\pi} V_m \sin\omega t \, d\omega t$$

$$= \frac{V_m}{\pi}$$

(b) Average output current

$$I_{dc} = \frac{V_{dc}}{R} = \frac{V_m}{\pi R}$$

(c) rms output voltage

$$V_{or} = \left[\frac{1}{2\pi} \int_0^{\pi} V_m^2 \sin^2\omega t \, d\omega t\right]^{1/2}$$

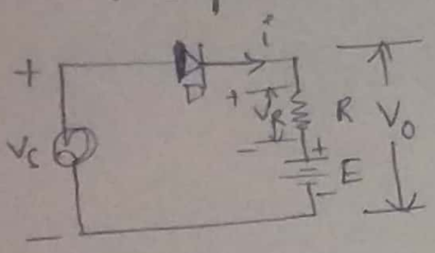
$$= \frac{V_m}{2}$$

(d) RMS output current

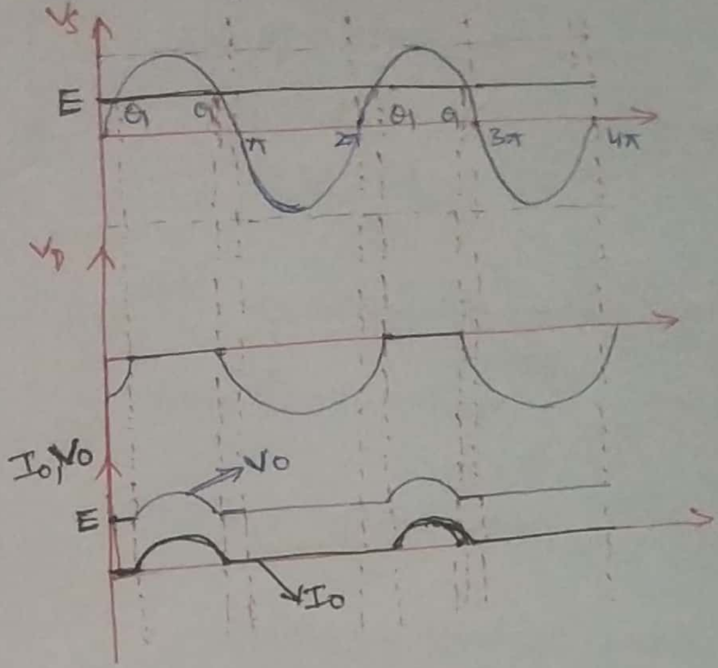
$$I_{or} = \frac{V_{or}}{R} = \frac{V_m}{2R}$$

1-ϕ Half wave uncontrolled Rectifier with RE-load.

Circuit diagram:



Waveform:



Circuit discription:

Mathematical Expressions:

During the conduction period of diode =

$$\omega t = \theta_1 \text{ to } \omega t = (\pi - \theta_1)$$

$$V_m \sin\omega t = E + i_0 R$$

$$i_0 = \frac{V_m \sin\omega t - E}{R}.$$

(a) Average value of current

$$I_0 = \frac{1}{2\pi R} \int_{\theta_1}^{\pi-\theta_1} (V_m \sin\omega t - E) \cdot d\omega t$$

$$I_0 = \frac{1}{2\pi R} \, 2V_m \cos\theta_1 - E(\pi - 2\theta_1)$$

(b) RMS value of load current

$$I_{or} = \left[\frac{1}{2\pi} \int_{\theta_1}^{\pi-\theta_1} \left[\frac{V_m \sin\omega t - E}{R}\right]^2 d\omega t\right]^{1/2}$$

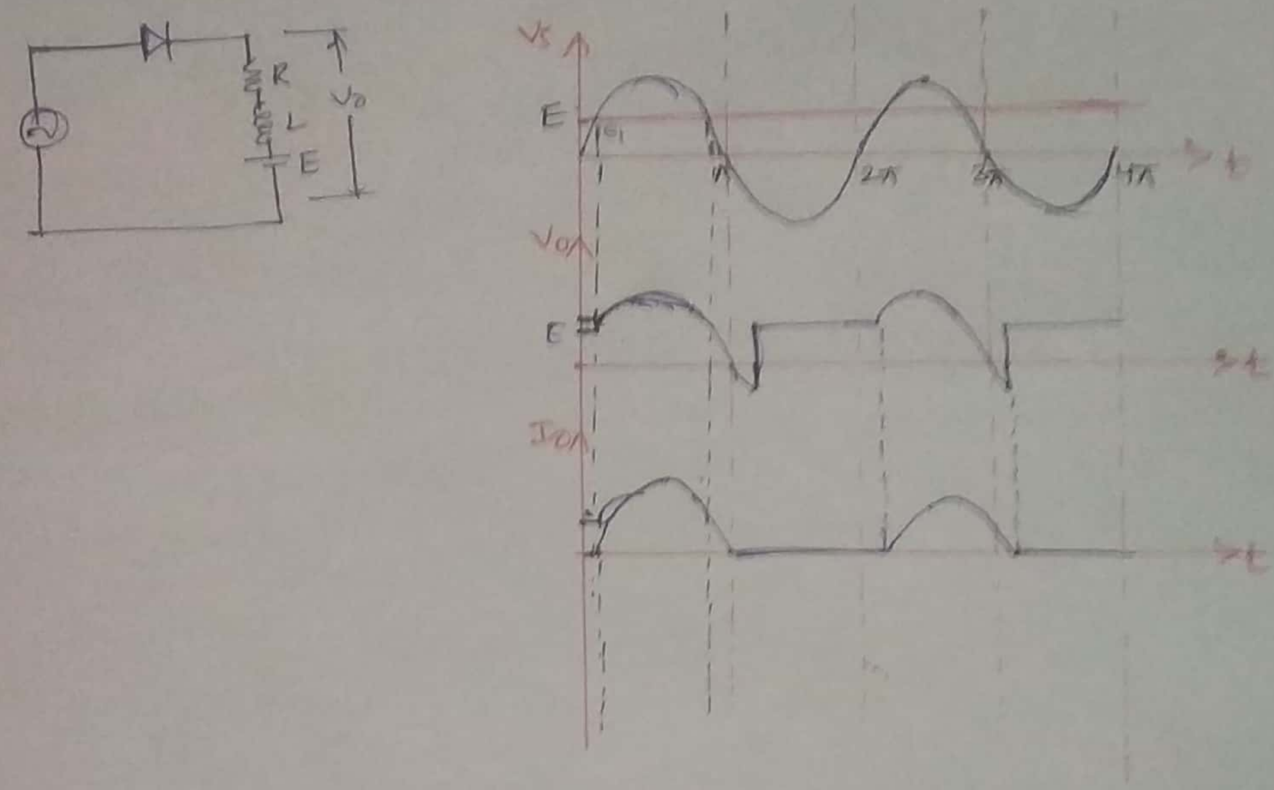
$$= \left[\frac{1}{2\pi R^2} \int_{\theta_1}^{\pi-\theta_1} (V_m^2 \sin^2\omega t + E^2 - 2V_m E \sin\omega t)\, d\omega t\right]^{1/2}$$

$$= \left[\frac{1}{2\pi R^2} \left\{(V_s^2 - E^2)(\pi - 2\theta_1) + V_s^2 \sin 2\theta_1 - 4V_m E \cos\theta_1\right\}\right]^{1/2}$$

(c) power deliver to load $= P = EI_0 + I_{or}^2 R$ watt.

(d) supply power factor $= \dfrac{EI_0 + I_{or}^2 R}{V_s \cdot I_{or}}$

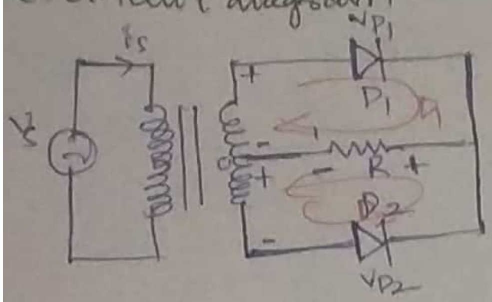
1-ϕ Half wave uncontrolled Rectifier with RLE - load

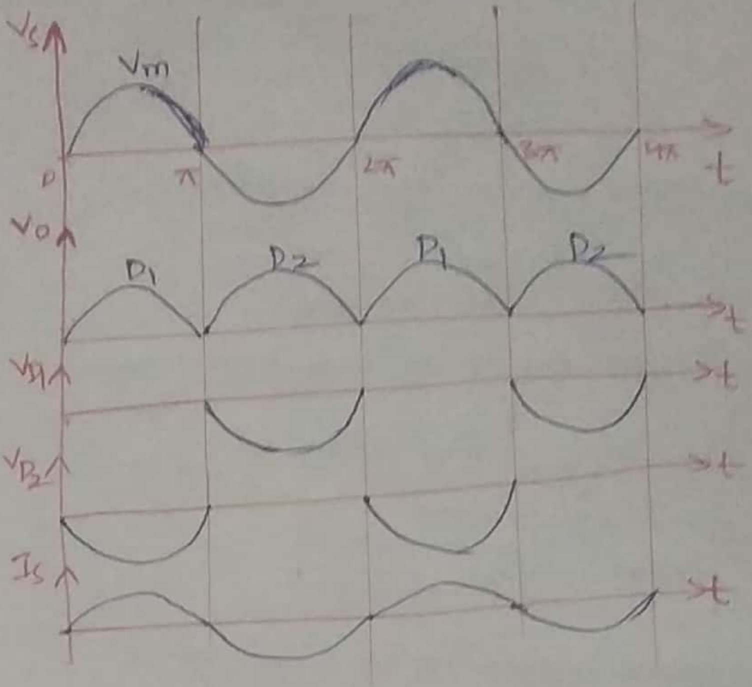


1-ϕ Half wave uncontrolled midpoint Rectifier with R-load

Midpoint or center tapped

(a) Circuit diagram





(b) Circuit description:

(c) operation :

- During +ve half cycle D_1 Conducts and $V_0 = V_s$

- During -ve half cycle D_2 Conducts and $V_0 = -V_s$

(a) Average output Voltage

$$V_{dc} = \frac{1}{\pi}\int_0^{\pi} V_m \sin\omega t \, d\omega t$$

$$= \frac{2V_m}{\pi}$$

(b) Average output current

$$I_{dc} = \frac{V_{dc}}{R} = \frac{2V_m}{\pi R}$$

(c) rms value of output voltage

$$V_{or} = \left[\frac{1}{\pi}\int_0^{\pi} V_m^2 \sin^2\omega t \, d\omega t\right]^{1/2}$$

$$V_{or} = \frac{V_m}{\sqrt{2}} = V_s$$

(d) rms value of load current

$$I_{or} = \frac{V_{or}}{R} = \frac{V_m}{\sqrt{2}R}$$

(e) power deliver to load

$$P_{dc} = V_{or} I_{or} = I_{or}^2 R$$

(f) input volt ampers $= V_s \cdot I_{or}$.

(g) power factor

$$p.f = \frac{V_{or} \cdot I_{or}}{V_s \cdot I_{or}} = 1$$

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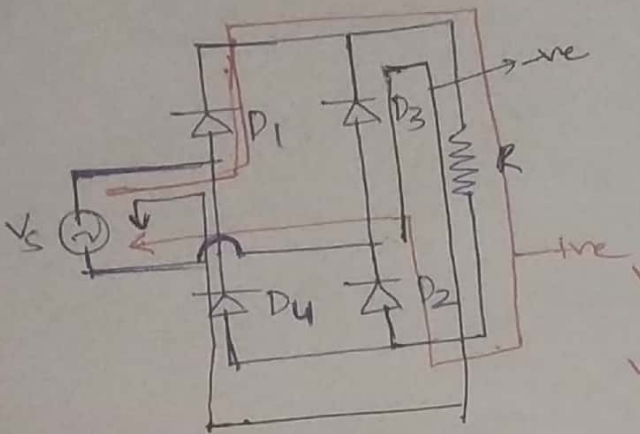
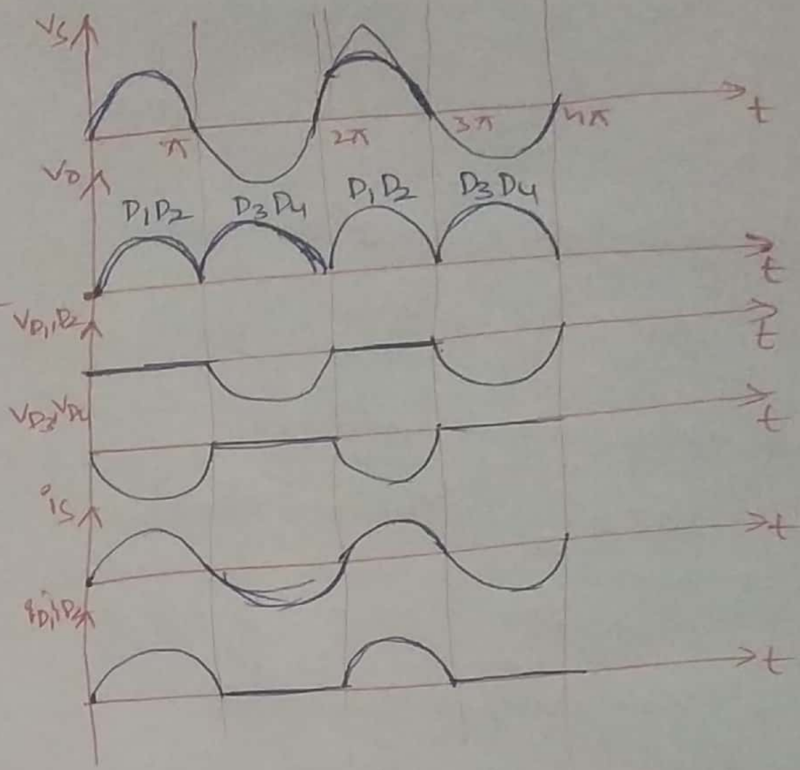
1-ϕ Full wave un controlled ~~Rectifier~~ Bridge Rectifier with R-load

(a) circuit diagram





(b) circuit discription

(c) operation:

_During +ve half cycle $D_1 D_2$ conducts and $V_0 = V_s$

|

_During -ve half cycle $D_3 D_4$ conducts and $V_0 = -V_s$

(a) average output voltage

$$V_{dc} = \frac{1}{\pi}\int_0^{\pi} V_m \sin\omega t \cdot d\omega t$$

$$= \frac{2V_m}{\pi}$$

(b) average output current

$$I_{dc} = \frac{V_{dc}}{R} = \frac{2V_m}{\pi R}$$

(c) rms output voltage

$$V_{or} = \sqrt{\frac{1}{\pi}\int_0^{\pi} (V_m \sin\omega t)^2 d\omega t)}$$

$$= \frac{V_m}{\sqrt{2}}$$

(d) rms output current

$$I_{or} = \frac{V_{or}}{R} = \frac{V_m}{\sqrt{2} \cdot R}$$

(e) $P_{dc} = V_{dc} I_{dc} = \frac{V_{dc}^2}{R}$

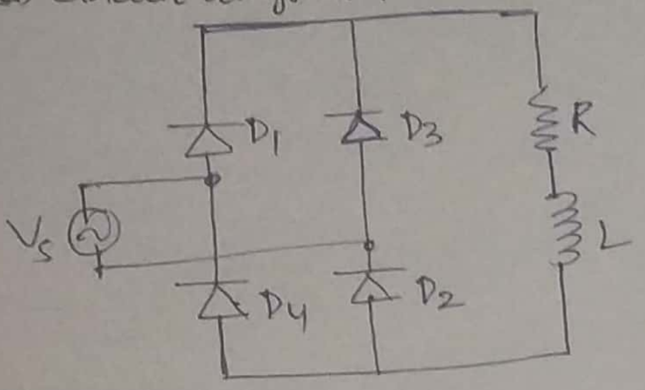
(f) $P_{ac} = V_{or} I_{or} = \frac{V_{or}^2}{R}$

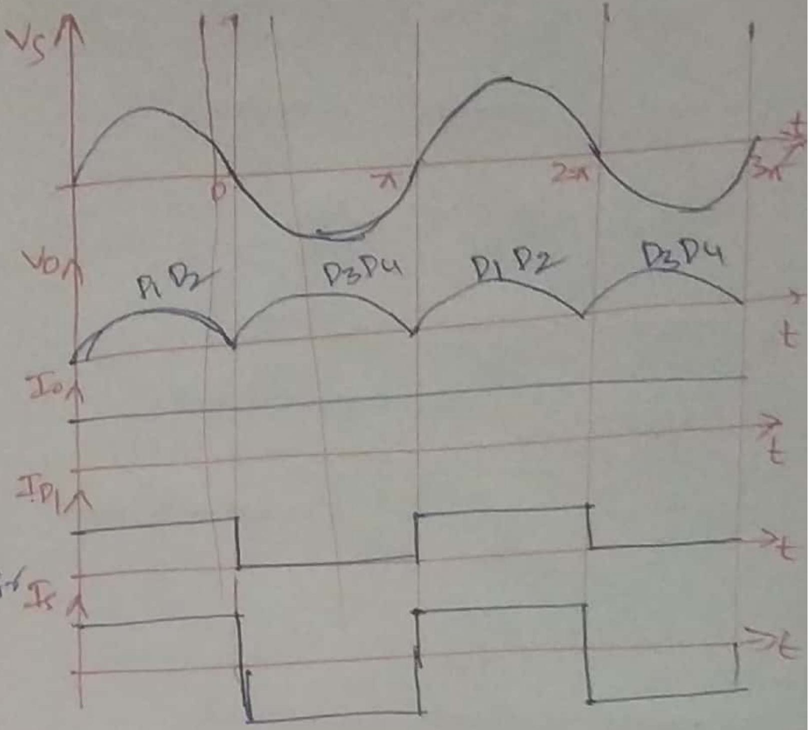
(g) $RF = \frac{\sqrt{V_{or}^2 - V_{dc}^2}}{V_{dc}} = 0.48$

1-ϕ Full wave uncontrolled Rectifier with RL load:

RL load or Highly inductive load

(a) Circuit diagram:





(b) Circuit discription:

- above circuit diagram is the connection for a 1-ϕ F.W rectifier loaded with highly inductive.
-

(c) operation:

- The load current assumed constant.
- During the conduction of D_1 & D_2 simultaneously the supply voltage appears directly across the load. It is same as the case of resistive load.

(a) average output voltage

$$V_{dc} = \frac{1}{\pi}\int_0^{\pi} V_m \sin\omega t \, d\omega t = \frac{2V_m}{\pi}$$

(b) Avg output current

$$I_{dc} = \frac{V_{dc}}{R} = \frac{2V_m}{\pi R}$$

(c) rms output voltage

$$V_{or} = \sqrt{\frac{1}{\pi}\int_0^{\pi} (V_m \sin\omega t)^2 d\omega t}$$

$$= \sqrt{\frac{V_m^2}{\pi}\int_0^{\pi} \frac{1}{2}(1-\cos 2\omega t)\, d\omega t}$$

$$= \frac{V_m}{\sqrt{2}}$$

(d) rms output current

$$I_{or} = \frac{V_{or}}{R} = \frac{V_m}{\sqrt{2} \cdot R}$$

(e) output ac power

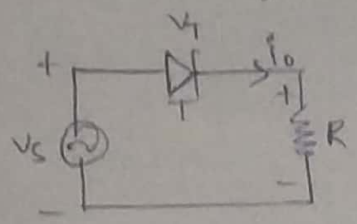
$$P_{ac} = V_{or} I_{or} = \frac{V_{or}^2}{R}$$

Controlled Rectifiers

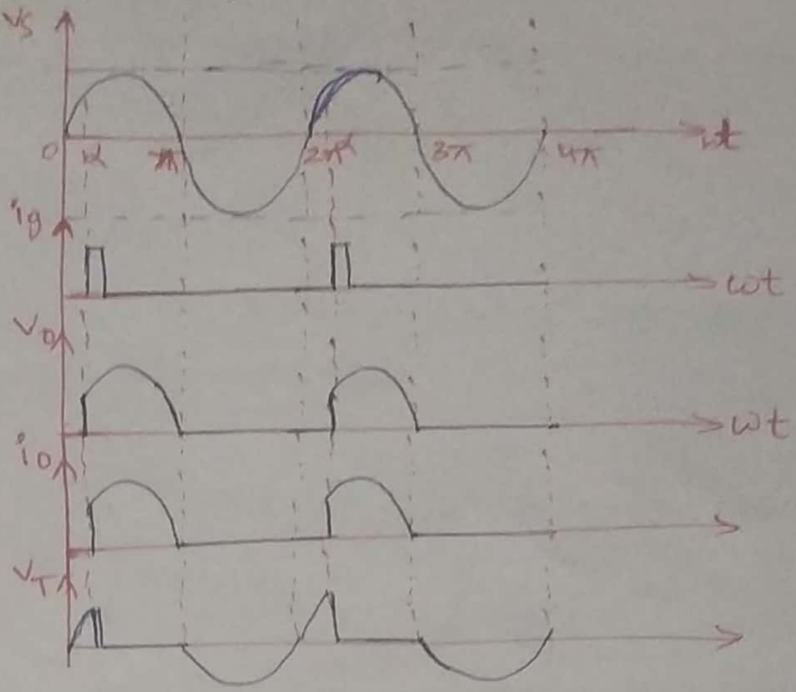
Controlled Rectifiers:

1-ϕ Halfwave Controlled Rectifier with R-load:

(a) Circuit diagram



Wave forms



$V_S = V_0 + V_T$

(b) Circuit discription:

- The basic circuit for a 1-ϕ halfwave, controlled rectifier loaded with a resistive load is shown in the figure

(c) operation:

- For this configuration, the thyristor will conducts, when triggered using gate pulses provided that the supply voltage (V_s) is positive.
- The thyristor is fired at $\omega t = \alpha$ and the input voltage appears across the load.
- At, $\omega t = \pi$, T_1 is reversed-biased by the negative supply voltage and is turned off.

★ α is termed as the delay or firing angle.

Mathematical Expressions:
Circuit turn off time $t_c = \pi/\omega$ sec

(a) Average output voltage $V_o = \frac{1}{2\pi}\int_0^{\pi} V_m \sin\omega t \cdot d\omega t$

$$= \frac{V_m}{2\pi}(1+\cos\alpha)$$

(b) Average load current

$$I_o = \frac{V_o}{R} = \frac{V_m}{2\pi R}(1+\cos\alpha)$$

(c) rms value of load current

$$V_{or} = \left[\frac{1}{2\pi}\int_0^{\pi} V_m \sin\omega t \cdot d\omega t\right]^{1/2}$$

$$= \frac{V_m}{2\sqrt{\pi}}\left[(\pi-\alpha) + \tfrac{1}{2}\sin 2\alpha\right]^{1/2}$$

(d) rms output current $I_{or} = \frac{V_{or}}{R}$

(e) power deliver to resistive load = (rms load voltage) $\cdot$ (rms load current)

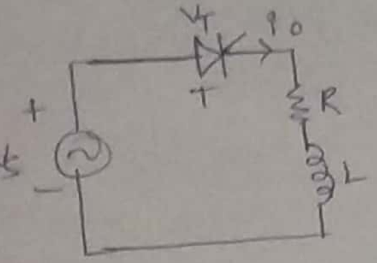
$$= V_{or} \cdot I_{or} = V_{or}^2 / R = I_{or}^2$$

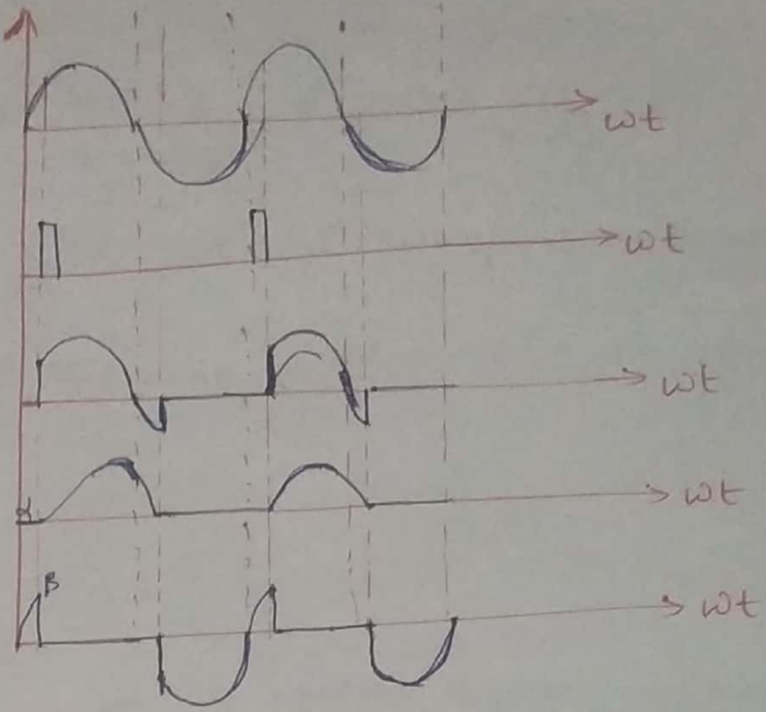
(f) Input power factor:

$$Pf = \frac{1}{\sqrt{2\pi}}\left[(\pi-\alpha) + \frac{1}{2}\sin 2\alpha\right]^{1/2}$$

1-ϕ Halfwave Controlled Rectifier with RL-load:

a) Circuit diagram:





b) Circuit discription:

Source Current

$$i_s = \frac{V_m}{\sqrt{R^2+X^2}} \sin(\omega t - \phi) \qquad \phi = \tan^{-1} X/R$$

and $X = \omega L$

Transient Component i_t = $R i_t + L \frac{di_t}{dt} = 0$ $\quad i_t = A e^{(-R/L)t}$

$$i_o = i_s + i_t = V_m/Z \sin(\omega t - \phi) + A e^{(-R/L)t}.$$

(a) Average output voltage ~~V_o~~

$$V_o = \frac{1}{2\pi} \int_{\alpha}^{\beta} V_m \sin\omega t \cdot d\omega t$$

$$= \frac{V_m}{2\pi} (\cos\alpha - \cos\beta)$$

(b) Average load current

$$I_o = \frac{V_o}{R} = \frac{V_m}{2\pi R} (\cos\alpha - \cos\beta)$$

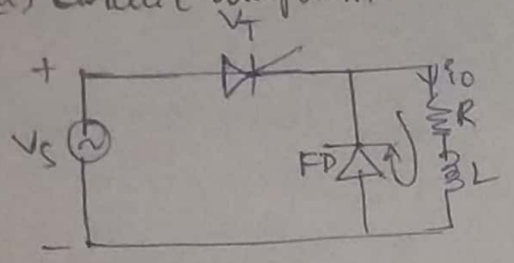
(c) rms ~~Average~~ load voltage ~~Current~~

$$V_{or} = \left[\frac{1}{2\pi} \int_{\alpha}^{\beta} V_m^2 \sin^2\omega t \cdot d\omega t\right]^{1/2}$$

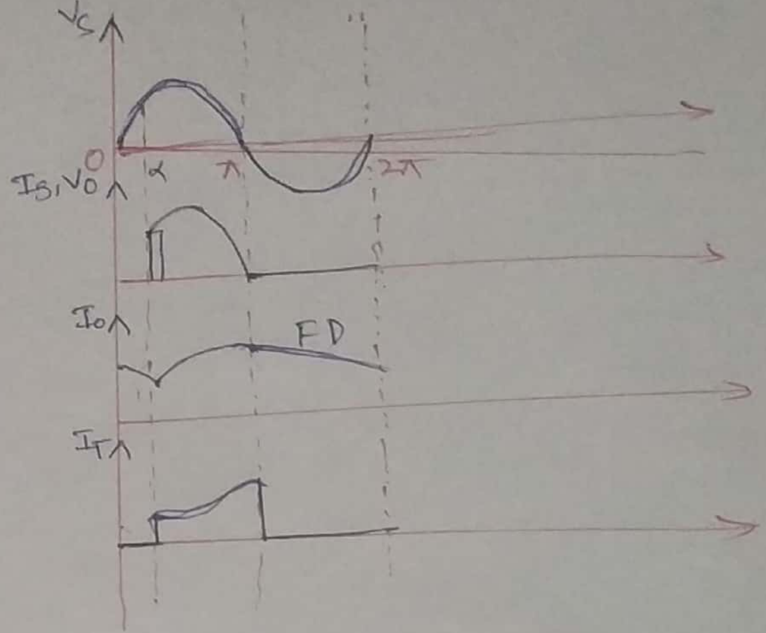
$$= \frac{V_m}{4\pi} \left[(\beta - \alpha) + \frac{1}{2} \sin 2\beta - \sin 2\alpha\right]^{1/2}$$

1-φ Halfwave Controlled Rectifier with RL-load and Freewheeling Diode

(a) Circuit diagram:



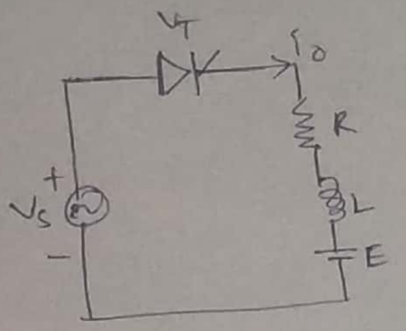
(b) Circuit discription:

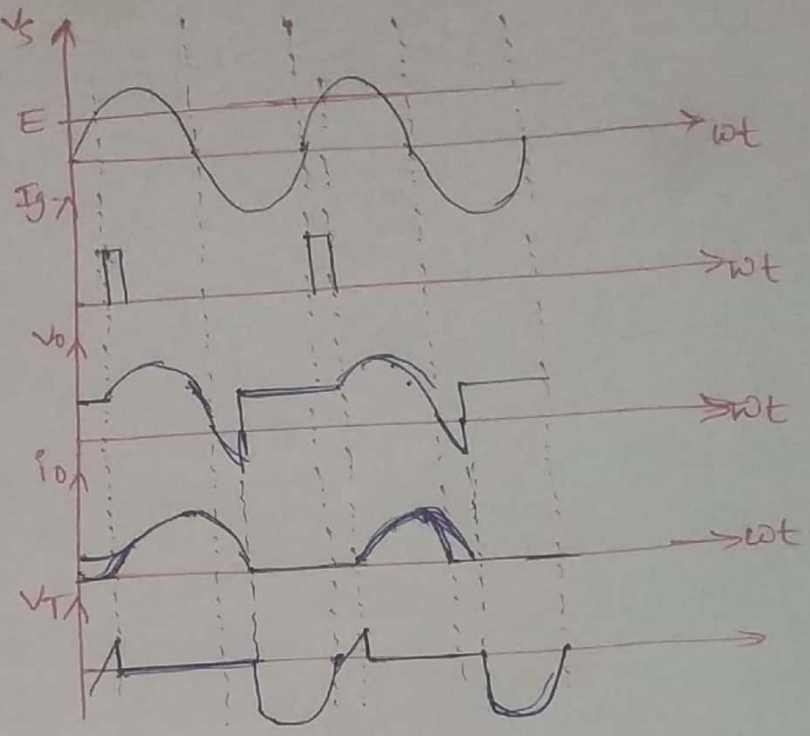


(c) operation:

- the freewheeling diode is connected in the circuit in such a way as to provide an alternative path for the decaying load current so that the thyristor current is allowed to become zero and the thyristor is allowed to switch off.
- The supply voltage is positive, from α to π, FD is reverse bias and passes no current, so that source and load current are equal ($i_s = i$).
- During negative half cycle, the load current i flows through the low resistance path provided by FD rather than against the negative supply voltage, so that $i_{FD} = i$. The current is driven by the energy stored in L. it decays according to the time constant of the circuit (R, L and FD)

1-ϕ Halfwave controlled Rectifier with RLE-load:





3. Single-phase controlled converter circuits

3.1 Single-phase, fully-controlled bridge rectifier (p = 2)

Single-phase, fully-controlled full- wave rectifier bridge is shown in Fig.6.3.In this this circuit, two thyristors must triggered simultaneously to permit current to flow. For example, with the instantaneous polarity indicated in Fig.6.3, T1 and T4 must be triggered, while in reverse, T3 and T2 must triggered at the same time. The output voltage waveform is shown in Fig.6.4 for the case of resistive load.

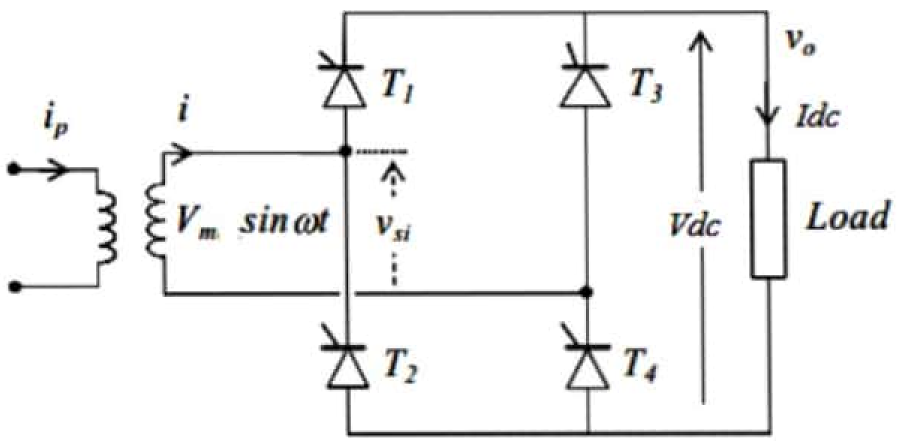


Fig. 6.3 Single-phase fully-controlled full- wave Rectifier Bridge

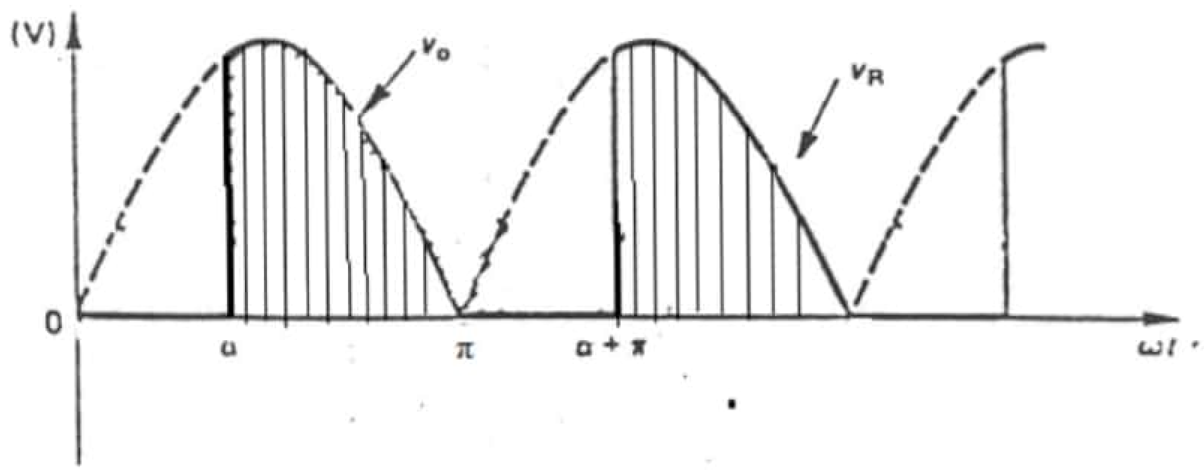


Fig.6.4 Output voltage waveform

- **Operation of the converter with Resistive load**

The dc output voltage of the converter with resistive load is given by

$$V_{dc} = \frac{1}{\pi}\int_{\alpha}^{\pi} V_m \sin\omega t \, d\omega t$$
$$= \frac{V_m}{\pi}(1+\cos\alpha)$$

$$\text{RMS output vol tage} = V_{rms} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi} (V_m \sin \omega t)^2 d\omega t}$$
$$= V_m\sqrt{\frac{\pi - \alpha + \frac{1}{2}\sin 2\alpha}{2\pi}}$$

- **Operation of the converter with R – L load**

(i) Case of R - L load with small L / R ratio : <u>Discontinuous load current</u> .
In this case the current will be discontinuous as shown in Fig.6.5.

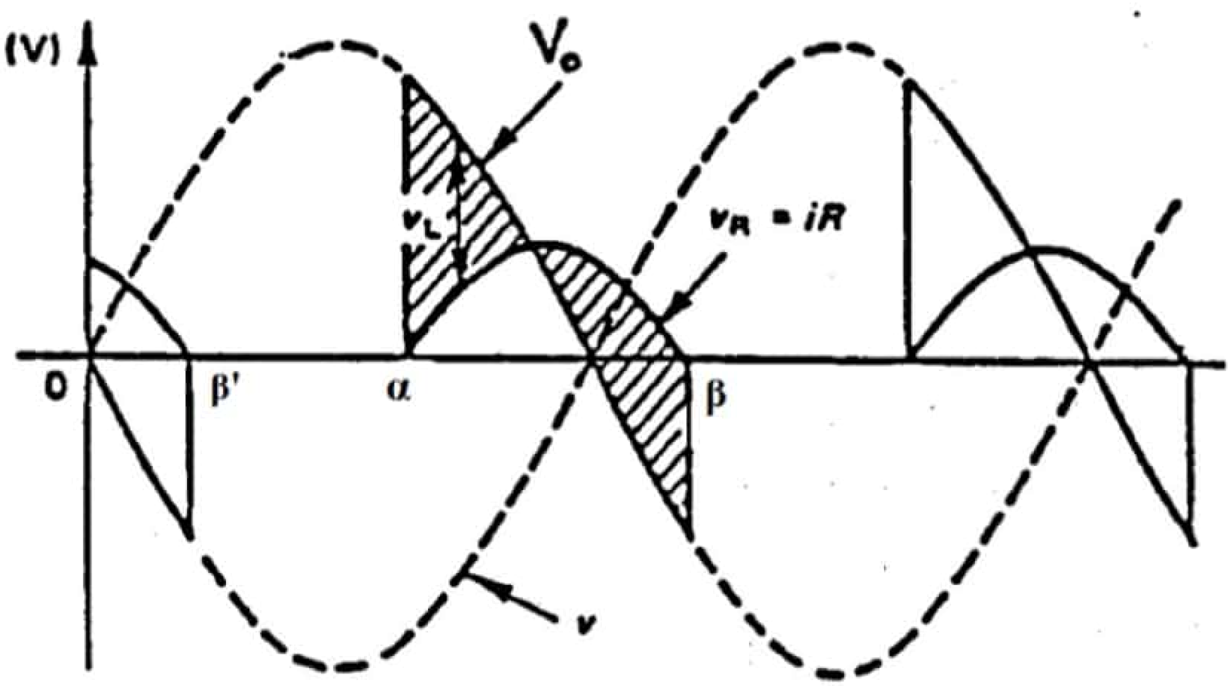


Fig.6.5

For $\alpha \le \omega t \le \beta$, the circuit equation is given by :-

$$\text{Average output voltage} = V_{dc} = \frac{1}{\pi}\int_{\alpha}^{\beta} V_m \sin \omega t \, d\omega t$$

$$= \frac{V_m}{\pi}(\cos\alpha - \cos\beta)$$

(ii) Case of R - L load with large L / R ratio: <u>Continuous load current</u> .

Under these conditions, a thyristor is still conducting when another is forward-biased and is turned on. The first device is instantaneously reverse-biased by the second device which has been turned on. The first device is commutated and load current is instantaneously transferred on the incoming device . In this case the current is continuous as shown in Fig.6.6.

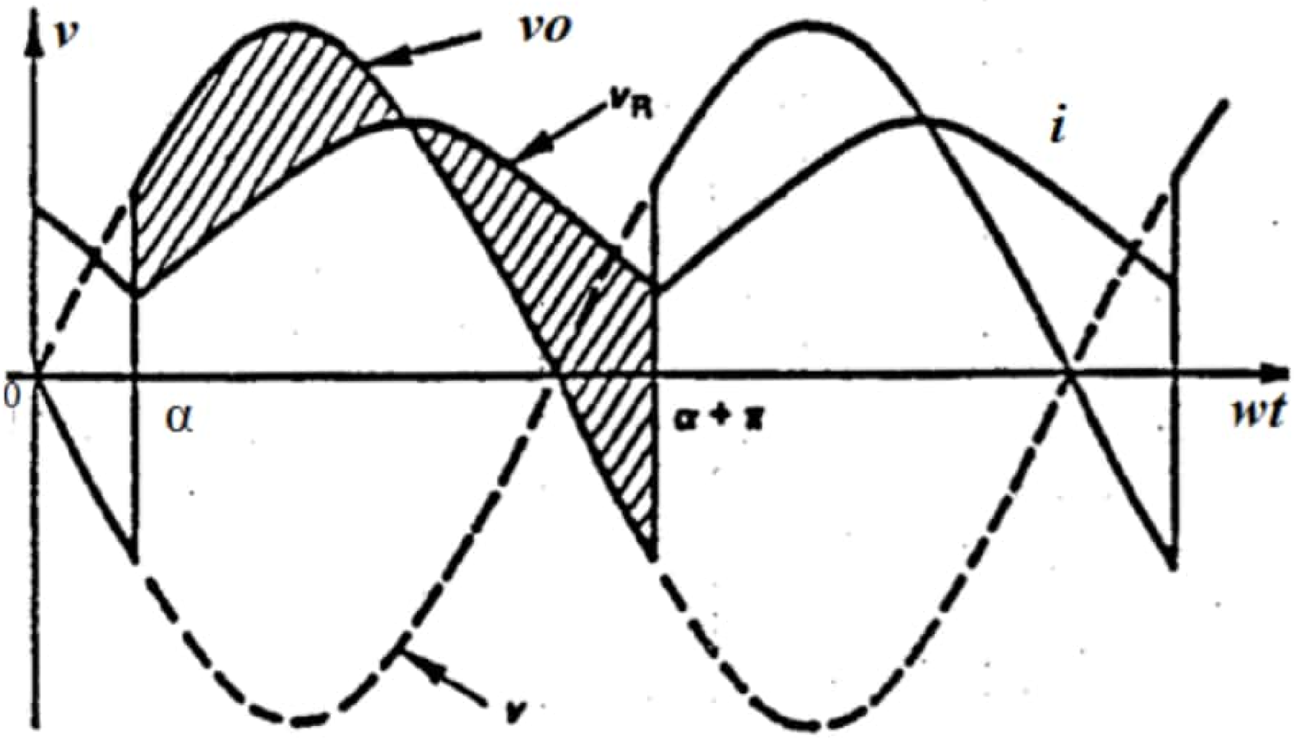


Fig. 6.6

$$V_{dc} = \frac{1}{\pi}\int_{\alpha}^{\alpha+\pi} V_m \sin \omega t \, d\omega t$$

$$V_{dc} = \frac{2V_m}{\pi}\cos\alpha$$

Under all delay angle condition, the average current is given by:

$$I_{dc} = \frac{V_{dc}}{R}$$

From the above voltage equation, if the firing angle is greater than 90°, the average voltage can be negative .Thus if the firing angle is suddenly increased to 170° , a large negative voltage will be applied to the load and the power is fed back to the supply. This process is known as 'INVERSION'.
The graph shown in Fig. 6.7 gives the relation between the firing angle and the output voltage in p.u. for the two modes of operation (continuous and discontinuous) for full-wave single- phase rectifier.

Rule – of- thumb: To find roughly the current is continuous or discontinuous:

If	$\pi + \alpha < \beta$	The current is continuous
If	$\pi + \alpha > \beta$	The current is discontinuous

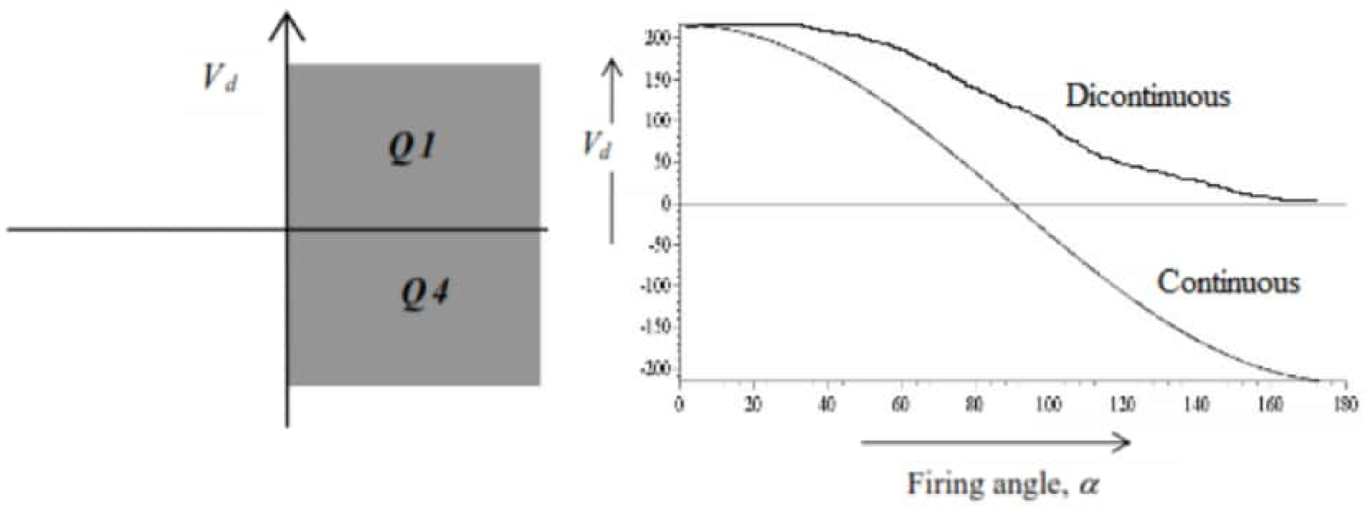
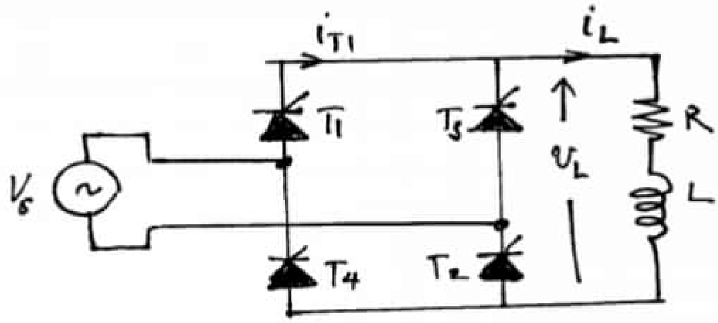


Fig. 6.7

Example : A single-phase controlled full-wave rectifier shown in Fig. A has a source of 120V rms at 50 Hz, and is feeding a load $R = 10\,\Omega$ and $L = 20\,mH$. The firing angle is $\alpha = 30°$ and the extinction angle is $\beta = 216°$.



1. Specify whether the current is continuous or discontinuous:

Since $\pi + \alpha = 210° < \beta$ then the current is continuous.

2. Determine the average load voltage and current

$$V_m = \sqrt{2} \times 120 = 169.7\ V.$$

$$V_{dc} = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} V_m \sin\omega t\, d\omega t = \frac{V_m}{\pi}\left(\cos\alpha - \cos(\alpha+\pi)\right)$$

$$= \frac{169.7}{\pi} \times 2\cos 30 = 93.56\,V.$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{93.56}{10} = 9.36\,A.$$

3. Determine the r.m.s load Voltage and current

$$V_{rms} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi+\alpha}(V_m \sin\omega t)^2 d\omega t} = V_m\sqrt{\frac{1}{\pi}\left[\frac{\omega t}{2} - \frac{\sin 2\omega t}{4}\right]_{\alpha}^{\pi+\alpha}}$$

$$= \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{\pi}\left[\pi + \alpha - \alpha - \frac{\sin 2\alpha - \sin 2\alpha}{2}\right.}$$

$$= \frac{V_m}{\sqrt{2}} = 120\,V.$$

$$I_{rms} = \frac{V_{rms}}{Z_L} = \frac{V_{rms}}{\sqrt{R^2 + (\omega L)^2}} = \frac{120}{\sqrt{10^2 + (314.16 \times 2 \times 10^{-2})^2}}$$

$$= \frac{120}{\sqrt{139.48}} = 10.16\,A.$$

5. Determine the ac power absorbed by the load

$$P_{ac} = V_{r.m.s}\, I_{r.m.s} = 120 \times 10.16 = 1219.2\,W$$

6. Determine the dc power absorbed by the load

$$P_{dc} = V_{dc}\, I_{dc} = 93.56 \times 9.36 = 875.7\ W$$

7. Determine the efficiency of the rectifier :

$$\eta = \frac{P_{dc}}{P_{ac}} = \frac{875.7}{1219.2} \times 100 = 71.82\ \%.$$

H.W : Repeat the example with a freewheeling diode connected across the load. Compare the new efficiency with that obtained in the above example. and conclude.

(iii) Case of highly inductive load. (L > > R)

Fig. 6.8 (a) shows the circuit connection for a single-phase, full-wave, controlled rectifier loaded with a highly inductive load. For one total period of operation of this circuit, the corresponding waveforms are shown in Fig. 6,8(c) .

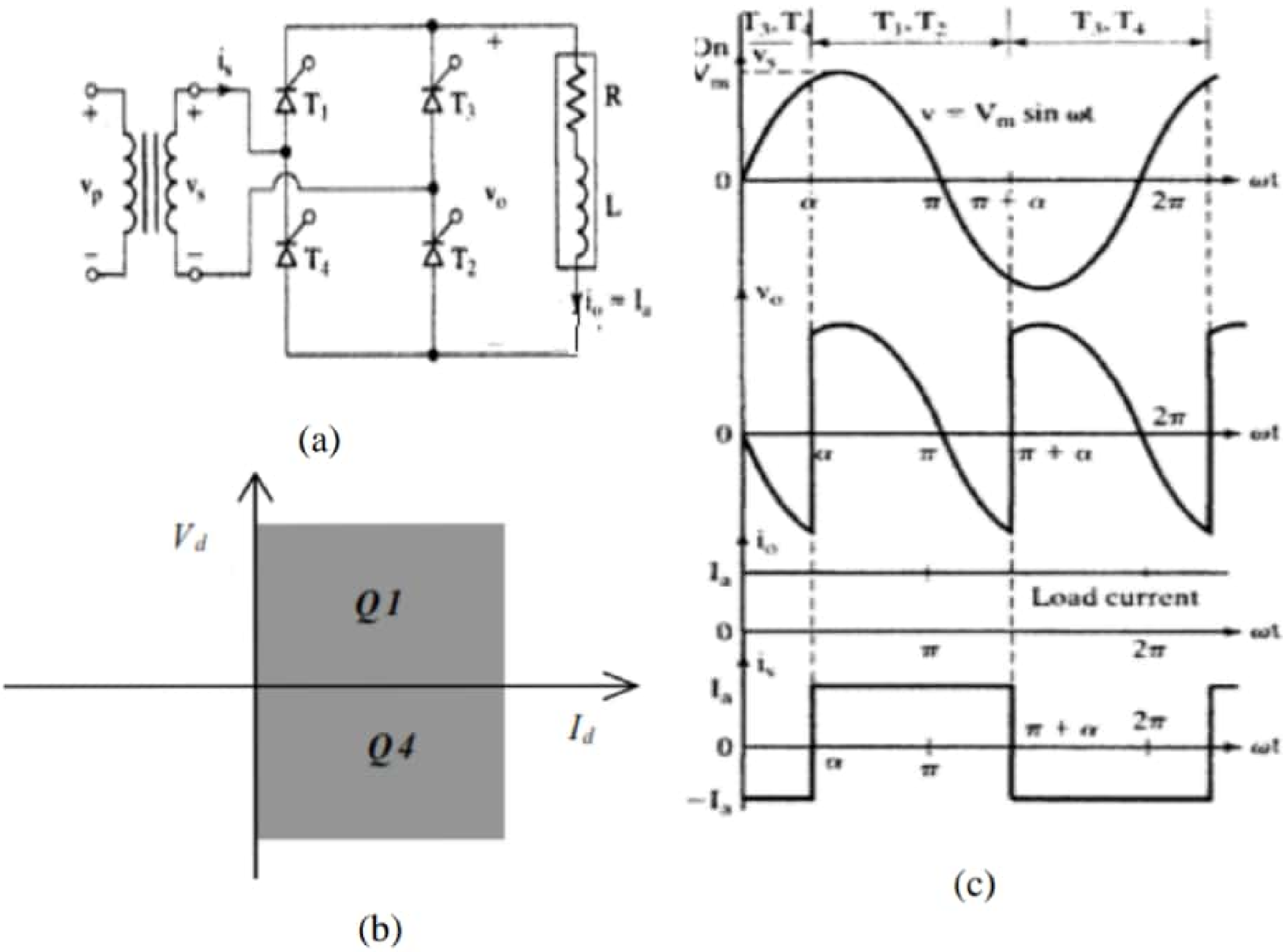


Fig. 6.8 Single phase full-wave rectifier loaded with highly inductive load

The average value of the load voltage V_{dc} can be calculated as follows,

$$V_{dc} = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} v_s(\omega t)\, d\omega t = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} V_m \sin(\omega t)\, d\omega t\; d\omega t$$

$$\therefore V_{dc} = \frac{2V_m}{\pi}\cos(\alpha)$$

Since the load is a highly inductive load. Then, the load current is considered constant (ripple free current) and equal to the average value of the load current I_{dc} as follows,

$$I_{dc} = I_a = \frac{V_{dc}}{R} = \frac{2V_m}{\pi R}\cos(\alpha)$$

Therefore, the average output voltage can vary from $+2V_m/\pi$ to $-2V_m/\pi$ when varying α from π to 0, respectively. Moreover, since the load voltage for this configuration can be positive or negative while the load current is always positive because the thyristors prevents a reverse current flow. Therefore, this converter operates in the first and the fourth quadrants as shown in Fig. 6.8(b).

The rms value of the load voltage V_{rms} can be calculated as follows,

$$V_{rms} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi+\alpha}\{v_s(\omega t)\}^2\, d\omega t} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi+\alpha}\{V_m \sin(\omega t)\}^2\, d\omega t}$$

$$\therefore V_{rms} = \frac{V_m}{\sqrt{2}} = 0.707\, V_m$$

Since the load current is constant over the studied period, therefore the rms value of the load current I_{rms} is

$$I_{rms} = I_{dc} = I_a$$

The PRV for any thyristor in this configuration is (V_m).

5.Poly- phase uncontrolled rectification

For most industrial applications poly – phase rectifier circuits are use. The circuit employed may gives either half – wave or full – wave, controlled or uncontrolled rectifier circuits.

1. Three –Phase Half -Wave Uncontrolled Rectifier

Fig.5.1 shows a 3-phase half-wave uncontrolled rectifier with resistive load. The rectifier is fed from an ideal 3 – phase supply through delta –star 3- phase transformer. The principle of operation of this convertor can be explained as follows:

- Diode 1 which has a more positive voltage at its anode conducts for the period from π/6 to 5 π / 6. In this period D2 and D3 are off. The neutral wire provides a return path to the load current.
- Similarly, diode 2, and 3, whichever has more positive voltage at its cathode conducts.
- The conduction pattern is: **D1 ,D2, D3.**

The output voltage and current waveforms are shown in Fig.5.2.

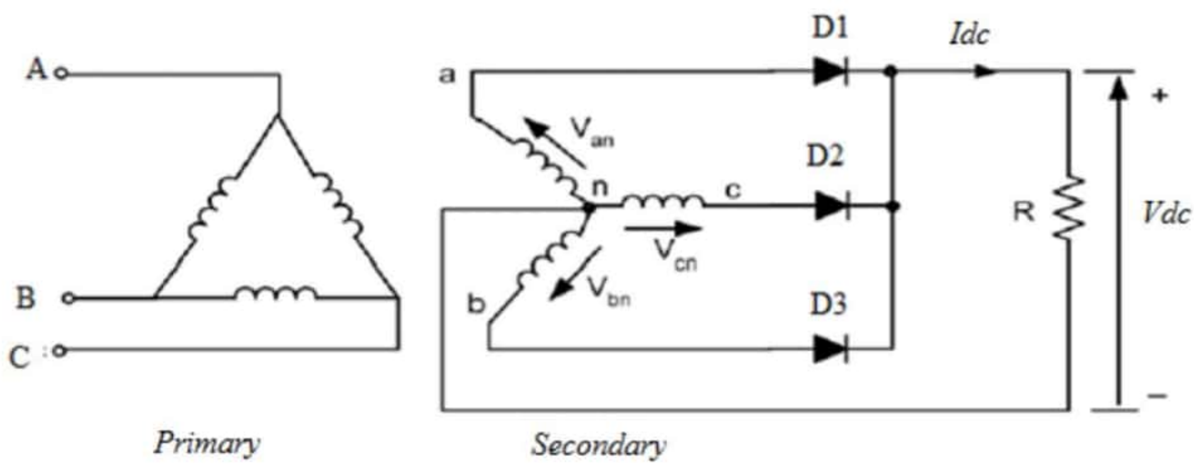


Fig.5.1 A 3-phase half-wave uncontrolled rectifier with resistive load.

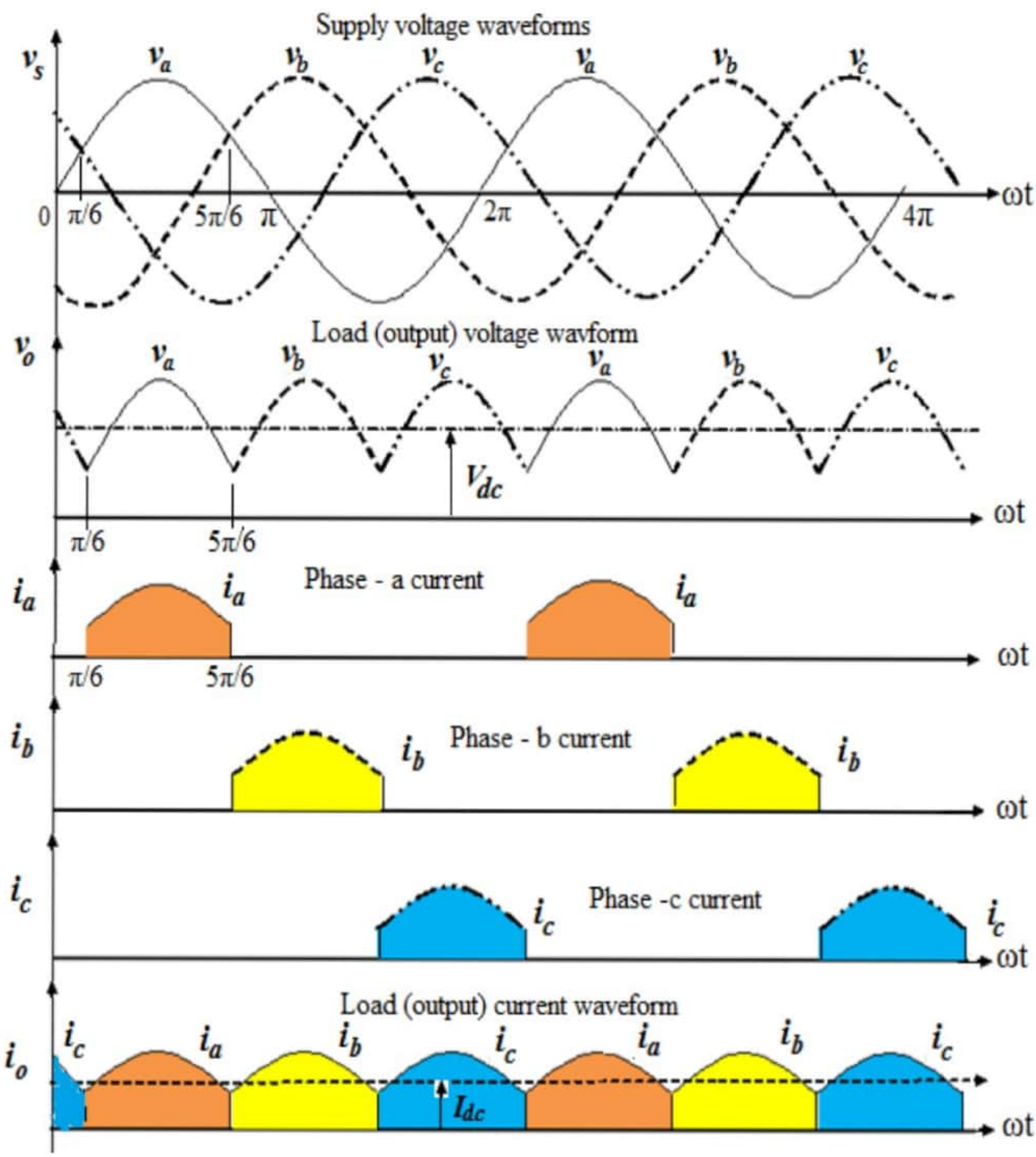
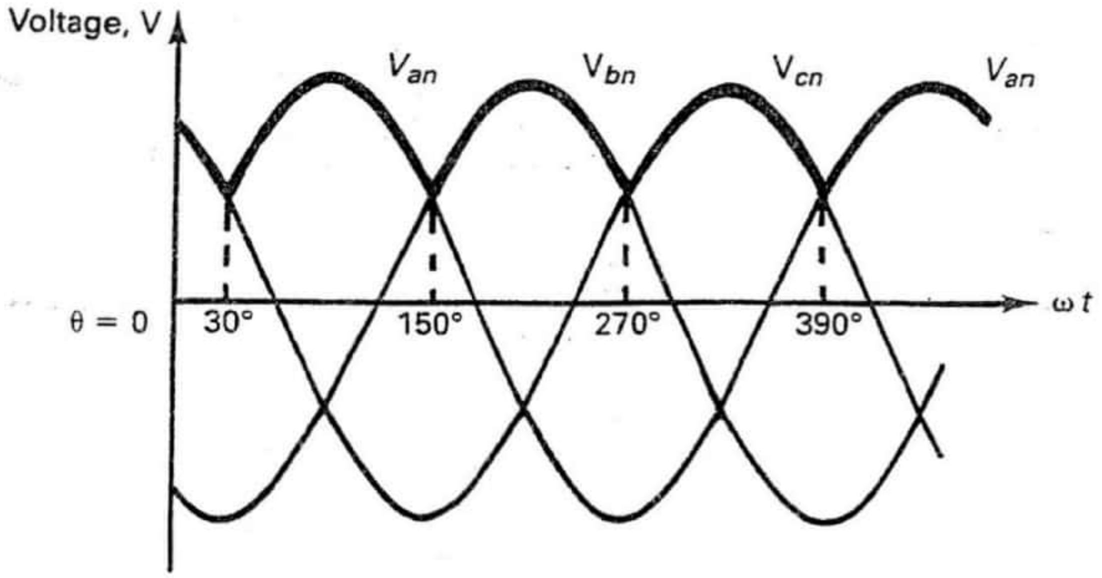


Fig .5.2 Load voltage and current waveforms for the 3 – phase hall – wave uncontrolled rectifier.

Analytical properties of the output voltage waveform

The average value of the voltage waveform in Fig.5.3 can be found as follows:



Let $v_{an} = V_m \sin \omega t$

$v_{bn} = V_m \sin(\omega t - 2\pi / 3)$

$v_{cn} = V_m \sin(\omega t - 4\pi / 3)$

The average value of the load voltage wave is

$$V_{dc} = \frac{1}{\frac{2\pi}{3}} \int_{\pi/6}^{5\pi/6} V_m \ sin\omega t \ d\omega t = \frac{3V_m}{2\pi} \left[-cos\omega t\right]_{\pi/6}^{5\pi/6}$$

$$= \frac{3V_m}{2\pi}\left[-(\cos\frac{5\pi}{6} - cos\frac{\pi}{6})\right] = \frac{3V_m}{2\pi}\left[-(-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2})\right] = \frac{3\sqrt{3}V_m}{2\pi}$$

The load current I_{dc} is:

$$I_{dc} = \frac{3\sqrt{3}.Vm}{2\pi R}$$

Note that the secondary windings of the supply transformer carry unidirectional currents, which leads dc magnetization of the transformer core. This implies that the transformer cores have dc flux, so that for the same ac voltage and hence flux swing, it must have larger core size than is necessary. This problem of dc magnetization is avoided using bridge rectifier circuit.

2.Three –Phase Full-Wave Uncontrolled Bridge Rectifier

Fig.5.4 shows a 3-phase full-wave uncontrolled bridge rectifier with resistive load. The rectifier is fed from an ideal 3 – phase supply through delta –star 3-phase transformer. The principle of operation of this convertor can be explained as follows:

Each three-phase line connects between pair of diodes .One to route power to positive (+) side of load, and other to route power to negative (-) side of load.

- Diode 1, 3 and 5, whichever has a more positive voltage at its anode conducts.
- Similarly, diode 2, 4 and 6, whichever has more negative voltage at its cathode return the load current.
- The conduction pattern is: **12-16-36-34-54-52-12.**
- Each diode conducts for 120˚ in each supply cycle as shown in Fig.5.5.

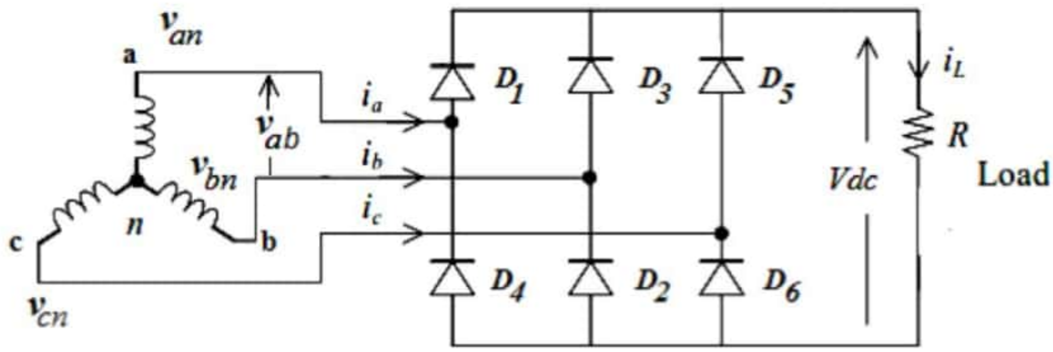


Fig.5.4 The rectifier

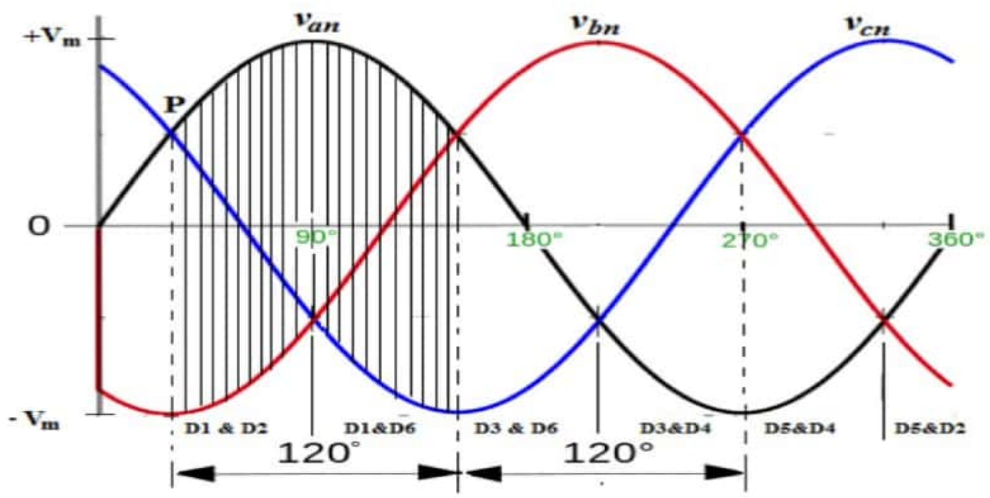


Fig.5.5 Each diode conducts for 120˚.

The output voltage is the instantaneous difference between two appropriate phases at each instant as depicted in Fig.5.6, and the resultant dc output voltage wave is shown in Fig.5.7.

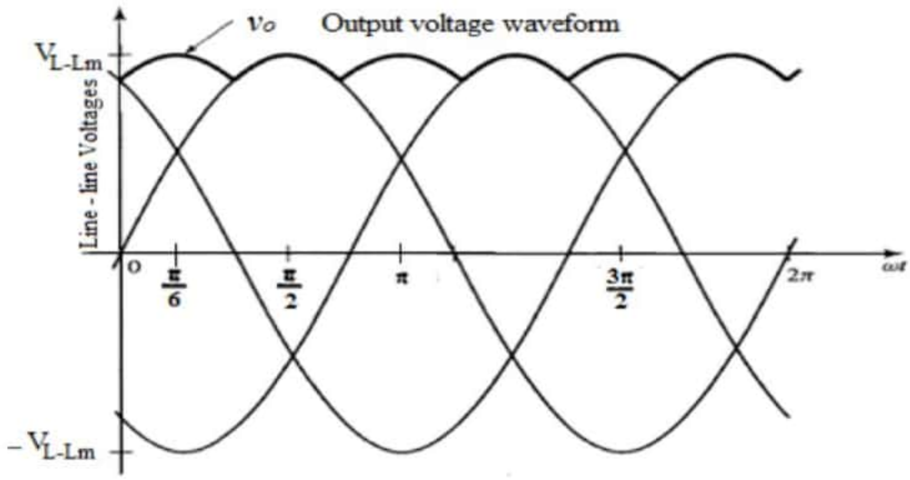


Fig.5.6 The line to line supply voltage and the output voltage waveforms.

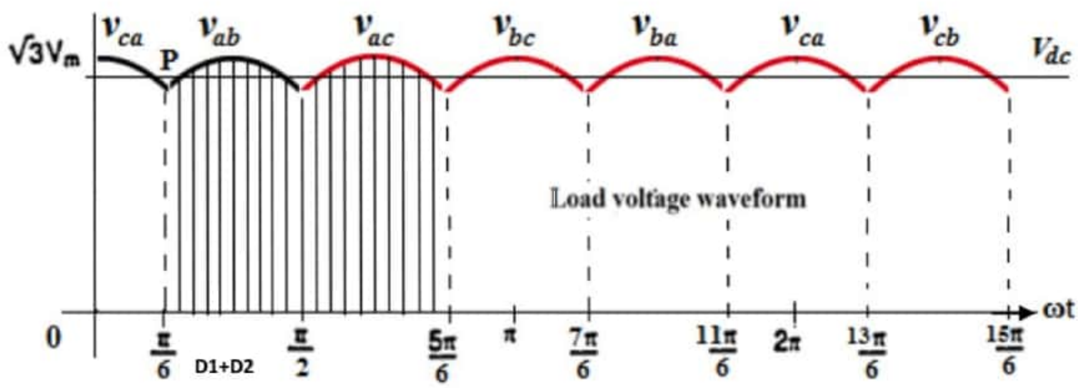


Fig.5.7 The dc output voltage waveform.

The average output voltage:

To find the average voltage V_{dc} on the load, assume that the line –to – line voltages are represented by the following equations,

$$v_{ab} = v_{an} - v_{bn} = V_m \sin(\omega t) - V_m \sin(\omega t - 2\pi/3),$$

hence

$$v_{ab}(\omega t) = \sqrt{3}\, V_m sin\left(\omega t + \frac{\pi}{6}\right)$$

Similarly,

$$v_{bc}(\omega t) = \sqrt{3}\;\, V_m sin\left(\omega t + \frac{\pi}{2}\right)$$

$$v_{ca}(\omega t) = \sqrt{3}\;\, V_m sin\left(\omega t + \frac{7\pi}{6}\right)$$

Hence: by integrating over 1/6 of a cycle,

$$V_{dc} = \frac{3}{\pi}\int_{\pi/6}^{\pi/2} v_{ab}(\omega t)\, d\omega t$$

$$= \frac{3}{\pi}\int_{\pi/6}^{\pi/2} \sqrt{3} V_m sin\left(\omega t + \frac{\pi}{6}\right) d\omega t$$

$$= \frac{3}{\pi}\int_{\pi/6}^{\pi/2} \sqrt{3} V_m (\sin \omega t \cos\frac{\pi}{6} + \cos \omega t \sin\frac{\pi}{6}) d\omega t$$

$$= \frac{3}{\pi}\int_{\pi/6}^{\pi/2} \sqrt{3} V_m \left(\sin \omega t\ \frac{\sqrt{3}}{2} + \cos \omega t \frac{1}{2}\right) d\omega t$$

$$= \frac{3\sqrt{3}.Vm}{\pi}[-\cos(\pi/2) + \cos(\pi/6)]x\frac{\sqrt{3}}{2} + [\sin(\pi/2) - \sin(\pi/6)]x\frac{1}{2}$$

$$= \frac{3\sqrt{3}V_m}{\pi}\left[0 + \frac{\sqrt{3}}{2}\right]x\frac{\sqrt{3}}{2} + \left[1 - \frac{1}{2}\right]x\frac{1}{2}$$

$$= \frac{3\sqrt{3}V_m}{\pi}\left[\frac{3}{4} + \frac{1}{4}\right] = \frac{3\sqrt{3}V_m}{\pi}$$

The load current I_{dc} is:

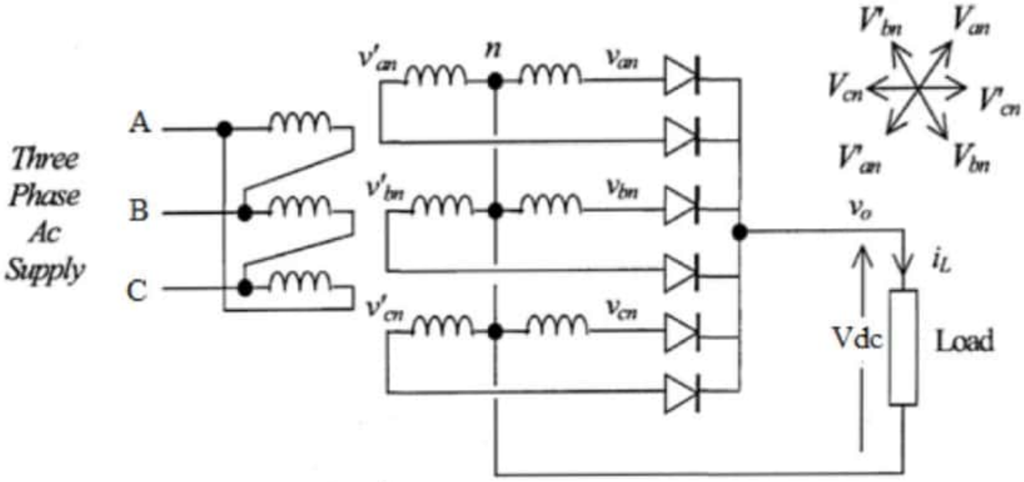
$$I_{dc} = \frac{3\sqrt{3}V_m}{\pi R}$$

The average power is:

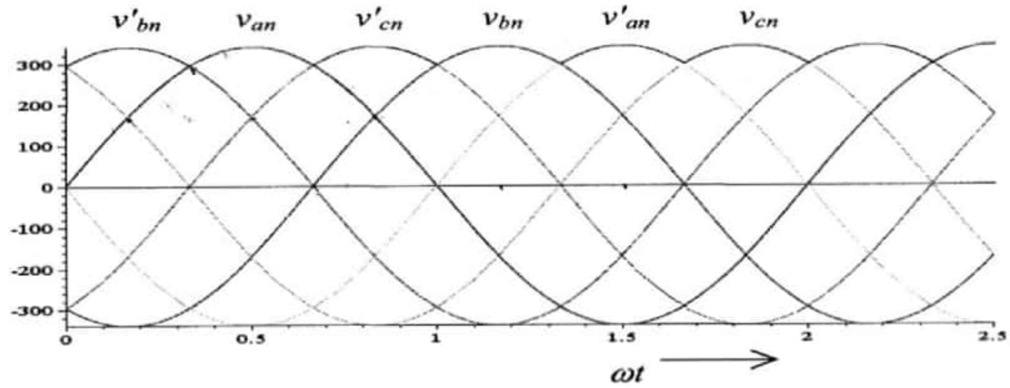
$$P_{dc} = V_{dc} I_{dc} = \frac{3\sqrt{3}V_m}{\pi} x\ \frac{3\sqrt{3}V_m}{\pi R} = \frac{27\ V_m}{\pi R}$$

3. Six –phase (hexa-phase) uncontrolled rectifier

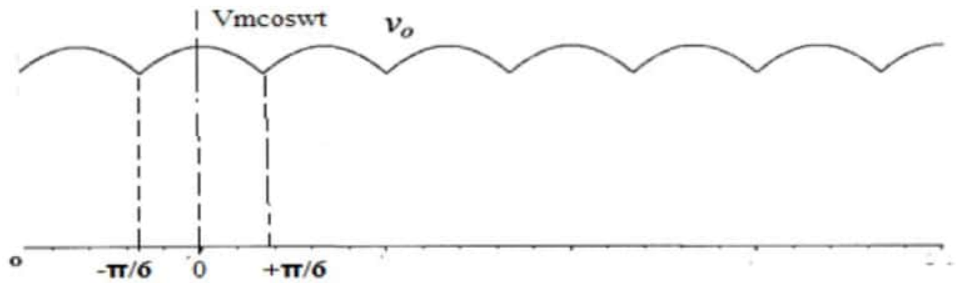
To get more smooth output voltage waveform, a six –phase supply is obtained from a three-phase system using transformer with centre tapped secondary winding as shown in the Fig.5.8 below:



(a)



(b)



(c)

Fig.5.8 Six – phase rectifier (a) Circuit, (b) Input voltages, (c) output voltage

The average output voltage is: $V_{dc} = \frac{3}{\pi}\int_{-\pi/6}^{\pi/6} V_m \cos\omega t \, d\omega t = \frac{3V_m}{\pi}$

THREE PHASE CONTROLLED RECTIFIERS

INTRODUCTION TO 3-PHASE CONTROLLED RECTIFIERS

Single phase half controlled bridge converters & fully controlled bridge converters are used extensively in industrial applications up to about 15kW of output power. The single phase controlled rectifiers provide a maximum dc output of $V_{dc(\max)} = \frac{2V_m}{\pi}$.

The output ripple frequency is equal to the twice the ac supply frequency. The single phase full wave controlled rectifiers provide two output pulses during every input supply cycle and hence are referred to as two pulse converters.

Three phase converters are 3-phase controlled rectifiers which are used to convert ac input power supply into dc output power across the load.

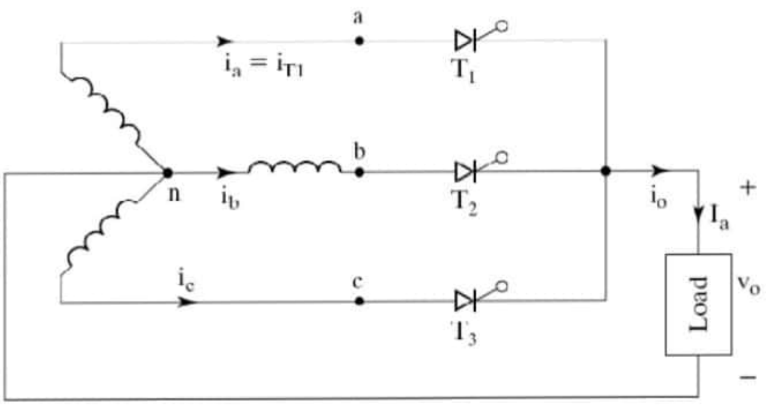
Features of 3-phase controlled rectifiers are

- Operate from 3 phase ac supply voltage.
- They provide higher dc output voltage and higher dc output power.
- Higher output voltage ripple frequency.
- Filtering requirements are simplified for smoothing out load voltage and load current

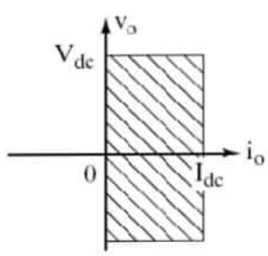
Three phase controlled rectifiers are extensively used in high power variable speed industrial dc drives.

3-PHASE HALF WAVE CONVERTER

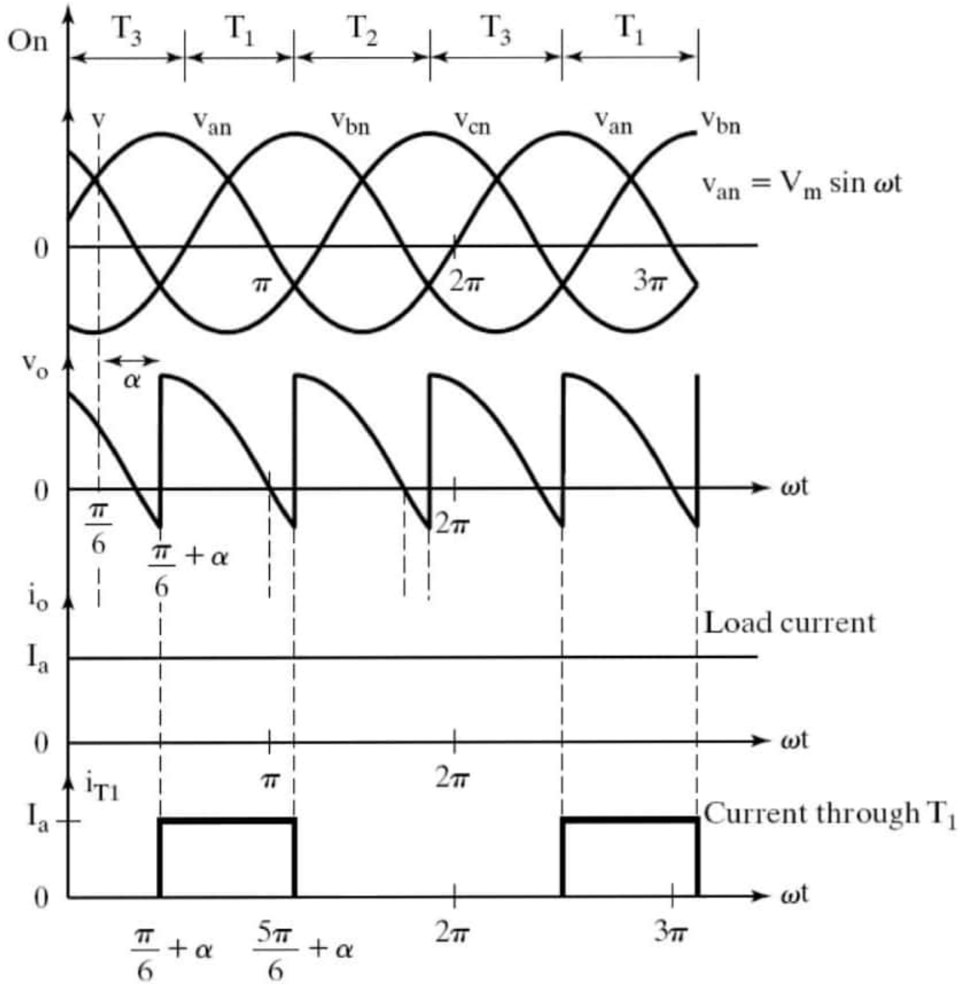
Three single phase half-wave converters are connected together to form a three phase half-wave converter as shown in the figure.



(a) Circuit



(b) Quadrant



(c) For inductive load

THEE PHASE SUPPLY VOLTAGE EQUATIONS

We define three line neutral voltages (3 phase voltages) as follows

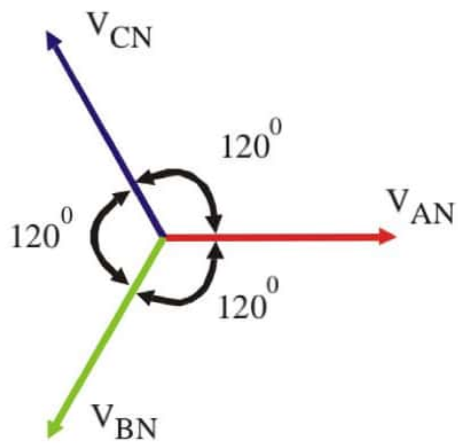
$v_{RN} = v_{an} = V_m \sin \omega t; \quad V_m =$ Max. Phase Voltage

$$v_{YN} = v_{bn} = V_m \sin\left(\omega t - \frac{2\pi}{3}\right)$$

$$v_{YN} = v_{bn} = V_m \sin\left(\omega t - 120^0\right)$$

$$v_{BN} = v_{cn} = V_m \sin\left(\omega t + \frac{2\pi}{3}\right)$$

$$v_{BN} = v_{cn} = V_m \sin\left(\omega t + 120^0\right)$$



Vector diagram of 3-phase supply voltages

$v_{BN} = v_{cn} = V_m \sin\left(\omega t - 240^0\right)$

The 3-phase half wave converter combines three single phase half wave controlled rectifiers in one single circuit feeding a common load. The thyristor T_1 in series with one of the supply phase windings $'a-n'$ acts as one half wave controlled rectifier. The second thyristor T_2 in series with the supply phase winding $'b-n'$ acts as the second half wave controlled rectifier. The third thyristor T_3 in series with the supply phase winding $'c-n'$ acts as the third half wave controlled rectifier.

The 3-phase input supply is applied through the star connected supply transformer as shown in the figure. The common neutral point of the supply is connected to one end of the load while the other end of the load connected to the common cathode point.

When the thyristor T_1 is triggered at $\omega t = \left(\frac{\pi}{6} + \alpha\right) = \left(30^0 + \alpha\right)$, the phase voltage v_{an} appears across the load when T_1 conducts. The load current flows through the supply phase winding $'a-n'$ and through thyristor T_1 as long as T_1 conducts.

When thyristor T_2 is triggered at $\omega t = \left(\frac{5\pi}{6} + \alpha\right) = \left(150^0 + \alpha\right)$, T_1 becomes reverse biased and turns-off. The load current flows through the thyristor T_2 and through the supply phase winding $'b-n'$. When T_2 conducts the phase voltage v_{bn} appears across the load until the thyristor T_3 is triggered .

When the thyristor T_3 is triggered at $\omega t = \left(\frac{3\pi}{2} + \alpha\right) = \left(270^0 + \alpha\right)$, T_2 is reversed biased and hence T_2 turns-off. The phase voltage v_{cn} appears across the load when T_3 conducts.

When T_1 is triggered again at the beginning of the next input cycle the thyristor T_3 turns off as it is reverse biased naturally as soon as T_1 is triggered. The figure shows the 3-phase input supply voltages, the output voltage which appears across the load, and the load current assuming a constant and ripple free load current for a highly inductive load and the current through the thyristor T_1.

For a purely resistive load where the load inductance 'L = 0' and the trigger angle $\alpha > \left(\frac{\pi}{6}\right)$, the load current appears as discontinuous load current and each thyristor is naturally commutated when the polarity of the corresponding phase supply voltage reverses. The frequency of output ripple frequency for a 3-phase half wave converter is $3f_S$, where f_S is the input supply frequency.

The 3-phase half wave converter is not normally used in practical converter systems because of the disadvantage that the supply current waveforms contain dc components (i.e., the supply current waveforms have an average or dc value).

TO DERIVE AN EXPRESSION FOR THE AVERAGE OUTPUT VOLTAGE OF A 3-PHASE HALF WAVE CONVERTER FOR CONTINUOUS LOAD CURRENT

The reference phase voltage is $v_{RN} = v_{an} = V_m \sin \omega t$. The trigger angle α is measured from the cross over points of the 3-phase supply voltage waveforms. When the phase supply voltage v_{an} begins its positive half cycle at $\omega t = 0$, the first cross over point appears at

$$\omega t = \left(\frac{\pi}{6}\right) \text{ radians} = 30^0 .$$

The trigger angle α for the thyristor T_1 is measured from the cross over point at $\omega t = 30^0$. The thyristor T_1 is forward biased during the period $\omega t = 30^0$ to 150^0, when the phase supply voltage v_{an} has a higher amplitude than the other phase supply voltages. Hence T_1 can be triggered between 30^0 to 150^0. When the thyristor T_1 is triggered at a trigger angle α, the average or dc output voltage for continuous load current is calculated using the equation

$$V_{dc} = \frac{3}{2\pi}\left[\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} v_O .d(\omega t)\right]$$

Output voltage $\quad v_O = v_{an} = V_m \sin \omega t$ for $\omega t = \left(30^0 + \alpha\right)$ to $\left(150^0 + \alpha\right)$

$$V_{dc} = \frac{3}{2\pi}\left[\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m \sin \omega t .d(\omega t)\right]$$

As the output load voltage waveform has three output pulses during the input cycle of 2π radians

$$V_{dc} = \frac{3V_m}{2\pi}\left[\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} \sin \omega t.d(\omega t)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[(-\cos \omega t)\Big/_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha}\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[-\cos\left(\frac{5\pi}{6}+\alpha\right)+\cos\left(\frac{\pi}{6}+\alpha\right)\right]$$

Note from the trigonometric relationship

$\cos(A+B) = (\cos A.\cos B - \sin A.\sin B)$

$$V_{dc} = \frac{3V_m}{2\pi}\left[-\cos\left(\frac{5\pi}{6}\right)\cos(\alpha)+\sin\left(\frac{5\pi}{6}\right)\sin(\alpha)+\cos\left(\frac{\pi}{6}\right).\cos(\alpha)-\sin\left(\frac{\pi}{6}\right)\sin(\alpha)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[-\cos(150^0)\cos(\alpha)+\sin(150^0)\sin(\alpha)+\cos(30^0).\cos(\alpha)-\sin(30^0)\sin(\alpha)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[-\cos(180^0-30^0)\cos(\alpha)+\sin(180^0-30^0)\sin(\alpha)+\cos(30^0).\cos(\alpha)-\sin(30^0)\sin(\alpha)\right]$$

Note: $\cos(180^0-30^0) = -\cos(30^0)$

$\sin(180^0-30^0) = \sin(30^0)$

Therefore

$$V_{dc} = \frac{3V_m}{2\pi}\left[+\cos(30^0)\cos(\alpha)+\sin(30^0)\sin(\alpha)+\cos(30^0).\cos(\alpha)-\sin(30^0)\sin(\alpha)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[2\cos(30^0)\cos(\alpha)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[2\times\frac{\sqrt{3}}{2}\cos(\alpha)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[\sqrt{3}\cos(\alpha)\right] = \frac{3\sqrt{3}V_m}{2\pi}\cos(\alpha)$$

$$V_{dc} = \frac{3V_{Lm}}{2\pi}\cos(\alpha)$$

Where

$V_{Lm} = \sqrt{3}V_m =$ Max. line to line supply voltage for a 3-phase star connected transformer.

The maximum average or dc output voltage is obtained at a delay angle $\alpha = 0$ and is given by

$$V_{dc(\max)} = V_{dm} = \frac{3\sqrt{3}\ V_m}{2\pi} \text{ Where } V_m \text{ is the peak phase voltage.}$$

And the normalized average output voltage is $\quad V_{dcn} = V_n = \dfrac{V_{dc}}{V_{dm}} = \cos\alpha$

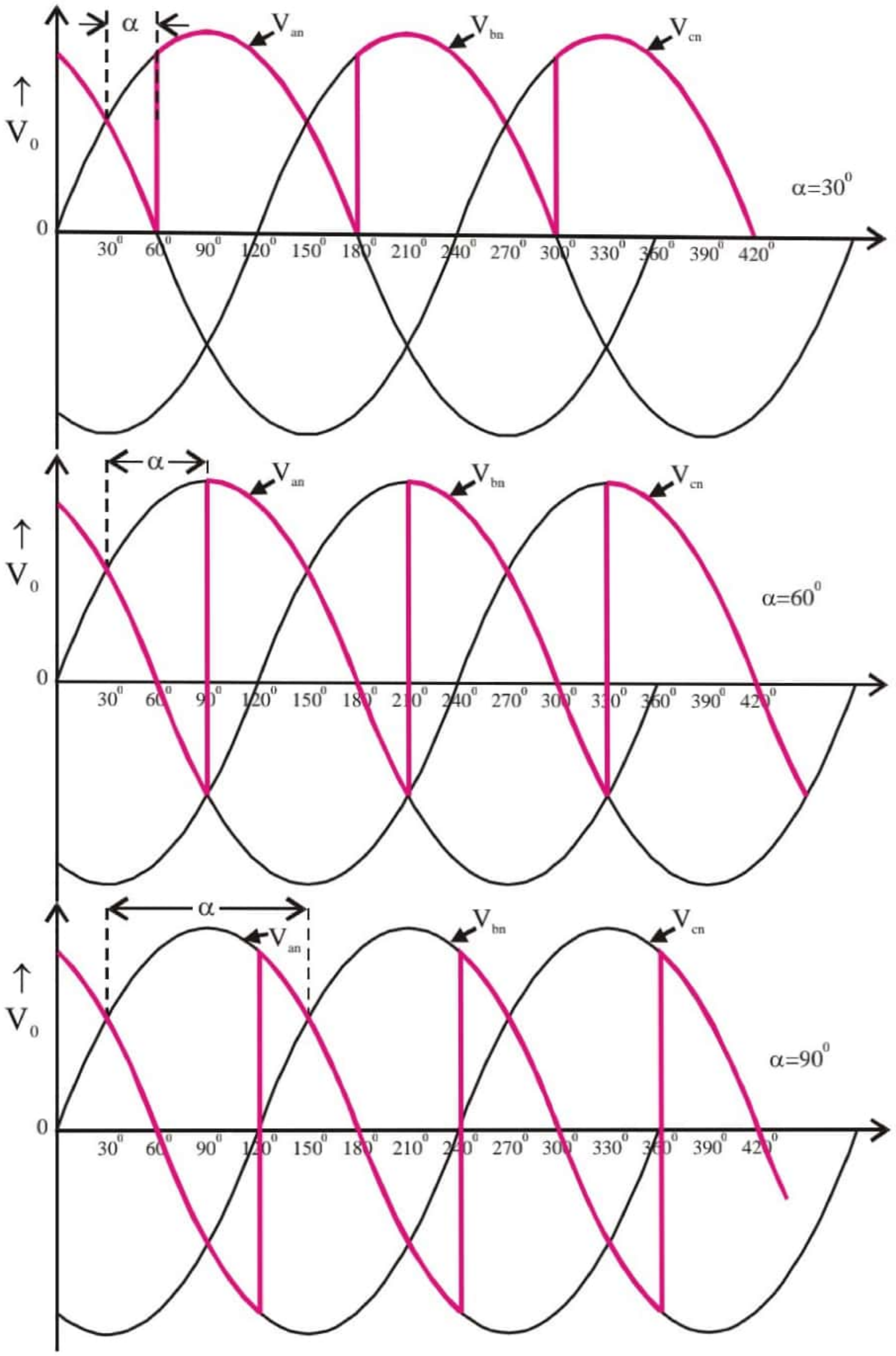
TO DERIVE AN EXPRESSION FOR THE RMS VALUE OF THE OUTPUT VOLTAGE OF A 3-PHASE HALF WAVE CONVERTER FOR CONTINUOUS LOAD CURRENT

The rms value of output voltage is found by using the equation

$$V_{O(RMS)} = \left[\frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_m^2 \sin^2 \omega t.d\left(\omega t\right)\right]^{\frac{1}{2}}$$

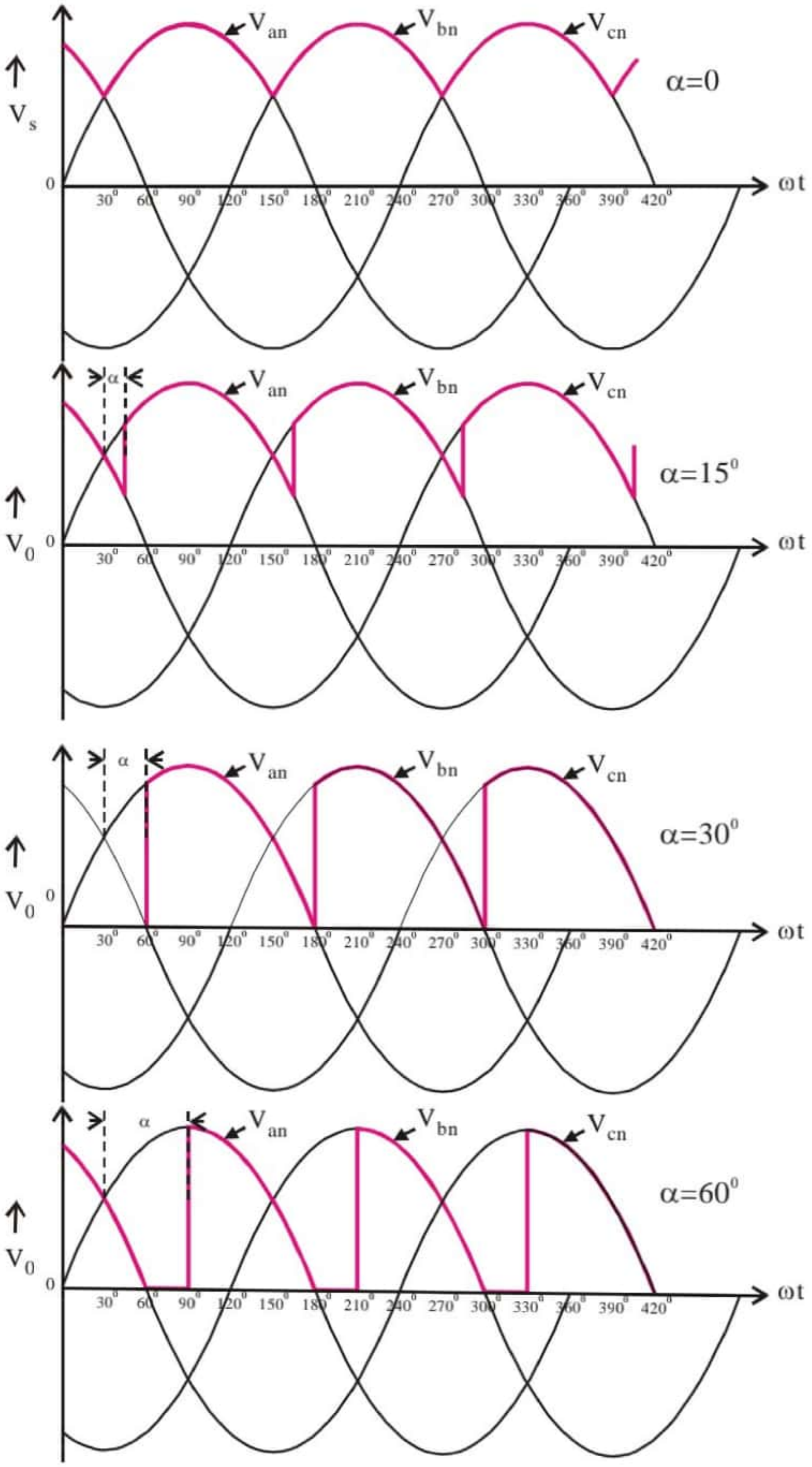
and we obtain $V_{O(RMS)} = \sqrt{3}V_m\left[\dfrac{1}{6}+\dfrac{\sqrt{3}}{8\pi}\cos 2\alpha\right]^{\frac{1}{2}}$

3 PHASE HALF WAVE CONTROLLED RECTIFIER OUTPUT VOLTAGE WAVEFORMS FOR DIFFERENT TRIGGER ANGLES WITH RL LOAD



α
V an
V bn
V cn
V 0
α=30⁰
0
ωt
30⁰ 60⁰ 90⁰ 120⁰ 150⁰ 180⁰ 210⁰ 240⁰ 270⁰ 300⁰ 330⁰ 360⁰ 390⁰ 420⁰
α=60⁰
α=90⁰

3 PHASE HALF WAVE CONTROLLED RECTIFIER OUTPUT VOLTAGE WAVEFORMS FOR DIFFERENT TRIGGER ANGLES WITH R LOAD



TO DERIVE AN EXPRESSION FOR THE AVERAGE OR DC OUTPUT VOLTAGE OF A 3 PHASE HALF WAVE CONVERTER WITH RESISTIVE LOAD OR RL LOAD WITH FWD.

In the case of a three-phase half wave controlled rectifier with resistive load, the thyristor T_1 is triggered at $\omega t = \left(30^0 + \alpha\right)$ and T_1 conducts up to $\omega t = 180^0 = \pi$ radians. When the phase supply voltage v_{an} decreases to zero at $\omega t = \pi$, the load current falls to zero and the thyristor T_1 turns off. Thus T_1 conducts from $\omega t = \left(30^0 + \alpha\right)$ to $\left(180^0\right)$.

Hence the average dc output voltage for a 3-pulse converter (3-phase half wave controlled rectifier) is calculated by using the equation

$$V_{dc} = \frac{3}{2\pi}\left[\int_{\alpha+30^0}^{180^0} v_O.d\left(\omega t\right)\right]$$

$$v_O = v_{an} = V_m \sin \omega t; \text{ for } \omega t = \left(\alpha + 30^0\right) \text{ to } \left(180^0\right)$$

$$V_{dc} = \frac{3}{2\pi}\left[\int_{\alpha+30^0}^{180^0} V_m \sin \omega t.d\left(\omega t\right)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[\int_{\alpha+30^0}^{180^0} \sin \omega t.d\left(\omega t\right)\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[-\cos \omega t \Big/_{\alpha+30^0}^{180^0}\right]$$

$$V_{dc} = \frac{3V_m}{2\pi}\left[-\cos 180^0 + \cos\left(\alpha + 30^0\right)\right]$$

Since $\cos 180^0 = -1$,

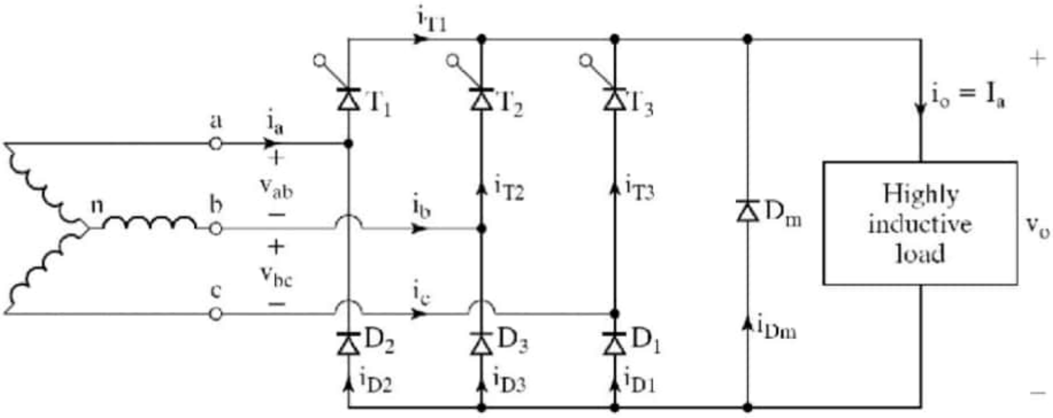
We get $$V_{dc} = \frac{3V_m}{2\pi}\left[1 + \cos\left(\alpha + 30^0\right)\right]$$

THREE PHASE SEMICONVERTERS

3-phase semi-converters are three phase half controlled bridge controlled rectifiers which employ three thyristors and three diodes connected in the form of a bridge configuration. Three thyristors are controlled switches which are turned on at appropriate times by applying appropriate gating signals. The three diodes conduct when they are forward biased by the corresponding phase supply voltages.

3-phase semi-converters are used in industrial power applications up to about 120kW output power level, where single quadrant operation is required. The power factor of 3-phase semi-converter decreases as the trigger angle α increases. The power factor of a 3-phase semi-converter is better than three phase half wave converter.

The figure shows a 3-phase semi-converter with a highly inductive load and the load current is assumed to be a constant and continuous load current with negligible ripple.



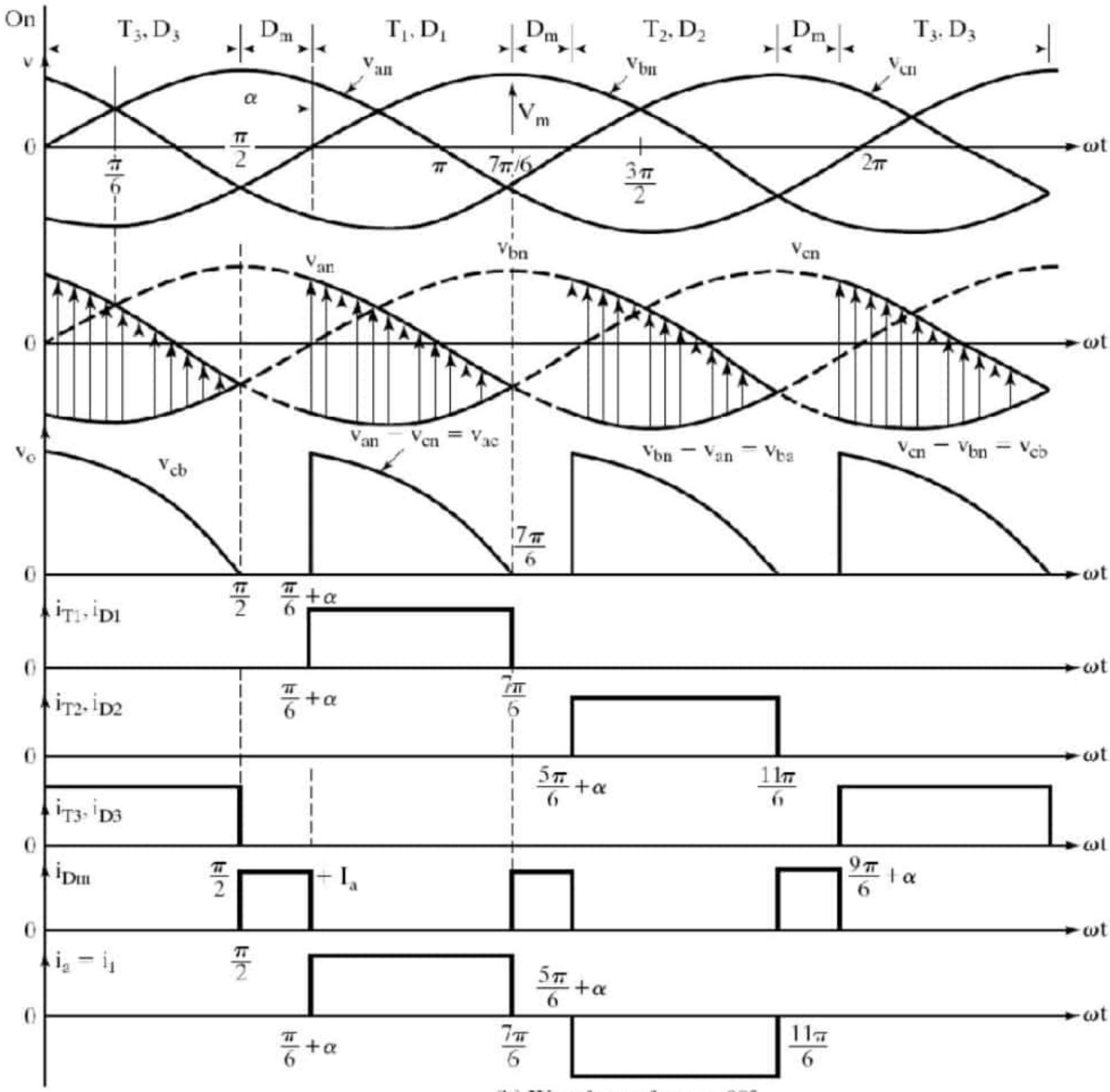
Thyristor T_1 is forward biased when the phase supply voltage v_{an} is positive and greater than the other phase voltages v_{bn} and v_{cn} . The diode D_1 is forward biased when the phase supply voltage v_{cn} is more negative than the other phase supply voltages.

Thyristor T_2 is forward biased when the phase supply voltage v_{bn} is positive and greater than the other phase voltages. Diode D_2 is forward biased when the phase supply voltage v_{an} is more negative than the other phase supply voltages.

Thyristor T_3 is forward biased when the phase supply voltage v_{cn} is positive and greater than the other phase voltages. Diode D_3 is forward biased when the phase supply voltage v_{bn} is more negative than the other phase supply voltages.

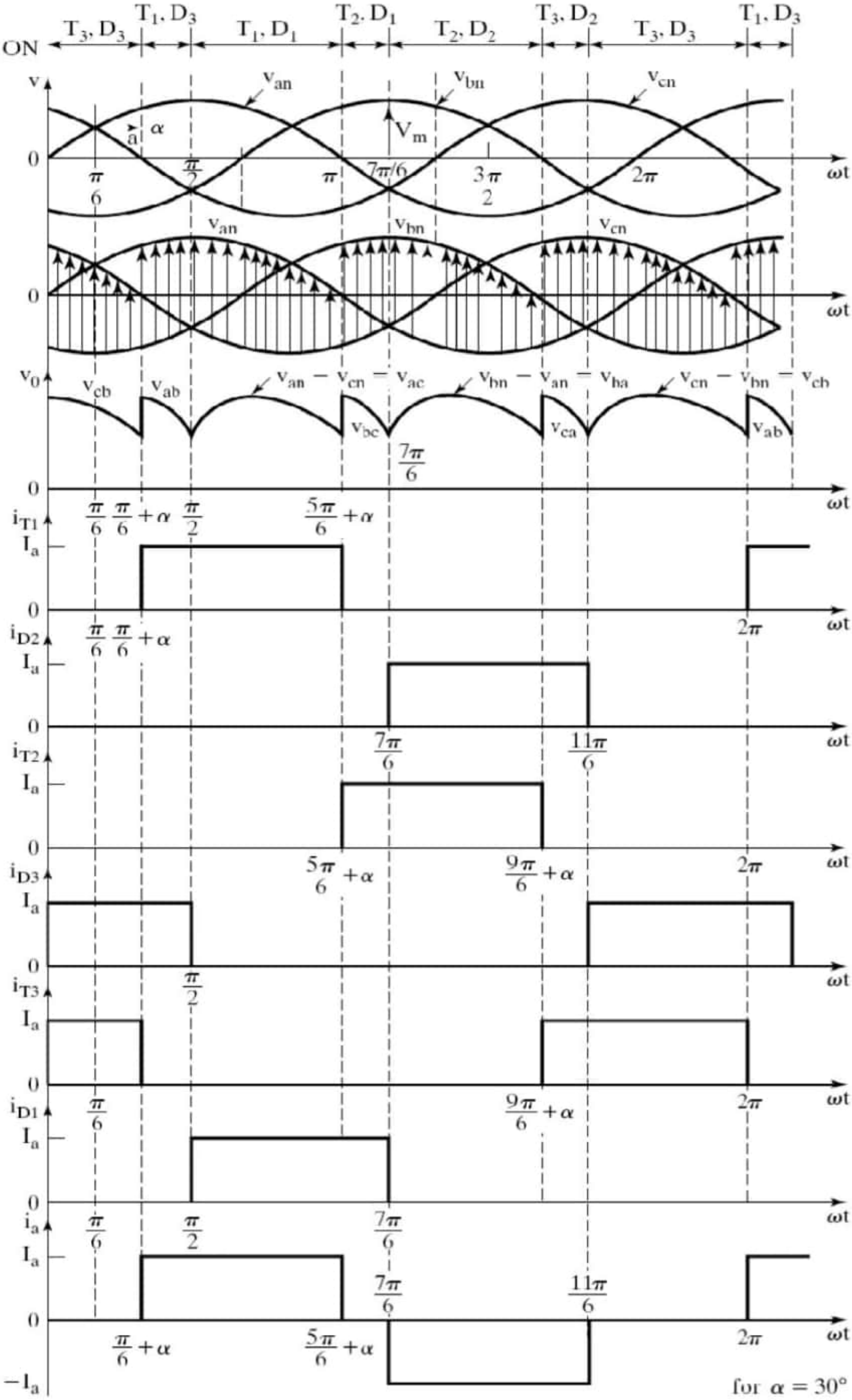
The figure shows the waveforms for the three phase input supply voltages, the output voltage, the thyristor and diode current waveforms, the current through the free wheeling diode D_m and the supply current i_a . The frequency of the output supply waveform is $3f_S$, where f_S is the input ac supply frequency. The trigger angle α can be varied from 0^0 to 180^0 .

During the time period $\left(\frac{\pi}{6}\right) \leq \omega t \leq \left(\frac{7\pi}{6}\right)$ i.e., for $30^0 \leq \omega t \leq 210^0$, thyristor T_1 is forward biased. If T_1 is triggered at $\omega t = \left(\frac{\pi}{6} + \alpha\right)$, T_1 and D_1 conduct together and the line to line supply voltage v_{ac} appears across the load. At $\omega t = \left(\frac{7\pi}{6}\right)$, v_{ac} starts to become negative and the free wheeling diode D_m turns on and conducts. The load current continues to flow through the free wheeling diode D_m and thyristor T_1 and diode D_1 are turned off.



(b) Waveforms for $\alpha = 90°$

If the free wheeling diode D_m is not connected across the load, then T_1 would continue to conduct until the thyristor T_2 is triggered at $\omega t = \left(\frac{5\pi}{6} + \alpha\right)$ and the free wheeling action is accomplished through T_1 and D_2, when D_2 turns on as soon as v_{an} becomes more negative at $\omega t = \left(\frac{7\pi}{6}\right)$. If the trigger angle $\alpha \leq \left(\frac{\pi}{3}\right)$ each thyristor conducts for $\frac{2\pi}{3}$ radians $\left(120^0\right)$ and the free wheeling diode D_m does not conduct. The waveforms for a 3-phase semi-converter with $\alpha \leq \left(\frac{\pi}{3}\right)$ is shown in figure


ON
T3, D3
T1, D3
T1, D1
T2, D1
T2, D2
T3, D2
T3, D3
T1, D3
v
van
vbn
vcn
Vm
α
ωt
v0
vcb
vab
van − vcn = vac
vbn − van = vba
vcn − vbn = vcb
vbc
vca
iT1
iD2
iT2
iD3
iT3
iD1
ia
Ia
−Ia
for α = 30°

We define three line neutral voltages (3 phase voltages) as follows

$$v_{RN} = v_{an} = V_m \sin \omega t; \quad V_m = \text{ Max. Phase Voltage}$$

$$v_{YN} = v_{bn} = V_m \sin\left(\omega t - \frac{2\pi}{3}\right), \; v_{YN} = v_{bn} = V_m \sin\left(\omega t - 120^0\right), \; v_{BN} = v_{cn} = V_m \sin\left(\omega t + \frac{2\pi}{3}\right)$$

$$v_{BN} = v_{cn} = V_m \sin\left(\omega t + 120^0\right), \; v_{BN} = v_{cn} = V_m \sin\left(\omega t - 240^0\right)$$

The corresponding line-to-line voltages are

$$v_{RB} = v_{ac} = \left(v_{an} - v_{cn}\right) = \sqrt{3} V_m \sin\left(\omega t - \frac{\pi}{6}\right), \quad v_{YR} = v_{ba} = \left(v_{bn} - v_{an}\right) = \sqrt{3} V_m \sin\left(\omega t - \frac{5\pi}{6}\right)$$

$$v_{BY} = v_{cb} = \left(v_{cn} - v_{bn}\right) = \sqrt{3} V_m \sin\left(\omega t + \frac{\pi}{2}\right), \quad v_{RY} = v_{ab} = \left(v_{an} - v_{bn}\right) = \sqrt{3} V_m \sin\left(\omega t + \frac{\pi}{6}\right)$$

Where V_m is the peak phase voltage of a star (Y) connected source.

TO DERIVE AN EXPRESSION FOR THE AVERAGE OUTPUT VOLTAGE OF THREE PHASE SEMICONVERTER FOR $\alpha > \left(\frac{\pi}{3}\right)$ AND DISCONTINUOUS OUTPUT VOLTAGE

For $\alpha \geq \frac{\pi}{3}$ and discontinuous output voltage: the average output voltage is found from

$$V_{dc} = \frac{3}{2\pi} \int_{\pi/6+\alpha}^{7\pi/6} v_{ac} . d\left(\omega t\right), \qquad V_{dc} = \frac{3}{2\pi} \int_{\pi/6+\alpha}^{7\pi/6} \sqrt{3}\, V_m \sin\left(\omega t - \frac{\pi}{6}\right) d\left(\omega t\right)$$

$$V_{dc} = \frac{3\sqrt{3} V_m}{2\pi}\left(1 + \cos\alpha\right)$$

$$V_{dc} = \frac{3V_{mL}}{2\pi}\left(1 + \cos\alpha\right)$$

The maximum average output voltage that occurs at a delay angle of $\alpha = 0$ is

$$V_{dm} = \frac{3\sqrt{3} V_m}{\pi}$$

The normalized average output voltage is

$$V_n = \frac{V_{dc}}{V_{dm}} = 0.5\left(1 + \cos\alpha\right)$$

The rms output voltage is found from

$$V_{O(RMS)} = \left[\frac{3}{2\pi} \int_{\pi/6+\alpha}^{7\pi/6} 3V_m^2 \sin^2\left(\omega t - \frac{\pi}{6}\right) d(\omega t) \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \sqrt{3}V_m \left[\frac{3}{4\pi}\left(\pi - \alpha + \frac{1}{2}\sin 2\alpha\right)\right]^{\frac{1}{2}}$$

For $\alpha \le \frac{\pi}{3}$, and continuous output voltage

Output voltage $v_O = v_{ab} = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right)$; for $\omega t = \left(\frac{\pi}{6} + \alpha\right)$ to $\left(\frac{\pi}{2}\right)$

Output voltage $v_O = v_{ac} = \sqrt{3}V_m \sin\left(\omega t - \frac{\pi}{6}\right)$; for $\omega t = \left(\frac{\pi}{2}\right)$ to $\left(\frac{5\pi}{6} + \alpha\right)$

The average or dc output voltage is calculated by using the equation

$$V_{dc} = \frac{3}{2\pi}\left[\int_{\pi/6+\alpha}^{\pi/2} v_{ab}.d(\omega t) + \int_{\pi/2}^{5\pi/6+\alpha} v_{ac}.d(\omega t) \right]$$

$$V_{dc} = \frac{3\sqrt{3}V_m}{2\pi}(1 + \cos\alpha)$$

$$V_n = \frac{V_{dc}}{V_{dm}} = 0.5(1 + \cos\alpha)$$

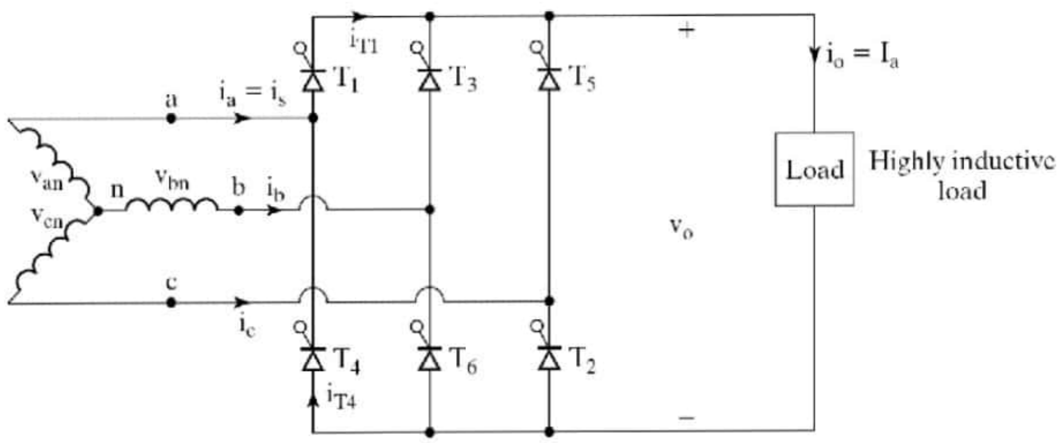
The RMS value of the output voltage is calculated by using the equation

$$V_{O(RMS)} = \left[\frac{3}{2\pi} \int_{\pi/6+\alpha}^{\pi/2} v_{ab}^2.d(\omega t) + \int_{\pi/2}^{5\pi/6+\alpha} v_{ac}^2.d(\omega t) \right]^{\frac{1}{2}}$$

$$V_{O(RMS)} = \sqrt{3}V_m \left[\frac{3}{4\pi}\left(\frac{2\pi}{3} + \sqrt{3}\cos^2\alpha\right)\right]^{\frac{1}{2}}$$

THREE PHASE FULL CONVERTER

Three phase full converter is a fully controlled bridge controlled rectifier using six thyristors connected in the form of a full wave bridge configuration. All the six thyristors are controlled switches which are turned on at a appropriate times by applying suitable gate trigger signals. The three phase full converter is extensively used in industrial power applications upto about 120kW output power level, where two quadrant operation is required. The figure shows a three phase full converter with highly inductive load. This circuit is also known as three phase full wave bridge or as a six pulse converter. The thyristors are triggered at an interval of $\left(\frac{\pi}{3}\right)$ radians (i.e. at an interval of 60^0). The frequency of output ripple voltage is $6f_S$ and the filtering requirement is less than that of three phase semi and half wave converters.



At $\omega t = \left(\frac{\pi}{6}+\alpha\right)$, thyristor T_6 is already conducting when the thyristor T_1 is turned on by applying the gating signal to the gate of T_1. During the time period $\omega t = \left(\frac{\pi}{6}+\alpha\right)$ to $\left(\frac{\pi}{2}+\alpha\right)$, thyristors T_1 and T_6 conduct together and the line to line supply voltage v_{ab} appears across the load. At $\omega t = \left(\frac{\pi}{2}+\alpha\right)$, the thyristor T_2 is triggered and T_6 is reverse biased immediately and T_6 turns off due to natural commutation. During the time period $\omega t = \left(\frac{\pi}{2}+\alpha\right)$ to $\left(\frac{5\pi}{6}+\alpha\right)$, thyristor T_1 and T_2 conduct together and the line to line supply voltage v_{ac} appears across the load. The thyristors are numbered in the circuit diagram corresponding to the order in which they are triggered. The trigger sequence (firing sequence) of the thyristors is 12, 23, 34, 45, 56, 61, 12, 23, and so on. The figure shows the waveforms of three phase input supply voltages, output voltage, the thyristor current through T_1 and T_4 , the supply current through the line 'a'.

We define three line neutral voltages (3 phase voltages) as follows

$v_{RN} = v_{an} = V_m \sin \omega t \quad ; \qquad V_m =$ Max. Phase Voltage

$$v_{YN} = v_{bn} = V_m \sin\left(\omega t - \frac{2\pi}{3}\right) = V_m \sin\left(\omega t - 120^0\right),$$

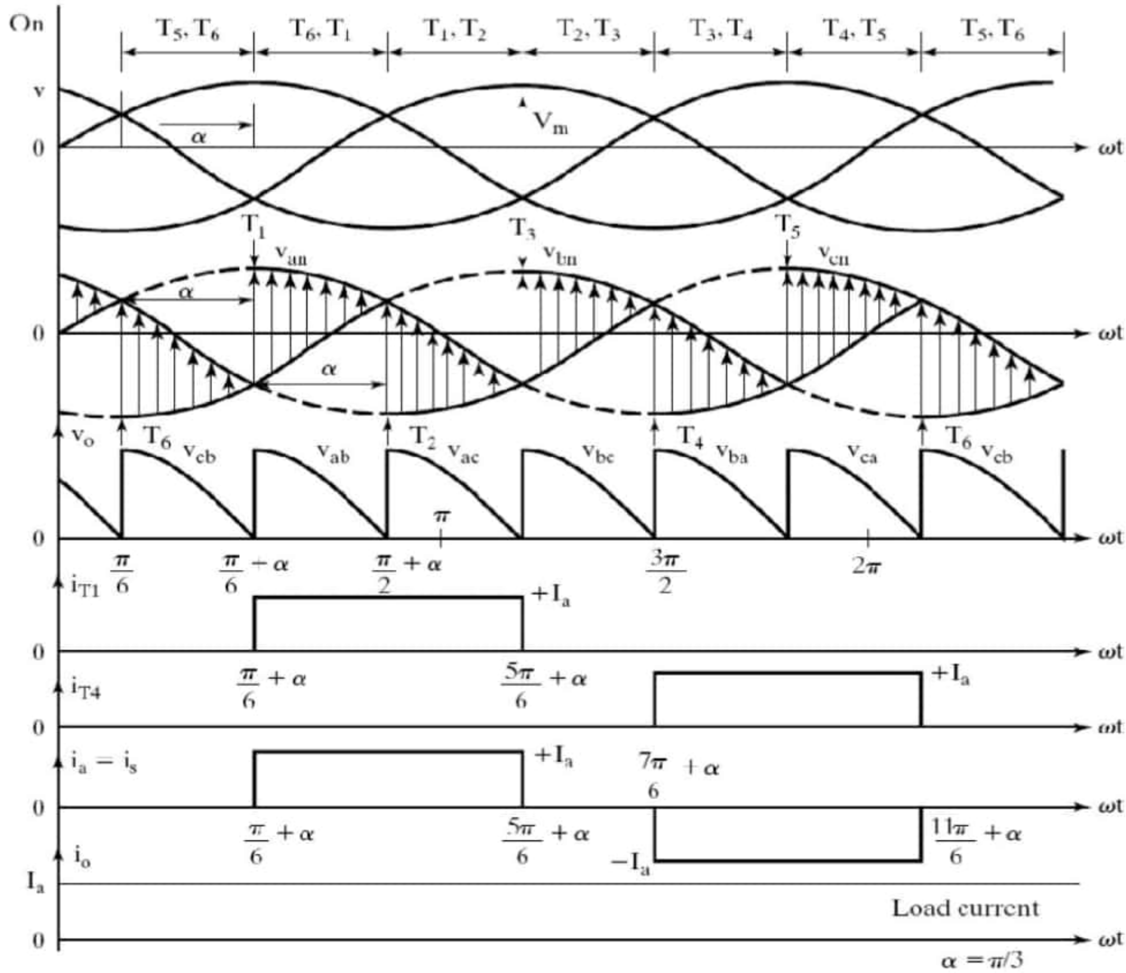
$$v_{BN} = v_{cn} = V_m \sin\left(\omega t + \frac{2\pi}{3}\right) = V_m \sin\left(\omega t + 120^0\right) == V_m \sin\left(\omega t - 240^0\right)$$

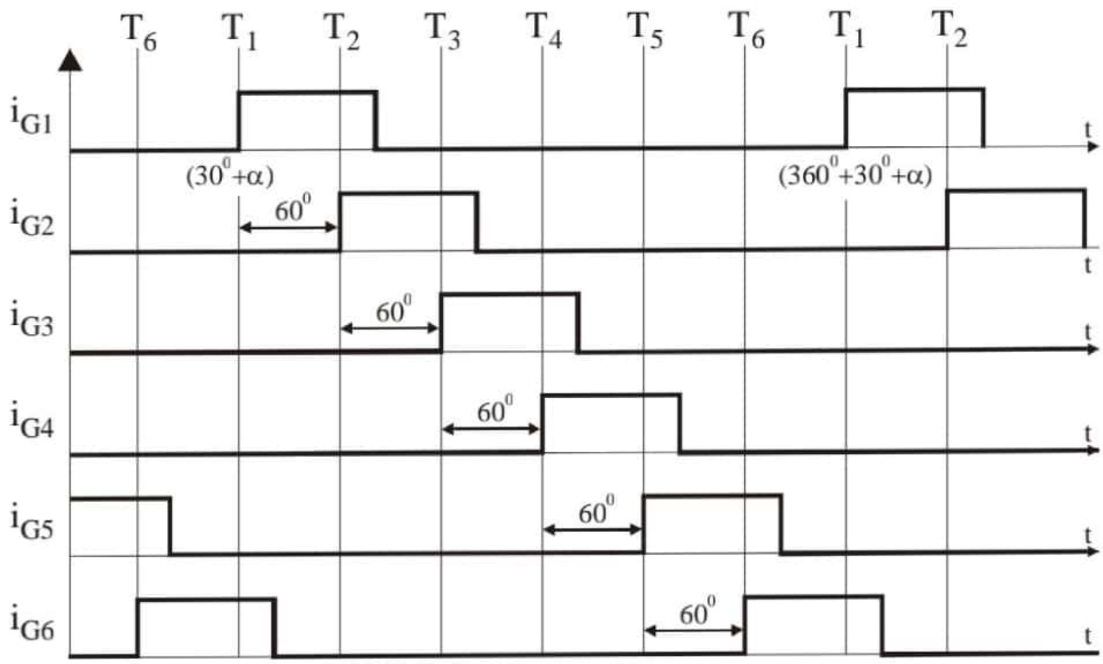
Where V_m is the peak phase voltage of a star (Y) connected source.

The corresponding line-to-line voltages are

$$v_{RY} = v_{ab} = \left(v_{an} - v_{bn}\right) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right), \; v_{YB} = v_{bc} = \left(v_{bn} - v_{cn}\right) = \sqrt{3}V_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$v_{BR} = v_{ca} = \left(v_{cn} - v_{an}\right) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{2}\right)$$





Gating (Control) Signals of 3-phase full converter

TO DERIVE AN EXPRESSION FOR THE AVERAGE OUTPUT VOLTAGE OF THREE PHASE FULL CONVERTER WITH HIGHLY INDUCTIVE LOAD ASSUMING CONTINUOUS AND CONSTANT LOAD CURRENT

The output load voltage consists of 6 voltage pulses over a period of 2π radians, hence the average output voltage is calculated as

$$V_{O(dc)} = V_{dc} = \frac{6}{2\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} v_O . d\omega t \quad ;$$

$$v_O = v_{ab} = \sqrt{3} V_m \sin\left(\omega t + \frac{\pi}{6}\right)$$

$$V_{dc} = \frac{3}{\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} \sqrt{3} V_m \sin\left(\omega t + \frac{\pi}{6}\right) . d\omega t$$

$$V_{dc} = \frac{3\sqrt{3} V_m}{\pi} \cos\alpha = \frac{3 V_{mL}}{\pi} \cos\alpha$$

Where $V_{mL} = \sqrt{3} V_m =$ Max. line-to-line supply voltage

The maximum average dc output voltage is obtained for a delay angle α = 0,

$$V_{dc(\max)} = V_{dm} = \frac{3\sqrt{3}V_m}{\pi} = \frac{3V_{mL}}{\pi}$$

The normalized average dc output voltage is

$$V_{dcn} = V_n = \frac{V_{dc}}{V_{dm}} = \cos\alpha$$

The rms value of the output voltage is found from

$$V_{O(rms)} = \left[\frac{6}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} v_O^2 .d\left(\omega t\right)\right]^{\frac{1}{2}}$$

$$V_{O(rms)} = \left[\frac{6}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} v_{ab}^2 .d\left(\omega t\right)\right]^{\frac{1}{2}}$$

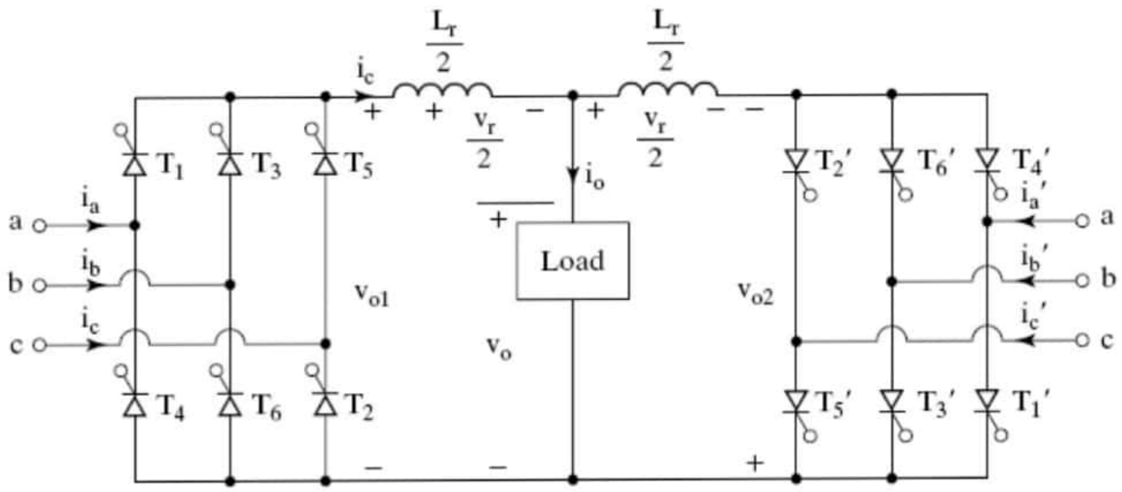
$$V_{O(rms)} = \left[\frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} 3V_m^2 \sin^2\left(\omega t + \frac{\pi}{6}\right).d\left(\omega t\right)\right]^{\frac{1}{2}}$$

$$V_{O(rms)} = \sqrt{3}V_m\left(\frac{1}{2}+\frac{3\sqrt{3}}{4\pi}\cos 2\alpha\right)^{\frac{1}{2}}$$

THREE PHASE DUAL CONVERTERS

In many variable speed drives, the four quadrant operation is generally required and three phase dual converters are extensively used in applications up to the 2000 kW level. Figure shows three phase dual converters where two three phase full converters are connected back to back across a common load. We have seen that due to the instantaneous voltage differences between the output voltages of converters, a circulating current flows through the converters. The circulating current is normally limited by circulating reactor, L_r. The two converters are controlled in such a way that if α_1 is the delay angle of converter 1, the delay angle of converter 2 is $\alpha_2 = \left(\pi - \alpha_1\right)$.

The operation of a three phase dual converter is similar that of a single phase dual converter system. The main difference being that a three phase dual converter gives much higher dc output voltage and higher dc output power than a single phase dual converter system. But the drawback is that the three phase dual converter is more expensive and the design of control circuit is more complex.



The figure below shows the waveforms for the input supply voltages, output voltages of converter1 and conveter2 , and the voltage across current limiting reactor (inductor) L_r. The operation of each converter is identical to that of a three phase full converter.

During the interval $\left(\frac{\pi}{6}+\alpha_1\right)$ to $\left(\frac{\pi}{2}+\alpha_1\right)$, the line to line voltage v_{ab} appears across the output of converter 1 and v_{bc} appears across the output of converter 2

We define three line neutral voltages (3 phase voltages) as follows

$$v_{RN} = v_{an} = V_m \sin \omega t \quad ; \quad V_m = \text{ Max. Phase Voltage}$$

$$v_{YN} = v_{bn} = V_m \sin\left(\omega t - \frac{2\pi}{3}\right) = V_m \sin\left(\omega t - 120^0\right)$$

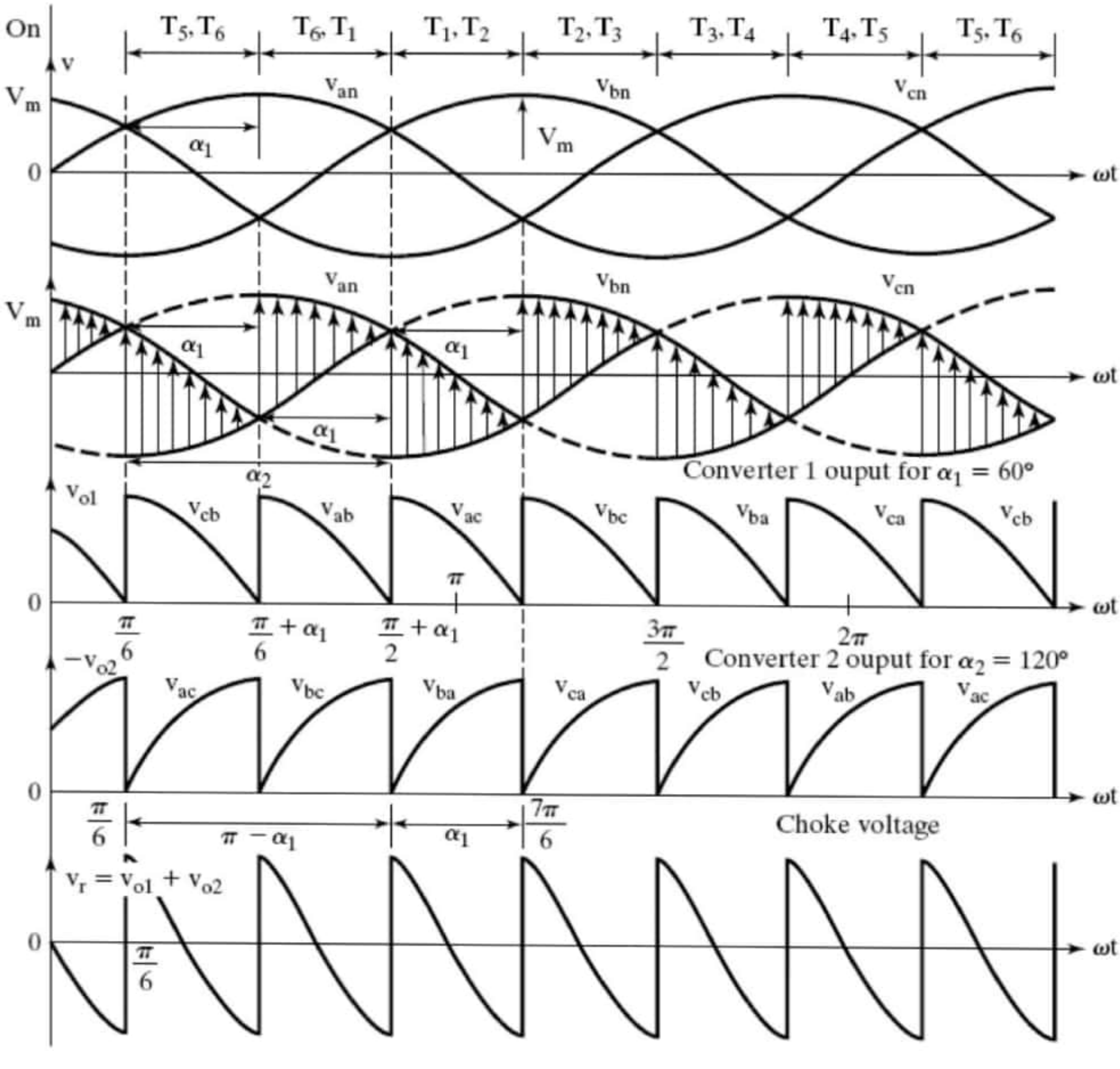
$$v_{BN} = v_{cn} = V_m \sin\left(\omega t + \frac{2\pi}{3}\right) = V_m \sin\left(\omega t + 120^0\right) = V_m \sin\left(\omega t - 240^0\right)$$

The corresponding line-to-line supply voltages are

$$v_{RY} = v_{ab} = (v_{an} - v_{bn}) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{6}\right)$$

$$v_{YB} = v_{bc} = (v_{bn} - v_{cn}) = \sqrt{3}V_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$v_{BR} = v_{ca} = (v_{cn} - v_{an}) = \sqrt{3}V_m \sin\left(\omega t + \frac{\pi}{2}\right)$$



TO OBTAIN AN EXPRESSION FOR THE CIRCULATING CURRENT

If v_{O1} and v_{O2} are the output voltages of converters 1 and 2 respectively, the instantaneous voltage across the current limiting inductor during the interval $\left(\frac{\pi}{6}+\alpha_1\right) \le \omega t \le \left(\frac{\pi}{2}+\alpha_1\right)$ is

$$v_r = \left(v_{O1}+v_{O2}\right) = \left(v_{ab}-v_{bc}\right)$$

$$v_r = \sqrt{3}V_m\left[\sin\left(\omega t+\frac{\pi}{6}\right)-\sin\left(\omega t-\frac{\pi}{2}\right)\right]$$

$$v_r = 3V_m\cos\left(\omega t-\frac{\pi}{6}\right)$$

The circulating current can be calculated by using the equation

$$i_r\left(t\right) = \frac{1}{\omega L_r}\int_{\frac{\pi}{6}+\alpha_1}^{\omega t} v_r.d\left(\omega t\right)$$

$$i_r\left(t\right) = \frac{1}{\omega L_r}\int_{\frac{\pi}{6}+\alpha_1}^{\omega t} 3V_m\cos\left(\omega t-\frac{\pi}{6}\right).d\left(\omega t\right)$$

$$i_r\left(t\right) = \frac{3V_m}{\omega L_r}\left[\sin\left(\omega t-\frac{\pi}{6}\right)-\sin\alpha_1\right]$$

$$i_{r(\max)} = \frac{3V_m}{\omega L_r}$$ = maximum value of the circulating current.

There are two different modes of operation of a three phase dual converter system.

- Circulating current free (non circulating) mode of operation
- Circulating current mode of operation

CIRCULATING CURRENT FREE (NON-CIRCULATING) MODE OF OPERATION

In this mode of operation only one converter is switched on at a time when the converter number 1 is switched on and the gate signals are applied to the thyristors the average output voltage and the average load current are controlled by adjusting the trigger angle α_1 and the gating signals of converter 1 thyristors.

The load current flows in the downward direction giving a positive average load current when the converter 1 is switched on. For $\alpha_1 < 90^0$ the converter 1 operates in the rectification mode V_{dc} is positive, I_{dc} is positive and hence the average load power P_{dc} is positive.

The converter 1 converts the input ac supply and feeds a dc power to the load. Power flows from the ac supply to the load during the rectification mode. When the trigger angle α_1 is increased above 90^0, V_{dc} becomes negative where as I_{dc} is positive because the thyristors of converter 1 conduct in only one direction and reversal of load current through thyristors of converter 1 is not possible.

For $\alpha_1 > 90^0$ converter 1 operates in the inversion mode & the load energy is supplied back to the ac supply. The thyristors are switched-off when the load current decreases to zero & after a short delay time of about 10 to 20 milliseconds, the converter 2 can be switched on by releasing the gate control signals to the thyristors of converter 2.

We obtain a reverse or negative load current when the converter 2 is switched ON. The average or dc output voltage and the average load current are controlled by adjusting the trigger angle α_2 of the gate trigger pulses supplied to the thyristors of converter 2. When α_2 is less than 90^0, converter 2 operates in the rectification mode and converts the input ac supply in to dc output power which is fed to the load.

When α_2 is less than 90^0 for converter 2, V_{dc} is negative & I_{dc} is negative, converter 2 operates as a controlled rectifier & power flows from the ac source to the load circuit. When α_2 is increased above 90^0, the converter 2 operates in the inversion mode with V_{dc} positive and I_{dc} negative and hence P_{dc} is negative, which means that power flows from the load circuit to the input ac supply. The power flow from the load circuit to the input ac source is possible if the load circuit has a dc source of appropriate polarity. When the load current falls to zero the thyristors of converter 2 turn-off and the converter 2 can be turned off.

CIRCULATING CURRENT MODE OF OPERATION

Both the converters are switched on at the same time in the mode of operation. One converter operates in the rectification mode while the other operates in the inversion mode. Trigger angles α_1 & α_2 are adjusted such that $(\alpha_1 + \alpha_2) = 180^0$

When $\alpha_1 < 90^0$, converter 1 operates as a controlled rectifier. When α_2 is made greater than 90^0, converter 2 operates in the inversion mode. V_{dc}, I_{dc}, P_{dc} are positive.

When $\alpha_2 < 90^0$, converter 2 operates as a controlled rectifier. When α_1 is made greater than 90^0, converter 1 operates as an Inverter. V_{dc} and I_{dc} are negative while P_{dc} is positive.