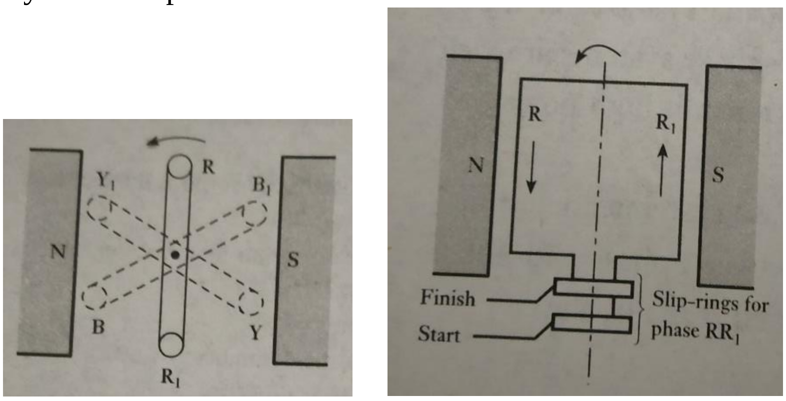
Three Phase System:

- The system utilizing one winding is referred to as single phase system. Similarly, the system utilizing two winding is referred to as two phase system and the system utilizing three winding is referred to as three phase system.

Generation of three phase emfs:

- RR_1, YY_1, BB_1 represent three similar loops/coils fixed to one another at an angle of 120^0 as shown in fig. 2.1 (a)
- Each loop terminating in a pair of slip-rings carried on the shaft as shown in fig. 2.1 (b)
- R, Y, B are the finishing ends and R_1, Y_1, B_1 are the starting ends of the loops.
- R stands for Red, Y stands for yellow and B stands for blue and these are the colors used to identify the three phases.



(a) (b)

Fig. 2.1 (a) Generation of three phase emfs and (b) Loop RR_1 at the instant of maximum emf with positive direction of voltages in coil sides.

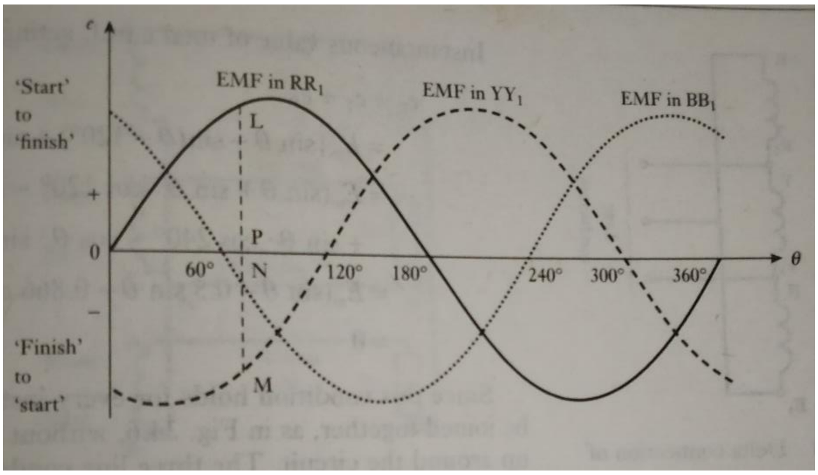


Fig. 2.2 Waveforms of three-phase emfs.

- The three coils are rotated anticlockwise at a uniform speed in the magnetic field due to the two poles and hence, alternating emf will be generated in the three coils.
- The instantaneous value of the three emfs induced in the coils are given by the following equations and the waveforms of the three emfs are shown in fig. 2.2.

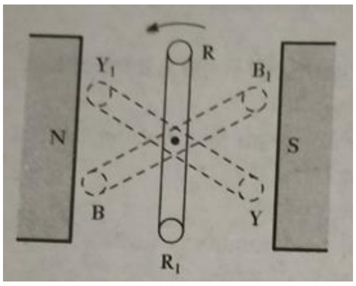
$$e_R = E_m Sin\theta$$
$$e_Y = E_m Sin(\theta - 120^\circ)$$
$$e_B = E_m Sin(\theta - 240^\circ) = E_m Sin(\theta + 120^\circ)$$

- The emfs being assumed positive when their directions round the loops are from start to finish of their respective loops. For instance, the assumed positive direction of emf for coil RR_1 is shown in fig 2.1 (b).
- The three phase emfs can be generated by any of the following arrangements:

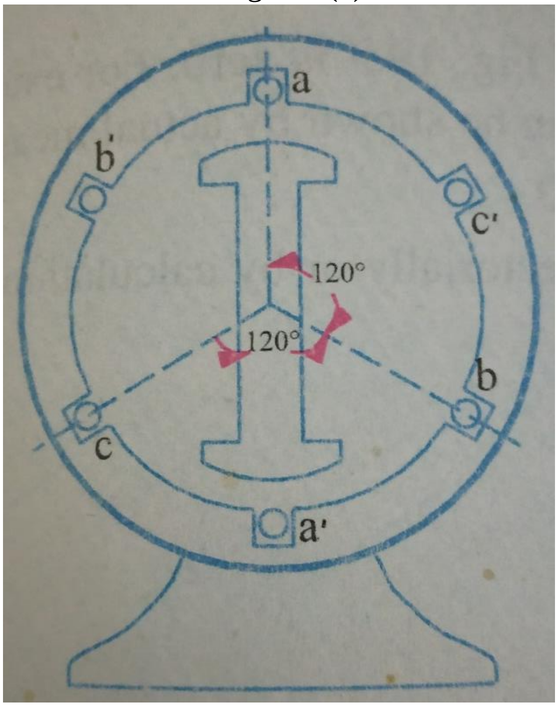
(i) stationary field and rotating coils as shown in fig. 2.3 (a)

or

(ii) stationary coils and rotating field as shown in fig. 2.3 (b)



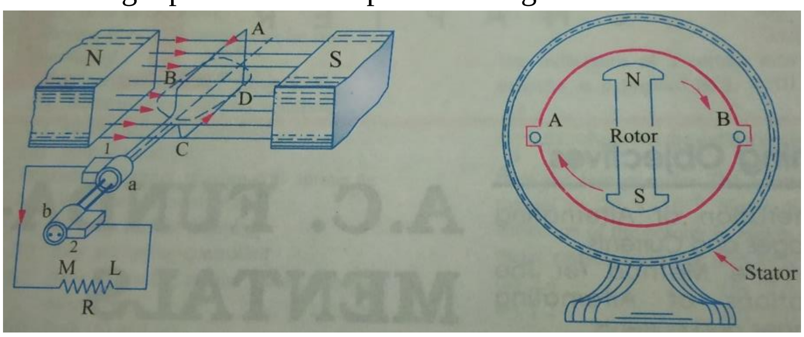
(a)



(b)

Fig. 2.3 Generation of three phase emfs (a) stationary field and rotating coils, (b) stationary coils and rotating field

- The generation of single phase emf is explained in fig. 2.4.



(a) (b)

Fig. 2.4 Generation of single phase emf (a) stationary field and rotating coils, (b) stationary coils and rotating field

- The phase displacement in a polyphase system is given by $360^\circ / n$, where $'n'$ is the number of phases or windings. This formula holds good for any polyphase system except then two-phase system in which case the phase displacement is 90^0.
- Hence, in a three-phase system, the phase displacement is $360^0/3 = 120^0$.
- Advantages of three phase system over single-phase system are:

(i) Its more efficient, (ii) it uses less conducting material for a given capacity, (iii) it costs less than single-phase system for a given capacity.

Interconnection of three phases:

- If the three coils are not interconnected then, it requires six conductors for transmission of power from source to load as shown in fig. 2.5 which is cumbersome and expensive.
- Hence, the three coils/phases are interconnected to save copper in which case the nos. of conductors are reduced to either 3 or 4.

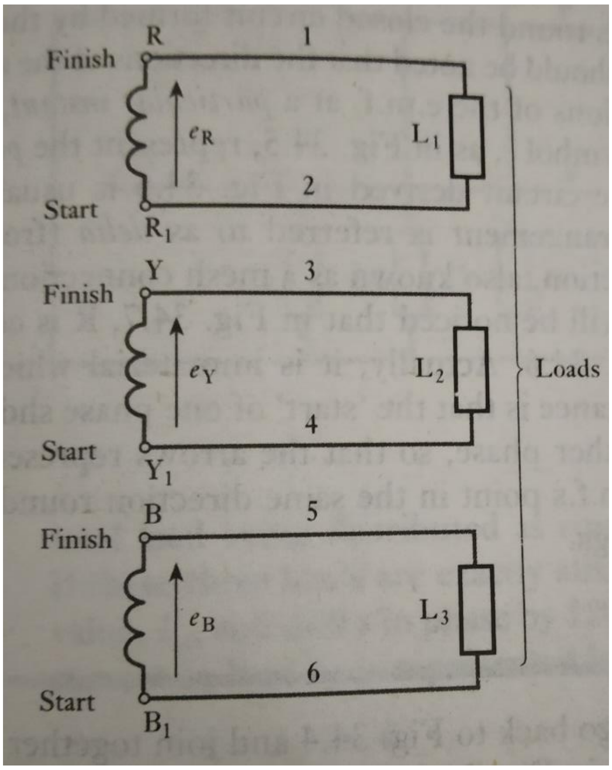


Fig. 2.5 Three phase windings with six-line conductors.

- The two methods of interconnection are:

(i) Delta (Δ) Connection and (ii) Star (Y) Connection.

Delta (Δ) Connection:

- *Dissimilar* ends of the coils are joined together to form a delta connection i.e., *'finish'* end of one coil is joined to the *'start'* end of the other coil and the power is transmitted from source to the load through the outer wires known as *'line conductors'* joined to the junctions where the dissimilar ends are joined as shown in fig. 2.7.
- The resultant emf around the loop is $e_R + e_Y + e_B$ as shown in fig. 2.6 and at any given instant, the resultant emf acting around the loop is zero and therefore there is no circulating current flowing in the loop as proved below.

$$
\begin{aligned}
e_R + e_Y + e_B &= E_m[Sin\theta + Sin(\theta - 120^\circ) + Sin(\theta - 240^\circ)\\
&= E_m(Sin\theta + Sin\theta.Cos120^\circ - Cos\theta.Sin120^\circ + Sin\theta.Cos240^\circ - Cos\theta.Sin240^\circ)\\
&= E_m(Sin\theta - 0.5Sin\theta - 0.866Cos\theta - 0.5Sin\theta + 0.866Cos\theta)\\
&= 0
\end{aligned}
$$

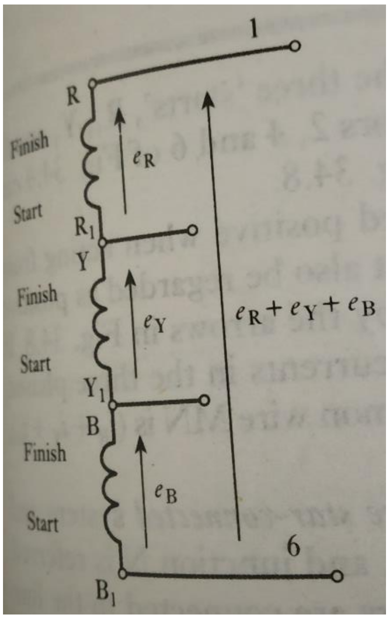
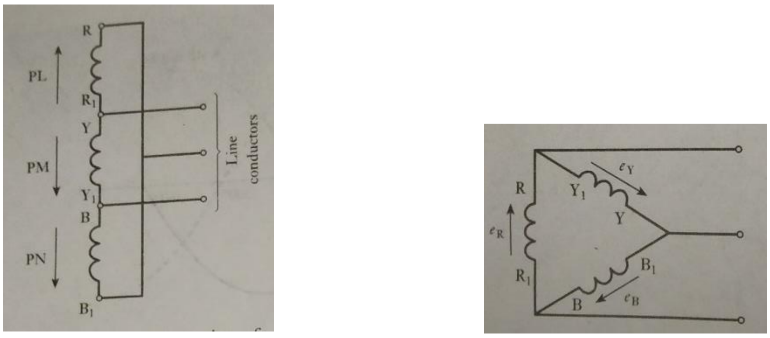


Fig. 2.6 Resultant emf in delta connection.



(a) (b)

Fig. 2.7 (a) Delta connection of three-phase winding, (b) Conventional representation of delta connection

Star (Y) Connection:

- As shown in fig. 2.8, similar ends of the coils are joined together to form a star connection i.e. either start or finish ends of the coils are joined together at one point and this point is known as *star* or *neutral point*.
- Power from source to load is transmitted through the outer wires known as ‘line conductors’ connected to the remaining end as shown in fig. 2.8.
- The wire connected to the neutral point is known as *common wire* or *neutral wire*. MN is common wire or neutral wire as shown in fig. 2.8.
- Since the generated emf has been assumed positive when acting from start to finish, the current in each phase must also be regarded as positive when flowing in that direction as represented by the arrows as shown in fig. 2.8.

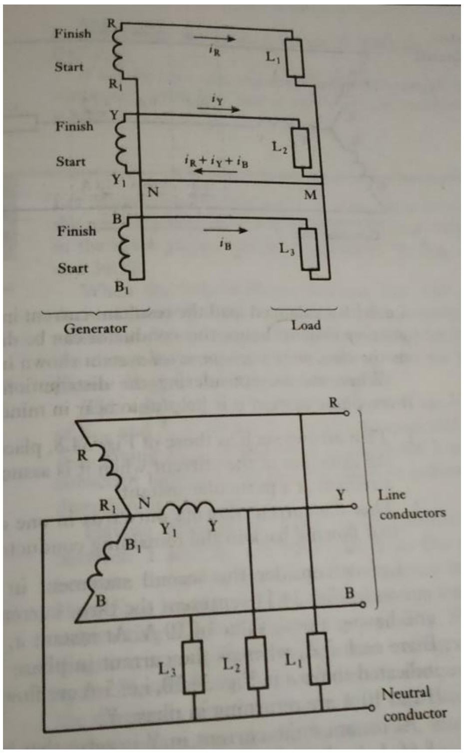


Fig. 2.8 Star connection of three phase windings (Three-phase four wire system)

- The summation of instantaneous value of the currents at the neutral point is $i_R + i_Y + i_B$.
- Three phase loads are connected between the line conductors and single-phase loads are connected between the line and the neutral conductor.
- If the three loads in a three-phase system are exactly alike then it is called as balanced load.
- The phase currents have the same peak value, and differ in phase by 120^0 if the three-phase system is balanced.
- For balanced load, the currents in the three phases are given by:

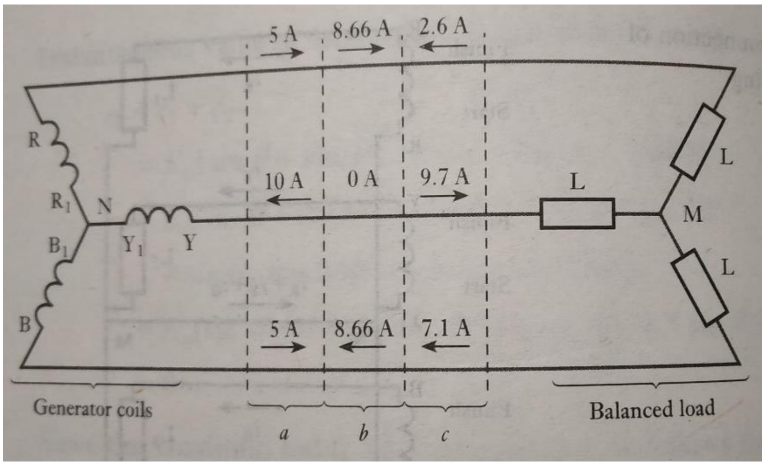
$$i_R = I_m Sin\theta$$
$$i_Y = I_m Sin(\theta - 120^\circ)$$
$$i_B = I_m Sin(\theta - 240^\circ) = I_m Sin(\theta + 120^\circ)$$

- Hence, the instantaneous value of the resultant current in neutral conductor MN is:

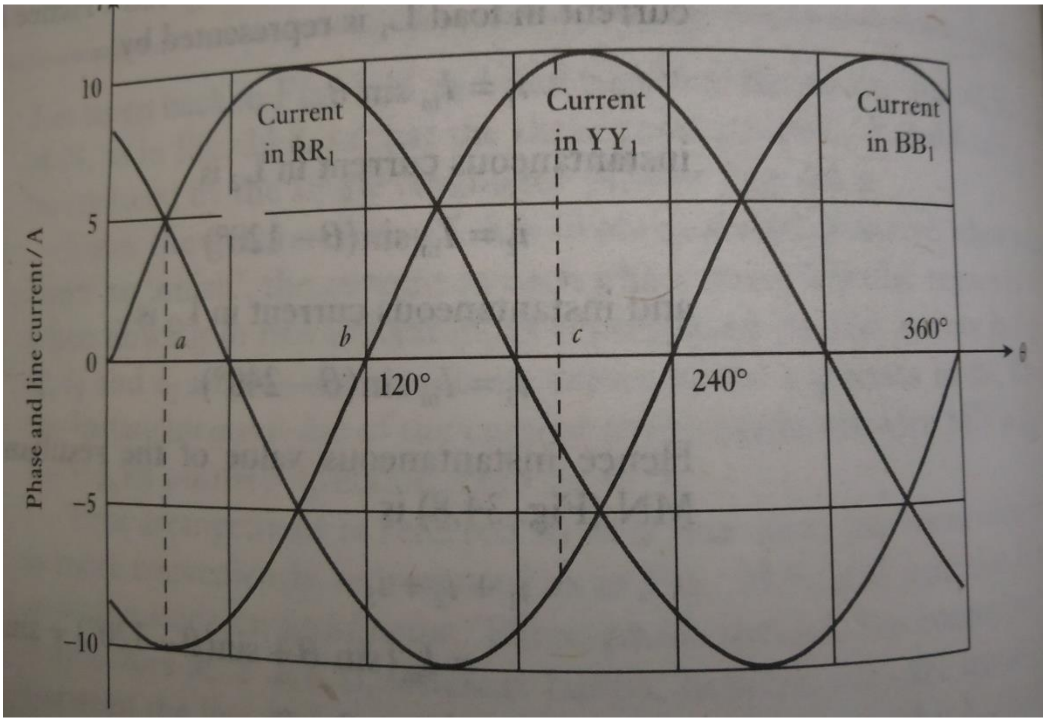
$$\begin{aligned} i_R + i_Y + i_B &= I_m[Sin\theta + Sin(\theta - 120^\circ) + Sin(\theta - 240^\circ)] \\ &= I_m(Sin\theta + Sin\theta.Cos120^\circ - Cos\theta.Sin120^\circ + Sin\theta.Cos240^\circ - Cos\theta.Sin240^\circ) \\ &= I_m(Sin\theta - 0.5Sin\theta - 0.866Cos\theta - 0.5Sin\theta + 0.866Cos\theta) \\ &= 0 \end{aligned}$$

- This means that for a balanced system, the current flowing outwards in one or two conductors is equal to that flowing back in the remaining conductor or conductors as shown in fig. 2.9.
- For a balanced load, the neutral conductor may be removed to supply power to a three-phase load and the system is now referred to as three phase three wire system as shown in fig. 2.9 (a).

- When the neutral conductor is present, the system is referred to as three phase four wire system as shown in fig. 2.8.



(a)



(b)

Fig. 2.9 (a) Three-phase three-wire star connected system with balanced load, (b) Waveforms of currents in a balanced three-phase system.

Phase voltage and phase current in 3-phase system:

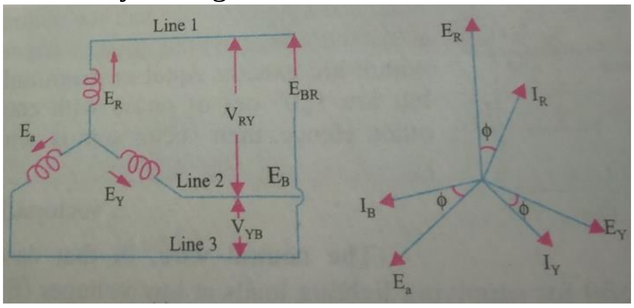
- Phase means winding. So, the voltage available across the winding is called phase voltage and the current flowing through the winding is called the phase current.

Line voltage and line current in 3-phase system:

- Line means outer wire or conductor through which the power transmission takes place from source to the load. Hence, the voltage available between any pair of line conductors is called the line voltage and the current flowing through the line conductor is called the line current.

Voltages and currents in a star-connected system:

- It is assumed that the emf in each phase to be positive when acting from the neutral point outwards, so that the rms values of the emfs generated in the three phases can be represented by $\boldsymbol{E_R}, \boldsymbol{E_Y}$ and $\boldsymbol{E_B}$ as shown in fig. 2.10 and 2.11.
- For a balanced system, $E_R = E_Y = E_B = V_{ph}$ and these three phase voltages will be displaced from each other by an angle 120^{0}.



(a) (b)

Fig. 2.10 (a) Star connection (b) Phasor diagram showing phase voltages and currents in star connection

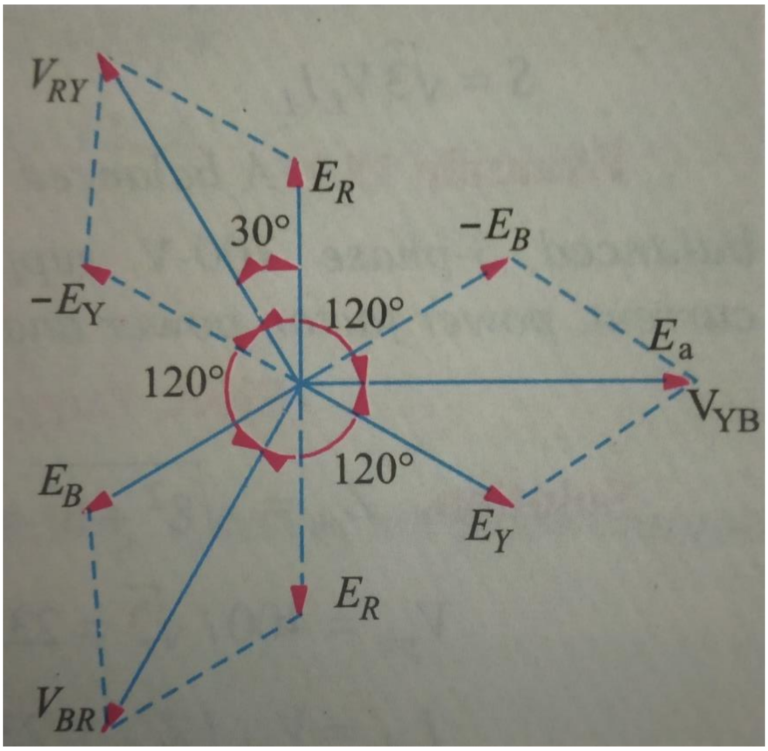


Fig. 2.11 Phasor diagram for computation of line voltage in star connection.

Voltage $\boldsymbol{V_{RY}}$ between line 1 and line 2 is given by $\boldsymbol{V_{RY}} = -\boldsymbol{E_Y} + \boldsymbol{E_R} = \boldsymbol{E_R} - \boldsymbol{E_Y}$

Voltage $\boldsymbol{V_{YB}}$ between line 2 and line 3 is given by $\boldsymbol{V_{YB}} = -\boldsymbol{E_B} + \boldsymbol{E_Y} = \boldsymbol{E_Y} - \boldsymbol{E_B}$

Voltage $\boldsymbol{V_{BR}}$ between line 3 and line 1 is given by $\boldsymbol{V_{BR}} = -\boldsymbol{E_R} + \boldsymbol{E_B} = \boldsymbol{E_B} - \boldsymbol{E_R}$

From the phasor diagram shown in fig. 2.11, the value of the line voltages can be calculated as follows.

$$\therefore V_{RY} = V_{YB} = V_{BR} = line\,voltage, V_L = 2\times V_{ph}\times Cos\,(60^\circ/2) = 2\times V_{ph}\times \frac{\sqrt{3}}{2} = \sqrt{3}V_{ph}$$

$$\therefore \textbf{\textit{In star connection,}}\ V_L = \sqrt{3}V_{ph}$$

Alternatively, the line voltages can be computed analytically as follows.

$$\boldsymbol{E_R} = V_{ph}\angle 0^\circ = V_{ph}(Cos0^\circ + jSin0^\circ) = V_{ph} + j0$$

$$\boldsymbol{E_Y} = V_{ph}\angle -120^\circ = V_{ph}[Cos(-120^\circ) + jSin(-120^\circ)] = -0.5V_{ph} - j0.866V_{ph}$$

$$\boldsymbol{E_B} = V_{ph}\angle 120^\circ = V_{ph}[Cos(120^\circ) + jSin(120^\circ)] = -0.5V_{ph} + j0.866V_{ph}$$

$\therefore$ *The three line voltages are :*

$$\boldsymbol{V_{RY}} = \boldsymbol{E_R} - \boldsymbol{E_Y} = V_{ph} + j0 + 0.5V_{ph} + j0.866V_{ph} = V_{ph}(1.5 + j0.866) = \sqrt{3}V_{ph}\angle 30^\circ$$

$$\boldsymbol{V_{YB}} = \boldsymbol{E_Y} - \boldsymbol{E_B} = -0.5V_{ph} - j0.866V_{ph} + 0.5V_{ph} - j0.866V_{ph} = V_{ph}(0 - j1.732) = \sqrt{3}V_{ph}\angle -90^\circ$$

$$\boldsymbol{V_{BR}} = \boldsymbol{E_B} - \boldsymbol{E_R} = -0.5V_{ph} + j0.866V_{ph} - V_{ph} - j0 = V_{ph}(-1.5 + j0.866) = \sqrt{3}V_{ph}\angle 150^\circ$$

$$\therefore \textbf{\textit{In star connection,}}\ V_L = \sqrt{3}V_{ph}$$

- From fig. 2.10 (a), it is clear that the same current flows through the phase and the line as the line conductors are connected in series with each phase which means line current I_L and phase current I_{ph} are same in star connection.

$$\therefore \textbf{\textit{In star connection,}}\ \boldsymbol{I_L = I_{ph}}$$

- The following points may be noted.

(i) Line voltages are 120^0 apart.

(ii) Line voltages are 30^0 ahead of their respective phase voltages.

(iii) Angle between the line voltage and corresponding line current is $(30+\Phi)$ for lagging pf and $(30-\Phi)$ for leading pf.

Power:

$$Power\,/\,Phase, \boldsymbol{S_{ph}} = \boldsymbol{V_{ph}I_{ph}}^{*} = (V_{ph}\angle 0^o)(I_{ph}\angle\pm\phi^o) = V_{ph}I_{ph}\angle\pm\phi^o = V_{ph}I_{ph}(Cos\phi^o \pm jSin\phi^o)$$

$$\Rightarrow Power\,/\,Phase, \boldsymbol{S_{ph}} = V_{ph}I_{ph}\angle\pm\phi^o = V_{ph}I_{ph}Cos\phi^o \pm jV_{ph}I_{ph}Sin\phi^o = P_{ph} \pm jQ_{ph}$$

$$\textbf{\textit{Apparent Power / Phase}} = S_{ph} = V_{ph}I_{ph}$$

$$\textbf{\textit{Active Power / Phase}} = P_{ph} = V_{ph}I_{ph}Cos\phi^o$$

$$\textbf{\textit{Reactive Power / Phase}} = Q_{ph} = \pm V_{ph}I_{ph}Sin\phi^o\ \text{('+' sign for lagging pf and '−' sign for leading pf)}$$

$$Total\ Power = \boldsymbol{S} = 3\boldsymbol{V_{ph}I_{ph}}^{*} = 3V_{ph}I_{ph}\angle\pm\phi^o = 3V_{ph}I_{ph}Cos\phi^o \pm j3V_{ph}I_{ph}Sin\phi^o = P \pm jQ$$

$$\textbf{\textit{Total Apparent Power}} = S = 3V_{ph}I_{ph} = 3\frac{V_L}{\sqrt{3}}\times I_L = \sqrt{3}V_L I_L$$

$$\textbf{\textit{Total Active Power}} = P = 3V_{ph}I_{ph}Cos\phi^o = 3\frac{V_L}{\sqrt{3}}\times I_L\times Cos\phi^o = \sqrt{3}V_L I_L Cos\phi^o$$

$$\textbf{\textit{Total Reactive Power}} = Q = \pm 3V_{ph}I_{ph}Sin\phi^o = \pm 3\frac{V_L}{\sqrt{3}}\times I_L\times Sin\phi^o$$

$$= \pm\sqrt{3}V_L I_L Sin\phi^o\ \text{('+' sign for lagging pf and '−' sign for leading pf)}$$

Currents and voltages in a delta-connected system:

- Let I_R, I_Y and I_B be the rms values of the phase currents having their positive directions as indicated by the arrows in fig. 2.12 and 2.13.
- For a balanced system, $I_R = I_Y = I_B = I_{ph}$ and these three phase currents will be displaced from each other by an angle 120^0.

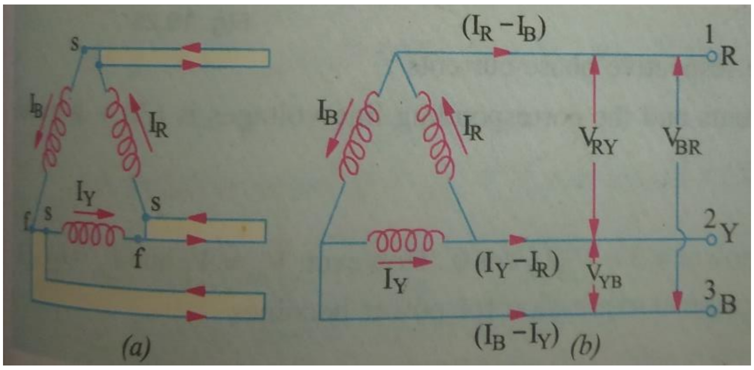


Fig. 2.12 Delta connection

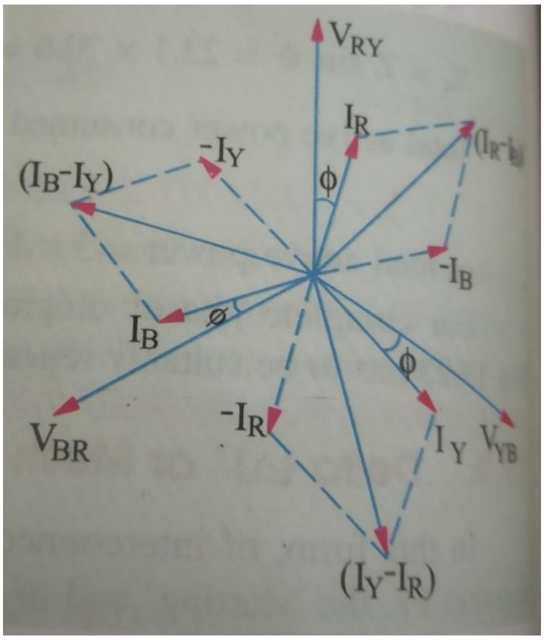


Fig. 2.13 Phasor diagram for computation of line current in delta connection

Current in line 1 is given by $\mathbf{I_1 = I_R - I_B}$
Current in line 2 is given by $\mathbf{I_2 = I_Y - I_R}$
Current in line 3 is given by $\mathbf{I_3 = I_B - I_Y}$
From the phasor diagram shown in fig. 2.13, the line current can be computed as follows.

$$\therefore I_1 = I_2 = I_3 = Line\,current, I_L = 2 \times I_{ph} \times Cos\,(60^\circ/2) = 2 \times I_{ph} \times \frac{\sqrt{3}}{2} = \sqrt{3}\, I_{ph}$$

$$\therefore \textbf{In delta connection, } I_L = \sqrt{3}\, I_{ph}$$

Alternatively, the line current can be computed analytically as follows.

$$\boldsymbol{I_R} = I_{ph}\angle 0^\circ = I_{ph}(Cos0^\circ + jSin0^\circ) = I_{ph} + j0$$
$$\boldsymbol{I_Y} = I_{ph}\angle -120^\circ = I_{ph}[Cos(-120^\circ) + jSin(-120^\circ)] = -0.5I_{ph} - j0.866I_{ph}$$
$$\boldsymbol{I_B} = I_{ph}\angle 120^\circ = I_{ph}[Cos(120^\circ) + jSin(120^\circ)] = -0.5I_{ph} + j0.866I_{ph}$$
$$\therefore \textit{The three line currents are:}$$
$$\boldsymbol{I_1} = \boldsymbol{I_R} - \boldsymbol{I_B} = I_{ph} + j0 + 0.5I_{ph} - j0.866I_{ph} = I_{ph}(1.5 - j0.866) = \sqrt{3}I_{ph}\angle -30^\circ$$
$$\boldsymbol{I_2} = \boldsymbol{I_Y} - \boldsymbol{I_R} = -0.5I_{ph} - j0.866I_{ph} - I_{ph} - j0 = I_{ph}(-1.5 - j0.866) = \sqrt{3}I_{ph}\angle -150^\circ$$
$$\boldsymbol{I_3} = \boldsymbol{I_B} - \boldsymbol{I_Y} = -0.5I_{ph} + j0.866I_{ph} + 0.5I_{ph} + j0.866I_{ph} = I_{ph}(0 + j1.732) = \sqrt{3}I_{ph}\angle 90^\circ$$

$\therefore$ ***In delta connection,*** $I_L = \sqrt{3} I_{ph}$

- From fig. 2.12, it is clear that the same voltage appears across the phase as well as in between the corresponding pair of line conductors which means line voltage V_L and phase voltage V_{ph} are same in delta connection.

$\therefore$ ***In delta connection,*** $V_L = V_{ph}$

The following points may be noted.

(iv) Line currents are 120^0 apart.
(v) Line currents are 30^0 behind their respective phase currents.
(vi) Angle between the line voltage and corresponding line current is $(30+\Phi)$ for lagging pf and $(30-\Phi)$ for leading pf.

Power:

$$\textit{Power / Phase}, \boldsymbol{S_{ph}} = \boldsymbol{V_{ph} I_{ph}}^* = (V_{ph}\angle 0^o)(I_{ph}\angle \pm\phi^o) = V_{ph}I_{ph}\angle \pm\phi^o = V_{ph}I_{ph}(Cos\phi^o \pm jSin\phi^o)$$
$$\Rightarrow \textit{Power / Phase}, \boldsymbol{S_{ph}} = V_{ph}I_{ph}\angle \pm\phi^o = V_{ph}I_{ph}Cos\phi^o \pm jV_{ph}I_{ph}Sin\phi^o = P_{ph} \pm jQ_{ph}$$
$$\textbf{\textit{Apparent Power / Phase}} = S_{ph} = V_{ph}I_{ph}$$
$$\textbf{\textit{Active Power / Phase}} = P_{ph} = V_{ph}I_{ph}Cos\phi^o$$
$$\textbf{\textit{Reactive Power / Phase}} = Q_{ph} = \pm V_{ph}I_{ph}Sin\phi^o \text{ ('+' \textit{sign for lagging pf and} '−' sign \textit{for leading pf})}$$
$$\textit{Total Power} = \boldsymbol{S} = 3\boldsymbol{V_{ph}I_{ph}}^* = 3V_{ph}I_{ph}\angle \pm\phi^o = 3V_{ph}I_{ph}Cos\phi^o \pm j3V_{ph}I_{ph}Sin\phi^o = P \pm jQ$$
$$\textbf{\textit{Total Apparent Power}} = S = 3V_{ph}I_{ph} = 3V_L \times \frac{I_L}{\sqrt{3}} = \sqrt{3}V_L I_L$$
$$\textbf{\textit{Total Active Power}} = P = 3V_{ph}I_{ph}Cos\phi^o = 3V_L \times \frac{I_L}{\sqrt{3}} \times Cos\phi^o = \sqrt{3}V_L I_L Cos\phi^o$$
$$\textbf{\textit{Total Reactive Power}} = Q = \pm 3V_{ph}I_{ph}Sin\phi^o = \pm 3V_L \times \frac{I_L}{\sqrt{3}} \times Sin\phi^o$$
$$= \pm\sqrt{3}V_L I_L Sin\phi^o \text{ ('+' \textit{sign for lagging pf and} '−' sign \textit{for leading pf})}$$

$\therefore$ ***For both star and delta connection*** **:**

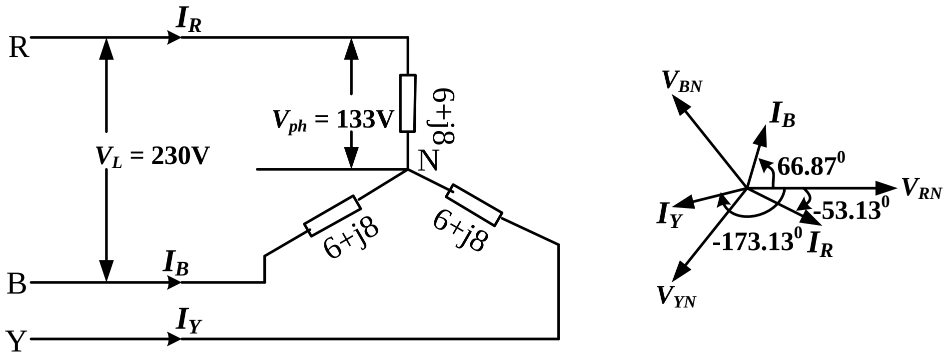
$$\boldsymbol{S_{ph}} = V_{ph}I_{ph}\angle \pm\phi^o = V_{ph}I_{ph}Cos\phi^o \pm jV_{ph}I_{ph}\phi^o = P_{ph} \pm jQ_{ph}$$
$$\textbf{\textit{Apparent Power / Phase}} = S_{ph} = V_{ph}I_{ph}$$
$$\textbf{\textit{Active Power / Phase}} = P_{ph} = V_{ph}I_{ph}Cos\phi^o$$
$$\textbf{\textit{Reactive Power / Phase}} = Q_{ph} = \pm V_{ph}I_{ph}Sin\phi^o \text{ ('+' \textit{sign for lagging pf and} '−' sign \textit{for leading pf})}$$
$$\textit{Total Power} = \boldsymbol{S} = 3\boldsymbol{V_{ph}I_{ph}}^* = 3V_{ph}I_{ph}\angle \pm\phi^o = 3V_{ph}I_{ph}Cos\phi^o \pm j3V_{ph}I_{ph}Sin\phi^o = P \pm jQ$$
$$\textbf{\textit{Total Apparent Power}} = S = 3V_{ph}I_{ph} = \sqrt{3}V_L I_L$$
$$\textbf{\textit{Total Active Power}} = P = 3V_{ph}I_{ph}Cos\phi^o = \sqrt{3}V_L I_L Cos\phi^o$$
$$\textbf{\textit{Total Reactive Power}} = Q = \pm 3V_{ph}I_{ph}Sin\phi^o$$
$$= \pm\sqrt{3}V_L I_L Sin\phi^o \text{ ('+' \textit{sign for lagging pf and} '−' sign \textit{for leading pf})}$$

Problem:

Given a balanced 3-phase, 3-wire system with star connected load for which line voltage is 230 V and impedance of each phase is $(6 + j8)\ \Omega$. Find the followings

(i) line current,
(ii) active, reactive and apparent power of each phase,
(iii) Total active, reactive and apparent power of the load.

Also draw the phasor diagram showing the phase voltages and line currents.
Solution:



(a) (b)

$$V_L = 230V$$
$$\Rightarrow V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{230}{\sqrt{3}} = 133V$$
$$Z_{ph} = 6 + j8 = 10\angle 53.13^\circ$$
$$Let,\ \boldsymbol{V_{RN}} = V_{ph}\angle 0^\circ = 133\angle 0^\circ = 133 + j0$$
$$\therefore \boldsymbol{V_{YN}} = V_{ph}\angle -120^\circ = 133\angle -120^\circ = -66.5 - j115$$
$$and,\ \boldsymbol{V_{BN}} = V_{ph}\angle +120^\circ = 133\angle +120^\circ = -66.5 + j115$$
Now,
$$\boldsymbol{I_R} = \frac{\boldsymbol{V_{RN}}}{Z_{ph}} = \frac{133\angle 0^\circ}{10\angle 53.13^\circ} = 13.3\angle -53.13^\circ$$
$$\boldsymbol{I_Y} = \frac{\boldsymbol{V_{YN}}}{Z_{ph}} = \frac{133\angle -120^\circ}{10\angle 53.13^\circ} = 13.3\angle -173.13^\circ$$
$$\boldsymbol{I_B} = \frac{\boldsymbol{V_{BN}}}{Z_{ph}} = \frac{133\angle +120^\circ}{10\angle 53.13^\circ} = 13.3\angle 66.87^\circ$$

Since it is a star connected load $\boldsymbol{I_R}, \boldsymbol{I_Y}$ *and* $\boldsymbol{I_B}$ *are the phase currents as well as line currents.*
Power absorbed by phase R is given by,
$$\boldsymbol{S_R} = \boldsymbol{V_{RN}}\boldsymbol{I_R}^* = (133\angle 0^\circ)(13.3\angle +53.13^\circ) = 1769\angle 53.13 = 1061 + j1415$$
Power absorbed by phase Y is given by,
$$\boldsymbol{S_Y} = \boldsymbol{V_{YN}}\boldsymbol{I_Y}^* = (133\angle -120^\circ)(13.3\angle +173.13^\circ) = 1769\angle 53.13 = 1061 + j1415$$
Power absorbed by phase B is given by,
$$\boldsymbol{S_B} = \boldsymbol{V_{BN}}\boldsymbol{I_B}^* = (133\angle +120^\circ)(13.3\angle -66.87^\circ) = 1769\angle 53.13 = 1061 + j1415$$
It is seen that, in each phase the power absorbed is same which is obvious as it is a balanced load

$\therefore$
Apparent power / phase, $S_{ph} = 1769VA$
Active power / phase, $P_{ph} = 1061W$
Reactive power / phase, $Q_{ph} = 1415VAR\,(lagging)$

Alternatively,
Apparent power / phase, $S_{ph} = V_{ph}I_{ph} = 133 \times 13.3 = 1769VA, Or$
$$S_{ph} = I_{ph}{}^2 Z_{ph} = 13.3^2 \times 10 = 1769VA$$

$$Active\ power\ /\ phase, P_{ph} = V_{ph} I_{ph} Cos\Phi = 133 \times 13.3 \times Cos 53.13^{\circ} = 1061W, Or$$
$$P_{ph} = I_{ph}{}^2 R_{ph} = 13.3^2 \times 6 = 1061W$$

$$Reactive\ power\ /\ phase, Q_{ph} = V_{ph} I_{ph} Sin\,\Phi = 133 \times 13.3 \times Sin 53.13^{\circ} = 1415 VAR(lagging), Or$$
$$Q_{ph} = I_{ph}{}^2 X = 13.3^2 \times 8 = 1415 VAR(lagging)$$

Total power can be obtained by multiplying the per phase power by 3 *as follows.*

$\therefore$

$Total\ Apparent\ power, S = 3V_{ph} I_{ph} = 3S_{ph} = 3 \times 1769 = 5307 VA$

$Total\ Active\ power, P = 3V_{ph} I_{ph} Cos\Phi = 3P_{ph} = 3 \times 1061 = 3183W$

$Total\ Reactive\ power, Q = 3V_{ph} I_{ph} Sin\,\Phi = 3Q_{ph} = 3 \times 1415 = 4245 VAR(lagging)$

Alternatively,

$Total\ Apparent\ power, S = \sqrt{3} V_L I_L = \sqrt{3} \times 230 \times 13.3 = 5298 VA$

$Total\ Active\ power, P = \sqrt{3} V_L I_L Cos\Phi = \sqrt{3} \times 230 \times 13.3 \times Cos 53.13^{\circ} = 3179W$

$Total\ Reactive\ power, Q = \sqrt{3} V_L I_L Sin\,\Phi = \sqrt{3} \times 230 \times 13.3 \times Sin 53.13^{\circ} = 4239 VAR(lagging)$

Problem:

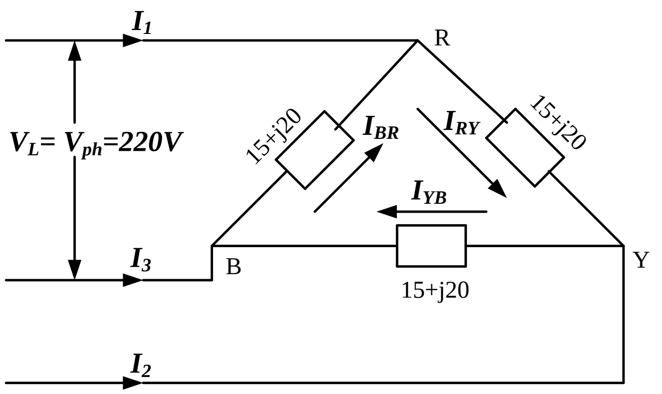
A 220 V, 3-phase voltage is applied to a balanced delta connected 3-phase load of phase impedance $(15 + j20)\ \Omega$. Determine the following:

(i) Phasor current in each line current,

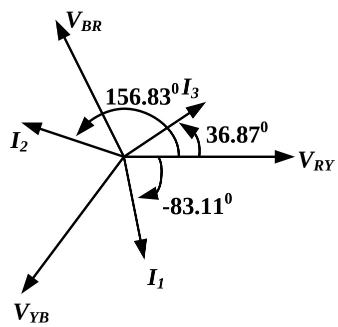
(ii) Active, reactive and apparent power of each phase

(iii) Total Active, reactive and apparent power of the load.

Also draw the phasor diagram showing the line voltages and line currents..



(a)



(b)

Solution:

$V_L = V_{ph} = 220V$

$Z_{ph} = 15 + j20 = 25\angle 53.13^{\circ}$

$Let, the\ phase\ voltage, \boldsymbol{V_{RY}} = V_{ph}\angle 0^{\circ} = 220\angle 0^{\circ} = 220 + j0$

$\therefore Phase\ voltage, \boldsymbol{V_{YB}} = V_{ph}\angle -120^{\circ} = 220\angle -120^{\circ} = -110 - j190.5$

$and, Phase\ voltage, \boldsymbol{V_{BR}} = V_{ph}\angle +120^{\circ} = 220\angle +120^{\circ} = -110 + j190.5$

$Now, the\ three\ phase\ currents\ are:$

$$\boldsymbol{I_{RY}} = \frac{\boldsymbol{V_{RY}}}{Z_{ph}} = \frac{220\angle 0^\circ}{25\angle 53.13^\circ} = 8.8\angle -53.13^\circ = 5.28 - j7.04$$

$$\boldsymbol{I_{YB}} = \frac{\boldsymbol{V_{YB}}}{Z_{ph}} = \frac{220\angle -120^\circ}{25\angle 53.13^\circ} = 8.8\angle -173.13^\circ = -8.74 - j1.05$$

$$\boldsymbol{I_{BR}} = \frac{\boldsymbol{V_{BR}}}{Z_{ph}} = \frac{220\angle +120^\circ}{25\angle 53.13^\circ} = 8.8\angle 66.87^\circ = 3.45 + j8.1$$

$\boldsymbol{I_R}, \boldsymbol{I_Y}$ *and* $\boldsymbol{I_B}$ *are the phase currents.*

The line currents are calculated as follows.

$$\boldsymbol{I_1} = \boldsymbol{I_{RY}} - \boldsymbol{I_{BR}} = 5.28 - j7.04 - 3.45 - j8.1 = 1.83 - j15.14 = 15.25\angle -83.11$$

$$\boldsymbol{I_2} = \boldsymbol{I_{YB}} - \boldsymbol{I_{RY}} = -8.74 - j1.05 - 5.28 + j7.04 = -14.02 + j6 = 15.25\angle 156.83$$

$$\boldsymbol{I_3} = \boldsymbol{I_{BR}} - \boldsymbol{I_{YB}} = 3.45 + j8.1 + 8.74 + j1.05 = 12.2 + j9.15 = 15.25\angle 36.87$$

Power absorbed by phase R is given by,

$$\boldsymbol{S_R} = \boldsymbol{V_{RY}}\boldsymbol{I_{RY}}^* = (220\angle 0^\circ)(8.8\angle 53.13^\circ) = 1936\angle 53.13 = 1162 + j1549$$

Power absorbed by phase Y is given by,

$$\boldsymbol{S_Y} = \boldsymbol{V_{YB}}\boldsymbol{I_{YB}}^* = (220\angle -120^\circ)(8.8\angle +173.13^\circ) = 1936\angle 53.13 = 1162 + j1549$$

Power absorbed by phase B is given by,

$$\boldsymbol{S_B} = \boldsymbol{V_{BR}}\boldsymbol{I_{BR}}^* = (220\angle +120^\circ)(8.8\angle -66.87^\circ) = 1936\angle 53.13 = 1162 + j1549$$

It is seen that, in each phase the power absorbed is same which is obvious as it is a balanced load.

$\therefore$

$Apparent\ power / phase, S_{ph} = 1936VA$

$Active\ power / phase, P_{ph} = 1162W$

$Reactive\ power / phase, Q_{ph} = 1549VAR\,(lagging)$

Alternatively,

$Apparent\ power / phase, S_{ph} = V_{ph}I_{ph} = 220 \times 8.8 = 1936VA, Or$

$$S_{ph} = I_{ph}^2 Z_{ph} = 8.8^2 \times 25 = 1936VA$$

$Active\ power / phase, P_{ph} = V_{ph}I_{ph}Cos\Phi = 220 \times 8.8 \times Cos53.13^\circ = 1162W, Or$

$$P_{ph} = I_{ph}^2 R_{ph} = 8.8^2 \times 15 = 1162W$$

$Reactive\ power / phase, Q_{ph} = V_{ph}I_{ph}Sin\,\Phi = 220 \times 8.8 \times Sin53.13^\circ = 1549VAR\,(lagging), Or$

$$Q_{ph} = I_{ph}^2 X = 8.8^2 \times 20 = 1549VAR\,(lagging)$$

Total power can be obtained by multiplying the per phase power by 3 *as follows.*

$\therefore$

$Total\ Apparent\ power, S = 3V_{ph}I_{ph} = 3S_{ph} = 3 \times 1936 = 5808VA$

$Total\ Active\ power, P = 3V_{ph}I_{ph}Cos\Phi = 3P_{ph} = 3 \times 1162 = 3486W$

$Total\ Reactive\ power, Q = 3V_{ph}I_{ph}Sin\,\Phi = 3Q_{ph} = 3 \times 1549 = 4647VAR\,(lagging)$

Alternatively,

$Total\ Apparent\ power, S = \sqrt{3}V_L I_L = \sqrt{3} \times 220 \times 15.25 = 5811VA$

$Total\ Active\ power, P = \sqrt{3}V_L I_L Cos\Phi = \sqrt{3} \times 220 \times 15.25 \times Cos53.13^\circ = 3486W$

$Total\ Reactive\ power, Q = \sqrt{3}V_L I_L Sin\,\Phi = \sqrt{3} \times 220 \times 15.25 \times Sin53.13^\circ = 4649VAR\,(lagging)$

Measurement of Power in 3-phase System:

In a three-phase system, power can be measured by any of the following methods.

(i) Three wattmeter method:

- In this method, three wattmeters are used to measure the total power and one wattmeter is connected in each phase as shown in fig. 2.14.

- The current coil of each wattmeter carries the phase current and the pressure coil measures the phase voltage of each phase. Hence, each wattmeter measures the power of the corresponding phase in which it is connected.
- The algebraic sum of all the wattmeters gives the total power consumed by the 3-phase load.
- The disadvantage of this method is that it may not be always possible to break into the phases in case of a delta connection and similarly in case of a star connection, it's difficult to find the neutral point for connections of wattmeters.

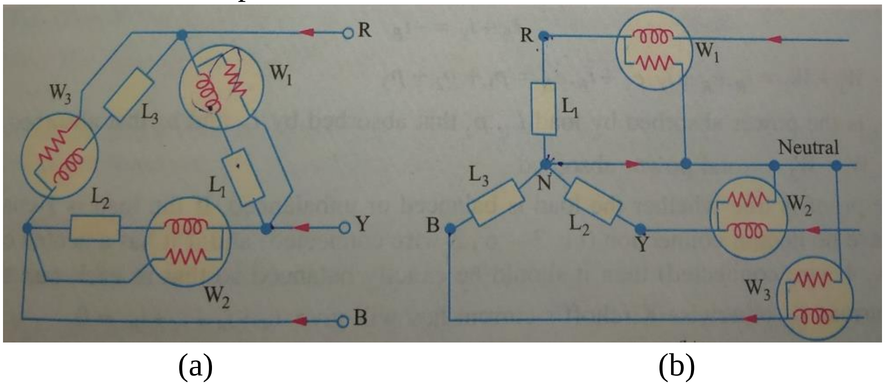


Fig. 2.14 Connection diagram for measurement of power by three wattmeter method for (a) delta connected load and (b) star connected load.

(ii) Two wattmeter method:

- This method can be applied for measurement of power for both balanced and unbalanced load. For unbalanced load, it must be a three-phase three wire system i.e. in case of star connected load, the neutral conductor must be absent because the neutral conductor carries current in unbalanced load.
- The connection diagram for measurement of 3-phase power by two wattmeter method for both the delta connected and star connected load is shown in fig. 2.15.
- In this method, the current coils of the two wattmeters are connected in any two line and the pressure coils of the wattmeters are connected to the third line as shown in the fig. 2.15.
- It can be proved that, the summation of the instantaneous powers indicated by the two wattmeters gives the total instantaneous powers absorbed by the three-phase load.

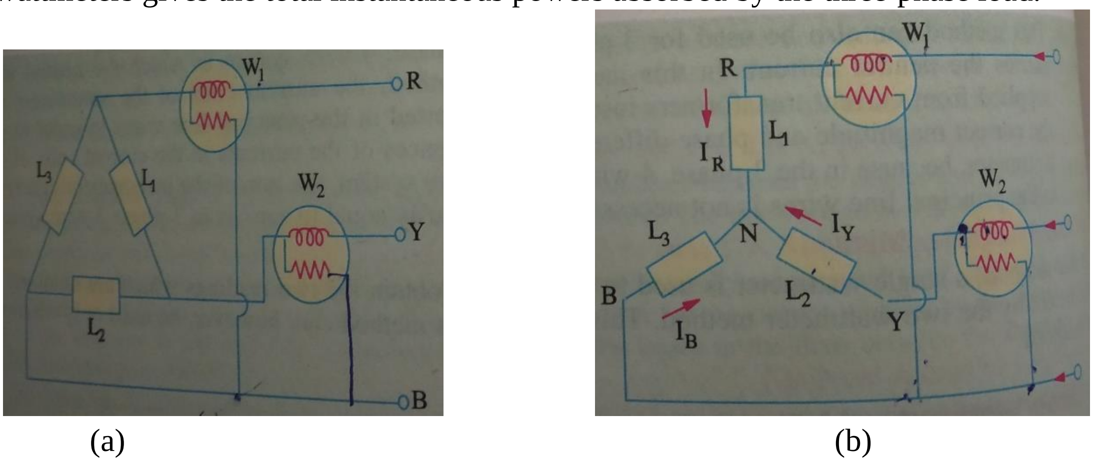


Fig. 2.15 Connection diagram for measurement of power by two wattmeter method for (a) delta connected load (b) star connected load

- A star connected load is considered to do the proof and the proof is valid for delta connection also because a delta connected load can always be replaced by an equivalent star connected load.
- The wattmeter reading is positive if the direction of both the current and voltage is taken along the same directions. The assumed positive directions of instantaneous currents in different phases are already shown in fig. 2.15 (b). Hence, the voltage measured by the pressure coil of W_1 will be $v_{RN} - v_{BN}$ *and not* $v_{BN} - v_{RN}$. Similarly, the voltage measured by the pressure coil of W_2 will be $v_{YN} - v_{BN}$ *and not* $v_{BN} - v_{YN}$.
- $\cdot \quad \text{Total instantaneous power of the star connected load} = v_{RN} i_R + v_{YN} i_Y + v_{BN} i_B$
- Referring to fig. 2.15 (b),

$$\text{Instantaneous current through the current coil of } W_1 = i_R$$
$$\text{Instantaneous voltge across the pressure coil of } W_1 = (v_{RN} - v_{BN})$$
$$\therefore \text{Instantaneous power measured by } W_1,\ P_1 = (v_{RN} - v_{BN}) i_R$$
$$\text{Instantaneous current through the current coil of } W_2 = i_Y$$
$$\text{Instantaneous voltge across the pressure coil of } W_2 = (v_{YN} - v_{BN})$$
$$\therefore \text{Instantaneous power measured by } W_2,\ P_2 = (v_{YN} - v_{BN}) i_Y$$
$$\text{Hence, the sum of the instantaneous power measured by } W_1 \text{ and } W_2 = P_1 + P_2$$
$$= (v_{RN} - v_{BN}) i_R + (v_{YN} - v_{BN}) i_Y$$
$$= v_{RN} i_R - v_{BN} i_R + v_{YN} i_Y - v_{BN} i_Y$$
$$= v_{RN} i_R + v_{YN} i_Y - v_{BN} (i_R + i_Y)$$
$$\text{Applying KCL at at } N,\ i_R + i_Y + i_B = 0$$
$$\Rightarrow i_R + i_Y = -i_B$$
$$\therefore P_1 + P_2 = v_{RN} i_R + v_{YN} i_Y - v_{BN} (i_R + i_Y) = v_{RN} i_R + v_{YN} i_Y + v_{BN} i_B \ (\text{putting } i_R + i_Y = -i_B)$$
$$= \text{Total instantaneous power (Proved)}$$

- The above proof holds good for both balanced and unbalanced load. But for unbalanced load, it must have no neutral connection, otherwise, KCL at point N will give $i_N + i_R + i_Y + i_B = 0$.
- The wattmeter readings give the average power in actual practice. The above proof is done by taking instantaneous power into consideration but it also holds true for average power since the average power is obtained by integrating the instantaneous power over a complete cycle and dividing it by the time base.

(iii) One wattmeter method:

- This method can be used to measure power of only balanced load. Instead of three wattmeters, only one wattmeter can be used in any phase of the fig. 2.14. The wattmeter gives the power of one phase. Hence, the total power can be obtained by multiplying three with the wattmeter reading.

Power factor measurement by two wattmeter method for balanced load:

(i) Lagging power factor

- For lagging pf, the phasor diagram for a balanced star connected load is shown in fig. 2.16.

  Let, V_{RN}, V_{YN}, V_{BN} *be the RMS values of phase voltages* and

  I_R, I_Y, I_B *be the RMS values of phase currents.*

$$\text{Current through the current coil of } W_1 = I_R$$
$$\text{Voltge across the pressure coil of } W_1 = V_{RB} = (V_{RN} - V_{BN})$$
$$\text{From the phasor diagram, Phase angle between } V_{RB} \text{ and } I_R \text{ is } (30 - \Phi)$$
$$\therefore \text{Reading of } W_1, P_1 = V_{RB} I_R \cos(30 - \Phi)$$

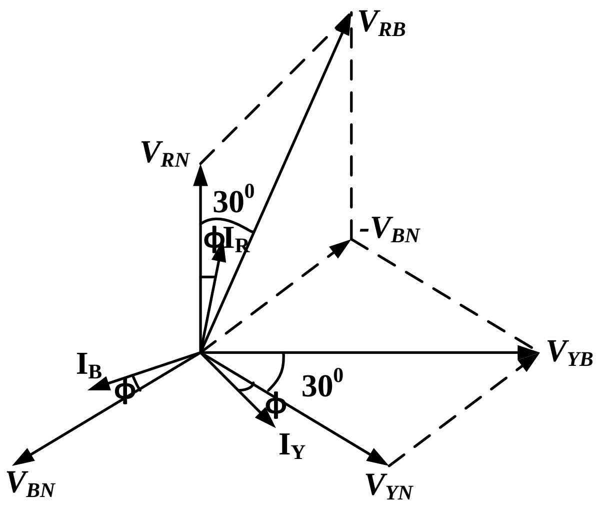


Fig. 2.16 Phasor diagram for balanced star connected load

Current through the current coil of $W_2 = I_Y$
Voltge across the pressure coil of $W_2 = V_{YB} = (V_{YN} - V_{BN})$
From the phasor diagram, Phase angle between V_{YB} *and* I_Y *is* $(30 + \Phi)$
$\therefore$ *Reading of* $W_2, P_2 = V_{YB} I_Y Cos(30 + \Phi)$
Since it is a balanced load, $V_{RB} = V_{YB} = V_L$
and $I_R = I_Y = I_L$ *(In star connection, phase currents and line currents are same)*
$\therefore P_1 = V_L I_L Cos(30 - \Phi)$ *and* $P_2 = V_L I_L Cos(30 + \Phi)$
Hence, the sum of the reading of W_1 *and* $W_2 = P_1 + P_2$

$$= V_L I_L Cos(30 - \Phi) + V_L I_L Cos(30 + \Phi)$$
$$= 2 V_L I_L Cos30 Cos\Phi$$
$$= \sqrt{3} V_L I_L Cos\Phi$$
$$= \text{Total power absorbed by the load}$$

Hence, the sum of the two wattmeter readings gives the total power absorbed by the three phase load

$$P_1 + P_2 = \sqrt{3} V_L I_L Cos\Phi$$
$$P_1 - P_2 = V_L I_L Cos(30 - \Phi) - V_L I_L Cos(30 + \Phi) = 2 V_L I_L Sin30 Sin\Phi$$
$$\Rightarrow P_1 - P_2 = V_L I_L Sin\Phi$$
$$\frac{P_1 - P_2}{P_1 + P_2} = \frac{1}{\sqrt{3}} tan\Phi$$
$$\therefore tan\Phi = \sqrt{3}\,\frac{P_1 - P_2}{P_1 + P_2}$$

$$\Phi = tan^{-1} \frac{\sqrt{3}(P_1 - P_2)}{(P_1 + P_2)}$$

$$tan\Phi = \sqrt{3}\,\frac{P_1 - P_2}{P_1 + P_2} = \frac{\sqrt{3}(1 - P_2 / P_1)}{(1 + P_2 / P_1)}$$

Taking $P_2 / P_1 = r$, *we have,*

$$\Rightarrow tan^2\Phi = \frac{3(1 - r)^2}{(1 + r)^2}$$

$$\Rightarrow Sec^2\Phi - 1 = \frac{3(1-r)^2}{(1+r)^2}$$

$$\Rightarrow Sec^2\Phi = 1 + \frac{3+3r^2-6r}{1+r^2+2r} = \frac{4+4r^2-4r}{1+r^2+2r}$$

$$\Rightarrow \frac{1}{Cos^2\Phi} = \frac{4+4r^2-4r}{1+r^2+2r}$$

$$\Rightarrow Cos^2\Phi = \frac{1+r^2+2r}{4+4r^2-4r}$$

$$\therefore Cos\Phi = \frac{1+r}{2\sqrt{1+r^2-r}}$$

The curve plotted between r and $Cos\Phi$ is known as watt-ratio curve and is shown in fig. 2.17.

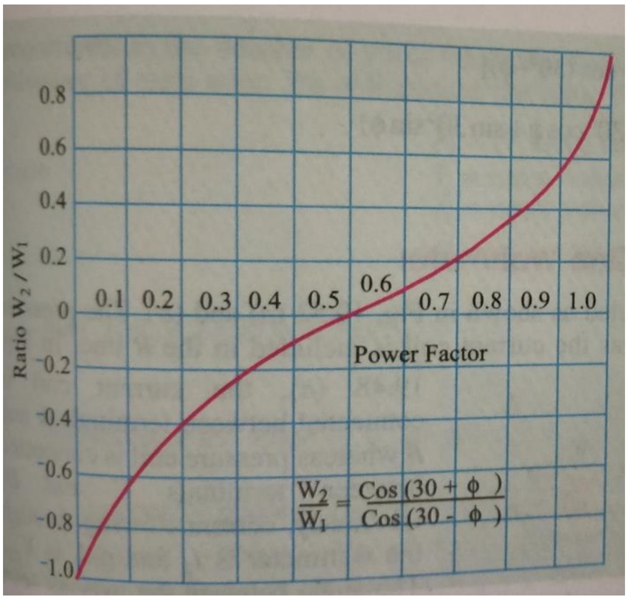


Fig. 2.17 Watt-ratio curve

Variations in wattmeter reading:

- The variations in wattmeter reading for various values of Φ is shown in the following table.

	$\Phi = 0^\circ$	$\Phi = 60^\circ$	$\Phi = 90^\circ$
P_1	*positive*	*positive*	*positive*
P_2	*positive*	*0*	*negative*
	$P_1 = P_2 = V_L I_L Cos30 = \frac{\sqrt{3}}{2} V_L I_L$	$P_1 = V_L I_L Cos30 = \frac{\sqrt{3}}{2} V_L I_L$	$P_1 = V_L I_L Sin30 = \frac{1}{2} V_L I_L$ $P_2 = -V_L I_L Sin30 = -\frac{1}{2} V_L I_L$ P_1 *and* P_2 *are equal in magnitude but of opposite sign* $P_1 + P_2 = 0$

- For $60^o < \Phi < 90^o$, P_1 is positive but P_2 is negative. In this condition the second wattmeter W_2 will read down scale. Hence, to obtain the reading, either the pressure coil or current coil of the wattmeter is reversed. But the reading so obtained after reversing the pressure coil or current coil of the wattmeter is taken as negative.

Notes:
In case of lagging power factor, the value of higher reading wattmeter is considered as P_1 and the value of lower reading wattmeter is considered as P_2. P_1 is always positive whereas P_2 may be negative.

(ii) Leading power factor:

- The derivation for P_1 and P_2 can also be done for leading pf by drawing the phasor diagram for leading pf.
- But, for leading pf, the angle Φ is negative. Hence, putting $-\Phi$ *in place of* Φ, *the readings of the two wattmeters can be obtained as follows.*

$$P_1 = V_L I_L Cos[30 - (-\Phi)] = V_L I_L Cos(30 + \Phi)$$
$$P_2 = V_L I_L Cos[30 + (-\Phi)] = V_L I_L Cos(30 - \Phi)$$

Hence, for leading power factors, the wattmeter readings are interchanged.

$$Here, P_2 - P_1 = V_L I_L Sin\Phi \text{ and } P_2 + P_1 = \sqrt{3} V_L I_L Cos\Phi$$
$$Hence, \frac{P_2 - P_1}{P_2 + P_1} = \frac{1}{\sqrt{3}} tan\Phi$$
$$\therefore tan\Phi = \sqrt{3}\frac{P_2 - P_1}{P_2 + P_1} = -\sqrt{3}\frac{P_1 - P_2}{P_1 + P_2}$$

Notes:
In case of leading power factor, the value of higher reading wattmeter is considered as P_2 and the value of lower reading wattmeter is considered as P_1 . P_2 is always positive whereas P_1 may be negative.

Problem:
The input power to a three-phase induction motor was measured by the two-wattmeter method. The readings were 5.2 KW and -1.7 KW, and the line voltage was 400 V. Calculate:

(i) The total active power,
(ii) The power factor,
(iii) The line current.

Solution:

$$(a) Total\ active\ power, P = P_1 + P_2 = 5.2 + (-1.7) = 3.5\,KW = 3500\,W$$
$$(b) tan\Phi = \sqrt{3}\frac{P_1 - P_2}{P_1 + P_2}$$
$$\Rightarrow tan\Phi = \sqrt{3}\frac{5.2 - (-1.7)}{5.2 + (-1.7)}$$
$$\Rightarrow \Phi = 73.67^\circ$$
$$\therefore Power\ factor, Cos\Phi = Cos 73.67^\circ = 0.281$$

$$(c) P = \sqrt{3} V_L I_L Cos\Phi$$
$$\Rightarrow 3500 = \sqrt{3} \times 400 \times I_L \times 0.281$$
$$\Rightarrow I_L = 18A$$

Problem:

Phase voltage and current of a star-connected inductive load is 150 V and 25 A respectively. Load power factor is 0.707 (lagging). Find the readings of the wattmeters if power is measured by two wattmeter method.

Solution:

$$(a) V_L = \sqrt{3} V_{ph} = \sqrt{3} \times 150 = 260V, I_L = I_{ph} = 25A$$
$$P = \sqrt{3} V_L I_L Cos\Phi = \sqrt{3} \times 260 \times 25 \times 0.707 = 7960W$$
$$Alternatively, P = 3V_{ph} I_{ph} Cos\Phi = 3 \times 150 \times 25 \times 0.707 = 7954W$$
$$Total\ active\ power, P = P_1 + P_2 = 7960$$
$$(b) tan\Phi = \sqrt{3} \frac{P_1 - P_2}{P_1 + P_2}$$
$$\Rightarrow tan\Phi = \sqrt{3} \frac{P_1 - P_2}{7960}$$
$$Cos\Phi = 0.707$$
$$\Rightarrow \Phi = 45^\circ$$
$$tan 45^\circ = \sqrt{3} \frac{P_1 - P_2}{7960}$$
$$\Rightarrow P_1 - P_2 = 4596W$$
$$(c) P_1 + P_2 = 7960\ and\ P_1 - P_2 = 4596W$$
$$\Rightarrow P_1 = 6278W\ and\ P_2 = 1682W$$

Problem:

Two wattmeters are used to measure the power input and the power factor of an over-excited synchronous motor. If the readings of the meters are -2KW and 7KW respectively, calculate the power input and power factor of the motor.

Solution:

$$Power\ input, P = P_2 + P_1 = (7) + (-2) = 5\ KW = 5000W$$
$$tan\Phi = \sqrt{3} \frac{P_2 - P_1}{P_2 + P_1}$$
$$\Rightarrow tan\Phi = \sqrt{3} \frac{7 - (-2)}{7 - 2} \Rightarrow \Phi = 72.21^\circ$$
$$\therefore Power\ factor, Cos\Phi = Cos 72.21^\circ = 0.305 (leading)$$

Problem:

The power in a 3-phase circuit is measured by two wattmeters. If the total power is 100 KW and power factor is 0.66 leading, then determine the reading of each wattmeter.

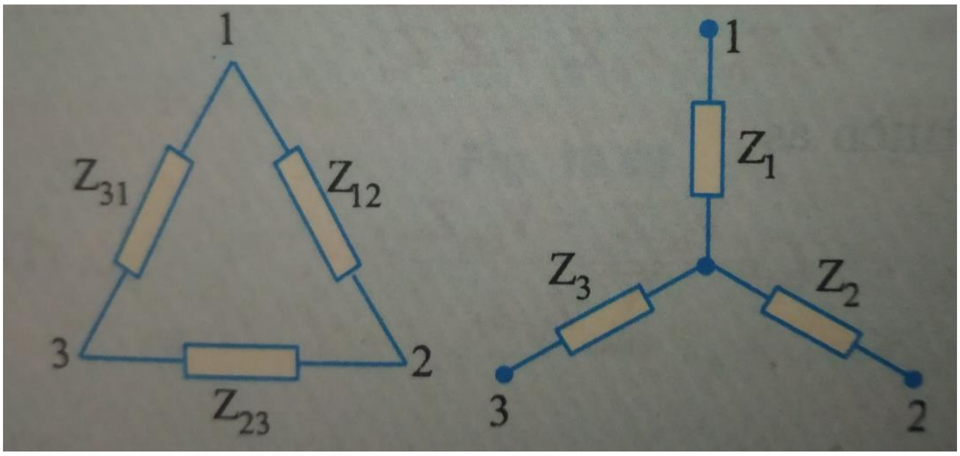
Solution:

$$Power\ input, P = 100\ KW$$
$$P_2 + P_1 = 100 \qquad (1)$$
$$Power\ factor, Cos\Phi = 0.66 (leading)$$
$$\Rightarrow \Phi = 48.7^\circ$$
$$tan 48.7^\circ = \sqrt{3} \frac{P_2 - P_1}{P_2 + P_1}$$
$$\Rightarrow 1.13 = \sqrt{3} \frac{P_2 - P_1}{100}$$

$\Rightarrow P_2 - P_1 = 65.71 \qquad (2)$
$\therefore From (1) and (2),$
$2P_2 = 165.71$
$\Rightarrow P_2 = 82.85 KW$
$P_1 = 100 - 82.85 = 17.14 KW$

Delta/Star and Star/Delta Conversion:

- If the two systems are to be equivalent, then the impedances between corresponding pairs of terminals of the two systems mut be the same.
- Also, if the two systems are equivalent, the corresponding line voltages and line currents in the two systems remain same.
- Let us consider the unbalanced delta and star connected load as shown in fig.2.18 for conversion of delta to star and vice-versa.



(a) (b)

Fig. 2.18 Delta/Star and Star/Delta Conversion (a) Delta connected load and (b) Star connected load

(i) Delta/Star Conversion:

- For star connected load, the equivalent impedance between terminals 1 and 2 is $Z_1 + Z_2$
- For delta connected load, let the equivalent impedance between terminals 1 and 2 is Z and the value of this equivalent impedance can be obtained as follows.

$$\frac{1}{Z} = \frac{1}{Z_{12}} + \frac{1}{Z_{23} + Z_{31}}$$
$$\Rightarrow Z = \frac{Z_{12}(Z_{23} + Z_{31})}{Z_{12} + Z_{23} + Z_{31}}$$

- Hence, for equivalency of the two systems, the equivalent impedance between terminals 1 and 2 for both the systems must be equal i.e.

$$Z_1 + Z_2 = \frac{Z_{12}(Z_{23} + Z_{31})}{Z_{12} + Z_{23} + Z_{31}} \qquad (1)$$

- Similarly, considering terminals 2 and 3 for equivalency, the following equation is obtained.

$$Z_2 + Z_3 = \frac{Z_{23}(Z_{31} + Z_{12})}{Z_{12} + Z_{23} + Z_{31}} \qquad (2)$$

- Again, considering terminals 3 and 1 for equivalency, the following equation is obtained.

$$Z_3 + Z_1 = \frac{Z_{31}(Z_{12} + Z_{23})}{Z_{12} + Z_{23} + Z_{31}} \tag{3}$$

- Subtracting equation (2) from equation (1) and adding the result to equation (3) i.e. [{$equation\ (1) -\ equation\ (2)\} + equation\ (3)$], gives the value of Z_1 as follows.

$$2Z_1 = \frac{Z_{12}Z_{23} + Z_{12}Z_{31} - Z_{23}Z_{31} - Z_{12}Z_{23} + Z_{31}Z_{12} + Z_{23}Z_{31}}{Z_{12} + Z_{23} + Z_{31}}$$

$$\Rightarrow Z_1 = \frac{Z_{12}Z_{31}}{Z_{12} + Z_{23} + Z_{31}} \tag{4}$$

$$= \frac{Product\ of\ impedances\ connected\ to\ terminal 1\ of\ delta\ connection}{Total\ of\ impedances\ of\ delta\ connection}$$

- Similarly, [{$equation\ (2) -\ equation\ (3)\} + equation\ (1)$], gives the value of Z_2 as follows.

$$\Rightarrow 2Z_2 = \frac{Z_{23}Z_{31} + Z_{12}Z_{23} - Z_{31}Z_{12} - Z_{23}Z_{31} + Z_{12}Z_{23} + Z_{12}Z_{31}}{Z_{12} + Z_{23} + Z_{31}}$$

$$\Rightarrow Z_2 = \frac{Z_{23}Z_{12}}{Z_{12} + Z_{23} + Z_{31}} \tag{5}$$

$$= \frac{Product\ of\ impedances\ connected\ to\ terminal 2\ of\ delta\ connection}{Sum\ of\ impedances\ of\ delta\ connection}$$

- Again, [{$equation\ (3) -\ equation\ (1)\} + equation\ (2)$], gives the value of Z_3 as follows.

$$2Z_3 = \frac{Z_{31}Z_{12} + Z_{23}Z_{31} - Z_{12}Z_{23} - Z_{31}Z_{12} + Z_{23}Z_{31} + Z_{12}Z_{23}}{Z_{12} + Z_{23} + Z_{31}}$$

$$\Rightarrow Z_3 = \frac{Z_{31}Z_{23}}{Z_{12} + Z_{23} + Z_{31}} \tag{6}$$

$$= \frac{Product\ of\ impedances\ connected\ to\ terminal 3\ of\ delta\ connection}{Sum\ of\ impedances\ of\ delta\ connection}$$

From the above, the conversion for balanced load can be found as follows,

$Z_Y = \frac{Z_\Delta}{3}$, Where, Z_Y and Z_Δ are the load in each phase of star and delta connection respectively.

- **Note: In the above formulas, all impedances are in complex forms.**

(ii) Star/Delta Conversion:

Rearranging equation (4), we have,

$$Z_1 = \frac{Z_{12}Z_{31}}{Z_{12} + Z_{23} + Z_{31}}$$

$$\Rightarrow Z_{12}Z_{31} = Z_1(Z_{12} + Z_{23} + Z_{31}) \tag{7}$$

Rearrangingequation (5), *we have,*

$$Z_2 = \frac{Z_{23}Z_{12}}{Z_{12}+Z_{23}+Z_{31}}$$

$$\Rightarrow Z_{23}Z_{12} = Z_2(Z_{12}+Z_{23}+Z_{31}) \qquad (8)$$

Rearranging equation (6), *we have,*

$$\Rightarrow Z_3 = \frac{Z_{31}Z_{23}}{Z_{12}+Z_{23}+Z_{31}}$$

$$\Rightarrow Z_{31}Z_{23} = Z_3(Z_{12}+Z_{23}+Z_{31}) \qquad (9)$$

equation (7) / *equation* (8) *gives* :

$$\frac{Z_{31}}{Z_{23}} = \frac{Z_1}{Z_2} \qquad (10)$$

equation (8) / *equation* (9) *gives* :

$$\frac{Z_{12}}{Z_{31}} = \frac{Z_2}{Z_3} \qquad (11)$$

equation (9) / *equation* (7) *gives* :

$$\frac{Z_{23}}{Z_{12}} = \frac{Z_3}{Z_1} \qquad (12)$$

Equation(7)is

$$Z_{12}Z_{31} = Z_1(Z_{12}+Z_{23}+Z_{31})$$

Putting the values of Z_{12} *and* Z_{23} *from equation*(11) *and equation*(10) respectively,

$$\frac{Z_2}{Z_3}\times Z_{31}\times Z_{31} = Z_1\left(\frac{Z_2}{Z_3}\times Z_{31}+\frac{Z_2}{Z_1}\times Z_{31}+Z_{31}\right)$$

$$\Rightarrow Z_{31} = Z_1+Z_3+\frac{Z_3Z_1}{Z_2}$$

$$\therefore Z_{31} = Z_3+Z_1+\frac{Z_3Z_1}{Z_2} = \frac{Z_1Z_2+Z_2Z_3+Z_3Z_1}{Z_2}$$

Equation(7)is

$$Z_{12}Z_{31} = Z_1(Z_{12}+Z_{23}+Z_{31})$$

Putting the values of Z_{31} *and* Z_{12} *from equation*(10) *and equation*(12) respectively,

$$\frac{Z_1}{Z_3}\times Z_{23}\times\frac{Z_1}{Z_2}\times Z_{23} = Z_1\left(\frac{Z_1}{Z_3}\times Z_{23}+Z_{23}+\frac{Z_1}{Z_2}\times Z_{23}\right)$$

$$\Rightarrow Z_{23} = Z_2+\frac{Z_2Z_3}{Z_1}+Z_3$$

$$\therefore Z_{23} = Z_2+Z_3+\frac{Z_2Z_3}{Z_1} = \frac{Z_1Z_2+Z_2Z_3+Z_3Z_1}{Z_1}$$

Equation(7)is

$$Z_{12}Z_{31} = Z_1(Z_{12}+Z_{23}+Z_{31})$$

Putting the values of Z_{23} *and* Z_{31} *from equation*(12) *and equation*(11) respectively,

$$Z_{12}\times\frac{Z_3}{Z_2}\times Z_{12} = Z_1\left(Z_{12}+\frac{Z_3}{Z_1}\times Z_{12}+\frac{Z_3}{Z_2}\times Z_{12}\right)$$

$$\Rightarrow Z_{12} = \frac{Z_1Z_2}{Z_3}+Z_2+Z_1$$

$$\therefore Z_{12} = Z_1+Z_2+\frac{Z_1Z_2}{Z_3} = \frac{Z_1Z_2+Z_2Z_3+Z_3Z_1}{Z_3}$$

Hence,

$$Z_{12} = Z_1+Z_2+\frac{Z_1Z_2}{Z_3} = \frac{Z_1Z_2+Z_2Z_3+Z_3Z_1}{Z_3}$$

$$Z_{23} = Z_2 + Z_3 + \frac{Z_2 Z_3}{Z_1} = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_1}$$
$$Z_{31} = Z_3 + Z_1 + \frac{Z_3 Z_1}{Z_2} = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_2}$$

From the above, the conversion for balanced load can be found as follows,

$Z_\Delta = 3Z_Y$ Where, Z_Δ and Z_Y are the load in each phase of delta and star connection respectively.

Note: In the above formulas, all impedances are in complex forms.

- Hence, a star connected system can be replaced by an equivalent delta connected system and vice versa using the conversion formula. The corresponding line voltages and line currents in the two equivalent system remain same.

Problem:

An unbalanced star-connected load has branch impedances of $Z_1 = 10\angle 30^\circ, Z_2 = 10\angle -45^\circ, Z_3 = 20\angle 60^\circ$ and is connected across a balanced 3-phase, 3-wire supply of 200 V. Determine the branch impedances of equivalent delta-connected load using Y/Δ conversion method. Also determine the line currents and voltages across each impedance of star-connected load using Y/Δ conversion method.

Solution:

Using Y/Δ conversion, the impedances of each branch of Δ connected load are found as follows.

$$Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1 = (10\angle 30^\circ)(10\angle -45^\circ) + (10\angle -45^\circ)(20\angle 60^\circ) + (20\angle 60^\circ)(10\angle 30^\circ)$$
$$\Rightarrow Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1 = (100\angle -15^\circ) + (200\angle 15^\circ) + (200\angle 90^\circ)$$
$$\Rightarrow Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1 = (96.6 - j25.88) + (193.18 + j51.76) + (0 + j200)$$
$$\Rightarrow Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1 = 290 + j226 = 368\angle 38^\circ$$

$$Z_{12} = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_3} = \frac{368\angle 38^\circ}{(20\angle 60^\circ)} = 18.4\angle -22^\circ = 17 - j7$$

$$Z_{23} = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_1} = \frac{368\angle 38^\circ}{(10\angle 30^\circ)} = 36.8\angle 8^\circ = 36 + j5$$

$$Z_{31} == \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_2} = \frac{368\angle 38^\circ}{10\angle -45^\circ} = 36.8\angle 83^\circ = 4.5 + j36.5$$

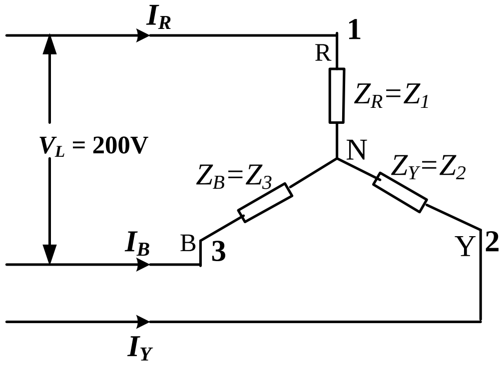
Let, $\boldsymbol{V_{RY}}, \boldsymbol{V_{YB}}$ and $\boldsymbol{V_{BR}}$ are the three line voltages of Δ connected load.
For equivalency, the line voltage apearing in Y connection will also appear as line voltages in Δ connection.

In Δ connection, line voltages and phase voltages are same and therefore,
$\boldsymbol{V_{RY}}, \boldsymbol{V_{YB}}$ and $\boldsymbol{V_{BR}}$ are also the three phase voltages of Δ connected load.
In view of the above fact, the three phase voltages of Δ connected load can be wriiten as follows.

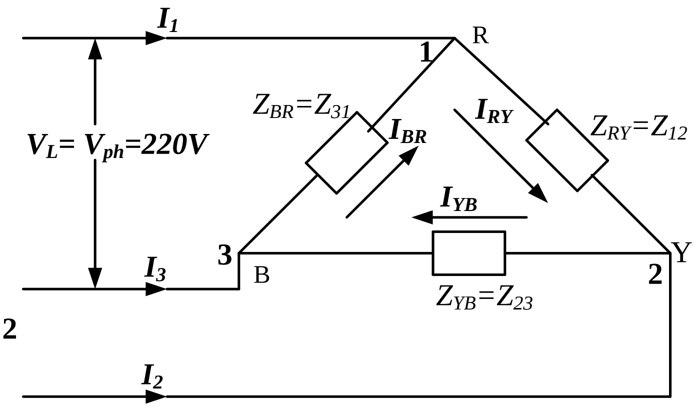
$$\boldsymbol{V_{RY}} = V_{ph}\angle 0^\circ = 200\angle 0^\circ, \boldsymbol{V_{YB}} = V_{ph}\angle -120^\circ = 200\angle -120^\circ \text{ and } \boldsymbol{V_{BR}} = V_{ph}\angle 120^\circ = 200\angle 120^\circ$$

Now the phase currents of Δ connections are determined as follows.

$$\boldsymbol{I_{RY}} = \frac{\boldsymbol{V_{RY}}}{Z_{RY}} = \frac{\boldsymbol{V_{RY}}}{Z_{12}} = \frac{200\angle 0^\circ}{18.4\angle -22^\circ} = 10.87\angle 22^\circ = (10 + j4)\,A$$



(a) Star connection



(b) Delta Connection

$$\boldsymbol{I_{YB}} = \frac{\boldsymbol{V_{YB}}}{Z_{YB}} = \frac{\boldsymbol{V_{YB}}}{Z_{23}} = \frac{200\angle -120^\circ}{36.8\angle 8^\circ} = 5.43\angle -128^\circ = (-3.34 - j4.28)\ A$$

$$\boldsymbol{I_{BR}} = \frac{\boldsymbol{V_{BR}}}{Z_{BR}} = \frac{\boldsymbol{V_{BR}}}{Z_{31}} = \frac{200\angle 120^\circ}{36.8\angle 83^\circ} = 5.43\angle 37^\circ = (4.33 + j3.27)\ A$$

The three line currents $\boldsymbol{I_1}, \boldsymbol{I_2}$ *and* $\boldsymbol{I_3}$ *in* Δ *connection are found as follows.*

Applying KCL at point1 of Δ *connection,*

$$\boldsymbol{I_1} + \boldsymbol{I_{BR}} - \boldsymbol{I_{RY}} = 0$$

$$\Rightarrow \boldsymbol{I_1} = \boldsymbol{I_{RY}} - \boldsymbol{I_{BR}} = (10 + j4) - (4.33 + j3.27) = 5.67 + j0.73 = 5.72\angle 7.33^\circ$$

Applying KCL at point2 of Δ *connection,*

$$\boldsymbol{I}_2 + \boldsymbol{I_{RY}} - \boldsymbol{I_{YB}} = 0$$

$$\Rightarrow \boldsymbol{I}_2 = \boldsymbol{I_{YB}} - \boldsymbol{I_{RY}} = (-3.34 - j4.28) - (10 + j4) = -13.34 - j8.28 = 15.7\angle -148.2^\circ$$

Applying KCL at point3 of Δ *connection,*

$$\boldsymbol{I}_3 + \boldsymbol{I_{YB}} - \boldsymbol{I_{BR}} = 0$$

$$\Rightarrow \boldsymbol{I}_3 = \boldsymbol{I_{BR}} - \boldsymbol{I_{YB}} = (4.33 + j3.27) - (-3.34 - j4.28) = 7.67 + j7.55 = 10.76\angle 44.54$$

The three line currents $\boldsymbol{I_1}, \boldsymbol{I_2}$ *and* $\boldsymbol{I_3}$ *in* Δ *connection are also the three line currents in Y connection because both the systems are equivalent.*

But, in Y connection line currents and phase currents are same.

Hence, $\boldsymbol{I_1}, \boldsymbol{I_2}$ *and* $\boldsymbol{I_3}$ *are the three phase currents in R, Y and B phase of Y connection.*

$\therefore \boldsymbol{I_1} = \boldsymbol{I_R}, \boldsymbol{I_2} = \boldsymbol{I_Y}$ *and* $\boldsymbol{I_3} = \boldsymbol{I_B}$ *in Y connected load.*

Phase voltages of Y connection are found as follows.

$$\boldsymbol{V_{RN}} = \boldsymbol{I_R} Z_R = \boldsymbol{I_1} Z_1 = (5.72\angle 7.33^\circ)(10\angle 30^\circ) = 57.2\angle 37.33^\circ = 45.48 + j34.68$$

$$\boldsymbol{V_{YN}} = \boldsymbol{I_Y} Z_Y = \boldsymbol{I_2} Z_2 = (15.7\angle -148.2^\circ)(10\angle -45^\circ) = 157\angle -193.2^\circ = -152.85 + j35.85$$

$$\boldsymbol{V_{BN}} = \boldsymbol{I_B} Z_B = \boldsymbol{I_3} Z_3 = (10.76\angle 44.54)(20\angle 60^\circ) = 215.2\angle 104.54^\circ = -54 + j208.3$$

The correctness of the above results can be checked by calculating the line voltages in Y connection as follows:

Voltage between line1 and line2, $\boldsymbol{V_{RY}} = \boldsymbol{V_{RN}} - \boldsymbol{V_{YN}} = (45.48 + j34.68) - (-152.85 + j35.85)$

$$= 198.33 - j1.17 = 200\angle 0^\circ$$

Voltage between line2 and line3, $\boldsymbol{V_{YB}} = \boldsymbol{V_{YN}} - \boldsymbol{V_{BN}} = (-152.85 + j35.85) - (-54 + j208.3)$

$$= -98.85 - j172.45 = 200\angle -120^\circ$$

Voltage between line3 and line1, $\boldsymbol{V_{BR}} = \boldsymbol{V_{BN}} - \boldsymbol{V_{RN}} = (-54 + j208.3) - (45.48 + j34.68)$

$$= -99.48 + j173.62 = 200\angle 120^\circ$$

It is found that, the supply voltages are balanced with a magnitude of 200 V as mentioned in the question. Hence, the solution is correct.

References:

1. Edward Hughes, “Electrical and Electronic Technology”, Pearson.
2. Giorgio Rizzoni, “Principles and Applications of Electrical Engineering”, McGraw Hill.
3. B.L. Theraja & A.K. Theraja, “A Text Book of Electrical Technology”, S. Chand.