COMPUTER SECURITY CONCEPTS

Computer Security

The protection afforded to an automated information system in order to attain the applicable objectives of preserving the integrity, availability, and confidentiality of information system resources (includes hardware, software, firmware, information / data, and telecommunications)

Confidentiality

- Data confidentiality
  - Assures that private or confidential information is not made available or disclosed to unauthorized
- Privacy
  - Assures that individuals control or influence what information related to them may be collected and stored and by whom and to whom that information may be disclosed.

Integrity

- Data integrity
  - Assures that information and programs are changed only in a specified and authorized manner.
- System integrity
  - Assures that a system performs its intended function in an unimpaired manner, free from deliberate or inadvertent unauthorized manipulation of the system.

Availability

- Assures that systems work promptly and service is not denied to authorized users.

CIA Triad

Confidentiality

- Preserving authorized restrictions on information access and disclosure, including means for protecting personal privacy and proprietary information.
- A loss of confidentiality is the unauthorized disclosure of information.

Integrity

- Guarding against improper information modification or destruction, including ensuring information nonrepudiation and authenticity.
- A loss of integrity is the unauthorized modification or destruction of information.

Availability

- Ensuring timely and reliable access to and use of information
- A loss of availability is the disruption of access to or use of information or an information system.

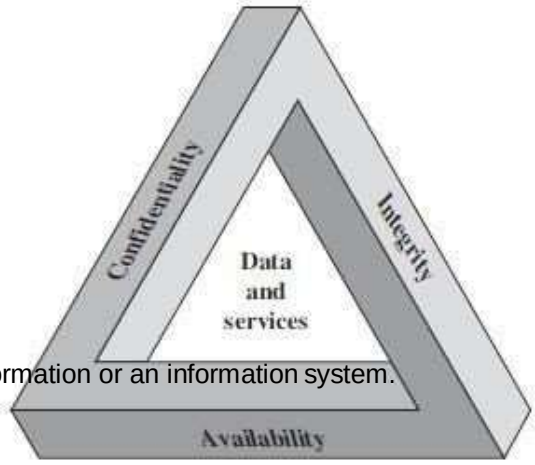


Figure 1.1 The Security Requirements Triad

Authenticity

- The property of being genuine and being able to be verified and trusted

Accountability

- The security goal that generates the requirement for actions of an entity to be traced uniquely to that entity

The OSI Security Architecture

- ITU-T Recommendation X.800, Security Architecture for OSI, defines such a systematic approach
- The OSI security architecture focuses on security attacks, mechanisms, and services.

Security attack

- Any action that compromises the security of information owned by an organization.

Security mechanism

- A process (or a device) that is designed to detect, prevent, or recover from a security attack.

Security service

- A processing or communication service that enhances the security of the data processing systems and the information transfers of an organization
- The services are intended to counter security attacks, and they make use of one or more security mechanisms to provide the service

Security Attacks

- means of classifying security attacks, used both in X.800 and RFC 2828
- A passive attack attempts to learn or make use of information but does not affect system resources.
- An active attack attempts to alter system resources or affect their operation.

Passive Attacks

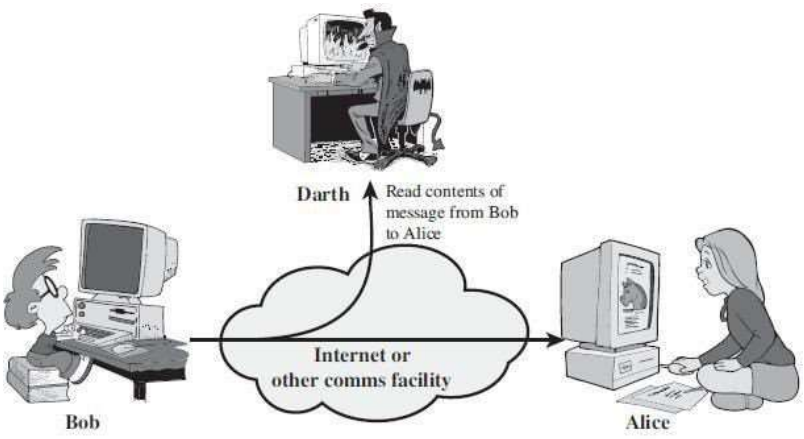
- in the nature of eavesdropping on, or monitoring of, transmissions.
- The goal is to obtain information that is being transmitted.
- very difficult to detect, because they do not involve any alteration of the data
- feasible to prevent the success of these attacks, usually by means of encryption
- emphasis in dealing with passive attacks is on prevention rather than detection

Two types of passive attacks

- Release of message contents
- Traffic analysis.

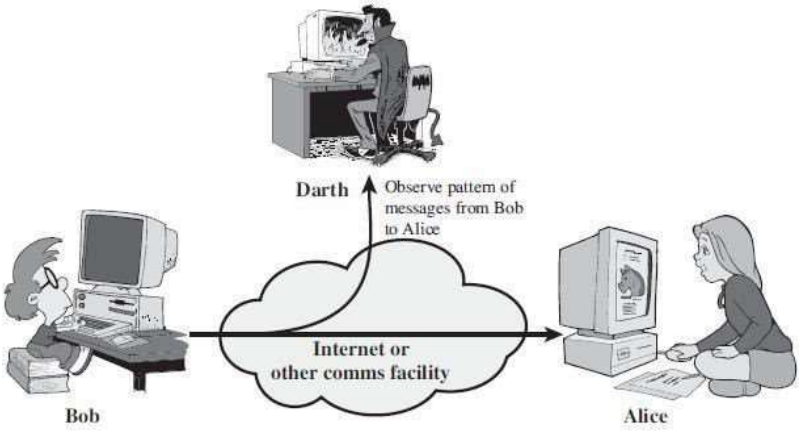
Release Of Message Contents

- A telephone conversation, an electronic mail message, and a transferred file may contain sensitive or confidential information
- prevent an opponent from learning the contents of these transmissions



Traffic Analysis

- observe the pattern of these messages
- The opponent could determine the location and identity of communicating hosts and could observe the frequency and length of messages being exchanged.
- This information might be useful in guessing the nature of the communication that was taking place



Active Attacks

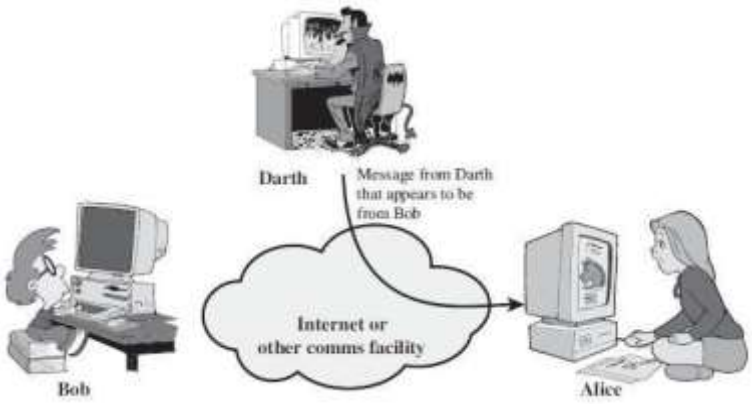
- Active attacks involve some modification of the data stream or the creation of a false stream
- detect and to recover from any disruption or delays caused by them
- can be subdivided into four categories:
  - masquerade,
  - replay,
  - modification of messages
  - denial of service

Masquerade

- one entity pretends to be a different entity
- usually includes one of the other forms of active attack

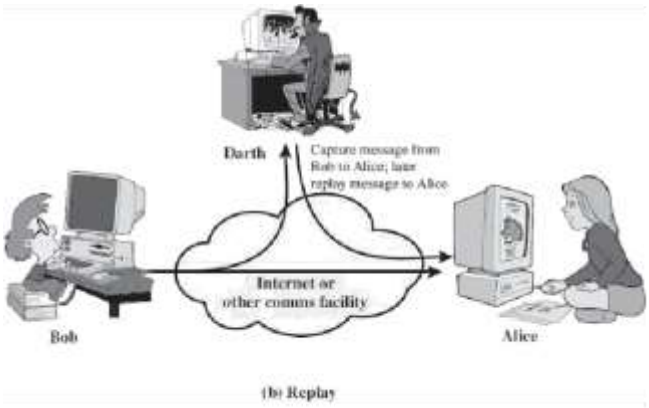
Example

- authentication sequences can be captured and replayed after a valid authentication sequence

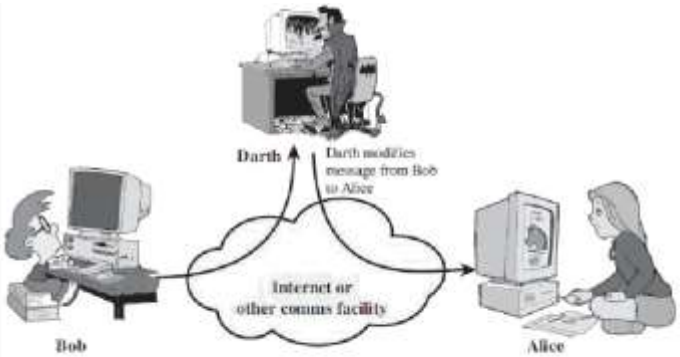


Replay

- passive capture of a data unit and its subsequent retransmission to produce an unauthorized effect



(b) Replay



(c) Modification of messages

Modification Of Messages

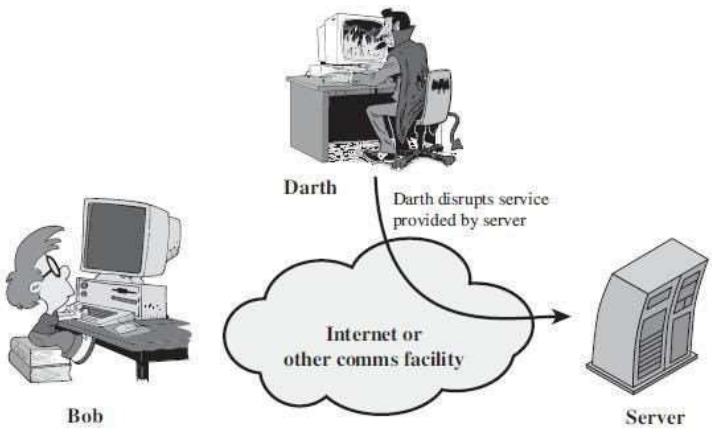
- some portion of a legitimate message is altered, or that messages are delayed or reordered, to produce an unauthorized effect

Example

- a message meaning “Allow John Smith to read confidential file accounts” is modified to mean “Allow Fred Brown to read confidential file accounts.”

Denial Of Service

- prevents or inhibits the normal use or management of communications facilities
- may have a specific target; for example, an entity may suppress all messages directed to a particular destination
- disruption of an entire network, either by disabling the network or by overloading it with messages so as to degrade performance



(d) Denial of service

Security Services in X.800

- X.800 defines a security service as a service that is provided by a protocol layer of communicating open systems and that ensures adequate security of the systems or of data transfers.
- RFC 2828, defines as a processing or communication service that is provided by a system to give a specific kind of protection to system resources;
  - security services implement security policies and are implemented by security mechanisms.

X.800

- divides these services into five categories and fourteen specific services

Authentication

- The assurance that the communicating entity is the one that it claims to be
- Two types
  - Peer Entity Authentication
  - Data-Origin Authentication

Access control

- The prevention of unauthorized use of a resource

Data confidentiality

- The protection of data from unauthorized disclosure.
- Four Types
  - Connection Confidentiality
  - Connectionless Confidentiality
  - Selective-Field Confidentiality
  - Traffic-Flow Confidentiality

Data integrity

- The assurance that data received are exactly as sent by an authorized entity (i.e., contain no modification, insertion, deletion, or replay).
- Five types
  - Connection Integrity with Recovery
  - Connection Integrity without Recovery
  - Selective-Field Connection Integrity
  - Connectionless Integrity
  - Selective-Field Connectionless Integrity

Nonrepudiation

- Provides protection against denial by one of the entities involved in a communication of having participated in all or part of the communication
- Two types
  - Nonrepudiation, Origin
  - Nonrepudiation, Destination

Security Mechanisms in X.800.

- feature designed to detect, prevent, or recover from a security attack
- no single mechanism that will support all services required

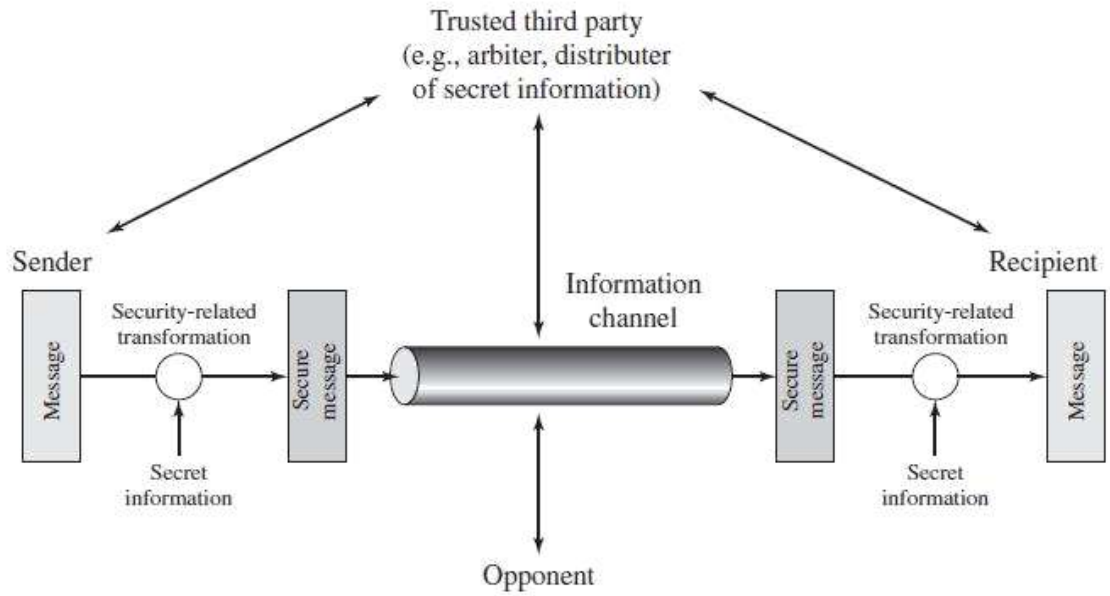
Specific security mechanisms:

- those that are implemented in a specific protocol layer, such as TCP or an application-layer protocol
- encipherment, digital signatures, access controls, data integrity, authentication exchange, traffic padding, routing control, notarization

pervasive security mechanisms:

- trusted functionality, security labels, event detection, security audit trails, security recovery
- those that are not specific to any particular protocol layer or security service

Model for Network Security



- A message is to be transferred from one party to another across some sort of Internet service.
- The two parties, who are the principals in this transaction, must cooperate for the exchange to take place.
- A logical information channel is established by defining a route through the Internet from source to destination and by the cooperative use of communication protocols (e.g., TCP/IP) by the two principals

All the techniques for providing security have two components:

- A security-related transformation on the information to be sent.
  - Examples: encryption of the message, addition of a code based on the contents
- Some secret information shared by the two principals, unknown to the opponent ○ Example: encryption key used in conjunction with the transformation

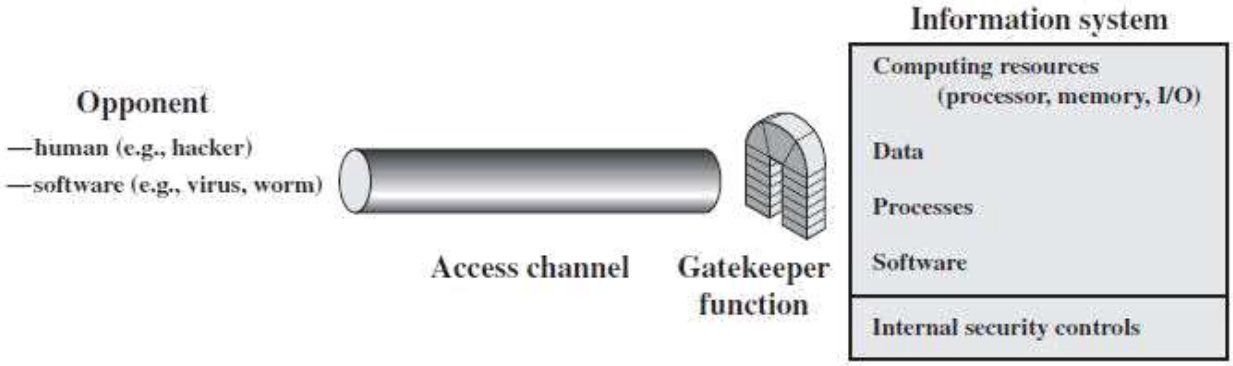
A trusted third party may be needed to achieve secure transmission.

- for distributing the secret information to the two principals
- to arbitrate disputes between the two principals concerning the authenticity of a message transmission

Four basic tasks in designing a particular security service:

1. Design an algorithm for performing the security-related transformation
   - such that an opponent cannot defeat its purpose.
2. Generate the secret information to be used with the algorithm.
3. Develop methods for the distribution and sharing of the secret information.
4. Specify a protocol to be used by the two principals that makes use of the security algorithm and the secret information to achieve a particular security service

Network Access Security Model



- protecting an information system from unwanted access from hacker, intruder
- hacker who, with no malign intent, simply gets satisfaction from breaking and entering a computer system.
- intruder can be a disgruntled employee who wishes to do damage or a criminal who seeks to exploit computer assets for financial gain
- placement in a computer system of logic that exploits vulnerabilities in the system and that can affect application programs as well as utility programs, such as editors and compilers
  - Two kinds of threats:
  - **Information access threats**: Intercept or modify data on behalf of users who should not have access
  - **Service threats**: Exploit service flaws in computers to inhibit use by legitimate users
  - Examples: Viruses and worms, spread using disks & inserted over network

Classical Encryption Techniques

- Symmetric Cipher Model
  - Cryptanalysis and Brute-Force Attack
- Substitution Techniques
  - Caesar Cipher
  - Monoalphabetic Ciphers
  - Playfair Cipher
  - Hill Cipher
  - Polyalphabetic Ciphers
  - One-Time Pad
- Transposition Techniques
- Rotor Machines
- Steganography

Introduction

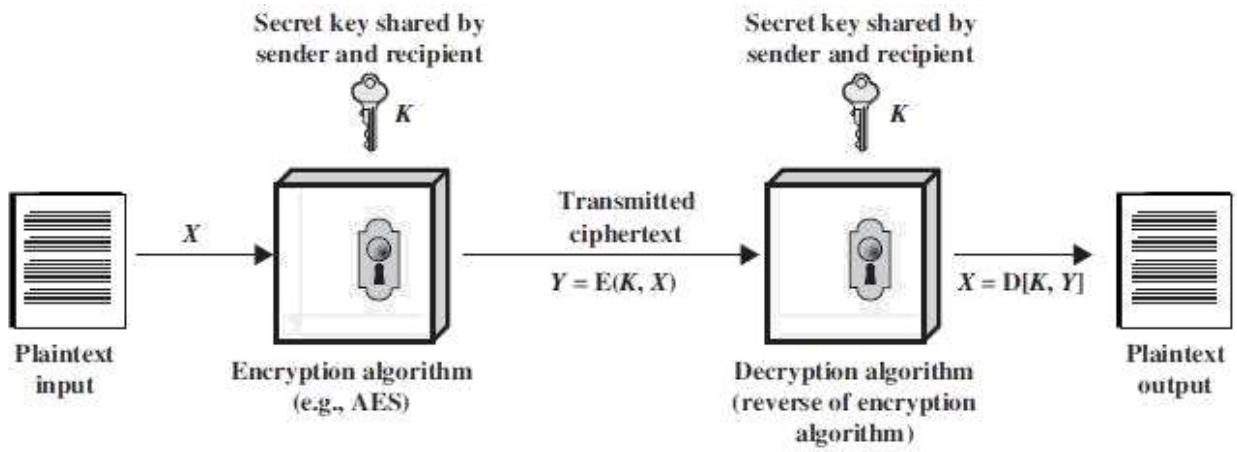
- **Symmetric encryption** is a form of cryptosystem in which encryption and decryption are performed using the **same key**. It is also known as **conventional encryption**.
- Symmetric encryption transforms plaintext into ciphertext using a secret key and an encryption algorithm. Using the same key and a decryption algorithm, the plaintext is recovered from the ciphertext.
- The two types of attack on an encryption algorithm are **cryptanalysis**,based on properties of the encryption algorithm, and **brute-force**, which involves trying all possible keys.
- Traditional (precomputer) symmetric ciphers use substitution and/or transposition techniques. Substitution techniques map plaintext elements (characters, bits) into ciphertext elements. Transposition techniques systematically transpose the positions of plaintext elements.
- Rotor machines are sophisticated precomputer hardware devices that use substitution techniques.
- Steganography is a technique for hiding a secret message within a larger one in such a way that others cannot discern the presence or contents of the hidden message.
- An original message is known as the **plaintext**, while the coded message is called the **ciphertext**.
- The process of converting from plaintext to ciphertext is known as **enciphering or encryption**; restoring the plaintext from the ciphertext is **deciphering or decryption**.
- The many schemes used for encryption constitute the area of study known as **cryptography**. Such a scheme is known as a **cryptographic system or a cipher**.
- Techniques used for deciphering a message without any knowledge of the enciphering details fall into the area of cryptanalysis. **Cryptanalysis** is what the layperson calls “breaking the code.”The areas of cryptography and cryptanalysis together are called **cryptology**

Symmetric Cipher Model

A symmetric encryption scheme has five ingredients

- Plaintext
- Encryption algorithm
  - performs various substitutions and transformations
- Secret key
  - another input to the encryption algorithm
  - a value independent of the plaintext and of the algorithm
- Ciphertext
  - For a given message, two different keys will produce two different ciphertexts
- Decryption algorithm
  - encryption algorithm run in reverse

Simplified Model of Symmetric Encryption



Two requirements for secure use of conventional / symmetric encryption

- need a strong encryption algorithm
  - The opponent should be unable to decrypt ciphertext or discover the key even if he or she is in possession of a number of ciphertexts together with the plaintext that produced each ciphertext
- Sender and receiver must have obtained copies of the secret key in a secure fashion and must keep the key secure.
  - If someone can discover the key and knows the algorithm, all communication using this key is readable
  - do not need to keep the algorithm secret; we need to keep only the key secret
  - the principal security problem is maintaining the secrecy of the key

Model of Symmetric Cryptosystem

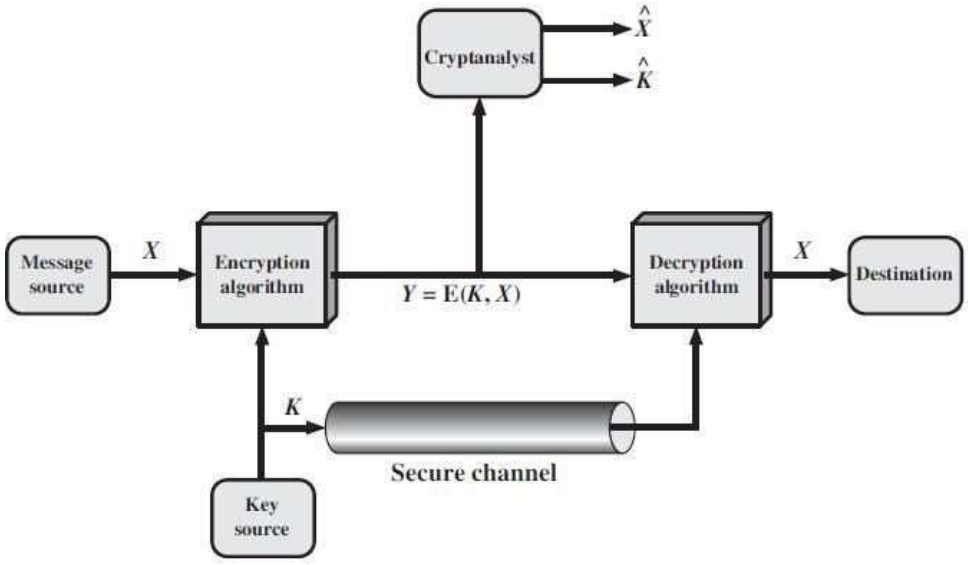
Plain Text: $X = [X_1, X_2, . , X_M]$

Key: $K = [K_1, K_2, . , K_J]$

Cipher text $Y = [Y_1, Y_2, . , Y_N]$

$Y = E(K, X)$

$X = D(K, Y)$



Cryptanalysis and Brute-Force Attack

Cryptanalysis

- Cryptanalytic attacks rely on the nature of the algorithm plus perhaps some knowledge of the general characteristics of the plaintext or even some sample plaintext–ciphertext pairs.
- This type of attack exploits the characteristics of the algorithm to attempt to deduce a specific plaintext or to deduce the key being used.
- various types of cryptanalytic attacks based on the amount of information known to the cryptanalyst

Type of Attack	Known to Cryptanalyst
• Ciphertext Only • Known Plaintext • Chosen Plaintext • Chosen Ciphertext • Chosen Text	• Encryption algorithm • Ciphertext • Plaintext message chosen by cryptanalyst, together with its corresponding ciphertext generated with the secret key • Ciphertext chosen by cryptanalyst, together with its corresponding decrypted plaintext generated with the secret key

Two schemes

- **unconditionally secure**
  - if the ciphertext generated by the scheme does not contain enough information to determine uniquely the corresponding plaintext, no matter how much ciphertext is available
- **computationally secure**
  - meets either of the following criteria:
  - The cost of breaking the cipher exceeds the value of the encrypted information.
  - The time required to break the cipher exceeds the useful lifetime of the information.

Brute-force attack

- The attacker tries every possible key on a piece of ciphertext until an intelligible translation into plaintext is obtained.
- On average, half of all possible keys must be tried to achieve success.

Cryptographic systems characterization

Three independent dimensions

- The type of operations used for transforming plaintext to ciphertext.
  - **substitution**
    - each element is mapped into another element
  - **transposition**
    - elements are rearranged
  - product systems, involve multiple stages of substitutions and transpositions
- The number of keys used
  - If both sender and receiver use the same key, the system is referred to as **symmetric**, single-key, secret-key, or conventional encryption.
  - If the sender and receiver use different keys, the system is referred to as **asymmetric**, two-key, or public-key encryption
- The way in which the plaintext is processed.
  - A **block cipher** processes the input one block of elements at a time, producing an output block for each
  - input block.
  - A **stream cipher** processes the input elements continuously, producing output one element at a time, as it goes along

Substitution Techniques

- A substitution technique is one in which the letters of plaintext are replaced by other letters or by numbers or symbols
- If the plaintext is viewed as a sequence of bits, then substitution involves replacing plaintext bit patterns with ciphertext bit patterns

Julius Caesar Cipher

- replacing each letter of the alphabet with the letter standing three places further down the alphabet
- alphabet is wrapped around, so that the letter following Z is A

can define transformation as:

a b c d e f g h i j k l m n o p q r s t u v w x y z D

E F G H I J K L M N O P Q R S T U V W X Y Z A B C

mathematically give each letter a number

a b c d e f g h i j k l m n o p q r s t u v w x y z

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25

then have Caesar cipher as:

c = E(p) = (p + k) mod (26) p

= D(c) = (c – k) mod (26)

Cryptanalysis of Caesar Cipher

- only have 26 possible ciphers
- A maps to A,B,..Z
- could simply try each in turn
- a brute force search
- given ciphertext, just try all shifts of letters
- do need to recognize when have plaintext

Monoalphabetic Ciphers

- rather than just shifting the alphabet shuffle (jumble) the letters arbitrarily
- each plaintext letter maps to a different random ciphertext letter
- hence key is 26 letters long
- the “cipher” line can be any permutation of the 26 alphabetic characters, then there are 26! or greater than $4x10^{26}$ possible keys.
- This is 10 orders of magnitude greater than the key space for DES and would seem to eliminate brute-force techniques for cryptanalysis
- Monoalphabetic ciphers are easy to break because they reflect the frequency data of the original alphabet
- A countermeasure is to provide multiple substitutes, known as **homophones**, for a single letter.
- For example, the letter e could be assigned a number of different cipher symbols, such as 16, 74, 35, and 21, with each homophone assigned to a letter in rotation or randomly

Language Redundancy and Cryptanalysis

- human languages are redundant
- eg "th lrd s m shphrd shll nt wnt"
- letters are not equally commonly used
- in English E is by far the most common letter
- followed by T,R,N,I,O,A,S
- other letters like Z,J,K,Q,X are fairly rare
- have tables of single, double & triple letter frequencies for various languages
- two-letter combinations, known as **digrams** (ex: th)

Playfair Cipher

- best-known multiple-letter encryption cipher
- treats digrams in the plaintext as single units and translates these units into ciphertext digrams

Playfair Key Matrix

M	O	N	A	R
C	H	Y	B	D
E	F	G	I/J	K
L	P	Q	S	T
U	V	W	X	Z

- 5 × 5 matrix of letters constructed using a keyword
- filling in the letters of the keyword (minus duplicates) from left to right and from top to bottom,
- filling in the remainder matrix with the remaining letters in alphabetic order.
- The letters I and J count as one letter
- Example matrix using the keyword MONARCHY

Plaintext is encrypted two letters at a time, according to the following rules

- Repeating plaintext letters that are in the same pair are separated with a filler letter, such as x,
  - Ex: balloon would be treated as ba lx lo on.
- Two plaintext letters that fall in the same row of the matrix are each replaced by the letter to the right, with the first element of the row circularly following the last.
  - Ex: ar is encrypted as RM.
- Two plaintext letters that fall in the same column are each replaced by the letter beneath, with the top element of the column circularly following the last.
  - Ex: mu is encrypted as CM.
- Otherwise, each plaintext letter in a pair is replaced by the letter that lies in its own row and the column occupied by the other plaintext letter.
  - Ex: hs becomes BP and ea becomes IM (or JM, as the encipherer wishes)

Example

Given the key MONARCHY apply Play fair cipher to plain text “FACTIONALISM”

Solution

(p) FA CT IO NA LI SM

(c) IO DL FA AR SE LA

(d) FA CT IO NA LI SM

Security of Playfair Cipher

- security much improved over monoalphabetic since have 26 x 26 = 676 digrams
- would need a 676 entry frequency table to analyse and correspondingly more ciphertext
- was widely used for many years eg. by US & British military in WW1
- it can be broken, given a few hundred letters since still has much of plaintext structure

Hill Cipher

Findin[illegible]

$$\text{3x3 Matrix} \begin{pmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{pmatrix}$$

$$|A| = k_{11}k_{22}k_{33} - k_{11}k_{23}k_{32} - k_{12}k_{21}k_{33} + k_{12}k_{23}k_{31} + k_{13}k_{21}k_{32} - k_{13}k_{22}k_{31}$$

Example:

$$\det\begin{pmatrix}5 & 8\\ 17 & 3\end{pmatrix} = (5\times 3) - (8\times 17) = -121 \bmod 26 = 9$$

We can show that $9^{-1} \bmod 26 = 3$, because $9 \times 3 = 27 \bmod 26 = 1$ (see Chapter 4 or Appendix E). Therefore, we compute the inverse of **A** as

$$\mathbf{A} = \begin{pmatrix}5 & 8\\ 17 & 3\end{pmatrix}$$

$$\mathbf{A}^{-1} \bmod 26 = 3\begin{pmatrix}3 & -8\\ -17 & 5\end{pmatrix} = 3\begin{pmatrix}3 & 18\\ 9 & 5\end{pmatrix} = \begin{pmatrix}9 & 54\\ 27 & 15\end{pmatrix} = \begin{pmatrix}9 & 2\\ 1 & 15\end{pmatrix}$$

The Hill algorithm

- This encryption algorithm takes m successive plaintext letters and substitutes for them m ciphertextletters.
- The substitution is determined by m linear equations in which each character is assigned a numerical value (a = 0, b = 1, . , z = 25)
- For m = 3, the system can be described as

$$c_1 = (k_{11}p_1 + k_{12}p_2 + k_{13}p_3) \bmod 26$$
$$c_2 = (k_{21}p_1 + k_{22}p_2 + k_{23}p_3) \bmod 26$$
$$c_3 = (k_{31}p_1 + k_{32}p_2 + k_{33}p_3) \bmod 26$$

This can be expressed in terms of row vectors and matrices:

$$(c_1\ c_2\ c_3) = (p\ \ p_2\ p_3)\begin{pmatrix}k_{11} & k_{12} & k_{13}\\ k_{21} & k_{22} & k_{23}\\ k_{31} & k_{32} & k_{33}\end{pmatrix} \bmod 26$$

or

$$\mathbf{C} = \mathbf{PK} \bmod 26$$

- where C and P are row vectors of length 3 representing the plaintext and ciphertext, and K is a 3x3 matrix representing the encryption key.
- Operations are performed mod 26.
- In general terms, the Hill system can be expressed as

$$\mathbf{C} = E(\mathbf{K}, \mathbf{P}) = \mathbf{PK} \bmod 26$$
$$\mathbf{P} = D(\mathbf{K}, \mathbf{C}) = \mathbf{CK}^{-1} \bmod 26 = \mathbf{PKK}^{-1} = \mathbf{P}$$

Example

Encrypt the message “meet me at the usual place at ten rather than eight oclock” using the Hill cipher with the key ( ). Show your calculations and the result.Show the calculations for the corresponding decryption of the ciphertext to recover the original plaintext.

1) mathematically give each letter a number

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

2) 1st pair from plain text “me” => $\begin{pmatrix}12\\4\end{pmatrix}$

$$\begin{pmatrix}9 & 4\\ 5 & 7\end{pmatrix}\begin{pmatrix}12\\4\end{pmatrix} => \begin{pmatrix}9x12+4x4\\ 5x12+7x4\end{pmatrix} = \begin{pmatrix}124\\88\end{pmatrix} => mod\ 26 => \begin{pmatrix}20\\10\end{pmatrix} => \begin{pmatrix}u\\k\end{pmatrix}$$

3) 2nd pair fro plain text “et”

$$\begin{pmatrix}9 & 4\\ 5 & 7\end{pmatrix}\begin{pmatrix}4\\19\end{pmatrix} => \begin{pmatrix}9x4+4x19\\ 5x4+7x19\end{pmatrix} = \begin{pmatrix}112\\153\end{pmatrix} => mod\ 26 => \begin{pmatrix}8\\23\end{pmatrix} => \begin{pmatrix}i\\x\end{pmatrix}$$

For example, to encipher the message "meet me after the toga party" with a rail fence of depth 2, we write the following

```
m e m a t r h t g p r y

 e t e f e t e o a a t
```

The encrypted message is

```
MEMATRHTGPRYETEFETEOAAT
```

Pure Transposition Cipher

write the message in a rectangle, row by row, and read the message off, column by column, but permute the order of the columns.

The order of the columns then becomes the key to the algorithm

Example

```
Key:        4 3 1 2 5 6 7

Plaintext: a t t a c k  p
           o s t p o n e
           d u n t i l t
           w o a m x y z

Ciphertext: TTNAAPTMTSUOAODWCOIXKNLYPETZ
```

Double Transposition

performing more than one stage of transposition

Example

if the foregoing message is reencrypted using the same algorithm

```
Key:       4 3 1 2 5 6 7

Input:     t t n a a p t
           m t s u o a o
           d w c o i x k
           n l y p e t z

Output: NSCYAUOPTTWLTMDNAOIEPAXTTOKZ
```

This is a much less structured permutation and is much more difficult to cryptanalyze

Rotor Machines (Skip)

The machine consists of a set of independently rotating cylinders through which electrical pulses can flow.

Each cylinder has 26 input pins and 26 output pins, with internal wiring that connects each input pin to a unique output pin

Steganography

A plaintext message may be hidden in one of two ways.

- The methods of steganography conceal the existence of the message
- The methods of cryptography render the message unintelligible to outsiders ○ by various transformations of the text

Various ways to conceal the message

arrangement of words or letters within an apparently innocuous text spells out the real message

Character marking

Selected letters of printed or typewritten text are overwritten in pencil. The marks are ordinarily not visible unless the paper is held at an angle to bright light.

Invisible ink

A number of substances can be used for writing but leave no visible trace until heat or some chemical is applied

Pin punctures

Small pin punctures on selected letters are ordinarily not visible unless the paper is held up in front of a light.

Typewriter correction ribbon

Used between lines typed with a black ribbon, the results of typing with the correction tape are visible only under a strong light

hiding a message by using the least significant bits of frames on a CD

- the Kodak Photo CD format's maximum resolution is 2048 by 3072 pixels, with each pixel containing 24 bits of RGB color information.
- The least significant bit of each 24-bit pixel can be changed without greatly affecting the quality of the image
- Thus you can hide a 2.3-megabyte message in a single digital snapshot

Number of drawbacks

- lot of overhead to hide a relatively few bits of information
- once the system is discovered, it becomes virtually worthless
- the insertion method depends on some sort of key
    - Alternatively, a message can be first encrypted and then hidden using steganography

Advantage of steganography

- can be employed by parties who have something to lose should the fact of their secret communication be discovered
- Encryption flags traffic as important or secret or may identify the sender or receiver as someone with something to hide

FINITE FIELDS AND NUMBER THEORY

Number Theory concepts

Divisibility and The Division Algorithm

Divisibility

- We say that a nonzero b **divides** a if a = mb for some m, where a, b and m are integers.
- That is, b divides a, if there is no remainder on division.
- The notation b|a is commonly used to mean b divides a.
- If b|a, we say that b is a **divisor** of a.

The positive divisors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24.
$13|182;\ -5|30;\ 17|289;\ -3|33;\ 17|0$

Properties of divisibility for integers

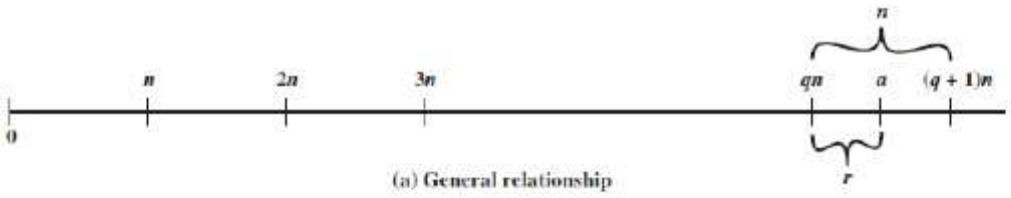
- If $a|1$, then $a = \pm 1$.
- If $a|b$ and $b|a$, then $a = \pm b$.
- Any $b \neq 0$ divides 0.
- If $a|b$ and $b|c$, then $a|c$: $11|66$ and $66|198 = 11|198$
- If $b|g$ and $b|h$, then $b|(mg + nh)$ for arbitrary integers m and n.

The Division Algorithm

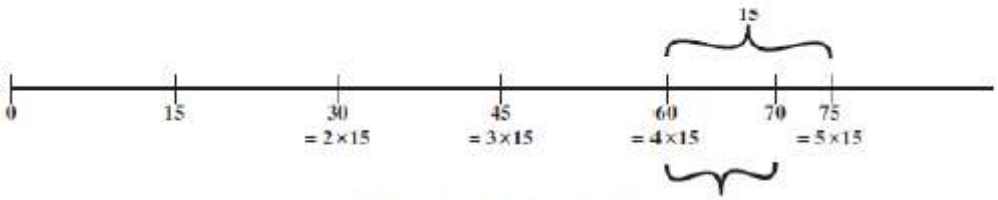
Given any positive integer n and any nonnegative integer a, if we divide a by n, we get an integer quotient an integer remainder r that obey the following relationship: The remainder r is often referred to as a residu

$$a = qn + r \qquad 0 \leq r < n; q = \lfloor a/n \rfloor$$

where $\lfloor x \rfloor$ is the largest integer less than or equal to x.



(a) General relationship



(b) Example: 70 = (4×15) + 10

The Euclidean Algorithm

- Simple procedure for determining the greatest common divisor of two positive integers.
- Two integers are relatively prime if their only common positive integer factor is 1

Greatest Common Divisor

- largest integer that divides both a and b
- gcd(0, 0) = 0.

the positive integer c is said to be the greatest common divisor of a and b if

1. c is a divisor of a and of b
2. Any divisor of a and b is a divisor of c

$$\gcd(a, b) = \max[k, \text{such that } k|a \text{ and } k|b] \quad \gcd(a, b) = \gcd(|a|, |b|).$$

$$\gcd(60, 24) = \gcd(60, -24) = 12$$

- a and b are relatively prime if gcd(a, b) = 1

Example:

8 and 15 are relatively prime because the positive divisors of 8 are 1, 2, 4, and 8, and the positive divisors of are 1, 3, 5, and 15. So 1 is the only integer on both lists.

Steps

$$\begin{aligned}
a &= q_1 b + r_1 && 0 < r_1 < b \\
b &= q_2 r_1 + r_2 && 0 < r_2 < r_1 \\
r_1 &= q_3 r_2 + r_3 && 0 < r_3 < r_2 \\
&\vdots && \vdots \\
r_{n-2} &= q_n r_{n-1} + r_n && 0 < r_n < r_{n-1} \\
r_{n-1} &= q_{n+1} r_n + 0 \\
d &= \gcd(a, b) = r_n
\end{aligned}$$

Example

To find $d = \gcd(a,b) = \gcd(1160718174, 316258250)$		
$a = q_1 b + r_1$	$1160718174 = 3 \times 316258250 + 211943424$	$d = \gcd(316258250, 211943424)$
$b = q_2 r_1 + r_2$	$316258250 = 1 \times 211943424 + 104314826$	$d = \gcd(211943424, 104314826)$
$r_1 = q_3 r_2 + r_3$	$211943424 = 2 \times 104314826 + 3313772$	$d = \gcd(104314826, 3313772)$

Dividend	Divisor	Quotient	Remainder
$a = 1160718174$	$b = 316258250$	$q_1 = 3$	$r_1 = 211943424$
$b = 316258250$	$r_1 = 211943434$	$q_2 = 1$	$r_2 = 104314826$
$r_1 = 211943424$	$r_2 = 104314826$	$q_3 = 2$	$r_3 = 3313772$

Euclidean Algorithm Revisited

- For any nonnegative integer a and any positive integer b,
- gcd(a, b) = gcd (b, a mod b)
  - Example: gcd(55, 22) = gcd(22, 55 mod 22) = gcd(22, 11) = 11

Recursive function.

```
Euclid(a,b)
      if (b=0) then return a;
      else return Euclid(b, a mod b);
```

The Extended Euclidean Algorithm

calculate the greatest common divisor but also two additional integers and that satisfy the following equation

$$ax + by = d = \gcd(a, b)$$

x and y will have opposite signs

(4.3), and we assume that at each step i we can find integers x_i and y_i that satisfy $r_i = ax_i + by_i$. We end up with the following sequence.

$$\begin{aligned}
a &= q_1 b + r_1 & r_1 &= ax_1 + by_1 \\
b &= q_2 r_1 + r_2 & r_2 &= ax_2 + by_2 \\
r_1 &= q_3 r_2 + r_3 & r_3 &= ax_3 + by_3 \\
&\cdots & &\cdots \\
r_{n-2} &= q_n r_{n-1} + r_n & r_n &= ax_n + by_n \\
r_{n-1} &= q_{n+1} r_n + 0
\end{aligned}$$

we can rearrange terms to write

$$r_i = r_{i-2} - r_{i-1} q_i$$

Also, in rows $i - 1$ and $i - 2$, we find the values

$$r_{i-2} = ax_{i-2} + by_{i-2} \quad \text{and} \quad r_{i-1} = ax_{i-1} + by_{i-1}$$

Substituting into Equation (4.8), we have

$$\begin{aligned}
r_i &= (ax_{i-2} + by_{i-2}) - (ax_{i-1} + by_{i-1})q_i \\
&= a(x_{i-2} - q_i x_{i-1}) + b(y_{i-2} - q_i y_{i-1})
\end{aligned}$$

But we have already assumed that $r_i = ax_i + by_i$. Therefore,

$$x_i = x_{i-2} - q_i x_{i-1} \quad \text{and} \quad y_i = y_{i-2} - q_i y_{i-1}$$

Let us now look at an example with relatively large numbers to see the power of this algorithm:

To find $d = \gcd(a,b) = \gcd(1160718174, 316258250)$		
$a = q_1 b + r_1$	$1160718174 = 3 \times 316258250 + 211943424$	$d = \gcd(316258250, 211943424)$
$b = q_2 r_1 + r_2$	$316258250 = 1 \times 211943424 + 104314826$	$d = \gcd(211943424, 104314826)$
$r_1 = q_3 r_2 + r_3$	$211943424 = 2 \times 104314826 + 3313772$	$d = \gcd(104314826, 3313772)$
$r_2 = q_4 r_3 + r_4$	$104314826 = 31 \times 3313772 + 1587894$	$d = \gcd(3313772, 1587894)$
$r_3 = q_5 r_4 + r_5$	$3313772 = 2 \times 1587894 + 137984$	$d = \gcd(1587894, 137984)$
$r_4 = q_6 r_5 + r_6$	$1587894 = 11 \times 137984 + 70070$	$d = \gcd(137984, 70070)$
$r_5 = q_7 r_6 + r_7$	$137984 = 1 \times 70070 + 67914$	$d = \gcd(70070, 67914)$
$r_6 = q_8 r_7 + r_8$	$70070 = 1 \times 67914 + 2156$	$d = \gcd(67914, 2156)$
$r_7 = q_9 r_8 + r_9$	$67914 = 31 \times 2516 + 1078$	$d = \gcd(2156, 1078)$
$r_8 = q_{10} r_9 + r_{10}$	$2156 = 2 \times 1078 + 0$	$d = \gcd(1078, 0) = 1078$
Therefore, $d = \gcd(1160718174, 316258250) = 1078$		

The Extended Euclidean Algorithm

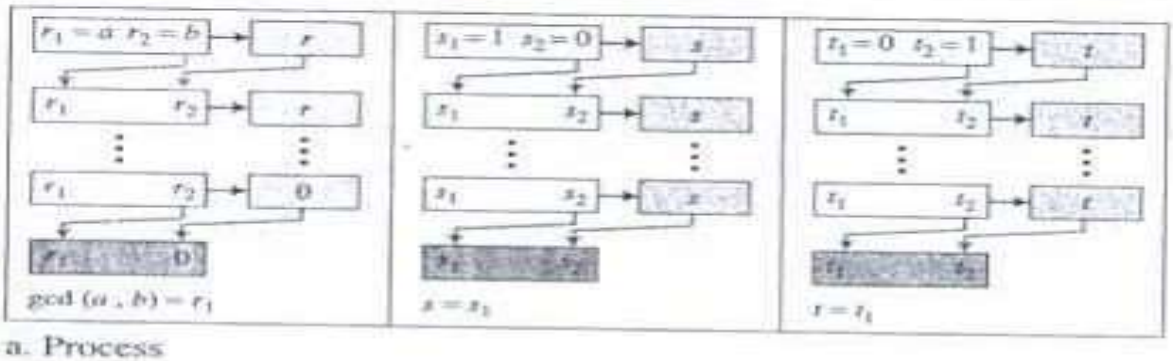
Given two integers a and b, we often need to find other two integers, s and t, such that

$$s \times a + t \times b = \gcd(a, b)$$

The **extended Euclidean algorithm** can calculate the gcd (a, b) and at the same time calculate the value of s and t. The algorithm and the process is shown in Figure 2.8.

As shown in Figure 2.8, the extended Euclidean algorithm uses the same number of steps as the Euclidean algorithm. However, in each step, we use three sets of calculations and exchanges instead of one. The algorithm uses three sets of variables, r's, s's, and t's.

Figure 2.8 *Extended Euclidean algorithm*



a. Process

```
r1 ← a; r2 ← b;
s1 ← 1; s2 ← 0;
t1 ← 0; t2 ← 1;
while (r2 > 0)
{
  q ← r1 / r2;
  r ← r1 - q × r2;
  r1 ← r2; r2 ← r;
  s ← s1 - q × s2;
  s1 ← s2; s2 ← s;
  t ← t1 - q × t2;
  t1 ← t2; t2 ← t;
}
gcd (a , b) ← r1; s ← s1; t ← t1
```

(Initialization)
(Updating r's)
(Updating s's)
(Updating t's)

b. Algorithm

Given $a = 161$ and $b = 28$, find gcd (a, b) and the values of s and t.

Solution

$$r = r_1 - q \times r_2 \qquad s = s_1 - q \times s_2 \qquad t = t_1 - q \times t_2$$

We use a table to follow the algorithm.

q	r_1	r_2	r	s_1	s_2	s	t_1	t_2	t
5	161	28	21	1	0	-1	0	1	-5
1	28	21	7	0	1	-1	1	-5	6
3	21	7	0	1	-1	4	-5	6	-23
	7	0		-1	4		6	-23	

We get gcd (161, 28) = 7, $s = -1$ and $t = 6$. The answers can be tested because we have

$$(-1) \times 161 + 6 \times 28 = 7$$

Primality

Prime Numbers

- An integer p > 1 is prime number, if its divisors are +/- 1 and +/1 p
- Any non negative integer a > 1 can be factored in the form as $a = p_1^{a_1} \times p_2^{a_2} \times \cdots \times p_t^{a_t}$
- where p1 < p2 < ... < pt are prime numbers and where each a is a positive integer.
- This is known as the fundamental theorem of arithmetic
- Examples: 91 = 7 x 13, 3600 = 24 x 32 x 52

Miller-Rabin Algorithm

- also known as Rabin-Miller algorithm, or the Rabin-Miller test, or the Miller-Rabin test
- typically used to test a large number for primality

Two Properties of Prime Numbers

- If p is prime and a is a positive integer less than p, then a^2 mod p = 1, if and only if either a mod p = 1 or a mod p = -1 mod p = p – 1
- Let p be a prime number greater than 2. We can then write p - 1 = 2^kq with k > 0, q odd

Algorithm

```
TEST (n)
1. Find integers k, q, with k > 0, q odd, so that
   (n − 1 = 2^k q);
2. Select a random integer a, 1 < a < n − 1;
3. if a^q mod n = 1 then return("inconclusive");
4. for j = 0 to k − 1 do
5. if a^(2^j q) mod n = n − 1 then return("inconclusive");
6. return("composite");
```

Chinese Remainder Theorem

the CRT says it is possible to reconstruct integers in a certain range from their residues modulo a set of pairwise relatively prime moduli

Formula

$$M = \prod_{i=1}^{k} m_i$$

where the m_i are pairwise relatively prime; that is, $\gcd(m_i, m_j) = 1$ for $1 \leq i, j \leq k$, and $i \neq j$.

Relatively Prime

- Two integers are relatively prime, if their only common positive integer factor is 1
- **Example**: 8, 15 are relatively prime because positive divisors of 8 are 1, 2, 4, 8. Positive divisors of 15 are 1, 3, 5, 15. Common positive factor = 1

TWO ASSERTION OF CRT

1. The mapping of Equation (8.7) is a one-to-one correspondence (called a **bijection**) between Z_M and the Cartesian product $Z_{m_1} \times Z_{m_2} \times \cdots \times Z_{m_k}$. That is, for every integer A such that $0 \leq A \leq M$, there is a unique k-tuple $(a_1, a_2, \ldots, a_k)$ with $0 \leq a_i < m_i$ that represents it, and for every such k-tuple $(a_1, a_2, \ldots, a_k)$, there is a unique integer A in Z_M.
2. Operations performed on the elements of Z_M can be equivalently performed on the corresponding k-tuples by performing the operation independently in each coordinate position in the appropriate system.

The following is an example of a set of equations with different moduli:

$$x \equiv 2 \pmod 3$$
$$x \equiv 3 \pmod 5$$
$$x \equiv 2 \pmod 7$$

The solution to this set of equations is given in the next section; for the moment, note that the answer to this set of equations is $x = 23$. This value satisfies all equations: $23 \equiv 2 \pmod 3$, $23 \equiv 3 \pmod 5$, and $23 \equiv 2 \pmod 7$.

Solution

The solution to the set of equations follows these steps:

1. Find $M = m_1 \times m_2 \times \cdots \times m_k$. This is the common modulus.
2. Find $M_1 = M/m_1$, $M_2 = M/m_2$, ..., $M_k = M/m_k$.
3. Find the multiplicative inverse of $M_1, M_2, \ldots, M_k$ using the corresponding moduli ($m_1, m_2, \ldots, m_k$). Call the inverses $M_1^{-1}, M_2^{-1}, \ldots, M_k^{-1}$.
4. The solution to the simultaneous equations is

$$x = (a_1 \times M_1 \times M_1^{-1} + a_2 \times M_2 \times M_2^{-1} + \cdots + a_k \times M_k \times M_k^{-1}) \bmod M$$

Note that the set of equations can have a solution even if the moduli are not relatively prime but meet other conditions. However, in cryptography, we are only interested in solving equations with coprime moduli.

From the previous example, we already know that the answer is $x = 23$. We follow the four steps.

1. $M = 3 \times 5 \times 7 = 105$
2. $M_1 = 105 / 3 = 35$, $M_2 = 105 / 5 = 21$, $M_3 = 105 / 7 = 15$
3. The inverses are $M_1^{-1} = 2$, $M_2^{-1} = 1$, $M_3^{-1} = 1$
4. $x = (2 \times 35 \times 2 + 3 \times 21 \times 1 + 2 \times 15 \times 1) \bmod 105 = 23 \bmod 105$

DISCRETE LOGARITHMS

More generally, we can say that the highest possible exponent to which a number can belong (mod n) is $\phi(n)$. If a number is of this order, it is referred to as a **primitive root** of n. The importance of this notion is that if a is a primitive root of n, then its powers

$$a, a^2, \ldots, a^{\phi(n)}$$

are distinct (mod n) and are all relatively prime to n. In particular, for a prime number p, if a is a primitive root of p, then

$$a, a^2, \ldots, a^{p-1}$$

are distinct (mod p). For the prime number 19, its primitive roots are 2, 3, 10, 13, 14, and 15.

Logarithms for Modular Arithmetic

With ordinary positive real numbers, the logarithm function is the inverse of exponentiation. An analogous function exists for modular arithmetic.

Let us briefly review the properties of ordinary logarithms. The logarithm of a number is defined to be the power to which some positive base (except 1) must be raised in order to equal the number. That is, for base x and for a value y,

$$y = x^{\log_x(y)}$$

The properties of logarithms include

$$\log_x(1) = 0$$

$$\log_x(x) = 1$$

$$\log_x(yz) = \log_x(y) + \log_x(z) \quad \textbf{(8.11)}$$

$$\log_x(y^r) = r \times \log_x(y) \quad \textbf{(8.12)}$$

Consider a primitive root a for some prime number p (the argument can be developed for nonprimes as well). Then we know that the powers of a from 1 through $(p - 1)$ produce each integer from 1 through $(p - 1)$ exactly once. We also know that any integer b satisfies

$$b = r \pmod p \quad \text{for some } r, \text{ where } 0 \le r \le (p - 1)$$

by the definition of modular arithmetic. It follows that for any integer b and a primitive root a of prime number p, we can find a unique exponent i such that

$$b = a^i \pmod p \quad \text{where } 0 \le i \le (p - 1)$$

This exponent i is referred to as the **discrete logarithm** of the number b for the base a (mod p). We denote this value as $\text{dlog}_{a,p}(b)$.[10]

Note the following:

$$\text{dlog}_{a,p}(1) = 0 \quad \text{because } a^0 \bmod p = 1 \bmod p = 1 \quad \textbf{(8.13)}$$

$$\text{dlog}_{a,p}(a) = 1 \quad \text{because } a^1 \bmod p = a \quad \textbf{(8.14)}$$

Here is an example using a nonprime modulus, $n = 9$. Here $\phi(n) = 6$ and $a = 2$ is a primitive root. We compute the various powers of a and find

$$2^0 = 1 \qquad 2^4 = 7 \pmod 9$$

$$2^1 = 2 \qquad 2^5 = 5 \pmod 9$$

$$2^2 = 4 \qquad 2^6 = 1 \pmod 9$$

$$2^3 = 8$$

This gives us the following table of the numbers with given discrete logarithms (mod 9) for the root $a = 2$:

Logarithm	0	1	2	3	4	5
Number	1	2	4	8	7	5

To make it easy to obtain the discrete logarithms of a given number, we rearrange the table:

Number	1	2	4	5	7	8
Logarithm	0	1	2	5	4	3

Now consider

$$x = a^{\text{dlog}_{a,p}(x)} \bmod p \qquad y = a^{\text{dlog}_{a,p}(y)} \bmod p$$
$$xy = a^{\text{dlog}_{a,p}(xy)} \bmod p$$

Using the rules of modular multiplication,

$$xy \bmod p = [(x \bmod p)(y \bmod p)] \bmod p$$
$$a^{\text{dlog}_{a,p}(xy)} \bmod p = \left[\left(a^{\text{dlog}_{a,p}(x)} \bmod p\right)\left(a^{\text{dlog}_{a,p}(y)} \bmod p\right)\right] \bmod p$$
$$= \left\{a^{[\text{dlog}_{a,p}(x)+\text{dlog}_{a,p}(y)]}\right\} \bmod p$$

But now consider Euler's theorem, which states that, for every a and n that are relatively prime,

$$a^{\phi(n)} = 1 \pmod n$$

Any positive integer z can be expressed in the form $z = q + k\phi(n)$, with $0 \le q < \phi(n)$. Therefore, by Euler's theorem,

$$a^z = a^q \pmod n \qquad \text{if } z = q \bmod \phi(n)$$

Applying this to the foregoing equality, we have

$$\text{dlog}_{a,p}(xy) = [\text{dlog}_{a,p}(x) + \text{dlog}_{a,p}(y)] (\text{mod}\, \phi(p))$$

and generalizing,

$$\text{dlog}_{a,p}(y^r) = [r \times \text{dlog}_{a,p}(y)] (\text{mod}\, \phi(p))$$

This demonstrates the analogy between true logarithms and discrete logarithms.

Table 8.4 Tables of Discrete Logarithms, Modulo 19

(a) Discrete logarithms to the base 2, modulo 19

a	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
$\text{dlog}_{2,19}(a)$	18	1	13	2	16	14	6	3	8	17	12	15	5	7	11	4	10	9

(b) Discrete logarithms to the base 3, modulo 19

a	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
$\text{dlog}_{3,19}(a)$	18	7	1	14	4	8	6	3	2	11	12	15	17	13	5	10	16	9

(c) Discrete logarithms to the base 10, modulo 19

a	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
$\text{dlog}_{10,19}(a)$	18	17	5	16	2	4	12	15	10	1	6	3	13	11	7	14	8	9

(d) Discrete logarithms to the base 13, modulo 19

a	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
$\text{dlog}_{13,19}(a)$	18	11	17	4	14	10	12	15	16	7	6	3	1	5	13	8	2	9

(e) Discrete logarithms to the base 14, modulo 19

a	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
$\text{dlog}_{14,19}(a)$	18	13	7	8	10	2	6	3	14	5	12	15	11	1	17	16	4	9

(f) Discrete logarithms to the base 15, modulo 19

a	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
$\text{dlog}_{15,19}(a)$	18	5	11	10	8	16	12	15	4	13	6	3	7	17	1	2	14	9

Modular Arithmetic

The Modulus

If a is an integer and n is a positive integer, we define $a \bmod n$ to be the remainder when a is divided by n. The integer n is called the **modulus**. Thus, for any integer a, we can rewrite Equation (4.1) as follows:

$$a = qn + r \qquad 0 \leq r < n;\ q = \lfloor a/n \rfloor$$

$$a = \lfloor a/n \rfloor \times n + (a \bmod n)$$

$$11 \bmod 7 = 4; \qquad -11 \bmod 7 = 3$$

Two integers a and b are said to be **congruent modulo** n, if $(a \bmod n) = (b \bmod n)$. This is written as $a \equiv b \pmod{n}$.[2]

$$73 \equiv 4 \pmod{23}; \qquad 21 \equiv -9 \pmod{10}$$

Note that if $a \equiv 0 \pmod{n}$, then $n|a$.

Properties of Congruences

1. $a \equiv b \pmod{n}$ if $n|(a - b)$.
2. $a \equiv b \pmod{n}$ implies $b \equiv a \pmod{n}$.
3. $a \equiv b \pmod{n}$ and $b \equiv c \pmod{n}$ imply $a \equiv c \pmod{n}$.

To demonstrate the first point, if $n|(a - b)$, then $(a - b) = kn$ for some k. So we can write $a = b + kn$. Therefore, $(a \bmod n) =$ (remainder when $b + kn$ is divided by n) = (remainder when b is divided by n) $= (b \bmod n)$.

$23 \equiv 8 \pmod{5}$	because	$23 - 8 = 15 = 5 \times 3$
$-11 \equiv 5 \pmod{8}$	because	$-11 - 5 = -16 = 8 \times (-2)$
$81 \equiv 0 \pmod{27}$	because	$81 - 0 = 81 = 27 \times 3$

Modular Arithmetic Operations

1. $[(a \bmod n) + (b \bmod n)] \bmod n = (a + b) \bmod n$
2. $[(a \bmod n) - (b \bmod n)] \bmod n = (a - b) \bmod n$
3. $[(a \bmod n) \times (b \bmod n)] \bmod n = (a \times b) \bmod n$

We demonstrate the first property. Define $(a \bmod n) = r_a$ and $(b \bmod n) = r_b$. Then we can write $a = r_a + jn$ for some integer j and $b = r_b + kn$ for some integer k. Then

$$\begin{aligned}(a + b) \bmod n &= (r_a + jn + r_b + kn) \bmod n \\ &= (r_a + r_b + (k + j)n) \bmod n \\ &= (r_a + r_b) \bmod n \\ &= [(a \bmod n) + (b \bmod n)] \bmod n\end{aligned}$$

Examples

11 mod 8 = 3; 15 mod 8 = 7
[(11 mod 8) + (15 mod 8)] mod 8 = 10 mod 8 = 2
(11 + 15) mod 8 = 26 mod 8 = 2
[(11 mod 8) − (15 mod 8)] mod 8 = −4 mod 8 = 4
(11 − 15) mod 8 = −4 mod 8 = 4
[(11 mod 8) × (15 mod 8)] mod 8 = 21 mod 8 = 5
(11 × 15) mod 8 = 165 mod 8 = 5

To find 11^7 mod 13, we can proceed as follows:

$11^2 = 121 = 4 \pmod{13}$
$11^4 = (11^2)^2 = 4^2 = 3 \pmod{13}$
$11^7 = 11 \times 4 \times 3 = 132 = 2 \pmod{13}$

Properties of Modular Arithmetic

set of residues, or **residue classes** (mod n)

Define the set Z_n as the set of nonnegative integers less than n:

$$Z_n = [0, 1, \ldots, (n-1)]$$

precise, each integer in Z_n represents a residue class. We can label the residue classes (mod n) as $[0]$, $[1]$, $[2]$, $\ldots$, $[n-1]$, where

$$[r] = [a: a \text{ is an integer}, a \equiv r \pmod{n}]$$

The residue classes (mod 4) are

$[0] = \{\ldots, -16, -12, -8, -4, 0, 4, 8, 12, 16, \ldots\}$
$[1] = \{\ldots, -15, -11, -7, -3, 1, 5, 9, 13, 17, \ldots\}$
$[2] = \{\ldots, -14, -10, -6, -2, 2, 6, 10, 14, 18, \ldots\}$
$[3] = \{\ldots, -13, -9, -5, -1, 3, 7, 11, 15, 19, \ldots\}$

reducing k modulo n.

Finding the smallest nonnegative integer to which k is congruent modulo n

if $(a + b) = (a + c) \pmod{n}$ **then** $b = c \pmod{n}$

$(5 + 23) = (5 + 7) \pmod{8}$; $23 = 7 \pmod{8}$

if $(a \times b) = (a \times c) \pmod{n}$ **then** $b = c \pmod{n}$ **if** a is relatively prime to n

Properties of Modular Arithmetic for Integers in Zn

Commutative Laws	$(w + x) \bmod n = (x + w) \bmod n$ $(w \times x) \bmod n = (x + w) \bmod n$
Associative Laws	$[(w + x) + y] \bmod n = [w + (x + y)] \bmod n$ $[(w \times x) \times y] \bmod n = [w \times (x \times y)] \bmod n$
Distributive Law	$[w \times (x + y)] \bmod n = [(w \times x) + (w \times y)] \bmod n$
Identities	$(0 + w) \bmod n = w \bmod n$ $(1 \times w) \bmod n = w \bmod n$
Additive Inverse ($-w$)	For each $w \in Z_n$, there exists a z such that $w + z = 0 \bmod n$

Table 4.2 Arithmetic Modulo 8

+	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	0
2	2	3	4	5	6	7	0	1
3	3	4	5	6	7	0	1	2
4	4	5	6	7	0	1	2	3
5	5	6	7	0	1	2	3	4
6	6	7	0	1	2	3	4	5
7	7	0	1	2	3	4	5	6

(a) Addition modulo 8

×	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7
2	0	2	4	6	0	2	4	6
3	0	3	6	1	4	7	2	5
4	0	4	0	4	0	4	0	4
5	0	5	2	7	4	1	6	3
6	0	6	4	2	0	6	4	2
7	0	7	6	5	4	3	2	1

(b) Multiplication modulo 8

w	−w	w^{-1}
0	0	—
1	7	1
2	6	—
3	5	3
4	4	—
5	3	5
6	2	—
7	1	7

(c) Additive and multiplicative inverses modulo 8

GROUPS, RINGS AND FIELDS

Groups, rings, and fields are the fundamental elements of a branch of mathematics known as abstract algebra, or modern algebra. In abstract algebra, we are concerned with sets on whose elements we can operate algebraically; that is, we can combine two elements of the set, perhaps in several ways, to obtain a third element of the set. These operations are subject to specific rules, which define the nature of the set. By convention, the notation for the two principal classes of operations on set elements is usually the same as the notation for addition and multiplication on ordinary numbers. However, it is important to note that, in abstract algebra, we are not limited to ordinary arithmetical operations. All this should become clear as we proceed.

Groups

A **group** G, sometimes denoted by $\{G, \cdot\}$, is a set of elements with a binary operation denoted by $\cdot$ that associates to each ordered pair (a, b) of elements in G an element $(a \cdot b)$ in G, such that the following axioms are obeyed:[4]

(A1) Closure:	If a and b belong to G, then $a \cdot b$ is also in G.
(A2) Associative:	$a \cdot (b \cdot c) = (a \cdot b) \cdot c$ for all a, b, c in G.
(A3) Identity element:	There is an element e in G such that $a \cdot e = e \cdot a = a$ for all a in G.
(A4) Inverse element:	For each a in G, there is an element a' in G such that $a \cdot a' = a' \cdot a = e$.

If a group has a finite number of elements, it is referred to as a **finite group**, and the **order** of the group is equal to the number of elements in the group. Otherwise, the group is an **infinite group**.

A group is said to be **abelian** if it satisfies the following additional condition:

(A5) Commutative:	$a \cdot b = b \cdot a$ for all a, b in G.

The set of integers (positive, negative, and 0) under addition is an abelian group. The set of nonzero real numbers under multiplication is an abelian group. The set S_n from the preceding example is a group but not an abelian group for $n > 2$.

When the group operation is addition, the identity element is 0; the inverse element of a is $-a$; and subtraction is defined with the following rule: $a - b = a + (-b)$.

CYCLIC GROUP We define exponentiation within a group as a repeated application of the group operator, so that $a^3 = a \cdot a \cdot a$. Furthermore, we define $a^0 = e$ as the identity element, and $a^{-n} = (a')^n$, where a' is the inverse element of a within the group. A group G is **cyclic** if every element of G is a power a^k (k is an integer) of a fixed element $a \in G$. The element a is said to **generate** the group G or to be a **generator** of G. A cyclic group is always abelian and may be finite or infinite.

The additive group of integers is an infinite cyclic group generated by the element 1. In this case, powers are interpreted additively, so that n is the nth power of 1.

Rings

A **ring** R, sometimes denoted by $\{R, +, \times\}$, is a set of elements with two binary operations, called *addition* and *multiplication*,[6] such that for all a, b, c in R the following axioms are obeyed.

(A1–A5)	R is an abelian group with respect to addition; that is, R satisfies axioms A1 through A5. For the case of an additive group, we denote the identity element as 0 and the inverse of a as $-a$.
(M1) Closure under multiplication:	If a and b belong to R, then ab is also in R.
(M2) Associativity of multiplication:	$a(bc) = (ab)c$ for all a, b, c in R.
(M3) Distributive laws:	$a(b + c) = ab + ac$ for all a, b, c in R. $(a + b)c = ac + bc$ for all a, b, c in R.

In essence, a ring is a set in which we can do addition, subtraction $[a - b = a + (-b)]$, and multiplication without leaving the set.

With respect to addition and multiplication, the set of all n-square matrices over the real numbers is a ring.

A ring is said to be **commutative** if it satisfies the following additional condition:

(M4) Commutativity of multiplication: $ab = ba$ for all a, b in R.

Let S be the set of even integers (positive, negative, and 0) under the usual operations of addition and multiplication. S is a commutative ring. The set of all n-square matrices defined in the preceding example is not a commutative ring.

The set Z_n of integers $\{0, 1, \ldots, n-1\}$, together with the arithmetic operations modulo n, is a commutative ring (Table 4.3).

Next, we define an **integral domain**, which is a commutative ring that obeys the following axioms.

(M5) Multiplicative identity: There is an element 1 in R such that $a1 = 1a = a$ for all a in R.

(M6) No zero divisors: If a, b in R and $ab = 0$, then either $a = 0$ or $b = 0$.

Let S be the set of integers, positive, negative, and 0, under the usual operations of addition and multiplication. S is an integral domain.

Fields

A **field** F, sometimes denoted by $\{F, +, \times\}$, is a set of elements with two binary operations, called *addition* and *multiplication*, such that for all a, b, c in F the following axioms are obeyed.

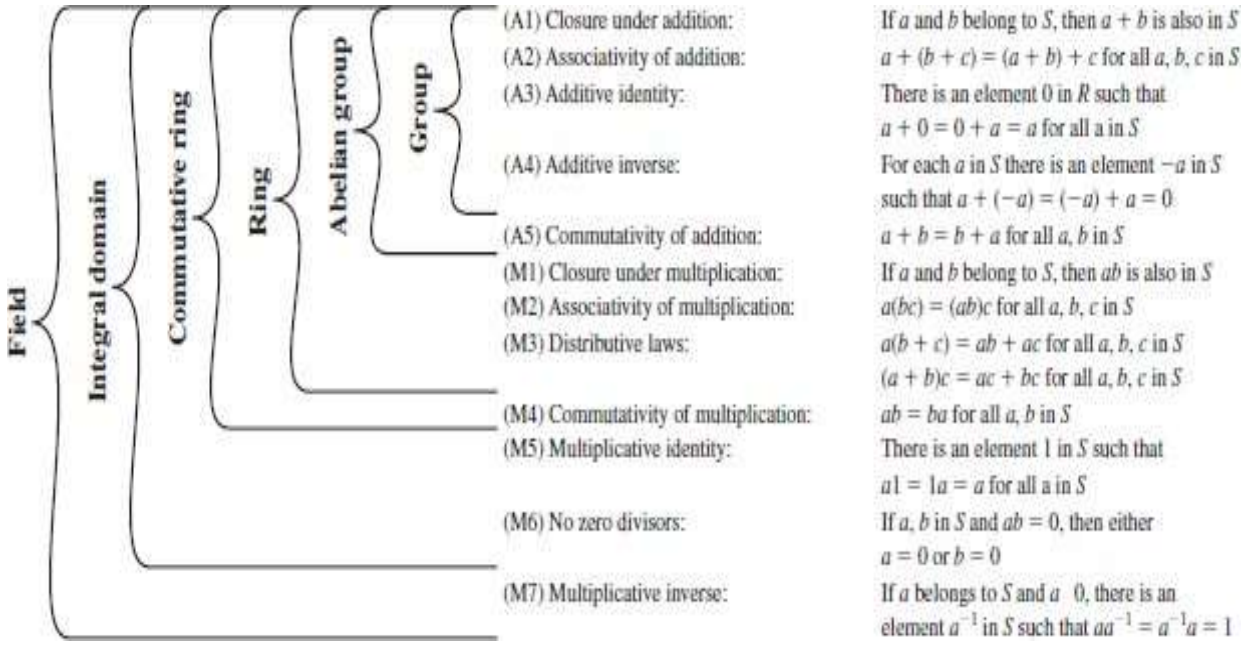
(A1–M6) F is an integral domain; that is, F satisfies axioms A1 through A5 and M1 through M6.

(M7) Multiplicative inverse: For each a in F, except 0, there is an element a^{-1} in F such that $aa^{-1} = (a^{-1})a = 1$.

In essence, a field is a set in which we can do addition, subtraction, multiplication, and division without leaving the set. Division is defined with the following rule: $a/b = a(b^{-1})$.

Familiar examples of fields are the rational numbers, the real numbers, and the complex numbers. Note that the set of all integers is not a field, because not every element of the set has a multiplicative inverse; in fact, only the elements 1 and –1 have multiplicative inverses in the integers.

Figure 4.2 summarizes the axioms that define groups, rings, and fields.



FINITE FIELDS OF THE FORM GF(p)

The finite field of order p^n is generally written $GF(p^n)$; GF stands for Galois field, in honor of the mathematician who first studied finite fields. Two special cases are of interest for our purposes. For $n = 1$, we have the finite field $GF(p)$; this finite field has a different structure than that for finite fields with $n > 1$ and is studied in this section. In Section 4.7, we look at finite fields of the form $GF(2^n)$.

Finite Fields of Order p

For a given prime, p, we define the finite field of order p, $GF(p)$, as the set Z_p of integers $\{0, 1, \ldots, p - 1\}$ together with the arithmetic operations modulo p.

Recall that we showed in Section 4.3 that the set Z_n of integers $\{0, 1, \ldots, n - 1\}$, together with the arithmetic operations modulo n, is a commutative ring (Table 4.3). We further observed that any integer in Z_n has a multiplicative inverse if and only if that integer is relatively prime to n [see discussion of Equation (4.5)].[7] If n is prime, then all of the nonzero integers in Z_n are relatively prime to n, and therefore there exists a multiplicative inverse for all of the nonzero integers in Z_n. Thus, for Z_p we can add the following properties to those listed in Table 4.3:

Multiplicative inverse (w^{-1})	For each $w \in Z_p$, $w \neq 0$, there exists a $z \in Z_p$ such that $w \times z \equiv 1 \pmod{p}$

The simplest finite field is GF(2). Its arithmetic operations are easily summarized:

+	0	1
0	0	1
1	1	0

Addition

×	0	1
0	0	0
1	0	1

Multiplication

w	$-w$	w^{-1}
0	0	—
1	1	1

Inverses

In this case, addition is equivalent to the exclusive-OR (XOR) operation, and multiplication is equivalent to the logical AND operation.

Finding the Multiplicative Inverse in $GF(p)$

It is easy to find the multiplicative inverse of an element in $GF(p)$ for small values of p. You simply construct a multiplication table, such as shown in Table 4.5b, and the desired result can be read directly. However, for large values of p, this approach is not practical.

Table 4.5 Arithmetic in GF(7)

+	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	0
2	2	3	4	5	6	0	1
3	3	4	5	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

(a) Addition modulo 7

×	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	4	6	1	3	5
3	0	3	6	2	5	1	4
4	0	4	1	5	2	6	3
5	0	5	3	1	6	4	2
6	0	6	5	4	3	2	1

(b) Multiplication modulo 7

w	$-w$	w^{-1}
0	0	—
1	6	1
2	5	4
3	4	5
4	3	2
5	2	3
6	1	6

(c) Additive and multiplicative inverses modulo 7

POLYNOMIAL ARITHMETIC

Three classes of polynomial arithmetic:

- Ordinary polynomial arithmetic, using the basic rules of algebra.
- Polynomial arithmetic in which the arithmetic on the coefficients is performed modulo p ; that is, the coefficients are in $GF(p)$.
- Polynomial arithmetic in which the coefficients are in $GF(p)$, and the polynomials are defined modulo a polynomial $m(x)$ whose highest power is some integer n.

$$\begin{array}{r} x^3 + x^2 \quad + 2 \\ +\ (x^2 - x + 1) \\ \hline x^3 + 2x^2 - x + 3 \end{array}$$

(a) Addition

$$\begin{array}{r} x^3 + x^2 \quad + 2 \\ -\ (x^2 - x + 1) \\ \hline x^3 \quad + x + 1 \end{array}$$

(b) Subtraction

$$\begin{array}{r} x^3 + x^2 \quad + 2 \\ \times\ (x^2 - x + 1) \\ \hline x^3 + x^2 \quad + 2 \\ -x^4 - x^3 \quad - 2x \\ \hline x^5 + x^4 \quad + 2x^2 \\ \hline x^5 \quad + 3x^2 - 2x + 2 \end{array}$$

(c) Multiplication

$$\begin{array}{r|l} & x + 2 \\ \hline x^2 - x + 1 & x^3 + x^2 \quad + 2 \\ & x^3 - x^2 + x \\ \hline & 2x^2 - x + 2 \\ & 2x^2 - 2x + 2 \\ \hline & x \end{array}$$

(d) Division

Figure 4.3 Examples of Polynomial Arithmetic

A polynomial $f(x)$ over a field F is called **irreducible** if and only if $f(x)$ cannot be expressed as a product of two polynomials, both over F, and both of degree lower than that of $f(x)$. By analogy to integers, an irreducible polynomial is also called a **prime polynomial**.

The polynomial[9] $f(x) = x^4 + 1$ over GF(2) is reducible, because

$$x^4 + 1 = (x + 1)(x^3 + x^2 + x + 1).$$

Consider the polynomial $f(x) = x^3 + x + 1$. It is clear by inspection that x is not a factor of $f(x)$. We easily show that $x + 1$ is not a factor of $f(x)$:

$$\begin{array}{r|l} & x^2 + x \\ \hline x + 1 & x^3 \quad + x + 1 \\ & x^3 + x^2 \\ \hline & x^2 + x \\ & x^2 + x \\ \hline & 1 \end{array}$$

Thus, $f(x)$ has no factors of degree 1. But it is clear by inspection that if $f(x)$ is reducible, it must have one factor of degree 2 and one factor of degree 1. Therefore, $f(x)$ is irreducible.

$$\begin{array}{r} x^7 \quad + x^5 + x^4 + x^3 \quad + x + 1 \\ +\ (x^3 \quad + x + 1) \\ \hline x^7 \quad + x^5 + x^4 \end{array}$$

(a) Addition

$$\begin{array}{r} x^7 \quad + x^5 + x^4 + x^3 \quad + x + 1 \\ -\ (x^3 \quad + x + 1) \\ \hline x^7 \quad + x^5 + x^4 \end{array}$$

(b) Subtraction

$$\begin{array}{r} x^7 \quad + x^5 + x^4 + x^3 \quad + x + 1 \\ \times\ (x^3 \quad + x + 1) \\ \hline x^7 \quad + x^5 + x^4 + x^3 \quad + x + 1 \\ x^8 \quad + x^6 + x^5 + x^4 \quad + x^2 + x \\ x^{10} + x^8 + x^7 + x^6 \quad + x^4 + x^3 \\ \hline x^{10} \quad + x^4 \quad + x^2 \quad + 1 \end{array}$$

(c) Multiplication

$$\begin{array}{r|l} & x^4 + 1 \\ \hline x^3 + x + 1 & x^7 \quad + x^5 + x^4 + x^3 \quad + x + 1 \\ & x^7 \quad + x^5 + x^4 \\ \hline & x^3 \quad + x + 1 \\ & x^3 \quad + x + 1 \\ \hline & \end{array}$$

(d) Division

Figure 4.4 Examples of Polynomial Arithmetic over GF(2)

FINITE FIELDS OF THE FORM GF(2^n)

Table 4.6 Arithmetic in GF(2^3)

	+	000	001	010	011	100	101	110	111
		0	1	2	3	4	5	6	7
000	0	0	1	2	3	4	5	6	7
001	1	1	0	3	2	5	4	7	6
010	2	2	3	0	1	6	7	4	5
011	3	3	2	1	0	7	6	5	4
100	4	4	5	6	7	0	1	2	3
101	5	5	4	7	6	1	0	3	2
110	6	6	7	4	5	2	3	0	1
111	7	7	6	5	4	3	2	1	0

(a) Addition

	×	000	001	010	011	100	101	110	111
		0	1	2	3	4	5	6	7
000	0	0	0	0	0	0	0	0	0
001	1	0	1	2	3	4	5	6	7
010	2	0	2	4	6	3	1	7	5
011	3	0	3	6	5	7	4	1	2
100	4	0	4	3	7	6	2	5	1
101	5	0	5	1	4	2	7	3	6
110	6	0	6	7	1	5	3	2	4
111	7	0	7	5	2	1	6	4	3

(b) Multiplication

w	−w	w^{-1}
0	0	—
1	1	1
2	2	5
3	3	6
4	4	7
5	5	2
6	6	3
7	7	4

(c) Additive and multiplicative inverses

Table 4.7 Polynomial Arithmetic Modulo ($x^3 + x + 1$)

	+	000 0	001 1	010 x	011 $x+1$	100 x^2	101 x^2+1	110 x^2+x	111 x^2+x+1
000	0	0	1	x	$x+1$	x^2	x^2+1	x^2+1	x^2+x+1
001	1	1	0	$x+1$	x	x^2+1	x^2	x^2+x+1	x^2+x
010	x	x	$x+1$	0	1	x^2+x	x^2+x+1	x^2	x^2+1
011	$x+1$	$x+1$	x	1	0	x^2+x+1	x^2+x	x^2+1	x^2
100	x^2	x^2	x^2+1	x^2+x	x^2+x+1	0	1	x	$x+1$
101	x^2+1	x^2+1	x^2	x^2+x+1	x^2+x	1	0	$x+1$	x
110	x^2+x	x^2+x	x^2+x+1	x^2	x^2+1	x	$x+1$	0	1
111	x^2+x+1	x^2+x+1	x^2+x	x^2+1	x^2	$x+1$	x	1	0

(a) Addition

	×	000 0	001 1	010 x	011 $x+1$	100 x^2	101 x^2+1	110 x^2+x	111 x^2+x+1
000	0	0	0	0	0	0	0	0	0
001	1	0	1	x	$x+1$	x^2	x^2+1	x^2+x	x^2+x+1
010	x	0	x	x^2	x^2+x	$x+1$	1	x^2+x+1	x^2+1
011	$x+1$	0	$x+1$	x^2+x	x^2+1	x^2+x+1	x^2	1	x
100	x^2	0	x^2	$x+1$	x^2+x+1	x^2+x	x	x^2+1	1
101	x^2+1	0	x^2+1	1	x^2	x	x^2+x+1	$x+1$	x^2+x
110	x^2+x	0	x^2+x	x^2+x+1	1	x^2+1	$x+1$	x	x^2
111	x^2+x+1	0	x^2+x+1	x^2+1	x	1	x^2+1	x^2	$x+1$

(b) Multiplication

Fermet & Euler Theorem

- play important roles in public-key cryptography

Fermat's Theorem

If p is prime and a is a positive integer not divisible by p, then

$$a^{p-1} \equiv 1 \pmod{p}$$

Proof:

- Consider the set of positive integers less than p: {1, 2, 3, ..., p-1}
- Multiply each element by a, modulo p to get the set
  - X = {a mod p, 2a mod p, ... (p - 1)a mod p}
- None of the elements of X is equal to zero because p does not divide a
- Therefore, we know that the (p - 1) elements of X are all positive integers with no two elements equal
- We can conclude the X consists of the set of integers {1, 2, ..., p - 1} in some order
- Multiplying the numbers in both sets (p and X) and taking the result mod p yields

$$a \times 2a \times \ldots \times (p-1)a = [(1 \times 2 \times \ldots \times (p-1)] \pmod{p}$$
$$a^{p-1}(p-1)! \equiv (p-1)! \pmod{p}$$

We can cancel the (p -1)! term because it is relatively prime to p, which prooves the theorem

Example:

$$a = 7, p = 19$$
$$7^2 = 49 \equiv 11 \pmod{19}$$
$$7^4 \equiv 121 \equiv 7 \pmod{19}$$
$$7^8 \equiv 49 \equiv 11 \pmod{19}$$
$$7^{16} = 121 = 7 \pmod{19}$$
$$a^{p-1} = 7^{18} = 7^{16} \times 7^2 = 7 \times 11 = 1 \pmod{19}$$

An alternative form of Fermat's theorem

If p is prime and a is a positive integer, then $a^p = a \pmod{p}$

$$p = 5, a = 3 \quad a^p = 3^5 = 243 \equiv 3 \pmod{5} = a \pmod{p}$$
$$p = 5, a = 10 \quad a^p = 10^5 = 100000 \equiv 10 \pmod{5} \equiv 0 \pmod{5} = a \pmod{p}$$

Euler's Theorem

Euler's totient function (ϕ (n))

- the number of positive integers less than n and relatively prime to n. ϕ (1) = 1
- for a prime number p, ϕ (p) = p - 1
- for two prime numbers where p # q,

$$\phi(n) = \phi(pq) = \phi(p) \times \phi(q) = (p-1) \times (q-1)$$

$$\phi(21) = \phi(3) \times \phi(7) = (3-1) \times (7-1) = 2 \times 6 = 12$$
where the 12 integers are $[1, 2, 4, 5, 8, 10, 11, 13, 16, 17, 19, 20]$.

To see that $\phi(n) = \phi(p) \times \phi(q)$, consider that the set of positive integers less that n is the set $\{1, \ldots, (pq - 1)\}$. The integers in this set that are not relatively prime to n are the set $\{p, 2p, \ldots, (q - 1)p\}$ and the set $\{q, 2q, \ldots, (p - 1)q\}$. Accordingly,

$$\begin{aligned}\phi(n) &= (pq - 1) - [(q - 1) + (p - 1)] \\ &= pq - (p + q) + 1 \\ &= (p - 1) \times (q - 1) \\ &= \phi(p) \times \phi(q)\end{aligned}$$

Euler's Theorem

- for every a and n that are relatively prime $a^{\phi(n)} = 1 (\text{mod } n)$
- alternative form $a^{\phi(n)+1} = a (\text{mod } n)$

Proof:

Consider the set of positive integers less thann that are relatively prime to n, labeled as

$$R = \{x_1, x_2, \ldots, x_{\phi(n)}\}$$

That is, each element x_i of R is a unique positive integer less than n with $\gcd(x_i, n) = 1$. Now multiply each element by a, modulo n:

$$S = \{(ax_1 \bmod n), (ax_2 \bmod n), \ldots, (ax_{\phi(n)} \bmod n)\}$$

The set S is a permutation[6] of R, by the following line of reasoning:

1. Because a is relatively prime to n and x_i is relatively prime to n, ax_i must also be relatively prime to n. Thus, all the members of S are integers that are less than n and that are relatively prime to n.
2. There are no duplicates in S. Refer to Equation (4.5). If $ax_i \bmod n = ax_j \bmod n$, then $x_i = x_j$.

Therefore,

$$\prod_{i=1}^{\phi(n)} (ax_i \bmod n) = \prod_{i=1}^{\phi(n)} x_i$$

$$\prod_{i=1}^{\phi(n)} ax_i \equiv \prod_{i=1}^{\phi(n)} x_i (\text{mod } n)$$

$$a^{\phi(n)} \times \left[\prod_{i=1}^{\phi(n)} x_i\right] = \prod_{i=1}^{\phi(n)} x_i (\text{mod } n)$$

$$a^{\phi(n)} = 1 (\text{mod } n)$$

which completes the proof. This is the same line of reasoning applied to the proof of Fermat's theorem.

Block Ciphers

- A block cipher is an encryption/decryption scheme in which **a block of plaintext is treated as a whole** and used to produce a ciphertext block of equal length.
- A **stream cipher** is one that encrypts a digital data stream **one bit or one byte at a time**.
  - Examples of classical stream ciphers are the autokeyed Vigenère cipher and the Vernam cipher.
- A **block cipher** is one in which a **block of plaintext** is treated as a whole and used to produce a ciphertext block of equal length
- Many block ciphers have a **Feistel structure**. Such a structure consists of a number of identical rounds of processing. In each round, a substitution is performed on one half of the data being processed, followed by a permutation that interchanges the two halves. The original key is expanded so that a different key is used for each round.
- The **Data Encryption Standard (DES)** has been the most widely used encryption algorithm until recently. It exhibits the classic Feistel structure. DES uses a 64-bit block and a 56-bit key.
- Two important methods of cryptanalysis are **differential cryptanalysis and linear cryptanalysis**. DES has been shown to be highly resistant to these two types of attack

Diffusion and Confusion

Shannon suggests two methods for frustrating statistical cryptanalysis: diffusion and confusion.

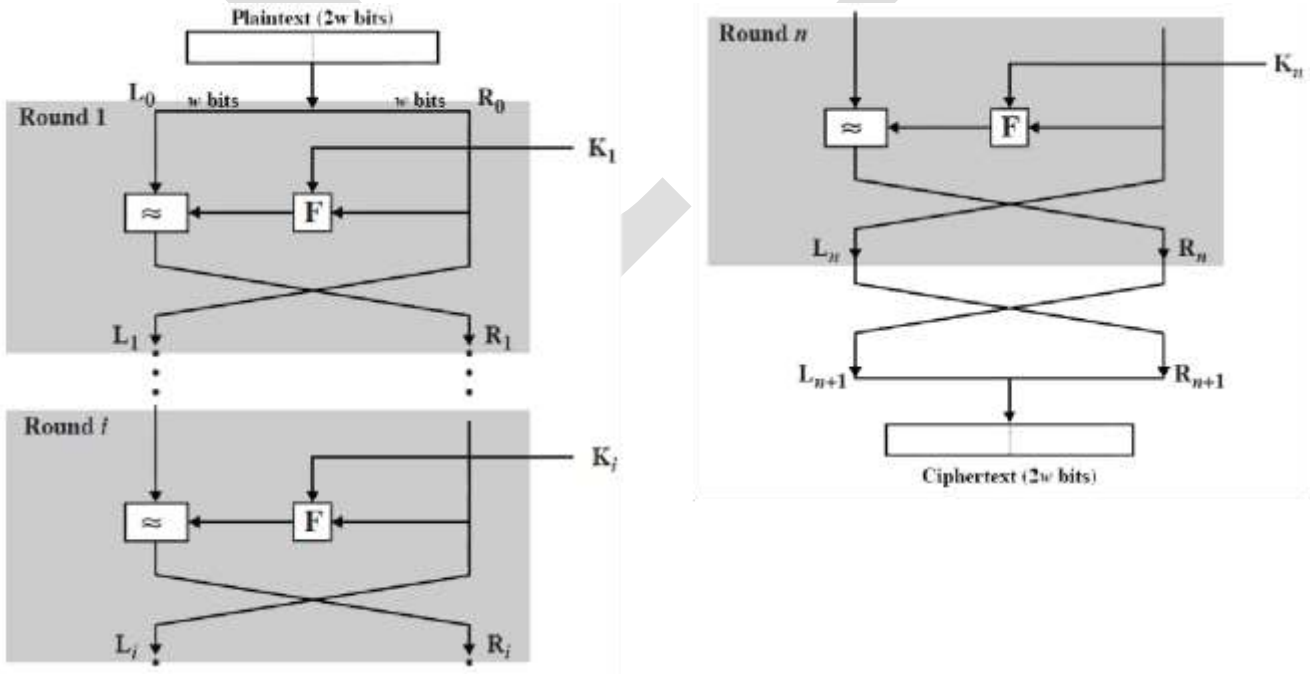
Diffusion

- the statistical structure of the plaintext is dissipated into long-range statistics of the ciphertext.
- This is achieved by having each plaintext digit affect the value of many ciphertext digits;
- generally this is equivalent to having each ciphertext digit be affected by many plaintext digits

Confusion

- seeks to make the relationship between the statistics of the ciphertext and the value of the encryption key as complex as possible, again to thwart attempts to discover the key.
- Thus, even if the attacker can get some handle on the statistics of the ciphertext, the way in which the key was used to produce that ciphertext is so complex as to make it difficult to deduce the key.

Feistel Cipher Structure



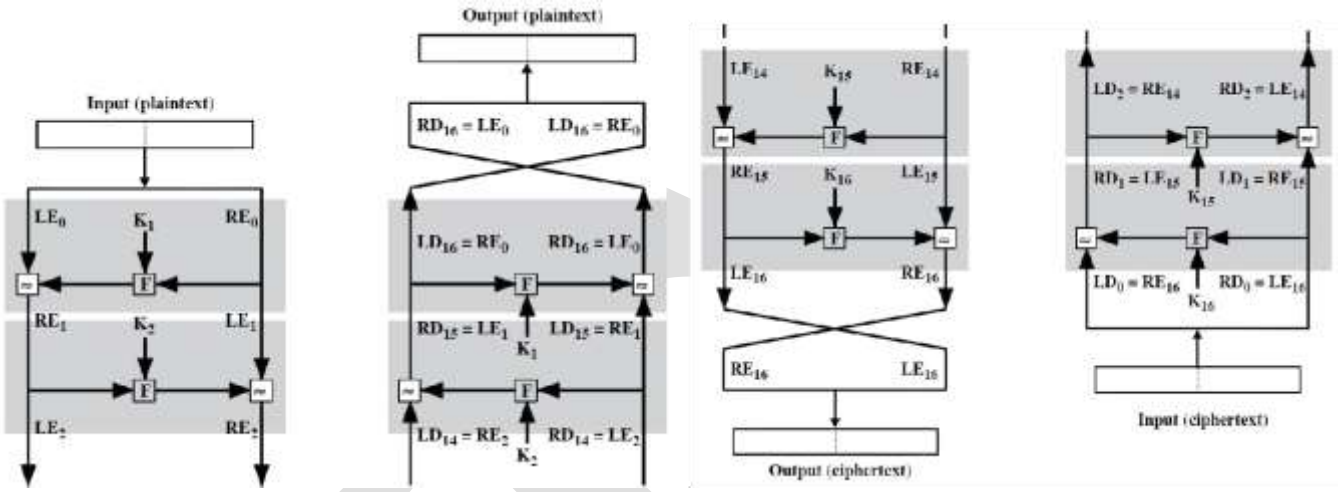
- All rounds have the same structure.
- A substitution is performed on the left half of the data.
- This is done by applying a round function F to the right half of the data and then taking the exclusive-OR of the output of that function and the left half of the data.
- The round function has the same general structure and parameterized by the round subkey Ki
- Following this, a

- This structure is a particular form of the substitution-permutation network (SPN)

Feistel network depends on the choice of the following parameters and design features

Block size, Key size, Number of rounds, Subkey generation algorithm, Round function, Fast software encryption/decryption, Ease of analysis

Feistel Encryption and Decryption

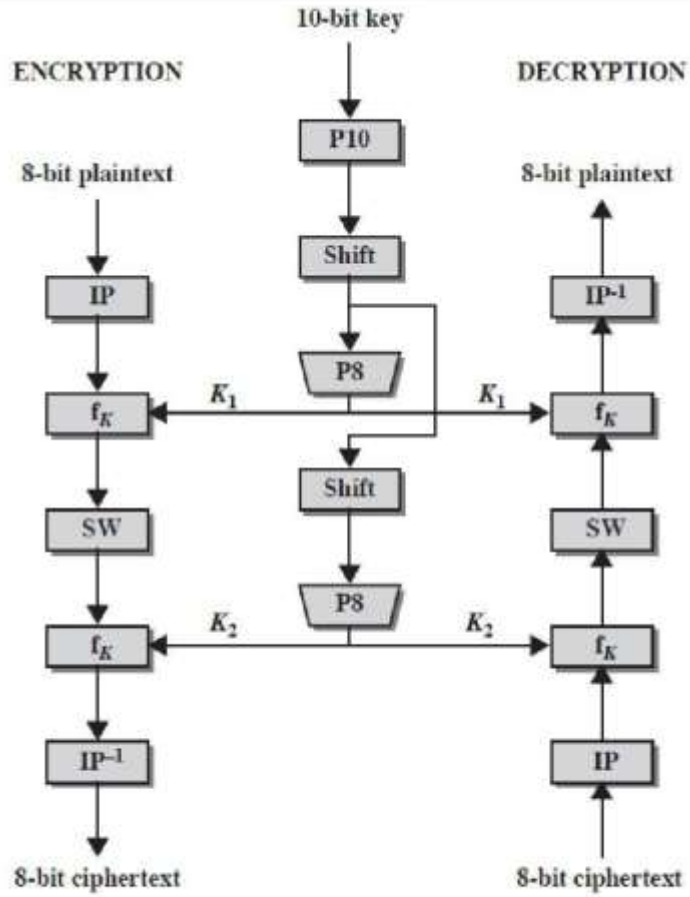


Simplified DES

- educational rather than a secure encryption algorithm.
- It has similar properties and structure to DES with much smaller parameters

Simplified DES Scheme

- The S-DES encryption algorithm takes an 8-bit block of plaintext (example: 10111101) and a 10-bit key as input and produces an 8-bit block of ciphertext as output.
- The S-DES decryption algorithm takes an 8-bit block of ciphertext and the same 10-bit key used to produce that ciphertext as input and produces the original 8-bit block of plaintext.



Involves five functions:

- an initial permutation (IP);
- a complex function labeled f_K, which involves both permutation and substitution operations and depends on a key input;
- a simple permutation function that switches (SW) the two halves of the data;
- the function f_K again;
- finally a permutation function that is the inverse of the initial permutation (IP^{-1}).

Algorithm

$$IP^{-1} \circ f_{K_2} \circ SW \circ f_{K_1} \circ IP$$

$$ciphertext = IP^{-1}\left(f_{K_2}\left(SW\left(f_{K_1}\left(IP(plaintext)\right)\right)\right)\right)$$

$$plaintext = IP^{-1}\left(f_{K_1}\left(SW\left(f_{K_2}\left(IP(ciphertext)\right)\right)\right)\right)$$

$$K_1 = P8\left(Shift\left(P10(key)\right)\right)$$

$$K_2 = P8\left(Shift\left(Shift\left(P10(key)\right)\right)\right)$$

$$f_K(L, R) = (L \oplus F(R, SK), R)$$

P10									
3	5	2	7	4	10	1	9	8	6

P8							
6	3	7	4	8	5	10	9

IP							
2	6	3	1	4	8	5	7

P4			
2	4	3	1

IP^{-1}							
4	1	3	5	7	2	8	6

E/P							
4	1	2	3	2	3	4	1

$$\begin{array}{c|cc|c} n_4 & n_1 & n_2 & n_3 \\ n_2 & n_3 & n_4 & n_1 \end{array}$$

$$\begin{array}{c|cc|c} n_4 \oplus k_{11} & n_1 \oplus k_{12} & n_2 \oplus k_{13} & n_3 \oplus k_{14} \\ n_2 \oplus k_{15} & n_3 \oplus k_{16} & n_4 \oplus k_{17} & n_1 \oplus k_{18} \end{array}$$

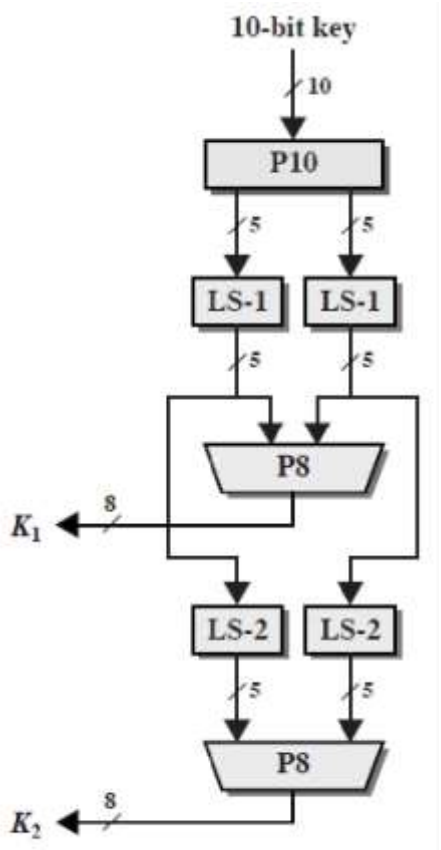
The 8-bit subkey $K_1 = (k_{11}, k_{12}, k_{13}, k_{14}, k_{15}, k_{16}, k_{17}, k_{18})$ is added

rename these 8 bits

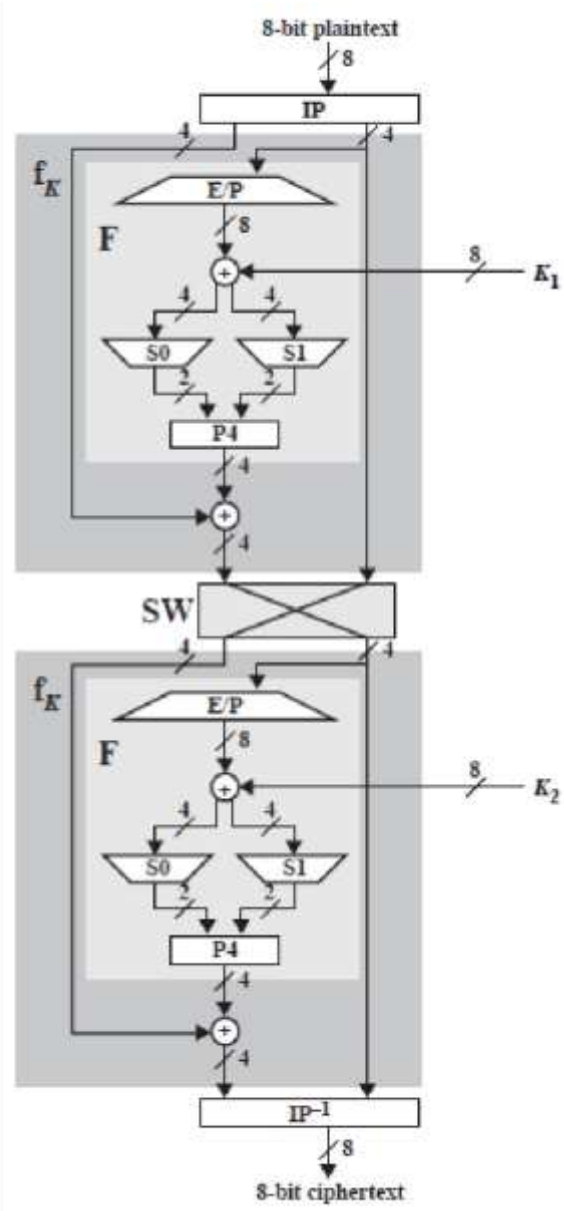
$$\begin{array}{c|c|c} p_{0,0} & p_{0,1} \quad p_{0,2} & p_{0,3} \\ p_{1,0} & p_{1,1} \quad p_{1,2} & p_{1,3} \end{array}$$

$$S0 = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 0 & 3 & 2 \\ 3 & 2 & 1 & 0 \\ 0 & 2 & 1 & 3 \\ 3 & 1 & 3 & 2 \end{bmatrix} \end{array} \qquad S1 = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 1 & 2 & 3 \\ 2 & 0 & 1 & 3 \\ 3 & 0 & 1 & 0 \\ 2 & 1 & 0 & 3 \end{bmatrix} \end{array}$$

Key Generation for Simplified DES



Simplified DES Encryption Detail



											1	2	3	4	5		6	7	8	9	10
Plain Text											0	0	1	0	1		0	0	0		
Key											1	1	0	0	0		1	1	1	1	0
P10	3	5	2	7	4	10	1	9	8	6	0	0	1	1	0		0	1	1	1	1
LS-1											0	1	1	0	0		1	1	1	1	0
P8 (K1)	6	3	7																		

LS-2		1 0 0 0 1	1 1 0 1 1
P8 (K2)		1 0 1 0 0	1 1 1
IP (PT)	2 6 3 1 4 8 5 7	0 0 1 0 0	0 1 0
R (IP)		0 0 1 0	
EP	4 1 2 3 2 3 4 1	0 0 0 1	0 1 0 0
K1		1 1 1 0	1 0 0 1
XOR		1 1 1 1	1 1 0 1
S0 (10 11)		S0 = 10	S1 = 0 0
P4		0 0 0 1	
P4 XOR L		0 0 1 1	
SWITCH		0 0 1 1	0 0 1 0
		0 0 1 0	0 0 1 1
EP(SW(R))		1 0 0 1	0 1 1 0
XOR K2		1 0 1 0	0 1 1 1
		0 0 1 1	0 0 0 1
S0 & S1		1 0	1 0
P4		0 0 1 1	
XOR L		0 0 1 0	
		0 0 0 1	
IP-1		0 0 0 1	0 0 1 1
CT		1 0 0 0	1 0 1 0

The Data Encryption Standard (DES)

- Overview
- DES Encryption
  - General Depiction of DES Encryption Algorithm
  - Initial Permutation
  - Permutation Tables for DES
  - Details of Single Round
  - Calculation of F(R, K)
  - Definition of DES S-Boxes
  - Key Generation
- DES Decryption
- The Avalanche Effect

Overview

- data are encrypted in 64-bit blocks using a 56-bit key.
- The algorithm transforms 64-bit input in a series of steps into a 64-bit output.
- The same steps, with the same key, are used to reverse the encryption

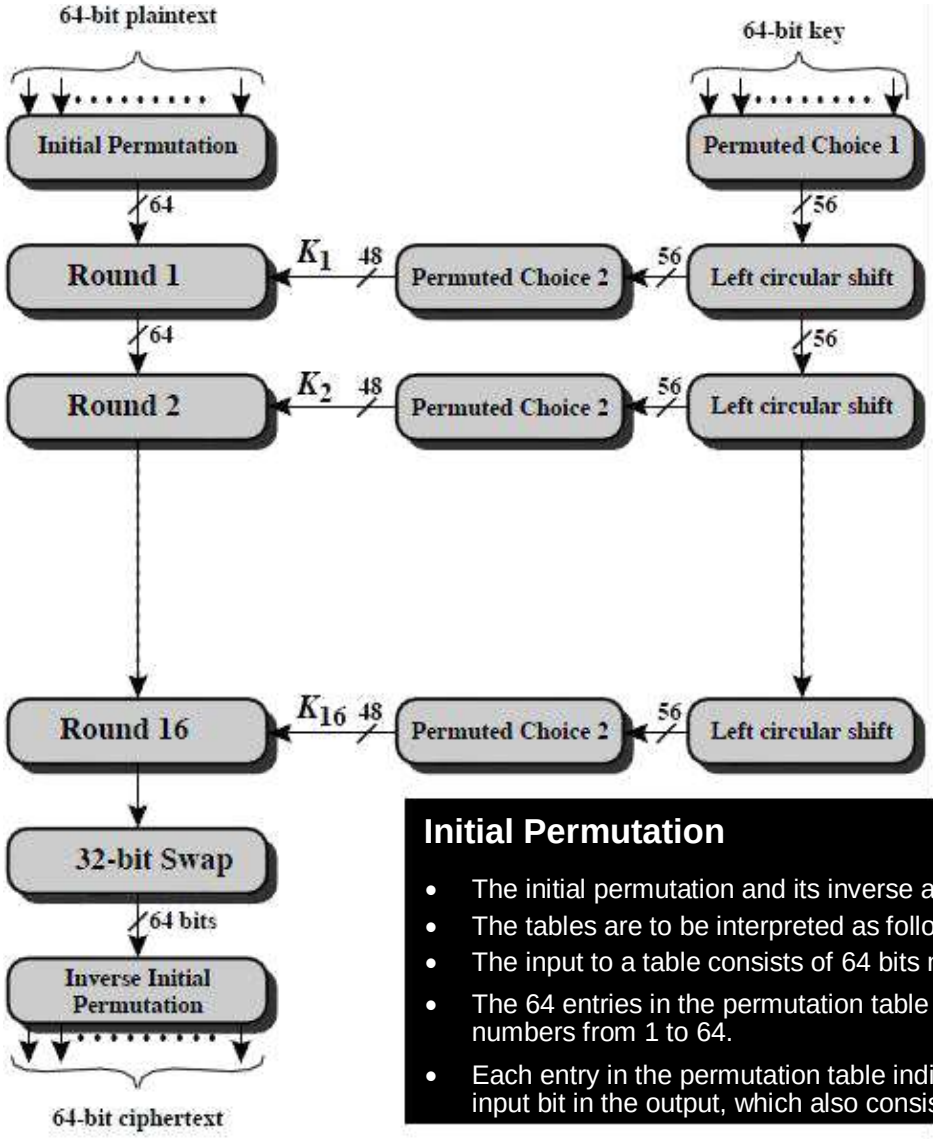
DES Encryption

- there are two inputs to the encryption function: the plaintext to be encrypted and the key.
- In this case, the plaintext must be 64 bits in length and the key is 56 bits in length

General Depiction of DES Encryption Algorithm

- processing of the plaintext proceeds **in three phases**.
- First, the 64-bit plaintext passes through an initial permutation (IP) that rearranges the bits to produce the permuted input.
- This is followed by a phase consisting of 16 rounds of the same function, which involves both permutation and substitution functions.
  - The output of the sixteenth round consists of 64 bits that are a function of the input plaintext and the key
  - The left and right halves of the output are swapped to produce the **preoutput**.

- Finally, the preoutput is passed through a permutation (IP-1) that is the inverse of the initial permutation function, to produce the 64-bit ciphertext.



Initial Permutation

- The initial permutation and its inverse are defined by tables
- The tables are to be interpreted as follows.
- The input to a table consists of 64 bits numbered from 1 to 64.
- The 64 entries in the permutation table contain a permutation of the numbers from 1 to 64.
- Each entry in the permutation table indicates the position of a numbered input bit in the output, which also consists of 64 bits

Permutation Tables for DES

Initial Permutation (IP)

58	50	42	34	26	18	10	2
60	52	44	36	28	20	12	4
62	54	46	38	30	22	14	6
64	56	48	40	32	24	16	8
57	49	41	33	25	17	9	1
59	51	43	35	27	19	11	3
61	53	45	37	29	21	13	5
63	55	47	39	31	23	15	7

Inverse Initial Permutation (IP^{-1})

40	8	48	16	56	24	64	32
39	7	47	15	55	23	63	31
38	6	46	14	54	22	62	30
37	5	45	13	53	21	61	29
36	4	44	12	52	20	60	28
35	3	43	11	51	19	59	27
34	2	42	10	50	18	58	26
33	1	41	9	49	17	57	25

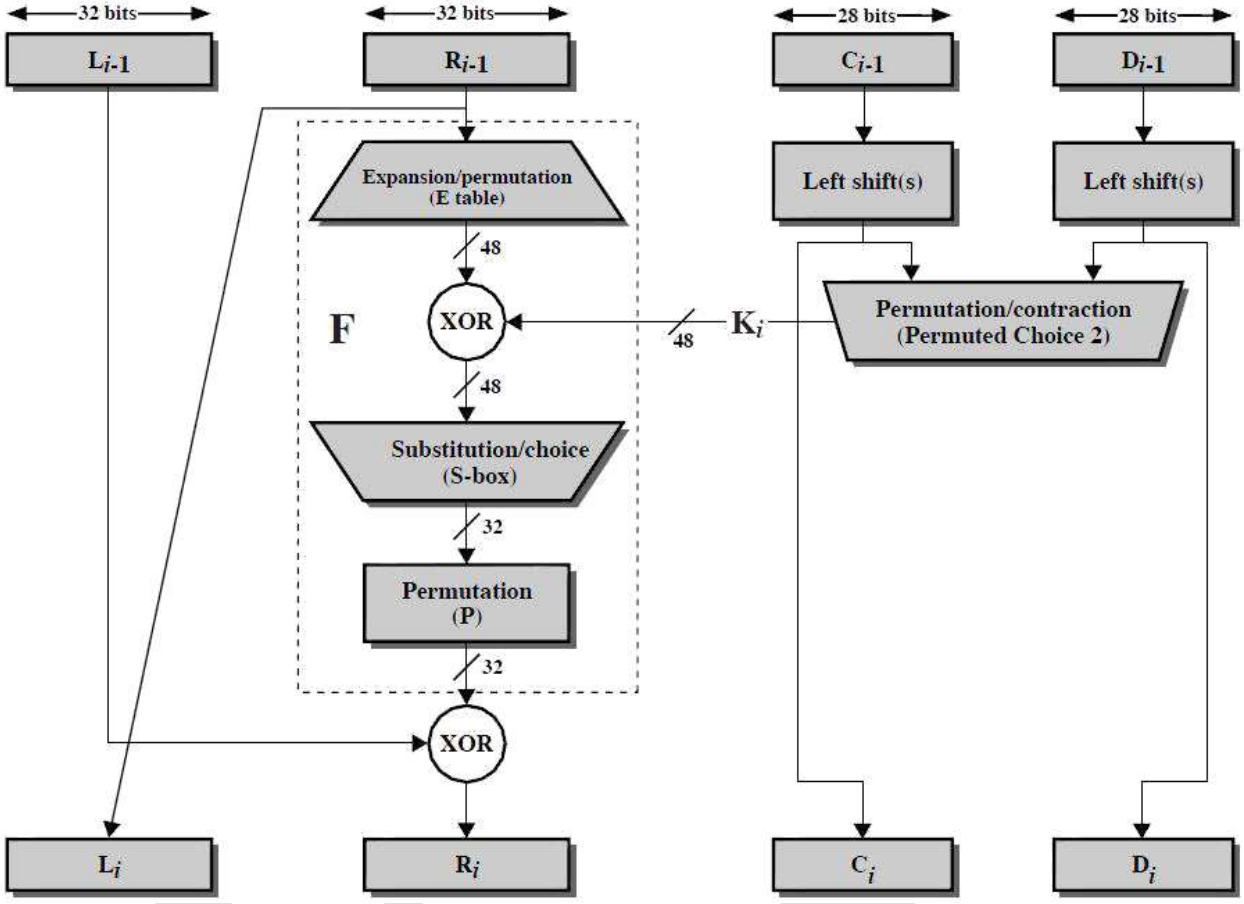
Expansion Permutation (E)

32	1	2	3	4	5
4	5	6	7	8	9
8	9	10	11	12	13
12	13	14	15	16	17
16	17	18	19	20	21
20	21	22	23	24	25
24	25	26	27	28	29
28	29	30	31	32	1

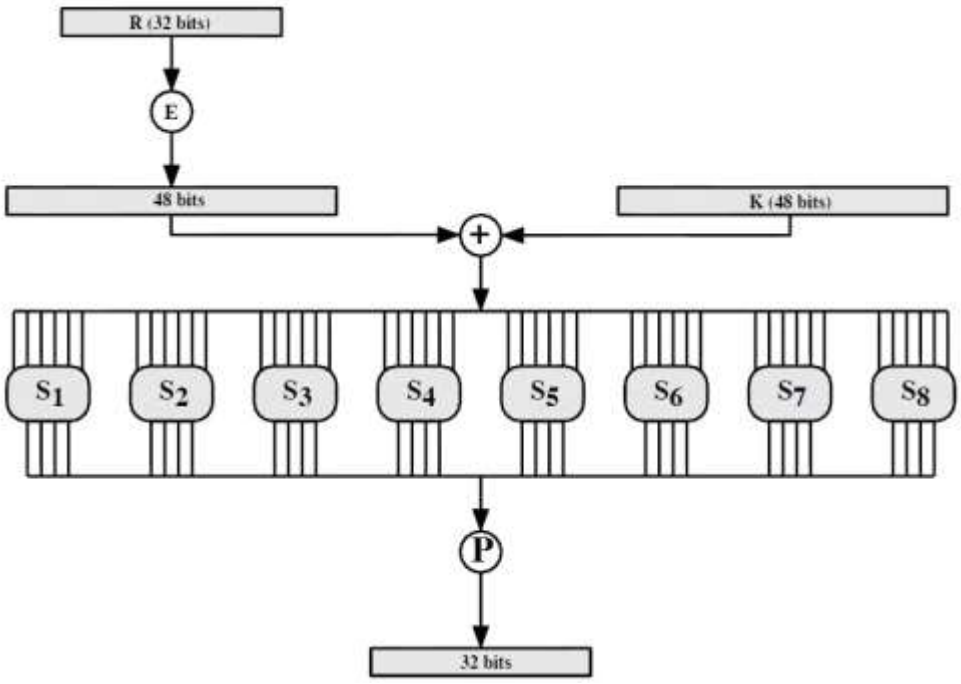
Permutation Function (P)

16	7	20	21	29	12	28	17
1	15	23	26	5	18	31	10
2	8	24	14	32	27	3	9
19	13	30	6	22	11	4	25

Details of Single Round



Calculation of F(R, K)



Definition of DES S-Boxes (S1 .. S8)

14	4	13	1	2	15	11	8	3	10	6	12	5	9	0	7
0	15	7	4	14	2	13	1	10	6	12	11	9	5	3	8
4	1	14	8	13	6	2	11	15	12	9	7	3	10	5	0
15	12	8	2	4	9	1	7	5	11	3	14	10	0	6	13

Key Generation

- a 64-bit key is used as input to the algorithm.
- The bits of the key are numbered from 1 through 64; every eighth bit is ignored
- The key is first subjected to a permutation governed by a table labeled Permuted Choice One
- The resulting 56-bit key is then treated as two 28-bit quantities, labeled C0 and D0.
- At each round, Ci-1 and Di-1 are separately subjected to a circular left shift, or rotation, of 1 or 2 bits
- These shifted values serve as input to the next round.
- They also serve as input to Permuted Choice Two , which produces a 48-bit output that serves as input to the function F(Ri-1, Ki).

DES Decryption

As with any Feistel cipher, decryption uses the same algorithm as encryption, except that the application of the subkeys is reversed

The Avalanche Effect

- a small change in either the plaintext or the key should produce a significant change in the ciphertext
- a change in one bit of the plaintext or one bit of the key should produce a change in many bits of the ciphertext
- DES exhibits a strong avalanche effect

Example

two plaintexts that differ by one bit were used

`00000000 00000000 00000000 00000000 00000000 00000000 00000000 00000000`

`10000000 00000000 00000000 00000000 00000000 00000000 00000000 00000000`

with the key

`0000001 1001011 0100100 1100010 0011100 0011000 0011100 0110010`

- after just three rounds, 21 bits differ between the two blocks.
- On completion, the two ciphertexts differ in 34 bit positions

similar test in which a single plaintext is input with two keys that differ in only one bit position

`01101000 10000101 00101111 01111010 00010011 01110110 11101011 10100100`

Keys

`1110010 1111011 1101111 0011000 0011101 0000100 0110001 11011100`

`0110010 1111011 1101111 0011000 0011101 0000100 0110001 11011100`

about half of the bits in the ciphertext differ and that the avalanche effect is pronounced after just a few rounds

Differential and Linear Cryptanalysis

Differential Cryptanalysis

- powerful method to analyse block ciphers
- a statistical attack against Feistel ciphers
- uses cipher structure not previously used
- the analysis compares differences between two related encryptions, and looks for a known difference in leading to a known difference out with some (pretty small but still significant) probability.
- If a number of such differences are determined, it is feasible to determine the subkey used in the function f.
- compares two related pairs of encryptions with a
  - known difference in the input and
  - searching for a known difference in output when same subkeys are used

In differential cryptanalysis, we start with two messages, m and m', with a known XOR difference $\Delta m = m \oplus m'$, and consider the difference between the intermediate message halves: $\Delta m_i = m_i \oplus m'_i$. Then we have

$$\begin{aligned}\Delta m_{i+1} &= m_{i+1} \oplus m'_{i+1} \\ &= [m_{i-1} \oplus \mathrm{f}(m_i, K_i)] \oplus [m'_{i-1} \oplus \mathrm{f}(m'_i, K_i)] \\ &= \Delta m_{i-1} \oplus [\mathrm{f}(m_i, K_i) \oplus \mathrm{f}(m'_i, K_i)]\end{aligned}$$

difference. Therefore, if we know Δm_{i-1} and Δm_i with high probability, then we know Δm_{i+1} with high probability. Furthermore, if a number of such differences are determined, it is feasible to determine the subkey used in the function f.

begin with two plaintext messages m and m'

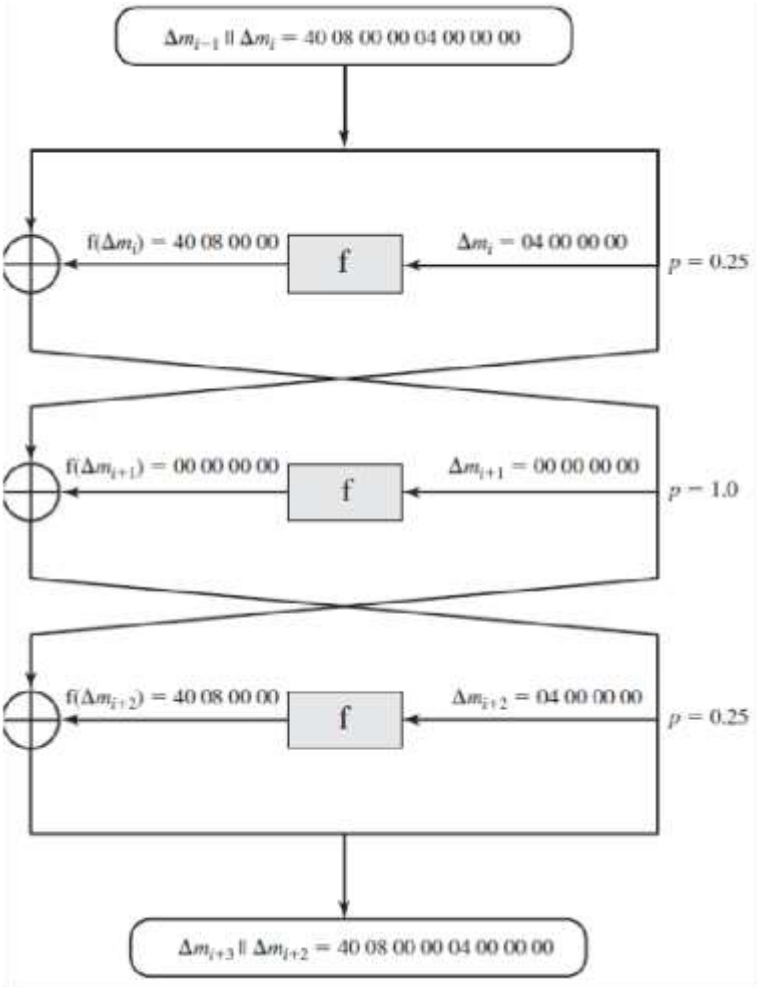
with a given difference and trace through a probable pattern of differences after each round to yield a probable difference for the ciphertext. Actually, there are two probable patterns of differences for the two 32-bit halves: $(\Delta m_{17} \| \Delta m_{16})$. Next, we submit m and m' for encryption to determine the actual difference under the unknown key and compare the result to the probable difference. If there is a match,

$$\mathrm{E}(K, m) \oplus \mathrm{E}(K, m') = (\Delta m_{17} \| \Delta m_{16})$$

then we suspect that all the probable patterns at all the intermediate rounds are correct. With that assumption, we can make some deductions about the key bits. This procedure must be repeated many times to determine all the key bits.

Differential Propagation through Three Rounds of DES

after three rounds, the probability that the output difference is as shown is equal to 0.25 * 1 * 0.25 = 0.0625



Linear Cryptanalysis

- This attack is based on finding linear approximations to describe the transformations performed in DES
- This method can find a DES key given 2^{43} known plaintexts, as compared to 2^{47} chosen plaintexts for differential cryptanalysis
- it may be easier to acquire known plaintext rather than chosen plaintext
- infeasible as an attack on DES
- For a cipher with nbit plaintext and ciphertext blocks and an m-bit key, let the plaintext block be labeled P[1], ... P[n], the cipher text block C[1], ... C[n], and the key K[1], ... K[m]

Then define $A[i, j, \ldots, k] = A[i] \oplus A[j] \oplus \ldots \oplus A[k]$

The objective of linear cryptanalysis is to find an effective linear equation of the form

$$P[\alpha_1, \alpha_2, \ldots, \alpha_a] \oplus C[\beta_1, \beta_2, \ldots, \beta_b] = K[\gamma_1, \gamma_2, \ldots, \gamma_c]$$

(where $x = 0$ or 1; $1 \leq a$; $b \leq n$; $c \leq m$; and where the α, β, and γ terms represent fixed, unique bit locations) that holds with probability $p \neq 0.5$. The further p is from

The further p is from 0.5, the more effective the equation

Once a proposed relation is determined, the procedure is to compute the results of the lefthand side of the preceding equation for a large number of plaintext-ciphertext pairs

If the result is 0 more than half the time, assume $K[\gamma_1, \gamma_2, \ldots, \gamma_c] = 0$.

If it is 1 most of the time, assume $K[\gamma_1, \gamma_2, \ldots, \gamma_c] = 1$. This gives us a linear equation on the key bits.

Modes of operation

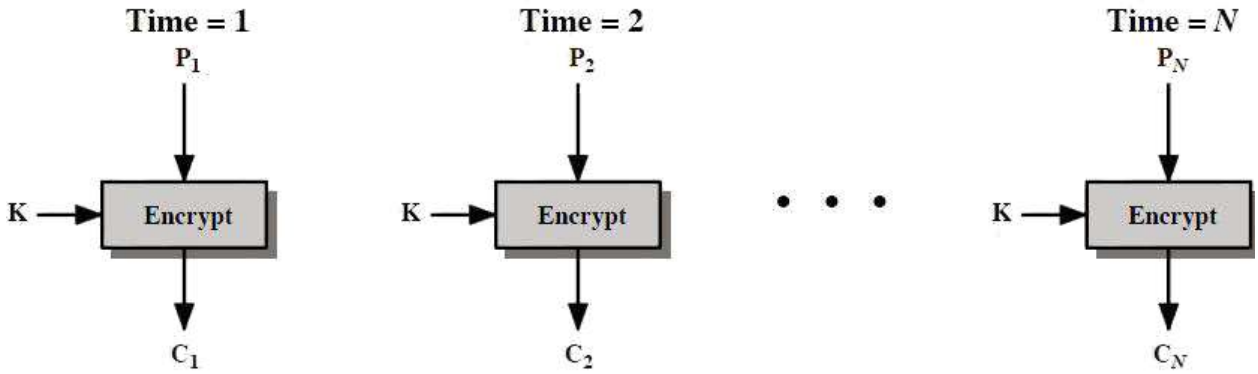
Mode	Description	Typical Application
Electronic Codebook (ECB)	Each block of 64 plaintext bits is encoded independently using the same key	Secure transmission of single values (e.g., an encryption key)
Cipher Block Chaining (CBC)	The input to the encryption algorithm is the XOR of the next 64 bits of plaintext and the preceding 64 bits of ciphertext	General-purpose block-oriented transmission Authentication
Cipher Feedback (CFB)	Input is processed j bits at a time. Preceding ciphertext is used as input to the encryption algorithm to produce pseudorandom output, which is XORed with plaintext to produce next unit of ciphertext	General-purpose stream-oriented transmission Authentication
Output Feedback (OFB)	Similar to CFB, except that the input to the encryption algorithm is the preceding DES output.	Stream-oriented transmission over noisy channel (e.g., satellite communication)
Counter (CTR)	Each block of plaintext is XORed with an encrypted counter. The counter is incremented for each subsequent block	General-purpose block-oriented transmission Useful for high-speed requirements

Various Modes

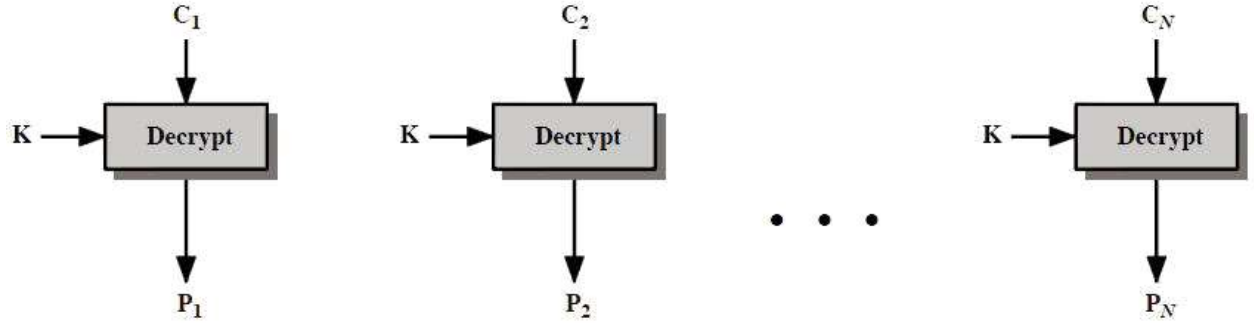
1. Electronic Codebook Mode
2. Cipher Block Chaining Mode
3. Cipher Feedback Mode
4. Output Feedback Mode
5. Counter Mode

Electronic Codebook Mode

Encryption

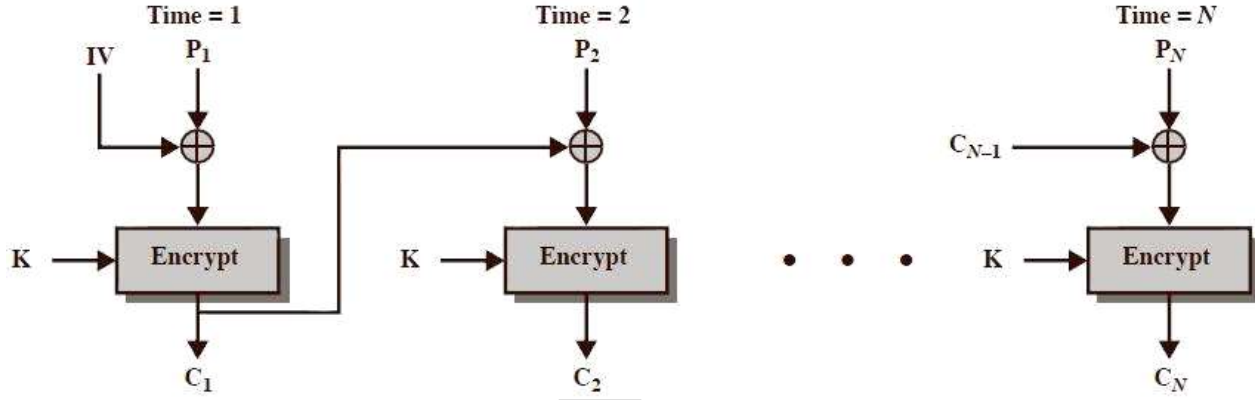


Decryption

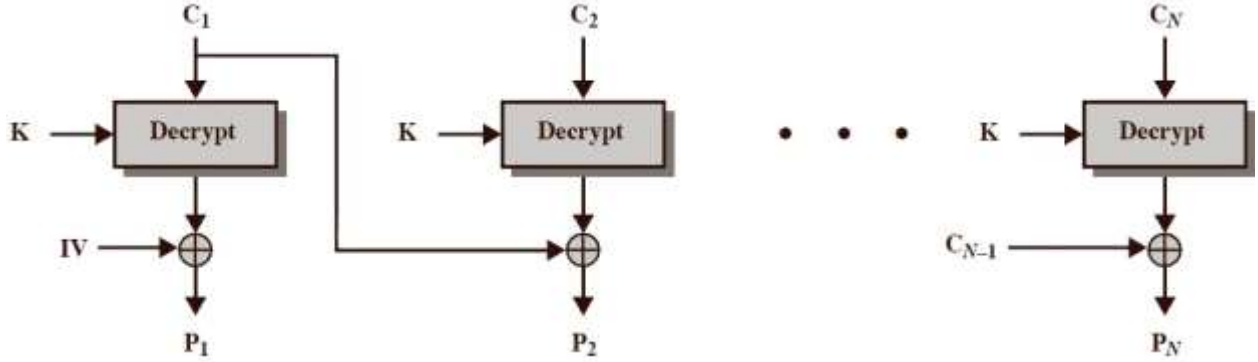


Cipher Block Chaining Mode

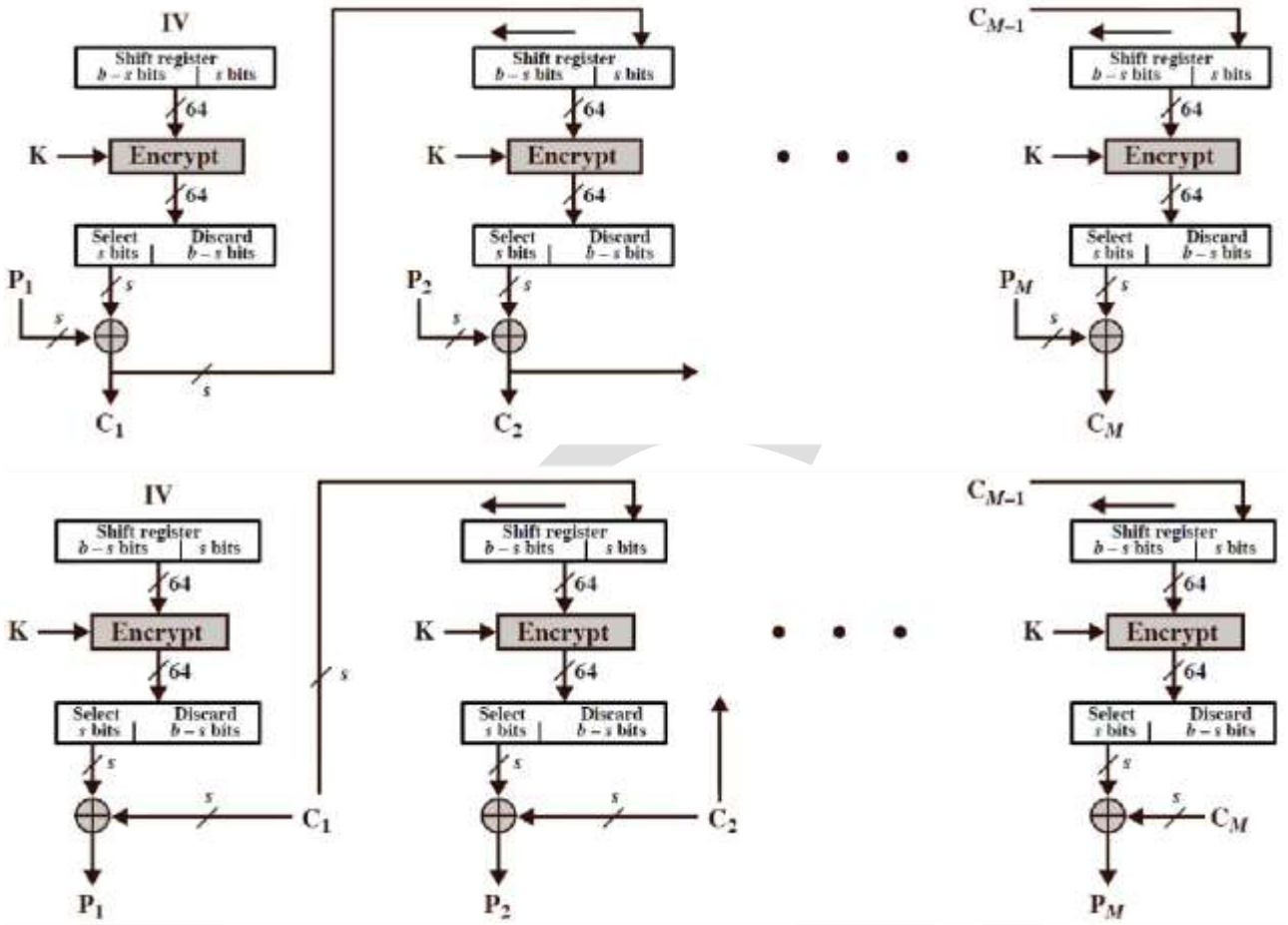
Encryption



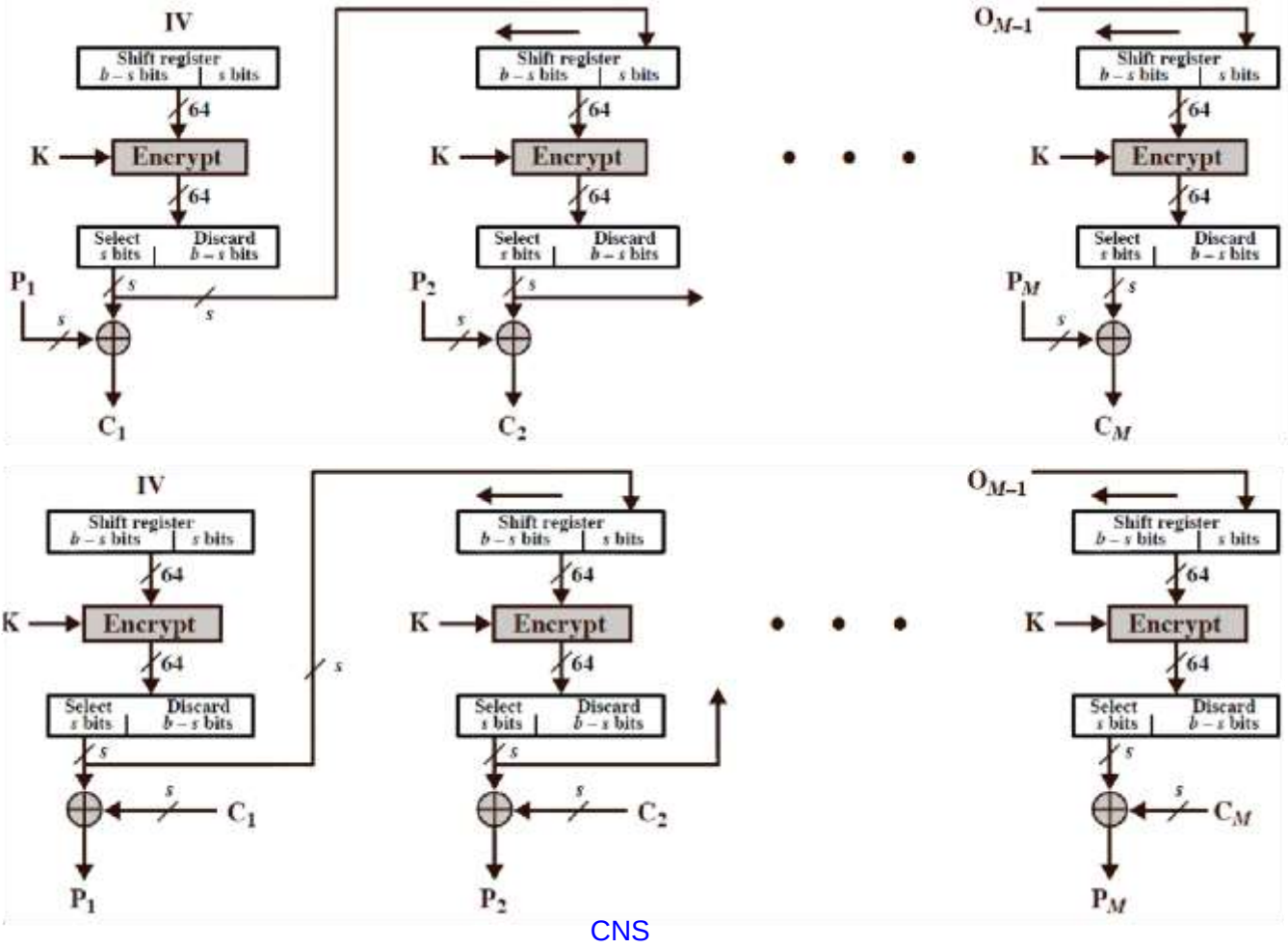
Decryption



Cipher Feedback Mode – Encryption / Decryption

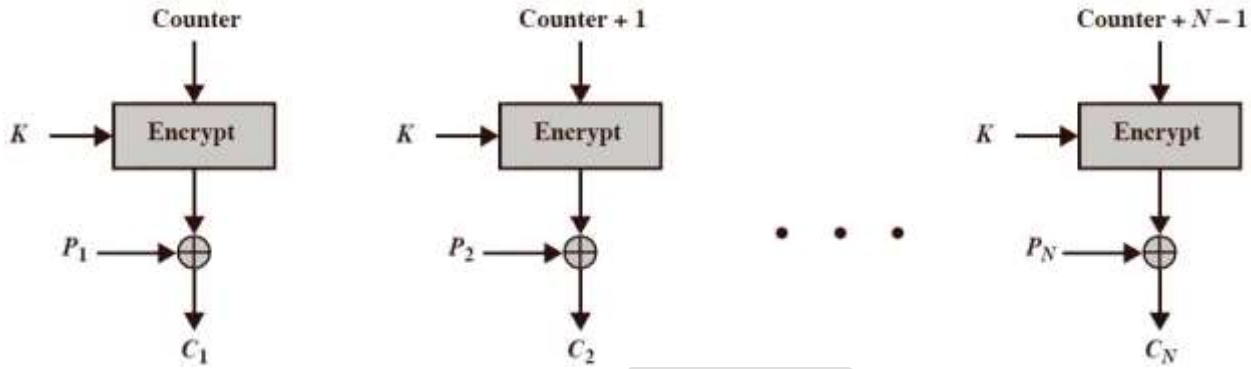


Output Feedback Mode– Encryption / Decryption

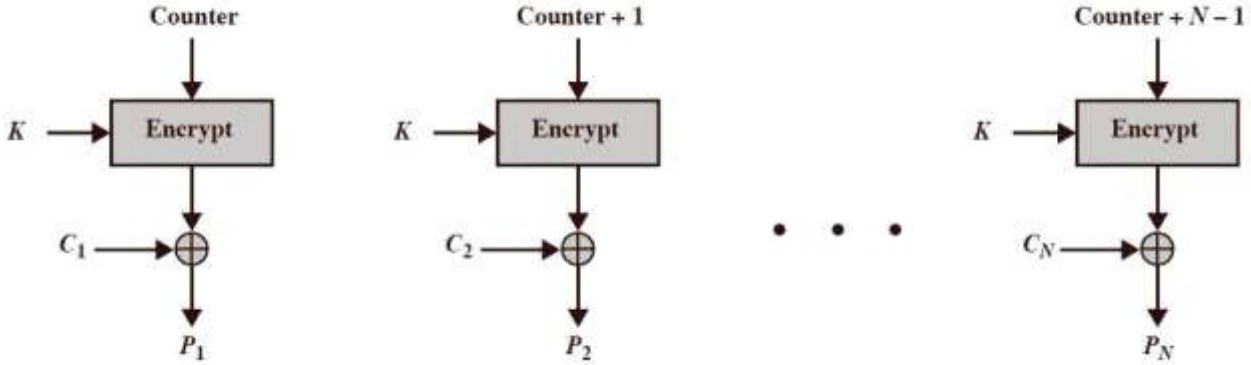


Counter Mode

Encryption



Decryption



- IV: initialization vector
- plaintext (padded as necessary) consists of a sequence of b-bit blocks, P1, P2,...,PN;
- the corresponding sequence of ciphertext blocks is C1, C2,..., CN.
- the unit of transmission is s bits; a common value is s = 8

Advantages Of CTR Mode

- Hardware efficiency
- Software efficiency
- Preprocessing
- Random access
- Provable security
- Simplicity

Encryption Algorithms

Advanced Encryption Standard

- The AES Cipher
- AES Parameters
- AES Encryption and Decryption
- AES Data Structures
- AES Encryption Round
- Substitute Bytes Transformation
- ShiftRows Transformation
- AddRoundKey Transformation
- AES Key Expansion

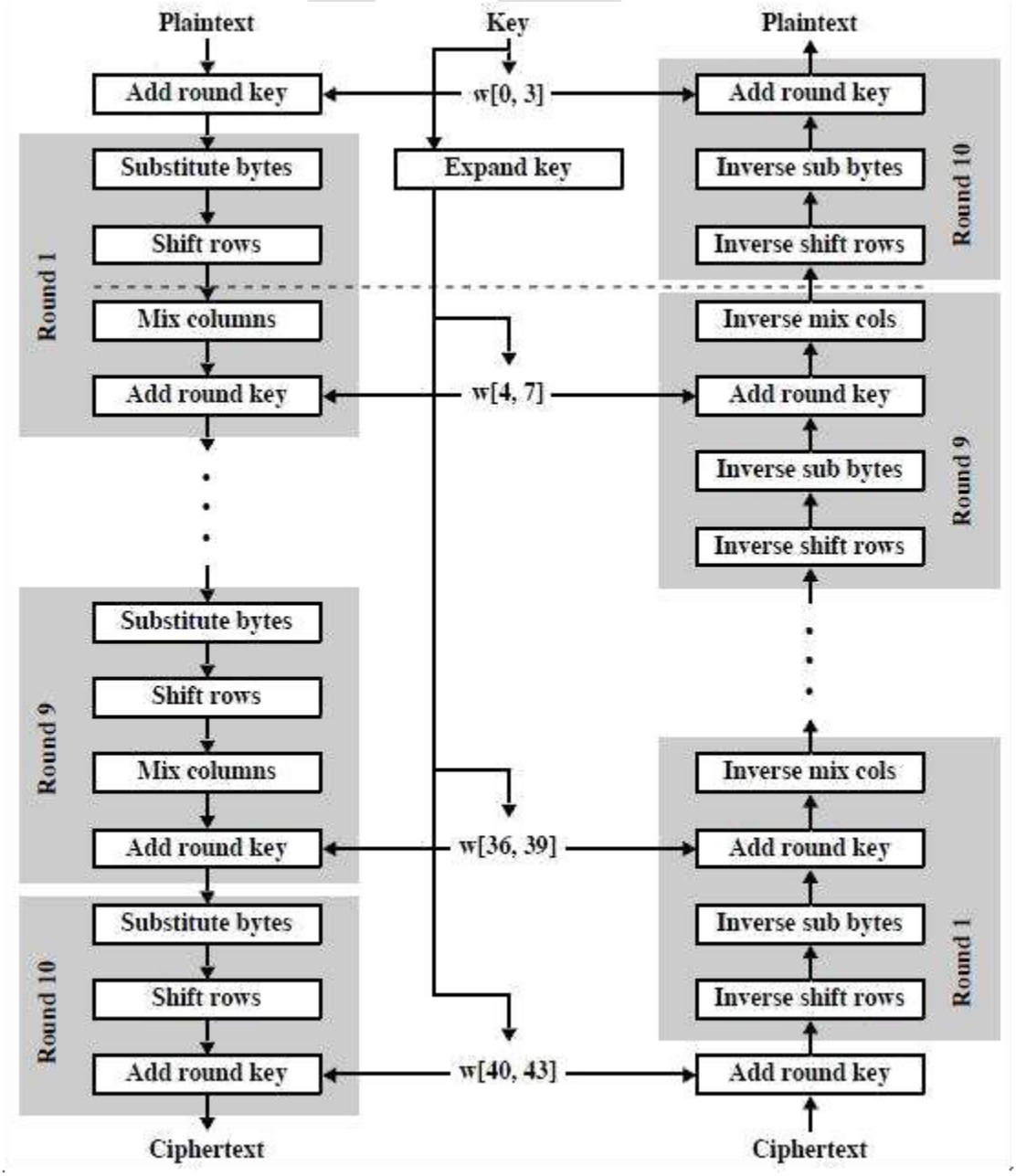
The AES Cipher

- The Rijndael proposal for AES defined a cipher in which the block length and the key length can be independently specified to be 128, 192, or 256 bits.
- The AES specification uses the same three key size alternatives but limits the block length to 128 bits.
- A number of AES parameters depend on the key length.
- In the description of this section, we assume a key length of 128 bits, which is likely to be the one most commonly implemented

AES Parameters

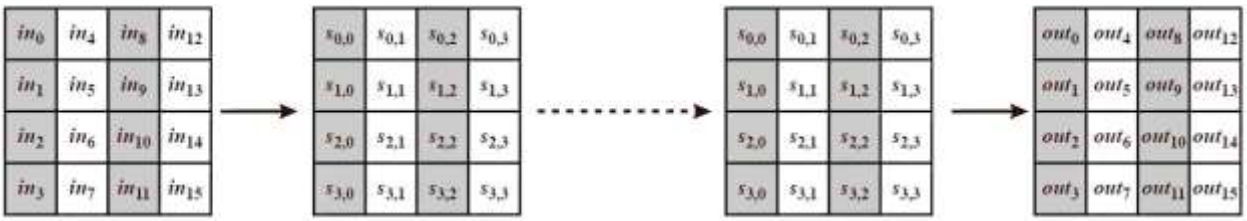
Key Size (words/bytes/bits)	4/16/128	6/24/192	8/32/256
Plaintext Block Size (words/bytes/bits)	4/16/128	4/16/128	4/16/128
Number of Rounds	10	12	14
Round Key Size (words/bytes/bits)	4/16/128	4/16/128	4/16/128
Expanded Key Size (words/bytes)	44/176	52/208	60/240

AES Encryption and Decryption



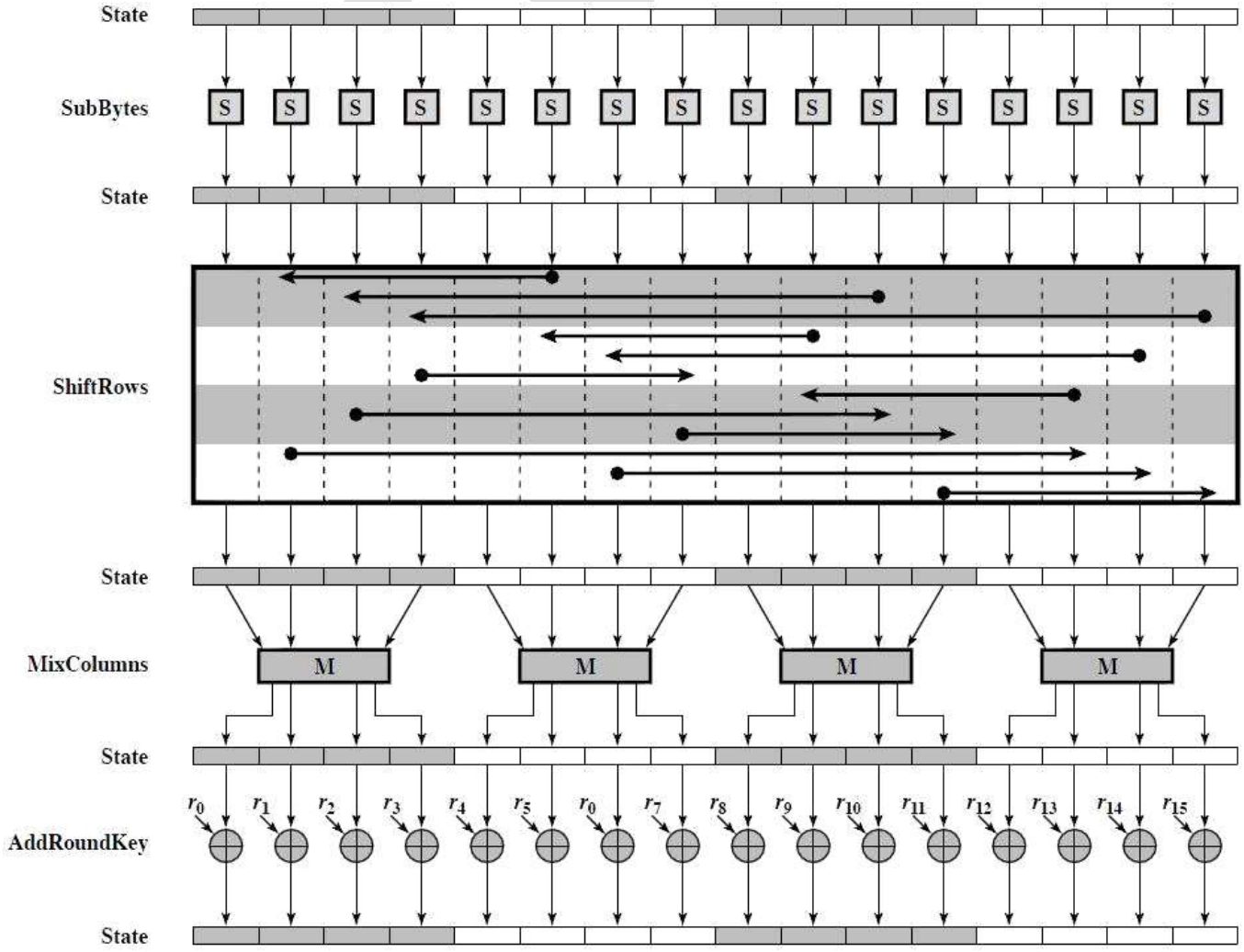
AES Data Structures

Input, state array, and output



Key and expanded key

AES Encryption Round



- Substitute Bytes Transformation
- ShiftRows Transformation
- AddRoundKey Transformation
- AES Key Expansion

Key Expansion Algorithm

The AES key expansion algorithm takes as input a four-word (16-byte) key and produces a linear array of 44 words (176 bytes). This is sufficient to provide a four-word round key for the initial AddRoundKey stage and each of the 10 rounds of the cipher. The pseudocode on the next page describes the expansion.

The key is copied into the first four words of the expanded key. The remainder of the expanded key is filled in four words at a time. Each added word **w**[i] depends on the immediately preceding word, **w**[i − 1], and the word four positions back, w[i − 4]. In three out of four cases, a simple XOR is used. For a word whose position in the **w** array is a multiple of 4, a more complex function is used. Figure 5.9 illustrates the generation of the expanded key, using the symbol g to represent that complex function. The function g consists of the following subfunctions.

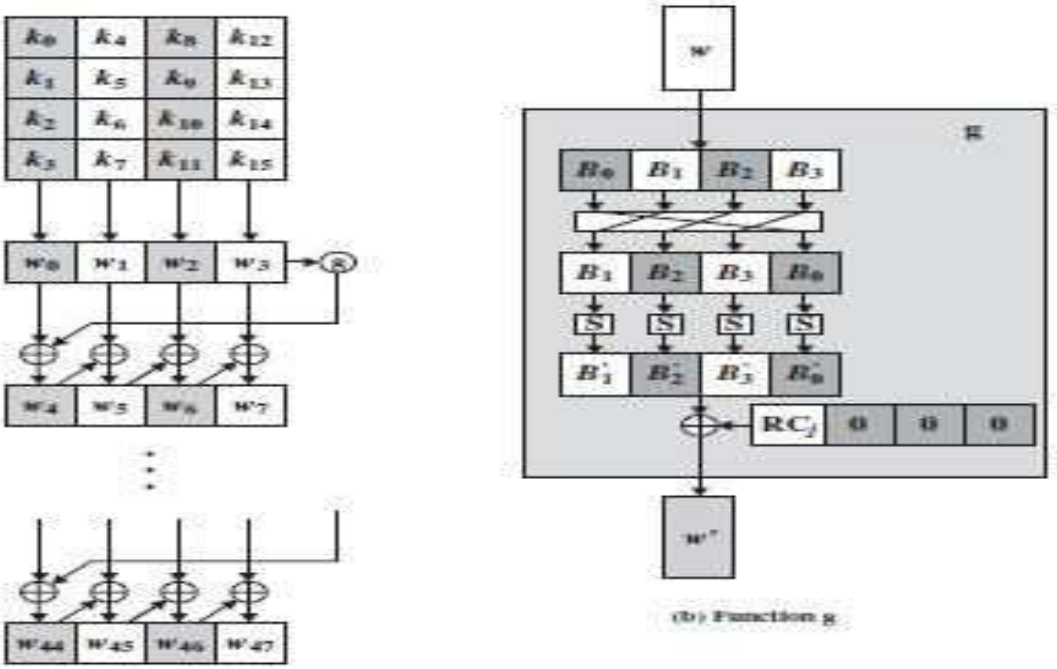
1. RotWord performs a one-byte circular left shift on a word. This means that an input word $[B_0, B_1, B_2, B_3]$ is transformed into $[B_1, B_2, B_3, B_0]$.
2. SubWord performs a byte substitution on each byte of its input word, using the S-box (Table 5.2a).
3. The result of steps 1 and 2 is XORed with a round constant, Rcon[j].

The round constant is a word in which the three rightmost bytes are always 0. Thus, the effect of an XOR of a word with Rcon is to only perform an XOR on the leftmost byte of the word. The round constant is different for each round and is defined as Rcon[j] = (RC[j], 0, 0, 0), with RC[1] = 1, RC[j] = 2 · RC[j−1] and with multiplication defined over the field $GF(2^8)$. The values of RC[j] in hexadecimal are

j	1	2	3	4	5	6	7	8	9	10
RC[j]	01	02	04	08	10	20	40	80	1B	36

```
KeyExpansion (byte key[16], word w[44])
{
    word temp
    for (i = 0; i < 4; i++)    w[i] = (key[4*i], key[4*i+1],
                                       key[4*i+2],
                                       key[4*i+3]);

    for (i = 4; i < 44; i++)
    {
     temp = w[i - 1];
     if (i mod 4 = 0)    temp = SubWord (RotWord (temp))
                                ⊕ Rcon[i/4];
     w[i] = w[i-4] ⊕ temp
    }
}
```

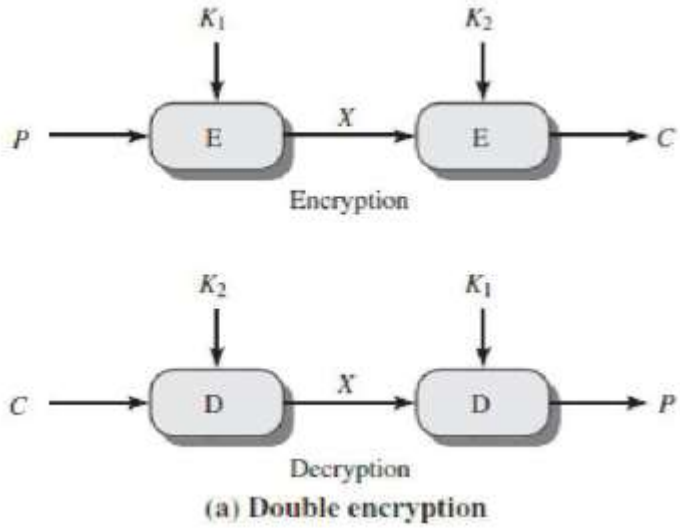


(a) Overall algorithm

(b) Function g

Figure 5.9 AES Key Expansion

Double DES



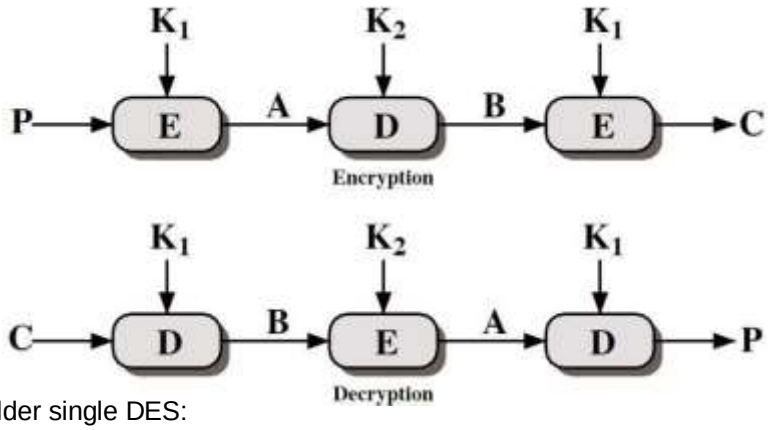
(a) Double encryption

- has two encryption stages and two keys
- Given a plaintext P and two encryption keys K1 and K2 and , ciphertext C is generated as C = E(K2, E(K1, P))
- Decryption requires that the keys be applied in reverse order P = D(K1, D(K2, C))
- this scheme apparently involves a key length of 56 * 2 = 112 bits, resulting in a dramatic increase in cryptographic strength

Meet-In-The-Middle Attack

- It is based on the observation that, if we have C = E(K2, E(K1, P)) then X = E(K1, P) = D(K2, C)
- Given a known pair, (P, C) the attack proceeds as follows
- First, encrypt P for all 2^{56} possible values of K1
- Store these results in a table and then sort the table by the values of X
- Next, decrypt C using all 2^{56} possible values of K2
- As each decryption is produced, check the result against the table for a match.
- If a match occurs, then test the two resulting keys against a new known plaintext–ciphertext pair.
- If the two keys produce the correct ciphertext, accept them as the correct keys.
- For any given plaintext P, there are 2^{64} possible ciphertext values that could be produced by double DES
- the foregoing procedure will produce about 2^{48} false alarms on the first (P,C) pair.
- With an additional 64 bits of known plaintext and ciphertext, the false alarm rate is reduced to $2^{48-64} = 2^{-16}$.
- If the meet-in-the-middle attack is performed on two blocks of known plaintext–ciphertext, the probability that the correct keys are determined is $1 - 2^{-16}$.
- The result is that a known plaintext attack will succeed against double DES, which has a key size of 112 bits, with an effort on the order of 2^{56}, which is not much more than the 2^{55} required for single DES

Triple DES



- triple encryption method that uses only two keys
- The function follows an encrypt-decrypt-encrypt (EDE) sequence
- C = E(K1, D(K2, E(K1, P)))
- There is no cryptographic significance to the use of decryption for the second stage.
- advantage is that it allows users of 3DES to decrypt data encrypted by users of the older single DES:
- C = E(K1, D(K1, E(K1, P))) = E(K1, P)

Attacks on TDES

Known-Plaintext Attack on Triple DES

Triple DES with Three Keys

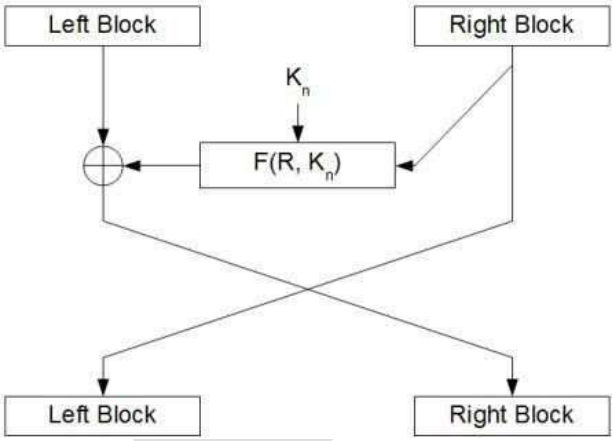
- Three-key 3DES has an effective key length of 168 bits and is defined as follows:
- C = E(K3, D(K2, E(K1, P)))
- Backward compatibility with DES is provided by putting K3 = K2 or K1 = K2.
- A number of Internet-based applications have adopted three-key 3DES, including PGP and S/MIME

Blowfish

- Symmetric block cipher that can be effectively used for encryption and safeguarding of data
- It takes a variable-length key, from 32 bits to 448 bits, making it ideal for securing data.
- fast, free alternative to existing encryption algorithms
- unpatented and license-free, and is available free for all uses
- **Blowfish Algorithm** is a Feistel Network, iterating a simple encryption function 16 times.
- The block size is 64 bits, and the key can be any length up to 448 bits.
- Although there is a complex initialization phase required before any encryption can take place, the actual encryption of data is very efficient on large microprocessors.
- Blowfish is a variable-length key block cipher.
- It is suitable for applications where the key does not change often, like a communications link or an automatic file encryptor.
- It is significantly faster than most encryption algorithms when implemented on 32-bit microprocessors with large data caches

Feistel Networks

- A Feistel network is a general method of transforming any function (usually called an Ffunction) into a permutation.
- It was invented by Horst Feistel and has been used in many block cipher designs.
- The working of a Feistal Network is given below:
  - Split each block into halves
  - Right half becomes new left half
  - New right half is the final result when the left half is XOR'd with the result of applying f to the right half and the key.
  - Note that previous rounds can be derived even if the function f is not invertible



The Blowfish Algorithm:

- Manipulates data in large blocks
- Has a 64-bit block size.
- Has a scalable key, from 32 bits to at least 256 bits.
- Uses simple operations that are efficient on microprocessors.
  - e.g., exclusive-or, addition, table lookup, modular- multiplication.
  - It does not use variable-length shifts or bit-wise permutations, or conditional jumps.
- Employs precomputable subkeys.
  - On large-memory systems, these subkeys can be precomputed for faster operation.
  - Not precomputing the subkeys will result in slower operation, but it should still be possible to encrypt data without any precomputations.
- Consists of a variable number of iterations.
- Uses subkeys that are a one-way hash of the key.
  - This allows the use of long passphrases for the key without compromising security.
- Has no linear structures that reduce the complexity of exhaustive search.
- Uses a design that is simple to understand.

Description Of The Algorithm

- Blowfish is a variable-length key, 64-bit block cipher.
- The algorithm consists of two parts:
  - a key-expansion part and
  - a data- encryption part.
- Key expansion converts a key of at most 448 bits into several subkey arrays totaling 4168 bytes.
- Data encryption occurs via a 16-round Feistel network.
- Each round consists of a keydependent permutation, and a key- and data-dependent substitution.
- All operations are XORs and additions on 32-bit words.

- The only additional operations are four indexed array data lookups per round

Subkeys

- Blowfish uses a large number of subkeys.
- These keys must be precomputed before any data encryption or decryption.
- The P-array consists of 18 32-bit subkeys:P1, P2,..., P18.
- There are four 32-bit S-boxes with 256 entries each:
  - S1,0, S1,1,..., S1,255;
  - S2,0, S2,1,..,, S2,255;
  - S3,0, S3,1,..., S3,255;
  - S4,0, S4,1,..,, S4,255.

Encryption

- Blowfish has 16 rounds.
- The input is a 64-bit data element, x.
- Divide x into two 32-bit halves: xL, xR.
- Then,

```
for i = 1 to 16:
      xL = xL XOR Pi
      xR = F(xL) XOR xR
      Swap xL and xR
```

- After the sixteenth round, swap xL and xR again to undo the last swap.
- Then, xR = xR XOR P17 and xL = xL XOR P18.
- Finally, recombine xL and xR to get the ciphertext.

Decryption

- Exactly the same as encryption, except that P1, P2,..., P18 are used in the reverse order

Generating the Subkeys

The subkeys are calculated using the Blowfish algorithm:

1. Initialize first the P-array and then the four S-boxes, in order, with a fixed string.

This string consists of the hexadecimal digits of pi (less the initial 3): P1 = 0x243f6a88, P2 = 0x85a308d3, P3 = 0x13198a2e, P4 = 0x03707344, etc.

2. XOR P1 with the first 32 bits of the key, XOR P2 with the second 32-bits of the key, and so on for all bits of the key (possibly up to P14). Repeatedly cycle through the key bits until the entire P-array has been XORed with key bits. (For every short key, there is at least one equivalent longer key; for example, if A is a 64-bit key, then AA, AAA, etc., are equivalent keys.)

3. Encrypt the all-zero string with the Blowfish algorithm, using the subkeys described in steps (1) and (2).

4. Replace P1 and P2 with the output of step (3).

5. Encrypt the output of step (3) using the Blowfish algorithm with the modified subkeys.

6. Replace P3 and P4 with the output of step (5).

7. Continue the process, replacing all entries of the P array, and then all four S-boxes in order, with the output of the continuously changing Blowfish algorithm.

- In total, 521 iterations are required to generate all required subkeys.
- Applications can store the subkeys rather than execute this derivation process multiple times.

RC5

Introduction

- a proprietary cipher owned by RSADSI
- designed by Ronald Rivest (of RSA fame)
- used in various RSADSI products
- can vary key size / data size / no rounds
- very clean and simple design
- easy implementation on various CPUs
- yet still regarded as secure

RC5 Ciphers

- RC5 is a family of ciphers RC5-w/r/b
  - w = word size in bits (16/32/64) nb data=2w
  - r = number of rounds (0..255)
  - b = number of bytes in key (0..255)
- nominal version is RC5-32/12/16
  - 32-bit words so encrypts 64-bit data blocks
  - using 12 rounds
  - with 16 bytes (128-bit) secret key

RC5 Key Expansion

- RC5 uses 2r+2 subkey words (w-bits)
- subkeys are stored in array S[i], i=0..t-1
- the key schedule consists of
  - initializing S to a fixed pseudorandom value, based on constants e and phi
  - the byte key is copied (little-endian) into a c-word array L
  - a mixing operation then combines L and S to form the final S array

RC5 Encryption

- split input into two halves A & B
- L0 = A + S[0];
- R0 = B + S[1];
- for i = 1 to r do
  - Li = ((Li-1 XOR Ri-1) <<< Ri-1) + S[2 x i];
  - Ri = ((Ri-1 XOR Li) <<< Li) + S[2 x i + 1];
- each round is like 2 DES rounds
- note rotation is main source of non-linearity
- need reasonable number of rounds (eg 12-16)

RC5 Modes

- RFC2040 defines 4 modes used by RC5
- RC5 Block Cipher, is ECB mode
- RC5-CBC, is CBC mode
- RC5-CBC-PAD, is CBC with padding by bytes with value being the number of padding bytes
- RC5-CTS, a variant of CBC which is the same size as the original message, uses ciphertext stealing to keep size same as original.

Public Key Cryptography

Principles of Public-Key Cryptosystems

- Public-Key Cryptosystems
- Applications for Public-Key Cryptosystems
- Requirements for Public-Key Cryptography
- Public-Key Cryptanalysis

Public-Key Cryptosystems

Introduction

- The concept evolved from an attempt to attack two of the most difficult problems associated with symmetric encryption
  - Key Distribution
  - The Digital Signatures
- Called as Asymmetric Cryptography
- Asymmetric algorithms make use of one key for encryption, another for decryption

Characteristics of Asymmetric algorithms

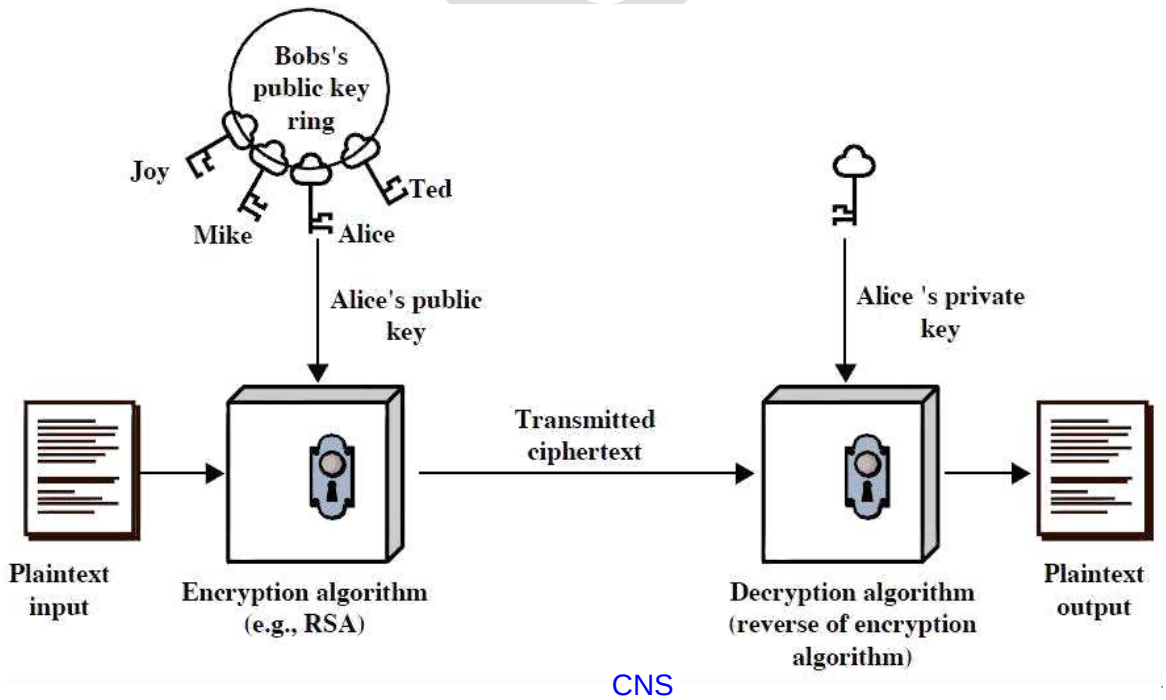
- It is computationally infeasible to determine the decryption key given only knowledge of the cryptographic algorithm and the encryption key
- Either of the two related keys can be used for encryption, with the other used for decryption

Public-Key Cryptography

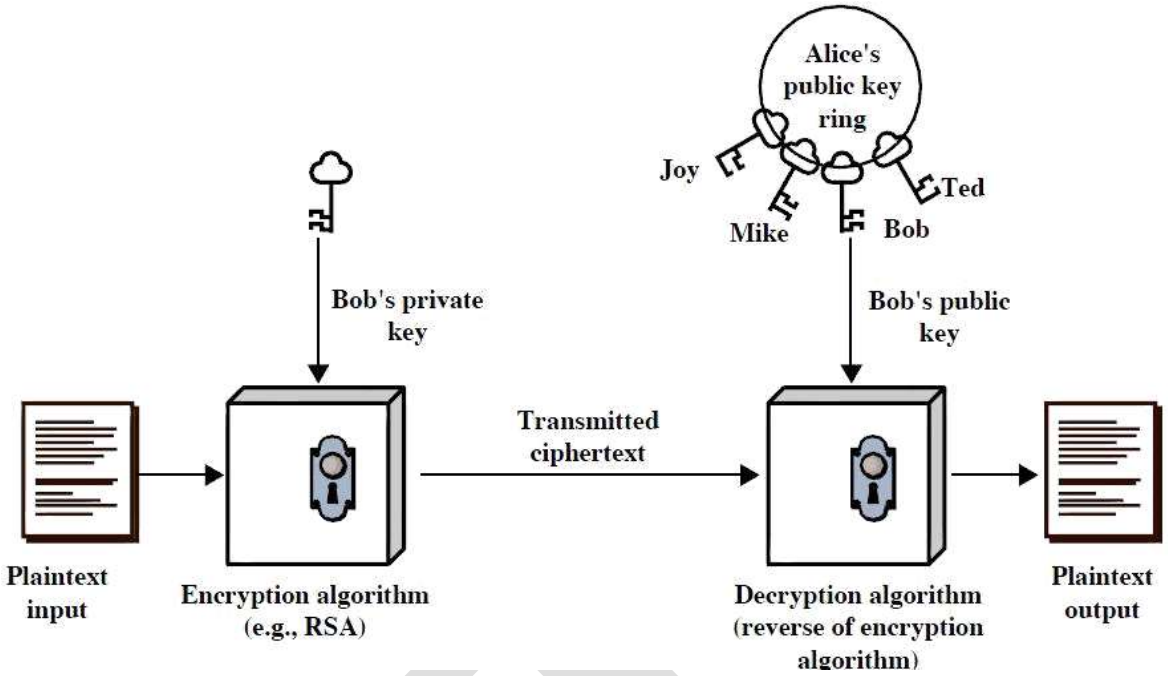
Six Ingredients

- **Plaintext**: This is the readable message or data that is fed into the algorithm as input.
- **Encryption algorithm**: The encryption algorithm performs various transformations on the plaintext.
- **Public and private keys**: This is a pair of keys that have been selected so that if one is used for encryption, the other is used for decryption. The exact transformations performed by the algorithm depend on the public or private key that is provided as input.
- **Ciphertext**: This is the scrambled message produced as output. It depends on the plaintext and the key. For a given message, two different keys will produce two different ciphertexts.
- **Decryption algorithm**: This algorithm accepts the ciphertext and the matching key and produces the original plaintext.

Encryption



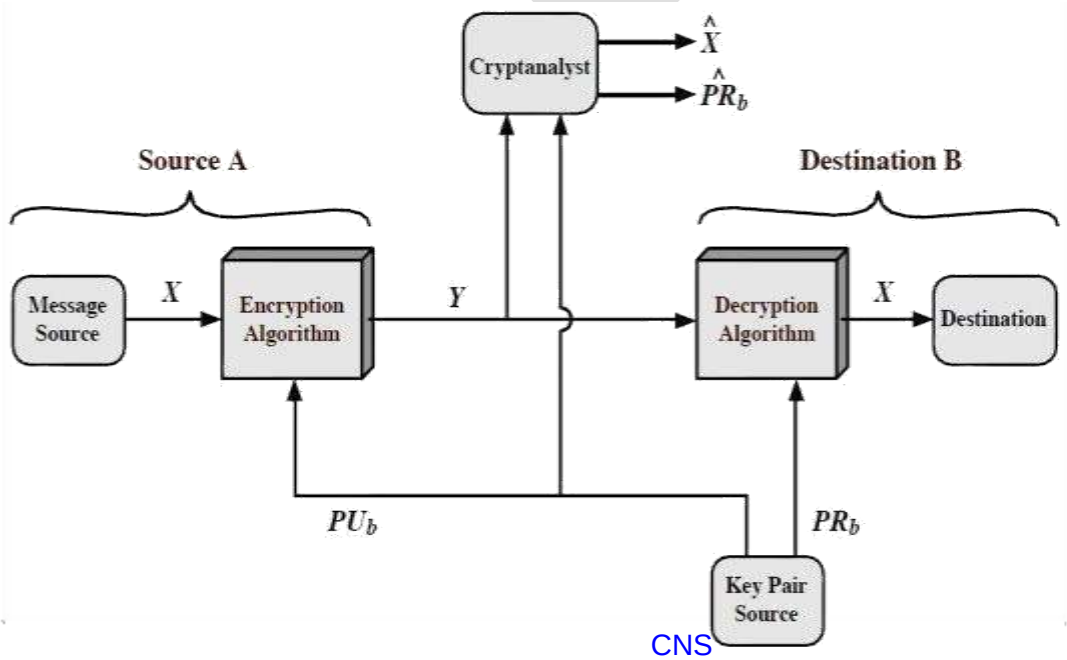
Authentication



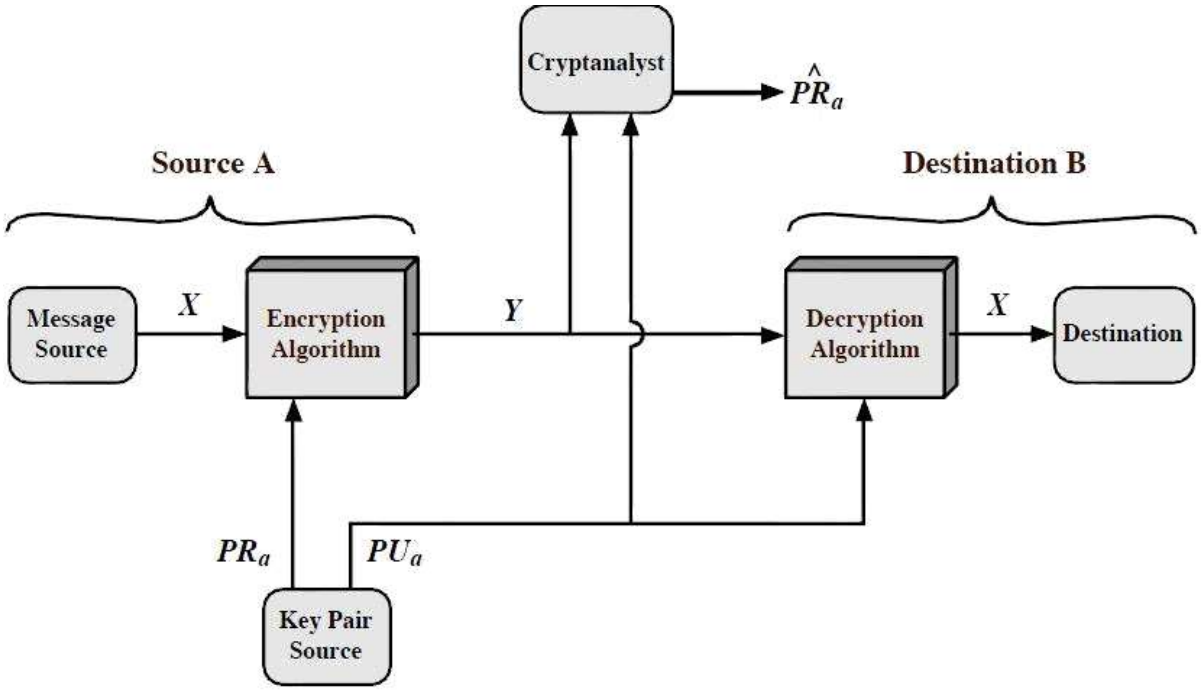
Comparison with Symmetric Key Encryption

Conventional Encryption	Public-Key Encryption
Needed to Work 1. The same algorithm with the same key is used for encryption and decryption. 2. The sender and receiver must share the algorithm and the key.	*Needed to Work* 1. One algorithm is used for encryption and decryption with a pair of keys, one for encryption and one for decryption. 2. The sender and receiver must each have one of the matched pair of keys (not the same one).
Needed for Security: 1. The key must be kept secret. 2. It must be impossible or at least impractical to decipher a message if no other information is available. 3. Knowledge of the algorithm plus samples of ciphertext must be insufficient to determine the key.	*Needed for Security:* 1. One of the two keys must be kept secret. 2. It must be impossible or at least impractical to decipher a message if no other information is available. 3. Knowledge of the algorithm plus one of the keys plus samples of ciphertext must be insufficient to determine the other key.

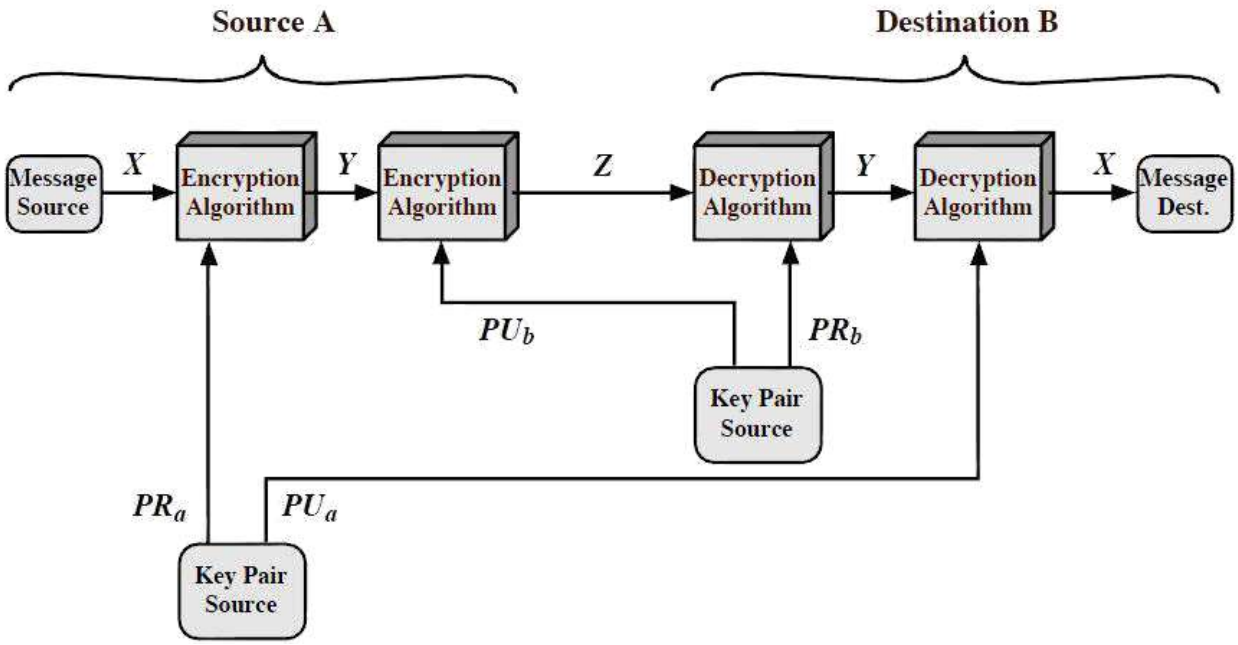
Public-Key Cryptosystem: Secrecy



Public-Key Cryptosystem: Authentication



Public-Key Cryptosystem: Authentication and Secrecy



Applications for Public-Key Cryptosystems

Encryption/decryption

The sender encrypts a message with the recipient's public key.

Digital signature

The sender "signs" a message with its private key. Signing is achieved by a cryptographic algorithm applied to the message or to a small block of data that is a function of the message.

Key exchange

Two sides cooperate to exchange a session key. Several different approaches are possible, involving the private key(s) of one or both parties.

- original message, M.
- The two keys can be applied in either order: M = D[PUb, E(PRb, M)] = D[PRb, E(PUb, M)]

Public-Key Cryptanalysis

Three types of attacks

- Brute force
- Deducing the private key
- Probale message attack

Brute force

- public-key encryption scheme is vulnerable to a brute-force attack
- countermeasure is to use large keys
- Public-key systems depend on the use of some sort of invertible mathematical function
- the key size must be large enough to make brute-force attack impractical but small enough for practical encryption and decryption

Deducing the private key

- find some way to compute the private key given the public key
- So far, not been mathematically proven that this is infeasible for a particular public-key algorithm
- Not been successful till date

Probale message attack

- peculiar to public-key systems
- Suppose, for example, that a message were to be sent that consisted solely of a 56-bit DES key.
- An adversary could encrypt all possible 56-bit DES keys using the public key and could discover the encrypted key by matching the transmitted ciphertext.
- Thus, no matter how large the key size of the public-key scheme, the attack is reduced to a brute-force attack on a 56-bit key.
- This attack can be thwarted by appending some random bits to such simple messages

Rivest-Shamir-Adleman (RSA) Algorithm

- block cipher in which the plaintext and ciphertext are integers between 0 and n - 1 for some n.
- A typical size for n is 1024 bits, or 309 decimal digits
- public-key encryption algorithm with a public key of PU = {e, n} and a private key of PR = {d, n}.

Key Generation

Select p, q	p and q both prime, $p \neq q$
Calculate $n = p \times q$	
Calcuate $\phi(n) = (p-1)(q-1)$	
Select integer e	$\gcd(\phi(n), e) = 1; 1 < e < \phi(n)$
Calculate d	$d \equiv e^{-1} \pmod{\phi(n)}$
Public key	$PU = \{e, n\}$
Private key	$PR = \{d, n\}$

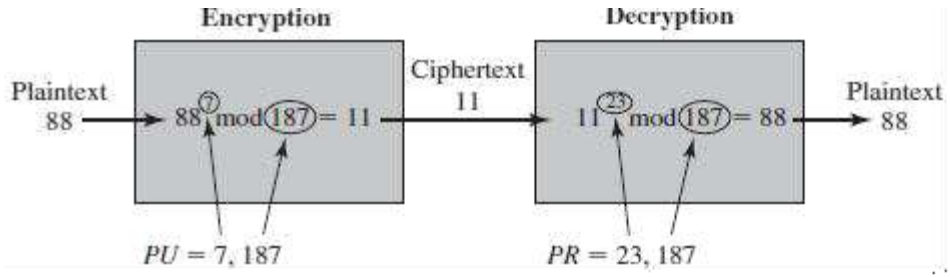
Encryption

Plaintext:	$M < n$
Ciphertext:	$C = M^e \bmod n$

Decryption

Ciphertext:	C
Plaintext:	$M = C^d \bmod n$

Example of RSA Algorithm



1. Select two prime numbers, $p = 17$ and $q = 11$.
2. Calculate $n = pq = 17 \times 11 = 187$.
3. Calculate $\phi(n) = (p-1)(q-1) = 16 \times 10 = 160$.
4. Select e such that e is relatively prime to $\phi(n) = 160$ and less than $\phi(n)$; we choose $e = 7$.
5. Determine d such that $de \equiv 1 \pmod{160}$ and $d < 160$. The correct value is $d = 23$, because $23 \times 7 = 161 = (1 \times 160) + 1$; d can be calculated using the extended Euclid's algorithm (Chapter 4).

The resulting keys are public key $PU = \{7, 187\}$ and private key $PR = \{23, 187\}$.

Encryption

we need to calculate $C = 88^7 \bmod 187$.

$$88^7 \bmod 187 = [(88^4 \bmod 187) \times (88^2 \bmod 187) \times (88^1 \bmod 187)] \bmod 187$$

$$88^1 \bmod 187 = 88$$

$$88^2 \bmod 187 = 7744 \bmod 187 = 77$$

$$88^4 \bmod 187 = 59{,}969{,}536 \bmod 187 = 132$$

$$88^7 \bmod 187 = (88 \times 77 \times 132) \bmod 187 = 894{,}432 \bmod 187 = 11$$

Decryption

For decryption, we calculate $M = 11^{23} \bmod 187$:

$$11^{23} \bmod 187 = [(11^1 \bmod 187) \times (11^2 \bmod 187) \times (11^4 \bmod 187) \times (11^8 \bmod 187) \times (11^8 \bmod 187)] \bmod 187$$

$$11^1 \bmod 187 = 11$$

$$11^2 \bmod 187 = 121$$

$$11^4 \bmod 187 = 14{,}641 \bmod 187 = 55$$

$$11^8 \bmod 187 = 214{,}358{,}881 \bmod 187 = 33$$

$$11^{23} \bmod 187 = (11 \times 121 \times 55 \times 33 \times 33) \bmod 187 = 79{,}720{,}245 \bmod 187 = 88$$

The Security of RSA

Four possible approaches to attacking the RSA algorithm are as follows:

Brute force

- This involves trying all possible private keys.

- The defense is to use a large key space
- the larger the number of bits in e and d, the better
- the larger the size of the key, the slower the system will run

Mathematical attacks

There are several approaches, all equivalent in effort to factoring the product of two primes.

Timing attacks

These depend on the running time of the decryption algorithm.

Chosen ciphertext attacks

This type of attack exploits properties of the RSA algorithm.

Mathematical Attacks

- Three approaches to attacking RSA mathematically:

1. Factor n into its two prime factors. This enables calculation of $\phi(n) = (p-1) \times (q-1)$, which in turn enables determination of $d \equiv e^{-1} \pmod{\phi(n)}$.
2. Determine $\phi(n)$ directly, without first determining p and q. Again, this enables determination of $d \equiv e^{-1} \pmod{\phi(n)}$.
3. Determine d directly, without first determining $\phi(n)$.

To avoid values of n that may be factored more easily, the algorithm's inventors suggest the following constraints on p and q.

1. p and q should differ in length by only a few digits. Thus, for a 1024-bit key (309 decimal digits), both p and q should be on the order of magnitude of 10^{75} to 10^{100}.
2. Both $(p-1)$ and $(q-1)$ should contain a large prime factor.
3. $\gcd(p-1, q-1)$ should be small.

if $e < n$ and $d < n^{1/4}$, then d can be easily determined

Timing Attack

- This attack is alarming for two reasons:
  - It comes from a completely unexpected direction
  - it is a ciphertext-only attack
- A timing attack is somewhat analogous to a burglar guessing the combination of a safe by observing how long it takes for someone to turn the dial from number to number.
- We can explain the attack using the modular exponentiation algorithm
- modular exponentiation is accomplished bit by bit, with one modular multiplication performed at each iteration and an additional modular multiplication performed for each 1 bit

Working of this attack

- The attack proceeds bit-by-bit starting with the leftmost bit, b_k
- Suppose that the first j bits are known
- For a given ciphertext, the attacker can complete the first j iterations of the for loop.
- The operation of the subsequent step depends on the unknown exponent bit.
- if the observed time to execute the decryption algorithm is always slow when this particular iteration is slow with a 1 bit, then this bit is assumed to be 1.
- If a number of observed execution times for the entire algorithm are fast, then this bit is assumed to be 0

Methods to overcome timing attacks

Constant exponentiation time

- Ensure that all exponentiations take the same amount of time before returning a result.
- This is a simple fix but does degrade performance

Message Authentication

- **Message authentication is concerned with:**
  - protecting the integrity of a message
  - validating identity of originator
  - non-repudiation of origin (dispute resolution)
- Will consider the security requirements
- **Three functions used:**
  - Message Encryption
  - Message Authentication Code (MAC)
  - Hash Function

Security Requirements

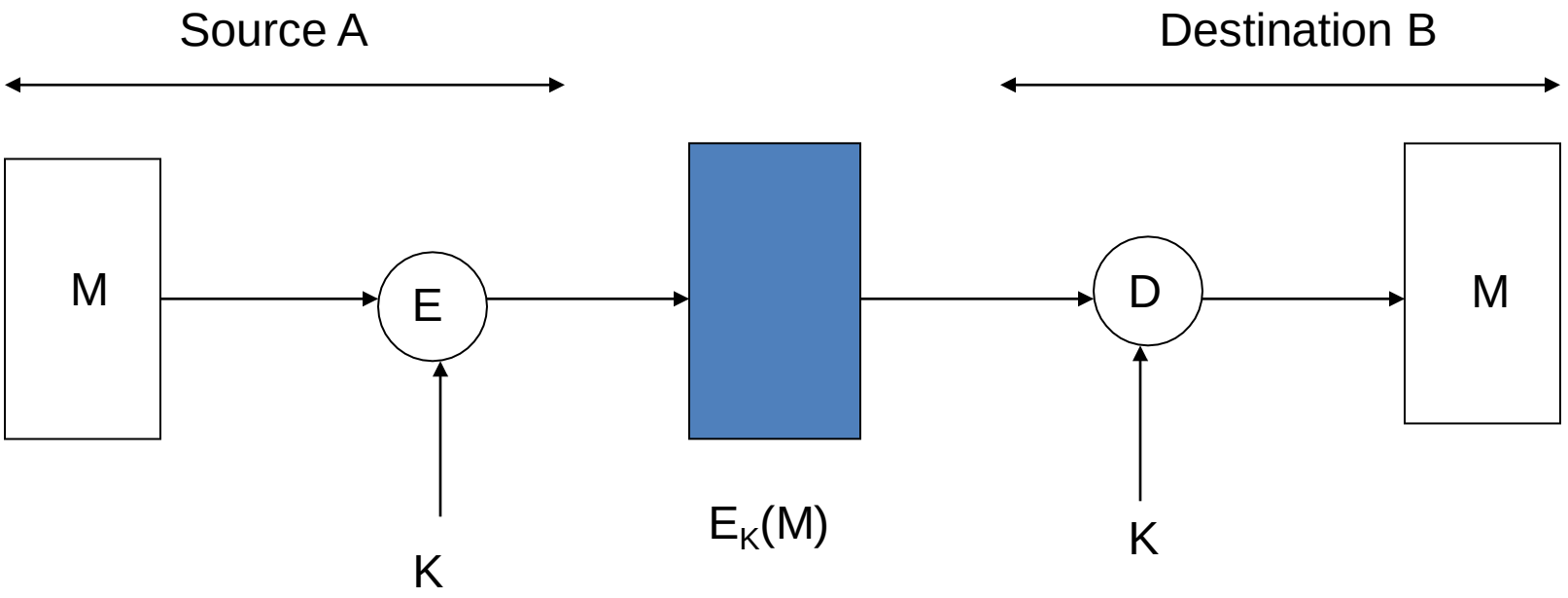
- Disclosure
- Traffic analysis
- Masquerade
- Content modification
- Sequence modification
- Timing modification
- Source repudiation
- Destination repudiation

Message Encryption

- Message encryption by itself also provides a measure of authentication
- If symmetric encryption is used then:
  - receiver knows sender must have created it
    since only sender and receiver know key used
  - knows content is not been altered
  - if message has suitable structure, redundancy or a checksum to detect any changes

Message Encryption

- Symmetric encryption: Confidentiality & Authentication

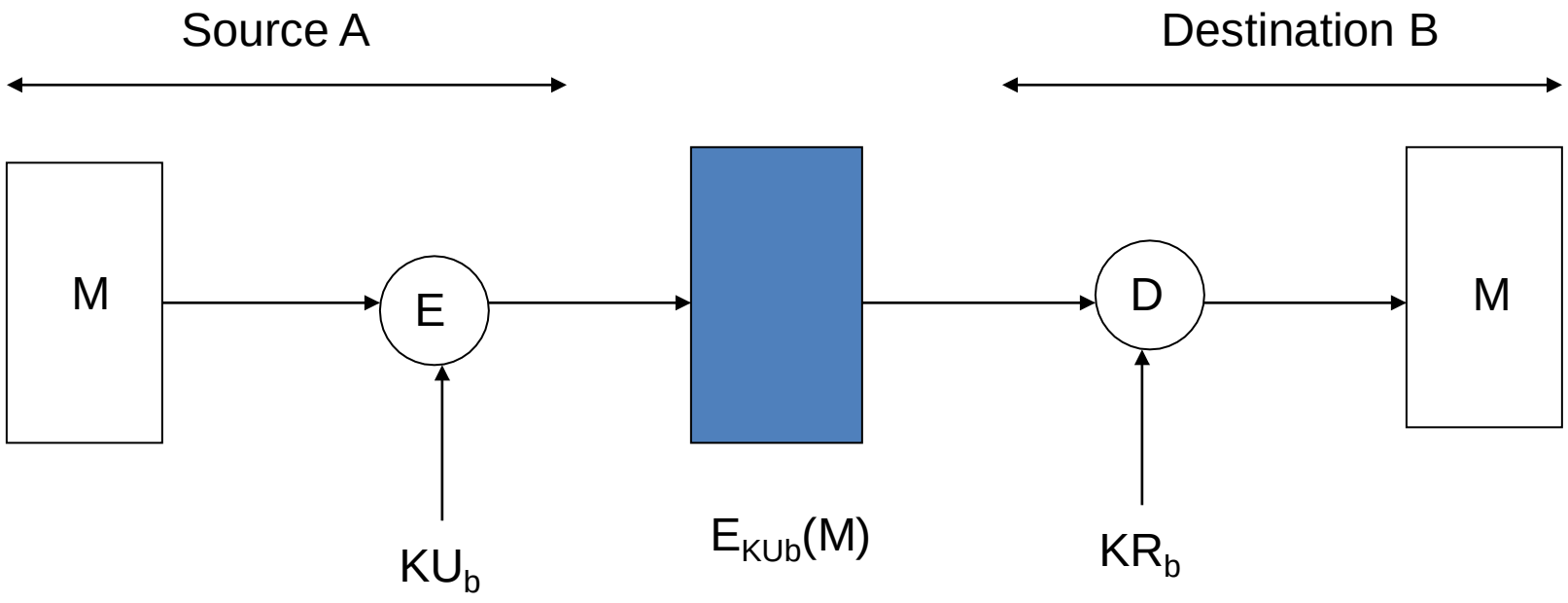


Message Encryption

- If public-key encryption is used:
  - encryption provides no confidence of sender
  - since anyone potentially knows public-key
  - however if
    - sender **signs** message using their private-key
    - then encrypts with recipients public key
    - have both secrecy and authentication

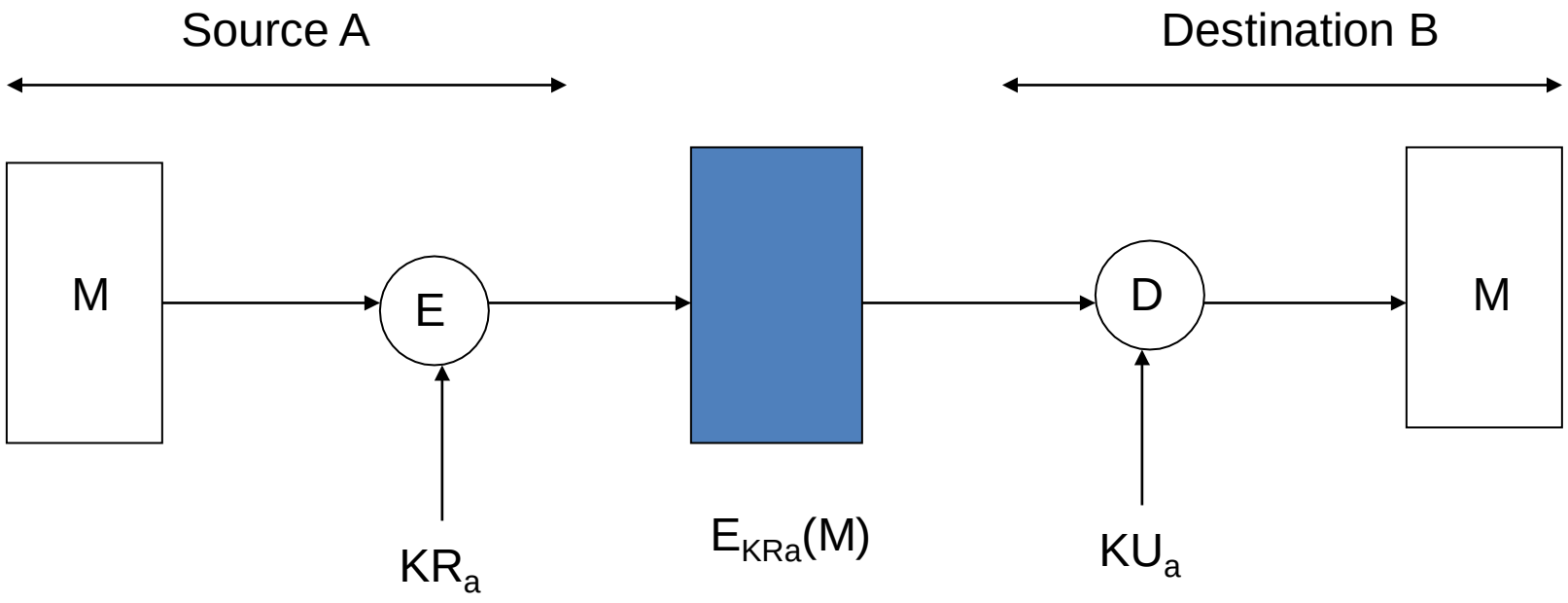
Message Encryption

- Public-key encryption : Confidentiality



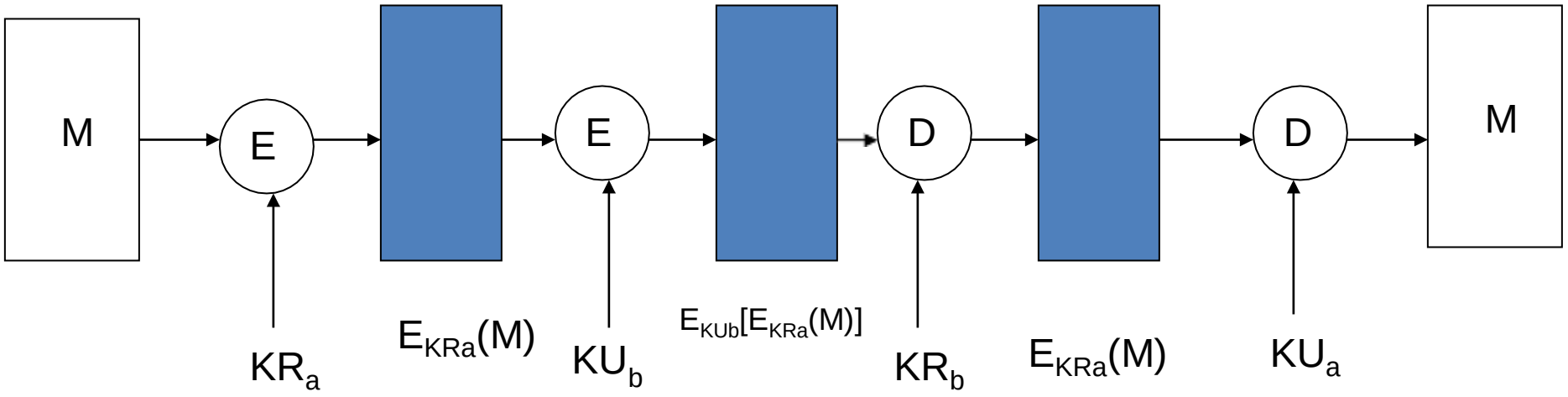
Message Encryption

- Public-key encryption : Authentication & Signature



Message Encryption

Public-key encryption : Confidentiality, Authentication & Signature



Message Authentication Code (MAC)

- Generated by an algorithm that creates a small fixed-sized block called **cryptographic checksum or MAC**
  - depending on both message and some key
  - need not be reversible like encryption
- Appended to message as a **signature**
- Receiver performs same computation on message and checks it matches the MAC
- Provides assurance that message is unaltered and comes from sender

Message Authentication Code

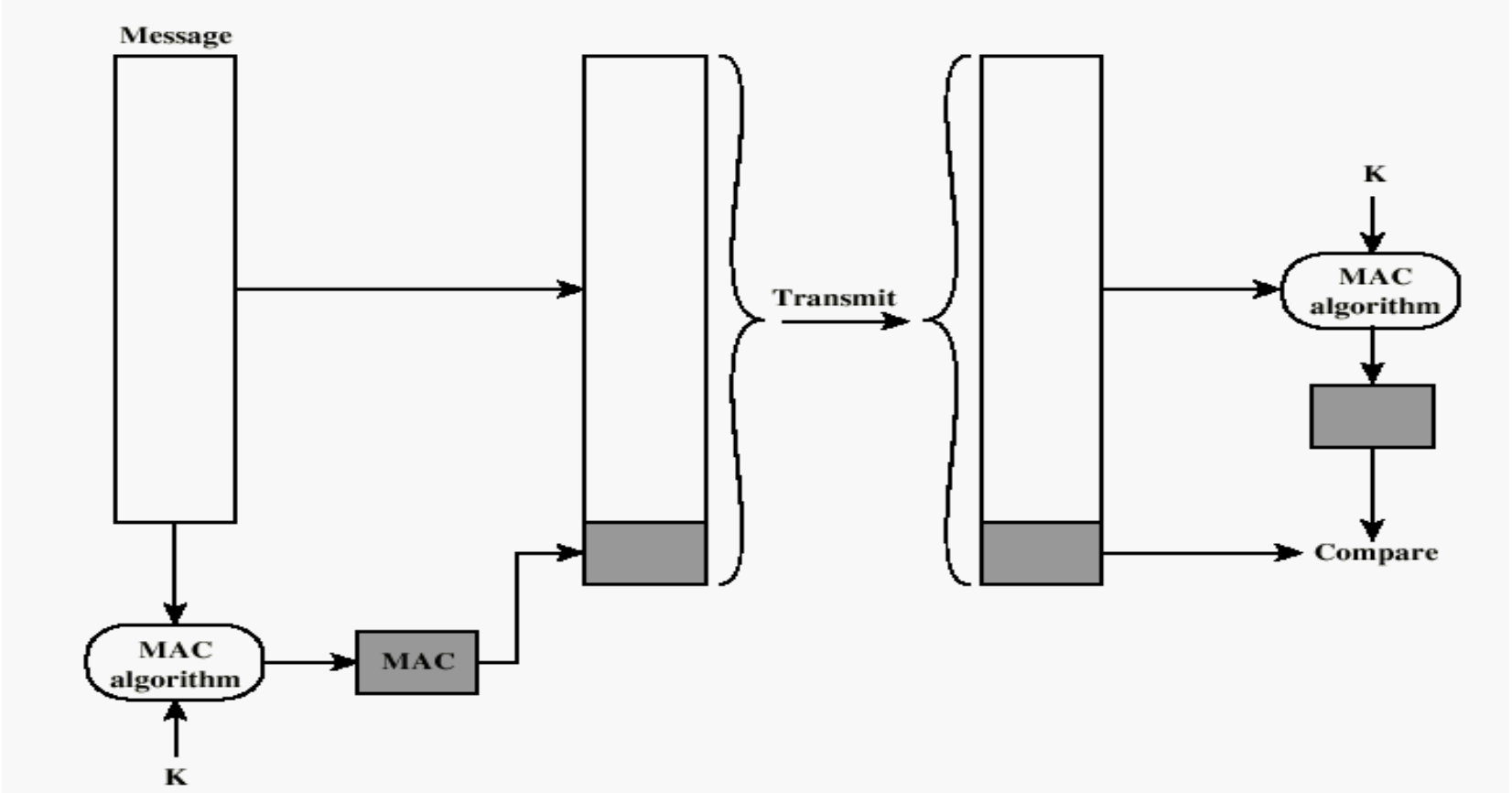


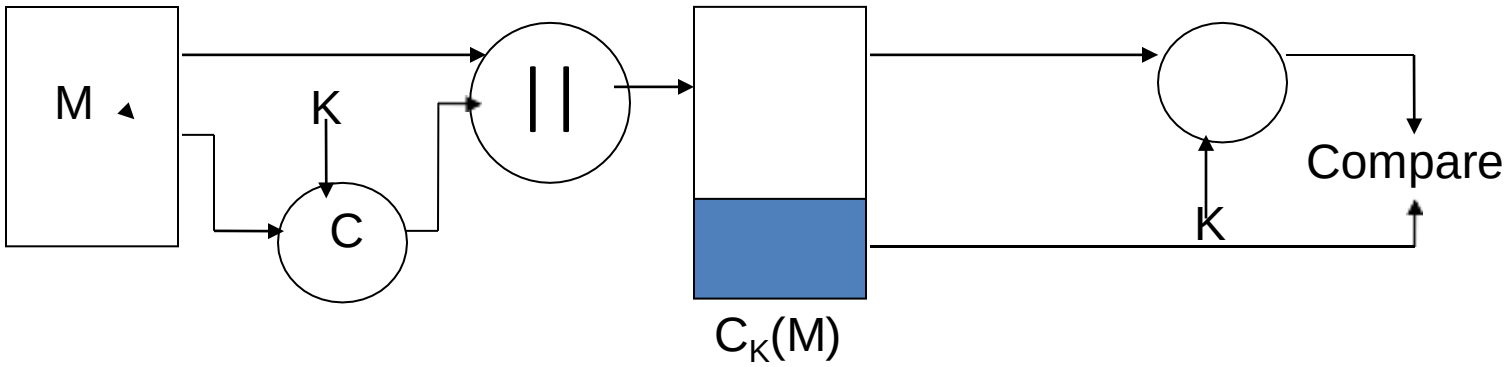
Figure 3.1 Message Authentication Using a Message Authentication Code (MAC)

Message Authentication Codes

- As shown the MAC provides authentication
- Can also use encryption for secrecy
  - generally use separate keys for each
  - can compute MAC either before or after encryption
  - is generally regarded as better done before
- If only sender and receiver only knows secret key and if MAC is matched then,
  - Receiver is assured that the message has not been altered
  - Receiver is assured that the message is from the alleged sender
  - If message includes Sequence no. then the receiver can be assured of the proper sequence

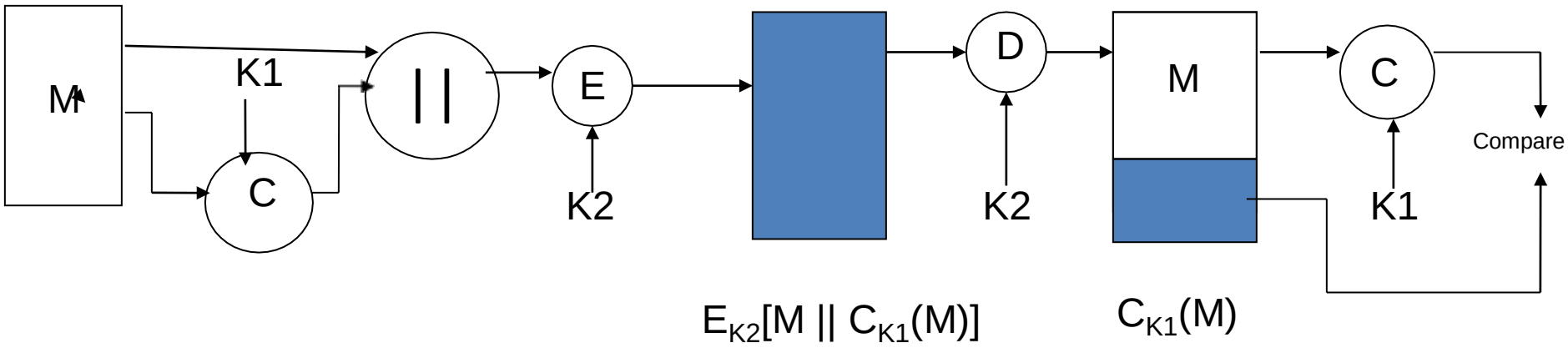
Message Authentication Codes

Message Authentication:



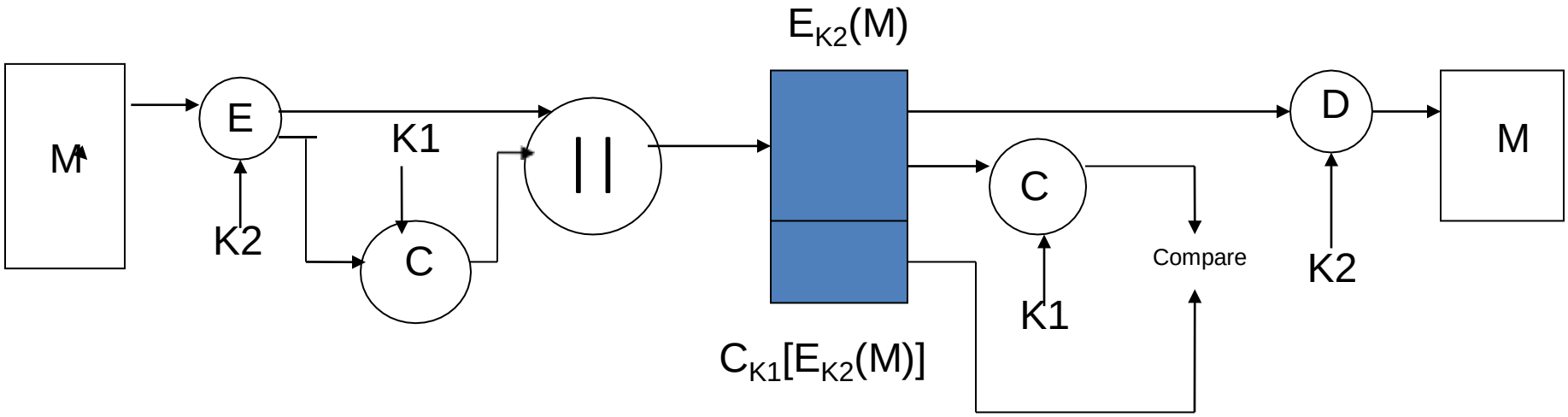
Message Authentication Codes

Message Authentication and Confidentiality: (Authentication Tied to Plaintext)



Message Authentication Codes

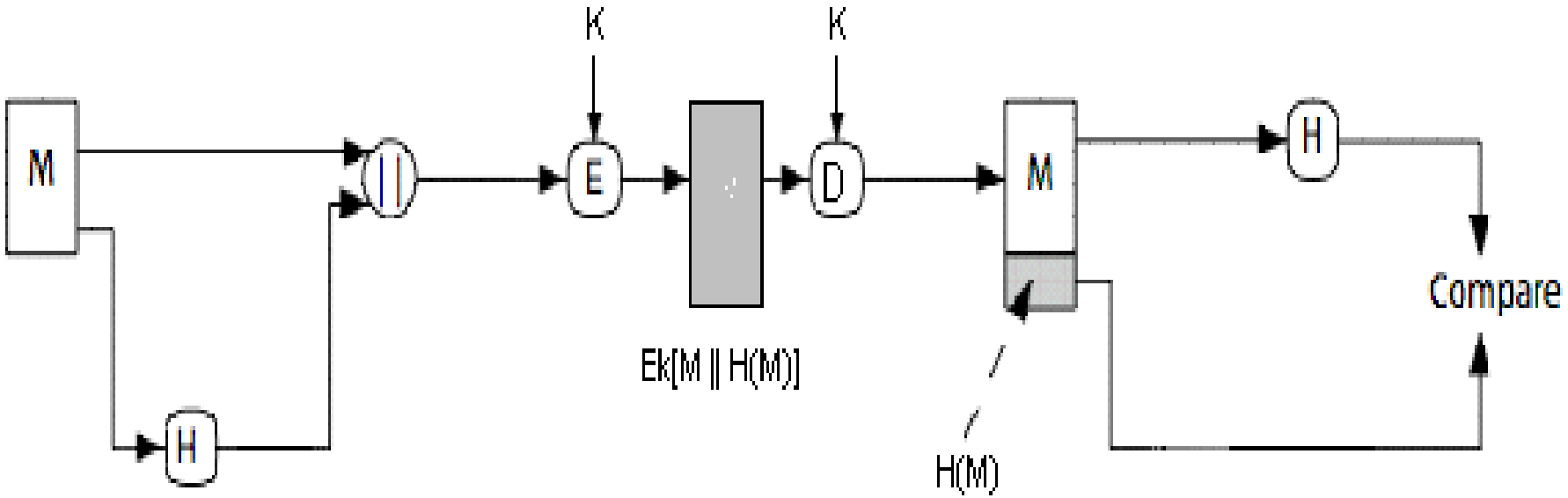
Message Authentication and Confidentiality: (Authentication Tied to Cipher text)



Hash Functions

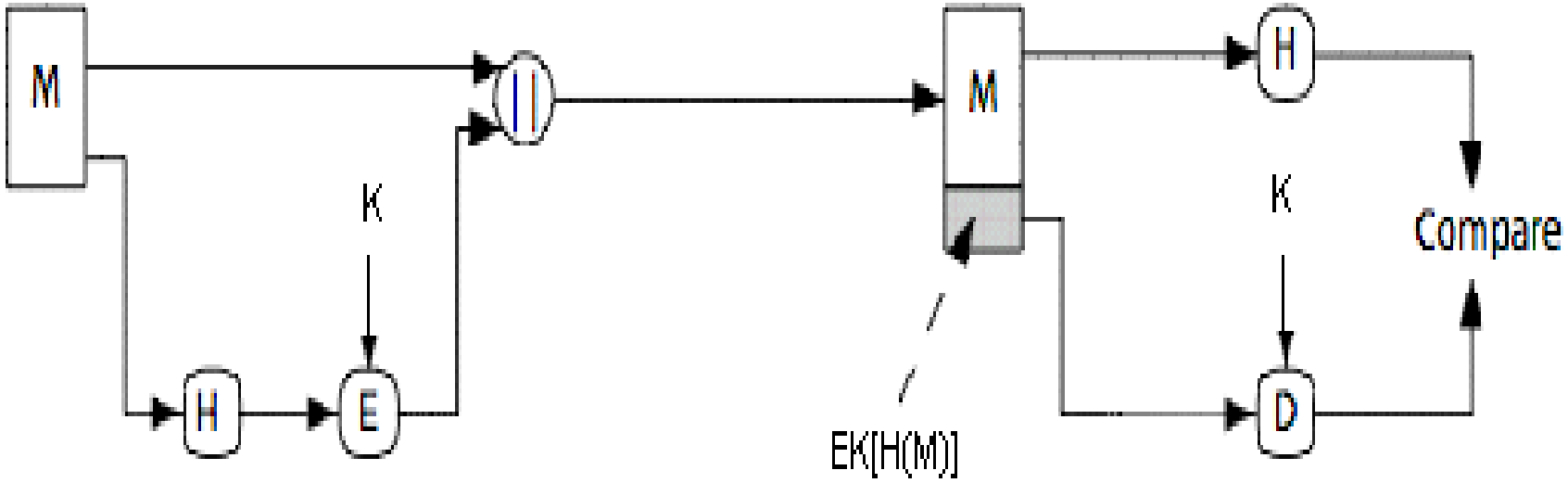
- Condenses arbitrary message to fixed size Hash code $h = H(M)$
- Also called Message digest or Hash value
- The hash function is public and not keyed (MAC is keyed)
- Hash code is a function of all bits of the message
- Change to any bit or bits in the message results in a change to the Hash code
- Most often to create a digital signature
- Can use in various ways with message

a. Message plus concatenated hash code is encrypted using symmetric encryption



Authentication and Confidentiality

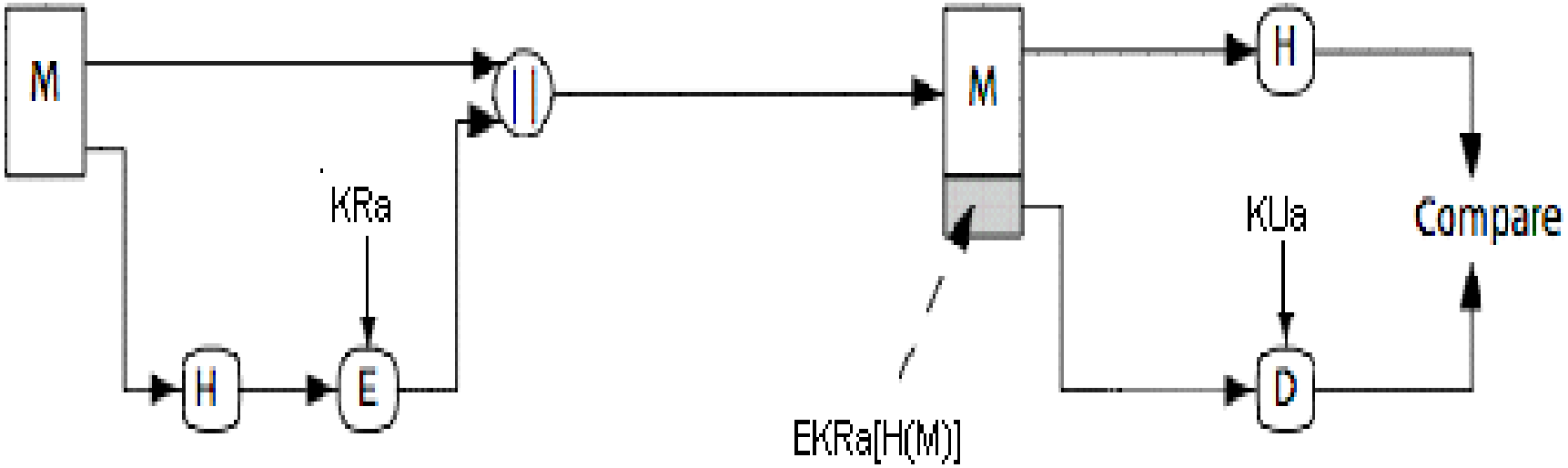
b. Only the hash code is encrypted using symmetric encryption



No Confidentiality only Authentication (Acts as MAC)

Basic Uses of Hash Function

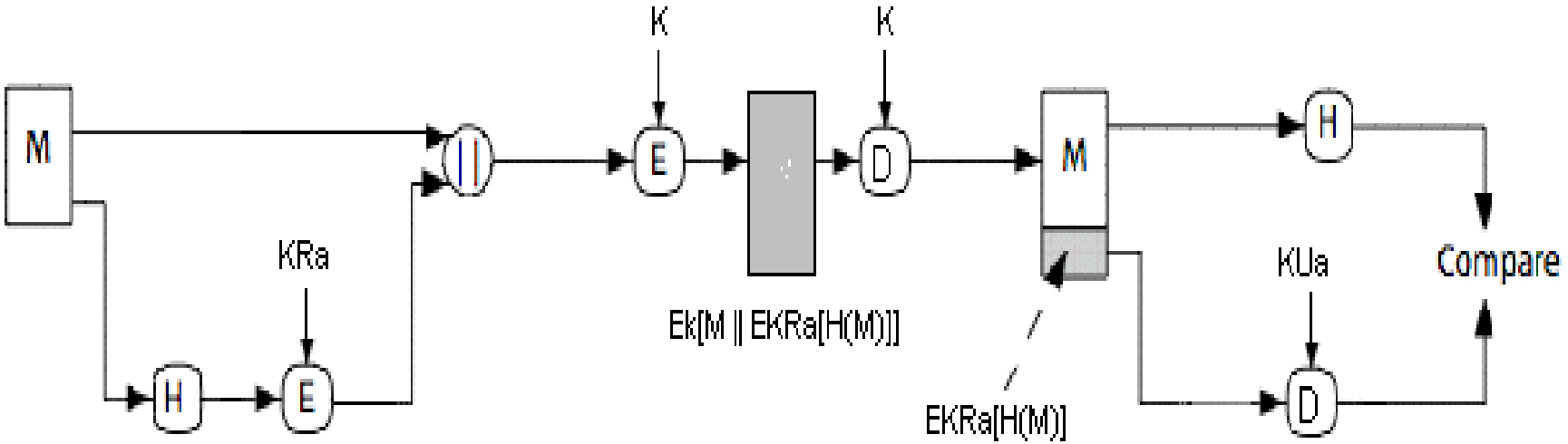
c. Only hash code is encrypted using public key encryption using sender's private key



No Confidentiality only Authentication

Basic Uses of Hash Function

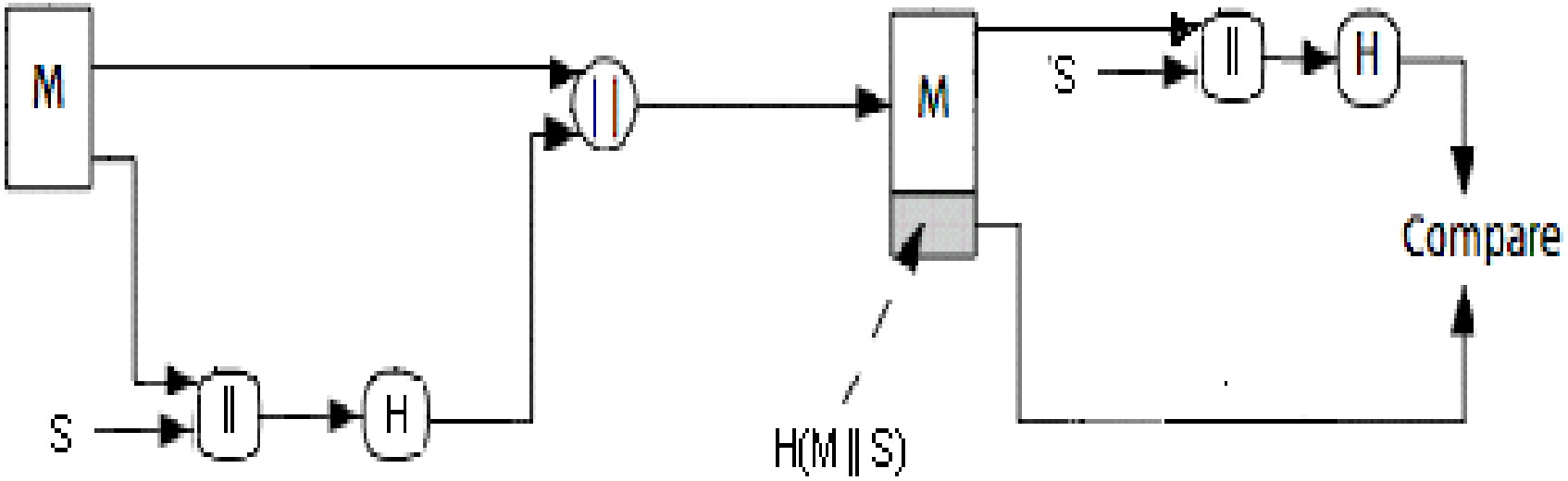
d. Message plus public-key-encrypted hash code is encrypted using a symmetric secret key



Confidentiality and digital signature

Basic Uses of Hash Function

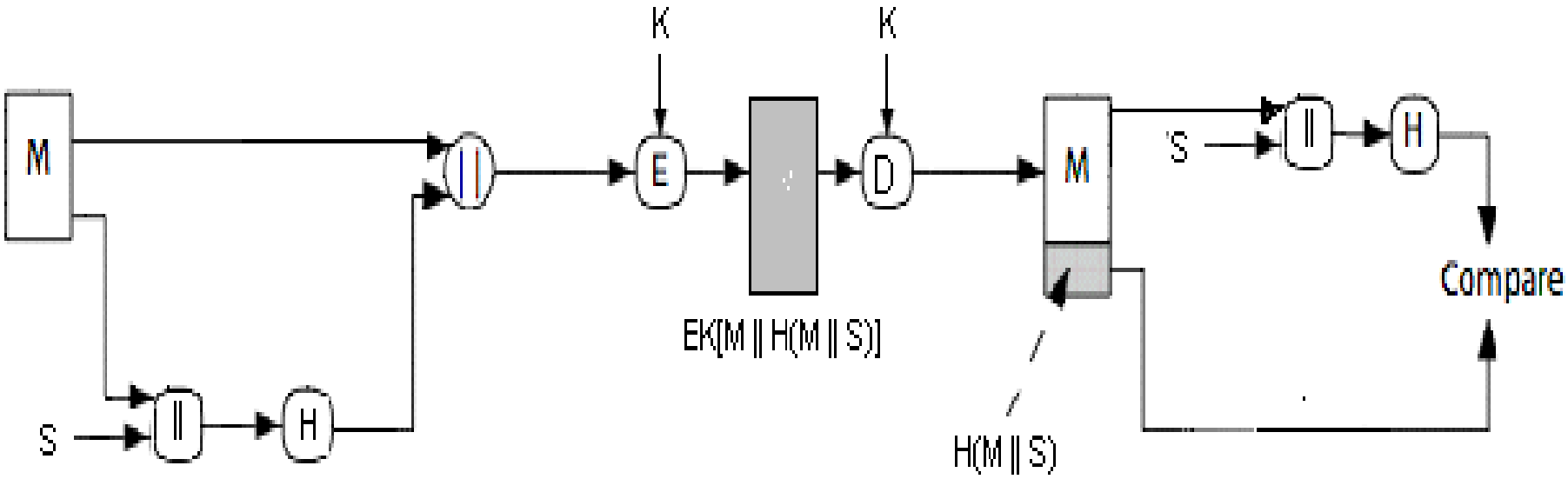
e. Hash value is computed over message plus secret value S



No Confidentiality only Authentication

Basic Uses of Hash Function

f. Message plus hash code is encrypted to the approach (e)



Confidentiality and Authentication

Requirements for Hash Functions

- The Purpose of Hash function is to produce the fingerprint of file, message or other block of data
- For message authentication, a hash function H must have the following properties:
  1. H can be applied to any sized message `M`
  2. H produces fixed-length output `h`
  3. It is easy to compute `h=H(M)` for any message `M`
  4. Given `h`, it is infeasible to find M s.t. `H(M)=h`
     - one-way property
  5. Given `x`, it is infeasible to find `y` s.t. `H(y)=H(x)`
     - weak collision resistance
  6. It is infeasible to find any `x, y` s.t. `H(y)=H(x)`
     - strong collision resistance

Birthday Attacks

- Might think a 64-bit hash is secure
- But by **Birthday Paradox** is not
- The **birthday paradox** can be stated as follows:
  - What is the minimum value of k such that the probability is greater than 0.5 that at least two people in a group of k people have the same birthday?
  - It turns out that the answer is 23 which is quite a surprising result.
  - In other words if there are 23 people in a room, the probability that two of them have the same birthday is approximately 0.5.
  - If there is 100 people (i.e. k=100) then the probability is .9999997, i.e. you are almost guaranteed that there will be a duplicate.

Birthday Attacks

- Digital signatures can be susceptible to a birthday attack.
- A message is typically signed by first computing , where is a cryptographic hash function, and then using some secret key to sign .
- Suppose Mallory wants to trick Bob into signing a fraudulent contract.
- Mallory prepares a fair contract and a fraudulent one .
- She then finds a number of positions where can be changed without changing the meaning, such as inserting commas, empty lines, one versus two spaces after a sentence, replacing synonyms, etc.

Birthday Attacks

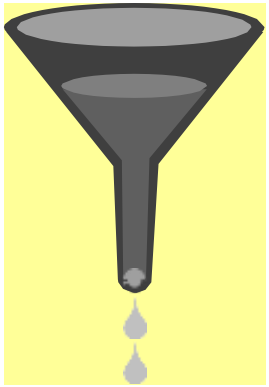
- By combining these changes, she can create a huge number of variations on which are all fair contracts.
- In a similar manner, Mallory also creates a huge number of variations on the fraudulent contract .
- She then applies the hash function to all these variations until she finds a version of the fair contract and a version of the fraudulent contract which have the same hash value, .
- She presents the fair version to Bob for signing.
- After Bob has signed, Mallory takes the signature and attaches it to the fraudulent contract.
- This signature then "proves" that Bob signed the fraudulent contract.

Birthday Attacks

- Might think a 64-bit hash is secure
- But by **Birthday Paradox** is not
- **Birthday attack** works thus:
  - opponent generates $2^{m/2}$ variations of a valid message all with essentially the same meaning
  - opponent also generates $2^{m/2}$ variations of a desired fraudulent message
  - two sets of messages are compared to find pair with same hash (probability > 0.5 by birthday paradox)
  - user sign the valid message, then substitute the forgery which will have a valid signature
- Conclusion is that need to use larger MAC/hash

MD5: Message Digest Version 5

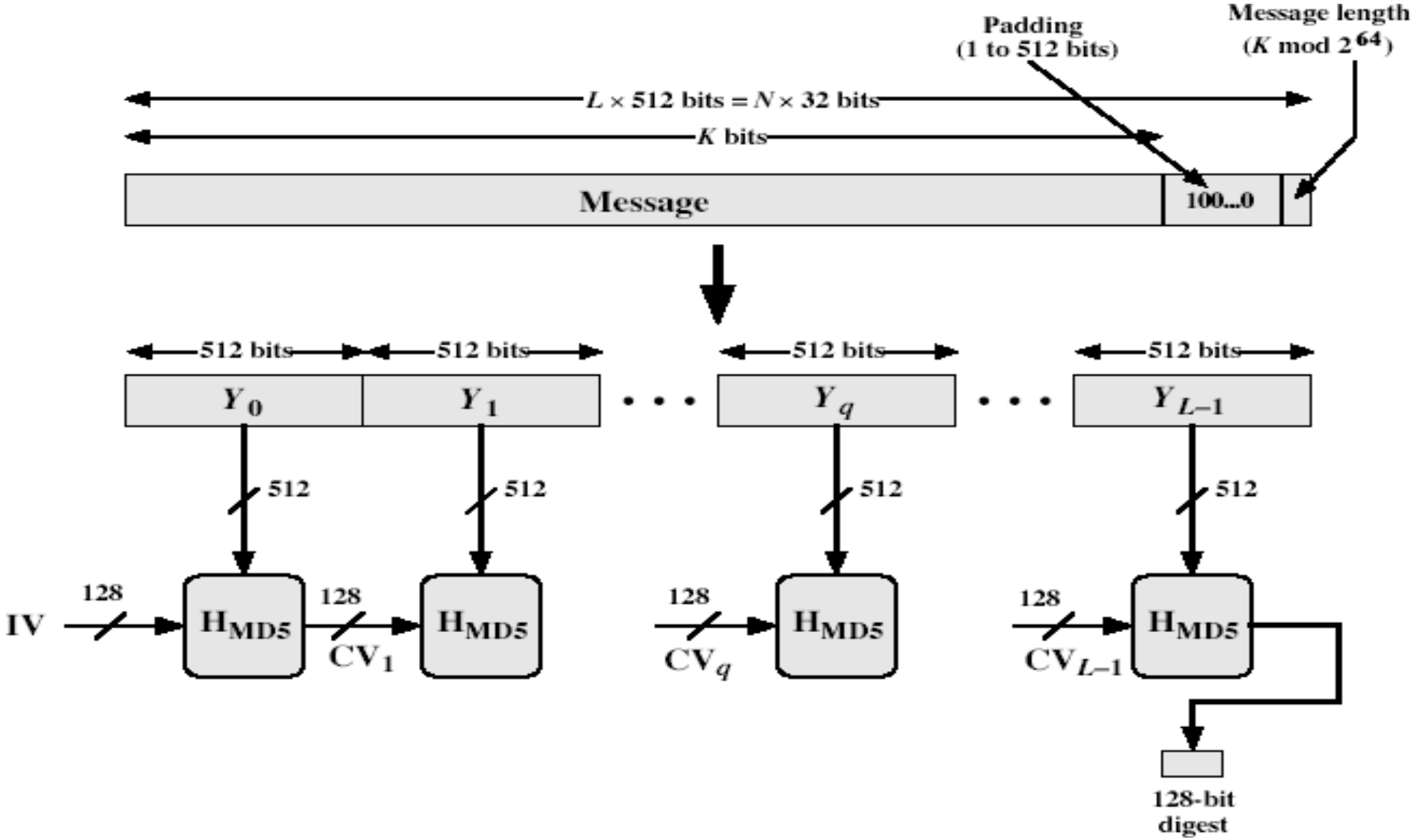
Input Message



Output 128 bits Digest

- Developed by Ron Rivest at MIT
- Until recently the most widely used hash algorithm
- Specified as Internet standard RFC1321

MD5 Overview



MD5

1. **<u>Append Padding Bits</u>:**

   - Pad message so its length is 448 mod 512

   - ie the length of padded message is 64 bits less than an integer multiple of 512 bits.

   - Padding is always added

   - For eg. If the message is 448 bits long, it is padded by 512 bits to a length of 960 bits

   - The number of padding bits is in the range of 1-512

   - Padding consists of single 1-bit followed by the necessary no of 0-bits

MD5

2. **<u>Append Length</u>:**

   -A 64-bit length value of original message is appended to the result of step 1.

   -If the original message is greater than 2^{64} then only the lower order 64 bits of the length are used.

   -Thus the field contains length of the original message

- The outcome of the first 2 steps yields the message that is an integer multiple of 512 bits in length.
- Expanded message is represented as the sequence of 512-bit blocks $Y_0, Y_1 \ldots Y_{L-1}$
- Total length of the expanded message is L x 512 bits

Initialize MD buffer:

- A 128-bit buffer is used to hold the intermediate and final results of the hash function.
- Buffer can be represented as four 32-bit registers (A,B,C,D)
- These registers are initialized to the following 32-bit integers:

A = 67 45 23 01 B = EF CD AB 89

C = 98 BA DC FE D = 10 32 54 76

- These values are stored in little-endian format

word A = 01 23 45 67 word B = 89 AB CD EF

word C = FE DC BA 98 word D = 76 54 32 10

MD5 Overview

3. <u>Process message in 512-bit (16-word) blocks</u>: (Compression Function)

- Using 4 rounds of 16 bit operations on message block & buffer
- These 4 rounds have similar structures but uses different logical functions, F, G, H and I
- Each round takes the current 512-bit block (Y_q) and 128-bit buffer value ABCD as input and updates the contents of the buffer.
- Each round also makes use of the one fourth of a 64-element taďle T[1…64] ĐoŶstruĐted froŵ siŶe fuŶĐtioŶ.
- The i^{th} element of T, T[i] = 2^{32} x abs(sin(i)) (i is in radians)
- The output of the fourth round is added to the input to the first round (CV_q) to produce CV_{q+1}.
- This addition is done using addition modulo 2^{32}.

MD5 Overview

4. **Output:**

- After all L 512-bit blocks have been processed, the output from the L^{th} stage is the 128-bit message digest.

- Behavior of MD5 can be summarized:

$CV_0 = IV$

$CV_{q+1} = SUM_{32}[CV_q, RF_I(Y_q, RF_H(Y_q, RF_G(Y_q, RF_F(Y_q, CV_q))))]$

where

IV = Initial value of ABCD buffer

Y_q = qth 512-bit block of the message

L = No. of blocks in the message

CV_q = Chaining Variable processed with qth block

RF_x = Round Function using primitive legal fn x

MD = Final message digest

SUM_{32} = Addition modulo 2^{32}

MD5 Compression Function

Each round has 16 steps of the form:

`b = b+((a + g(b,c,d)+ X[k]+T[i])<<< s)`

Where

a,b,c,d = the 4 words of the buffer

g = one of the primitive fn F, G, H, I

<<<s = circular left shift of the 32-bit argument by s Bits

X[k] = M[q x 16 + k] = kth 32-bit word in the qth 512-bit block of the message

X[i] = In the first round – used in their original order

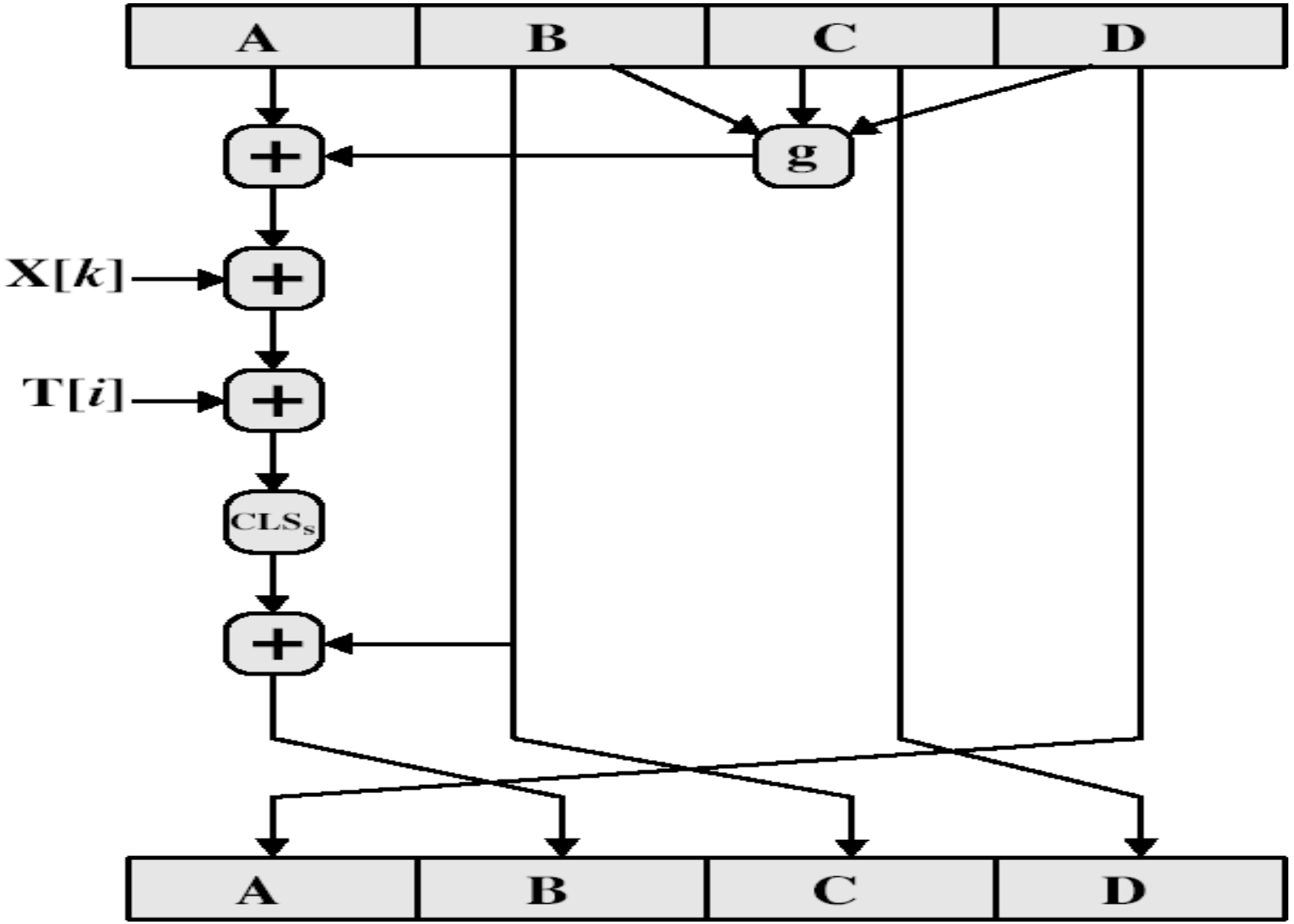
p2[i] = (1 + 5i) mod 16

p3[i] = (5 + 3i) mod 16

p4[i] = 7i mod 16

T[i] = ith 32-bit word in matrix T += Addition modulo 2^{32}

MD5 Compression Function



Functions F, G, H and I

Round	Primitive Fn. g	G(b, c, d)
1	F(b,c,d)	$(b \wedge c) \vee (\sim b \wedge d)$
2	G(b,c,d)	$(b \wedge d) \vee (c \wedge \sim d)$
3	H(b,c,d)	$b \oplus c \oplus d$
4	I(b,c,d)	$c \oplus (b \wedge \sim d)$

Truth Table of Logical Functions

b	c	d	F	G	H	I
0	0	0	0	0	0	1
0	0	1	1	0	1	0
0	1	0	0	1	1	0
0	1	1	1	0	0	1
1	0	0	0	0	1	1
1	0	1	0	1	0	1
1	1	0	1	1	0	0
1	1	1	1	1	1	0

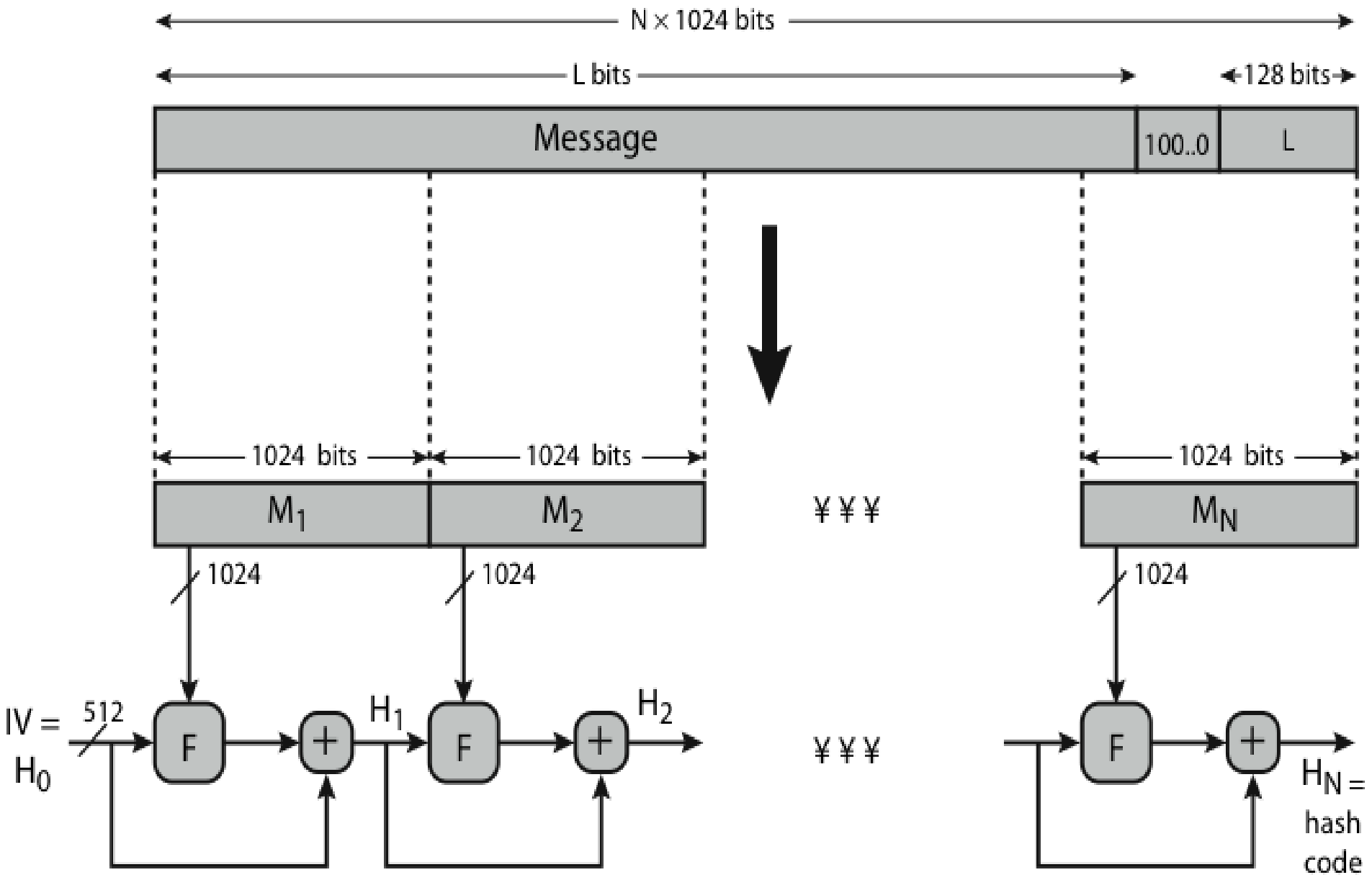
Secure Hash Algorithm

- SHA originally designed by NIST & NSA in 1993 and was revised in 1995 as SHA-1
- US standard for use with DSA signature scheme
  - standard is FIPS 180-1 1995, also Internet RFC3174
  - nb. the algorithm is SHA, the standard is SHS
- Based on design of MD4 with key differences
- Produces 160-bit hash values
- Recent 2005 results on security of SHA-1 have raised concerns on its use in future applications

Revised Secure Hash Standard

- NIST issued revision FIPS 180-2 in 2002 and added 3 additional versions of SHA
  - SHA-256, SHA-384, SHA-512
- Designed for compatibility with increased security provided by the AES cipher
- Structure & detail is similar to SHA-1
- Hence analysis should be similar
- But security levels are rather higher

SHA-512 Overview



SHA-512 Overview

1. **<u>Append Padding Bits</u>:**

   - Pad message so its length is congruent to 1024

   - Padding is always added

   - Padding consists of single 1-bit followed by the necessary no of 0-bits

2. **<u>Append Length</u>:**

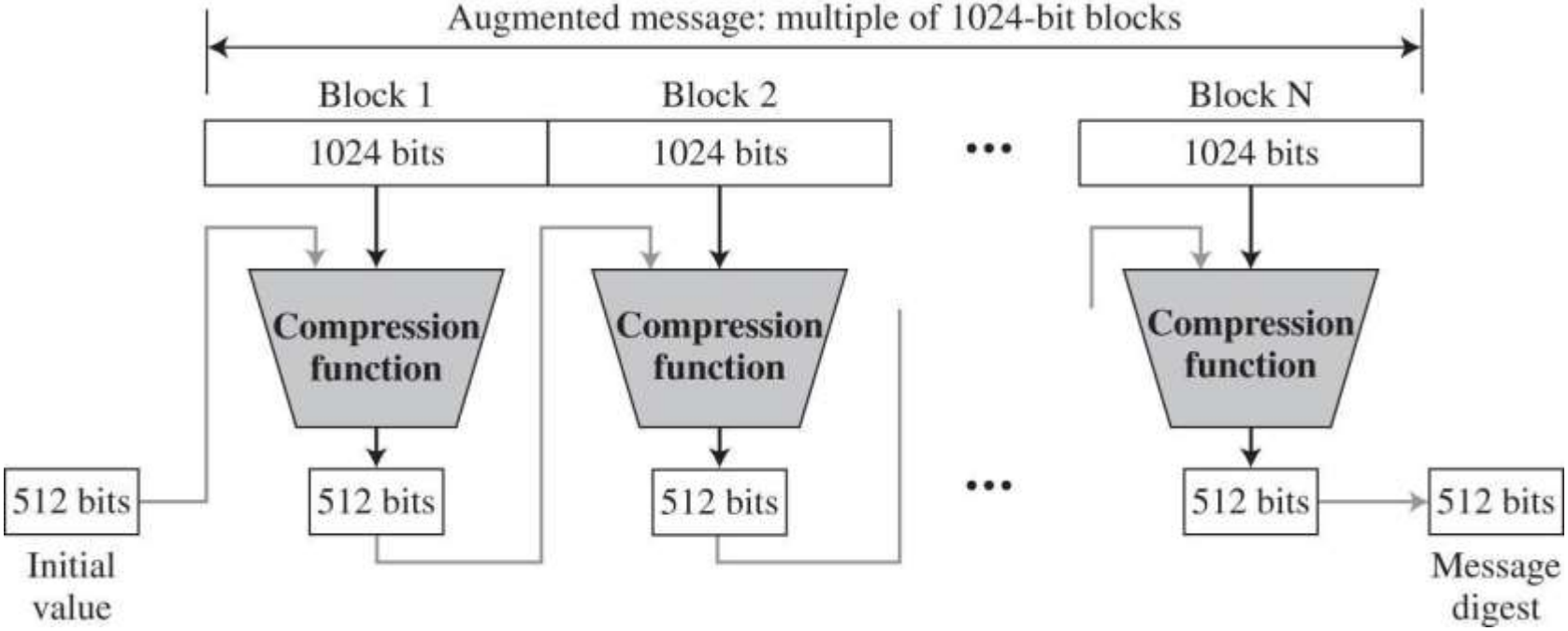
   - A 128-bit length value of original message is appended to the result of step 1.

   - If the original message is greater than 2^{128} then only the lower order 128 bits of the length are used.

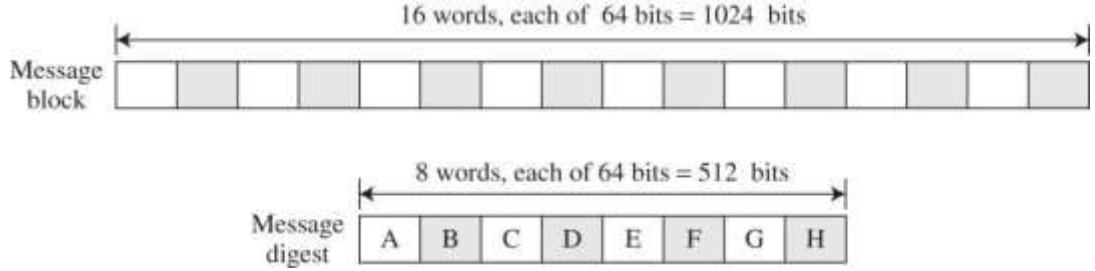
   - Thus the field contains length of the original message

SHA-512

- **512** bit message digest (secure against brute force attack)
  - Block size: **1024** bits



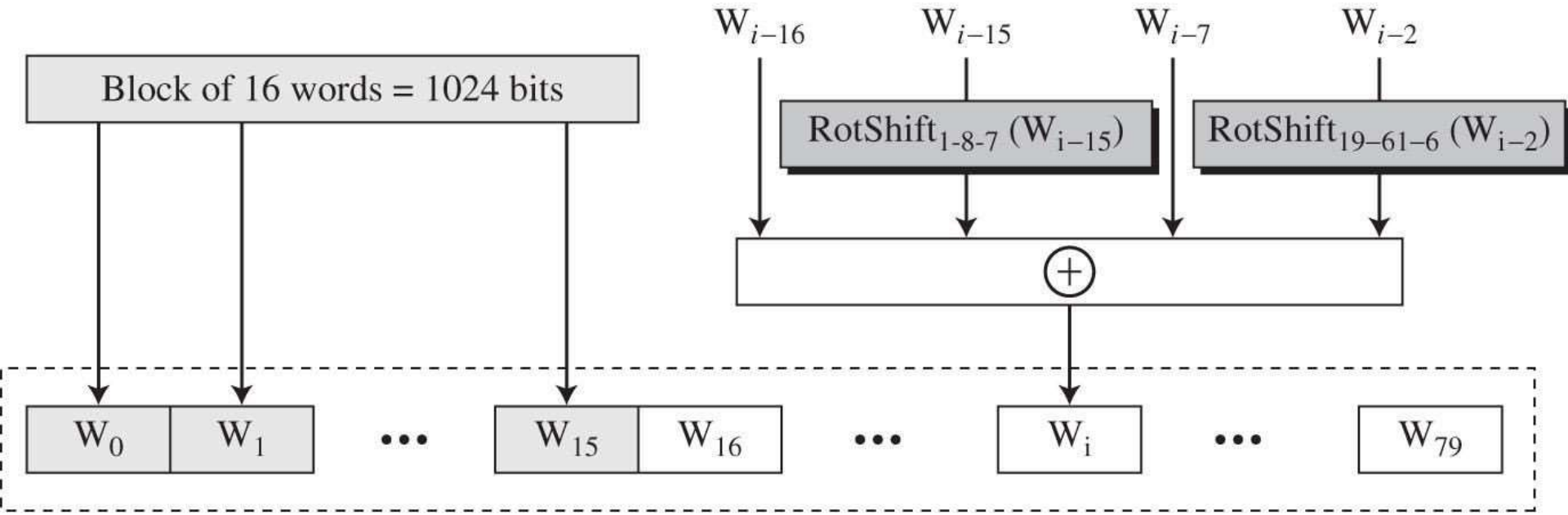
  - Digest ďrokeŶ dowŶ iŶto 64 ďiŧ words Đalled **A** – **H**



Word Expansion in SHA-512

3. Word Expansion:

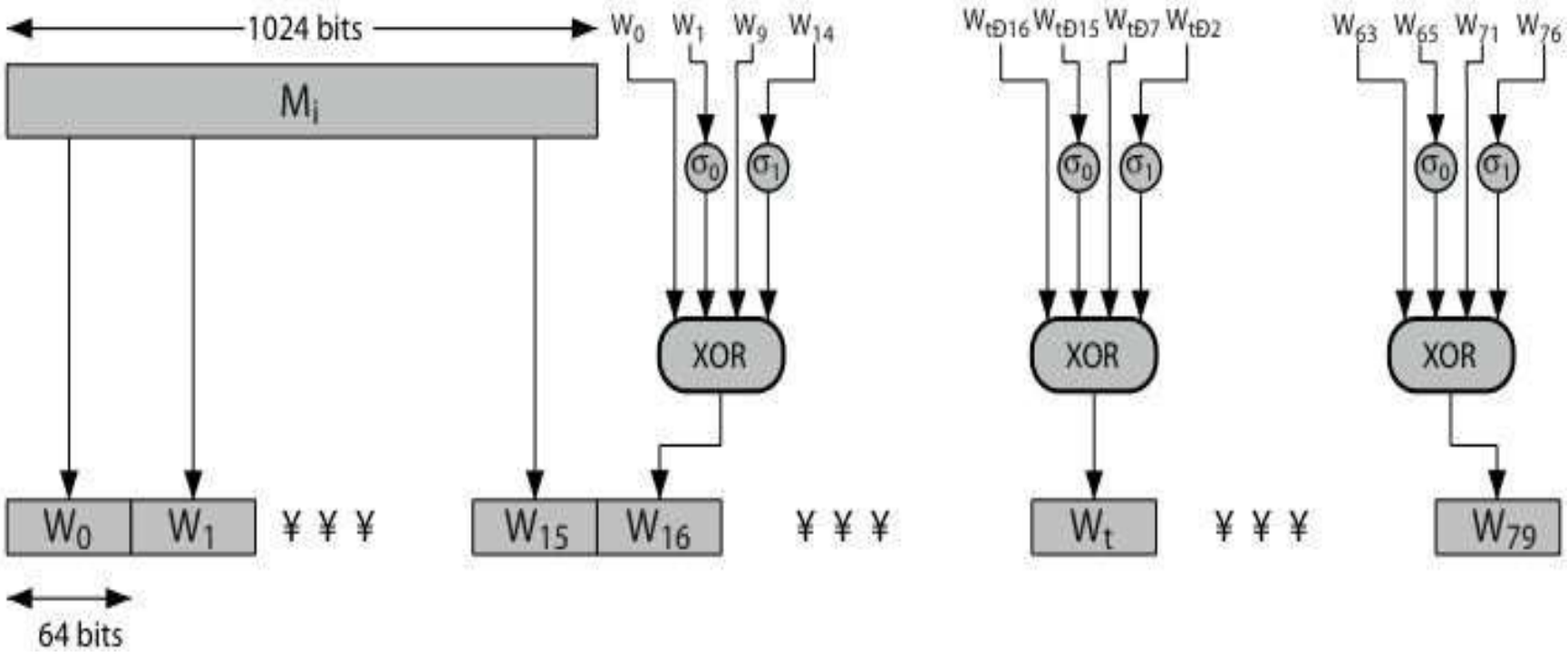
- Block of 16 words expanded to 80 words
  - Used by 80-round compression function



$RotShift_{l\text{-}m\text{-}n}(x)$: $\boldsymbol{RotR}_l(x) \oplus \boldsymbol{RotR}_m(x) \oplus \boldsymbol{ShL}_n(x)$

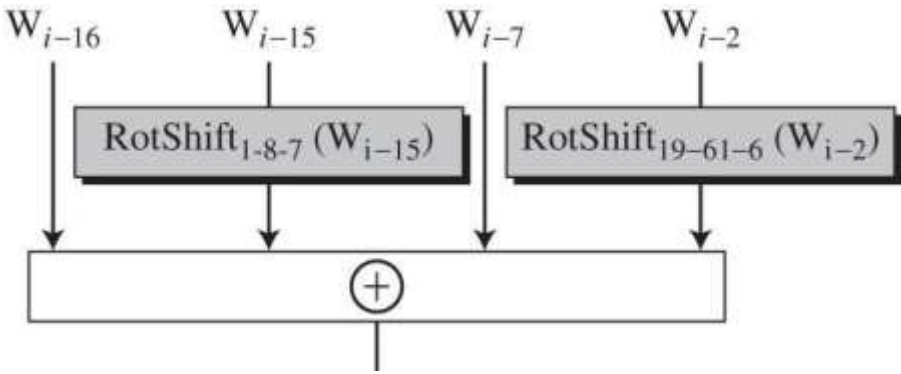
$\boldsymbol{RotR}_i(x)$: Right-rotation of the argument x by i bits
$\boldsymbol{ShL}_i(x)$: Shift-left of the argument x by i bits and padding the left by 0's.

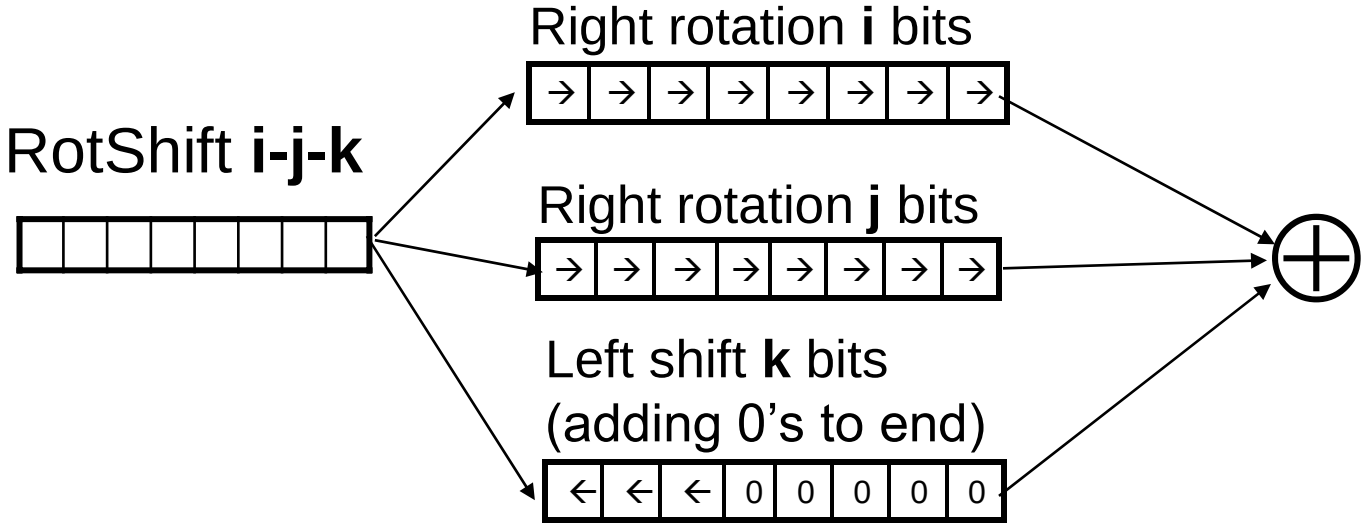
SHA-512 Round Function



Word Expansion in SHA-512

- Each word function of previous 4 words
  - Combined with XOR
  - Confusion added with rotation and shifting (not invertible)

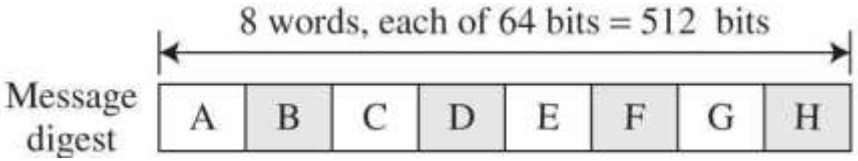




SHA-512 Initial Digest

5. Initializing values of Buffers

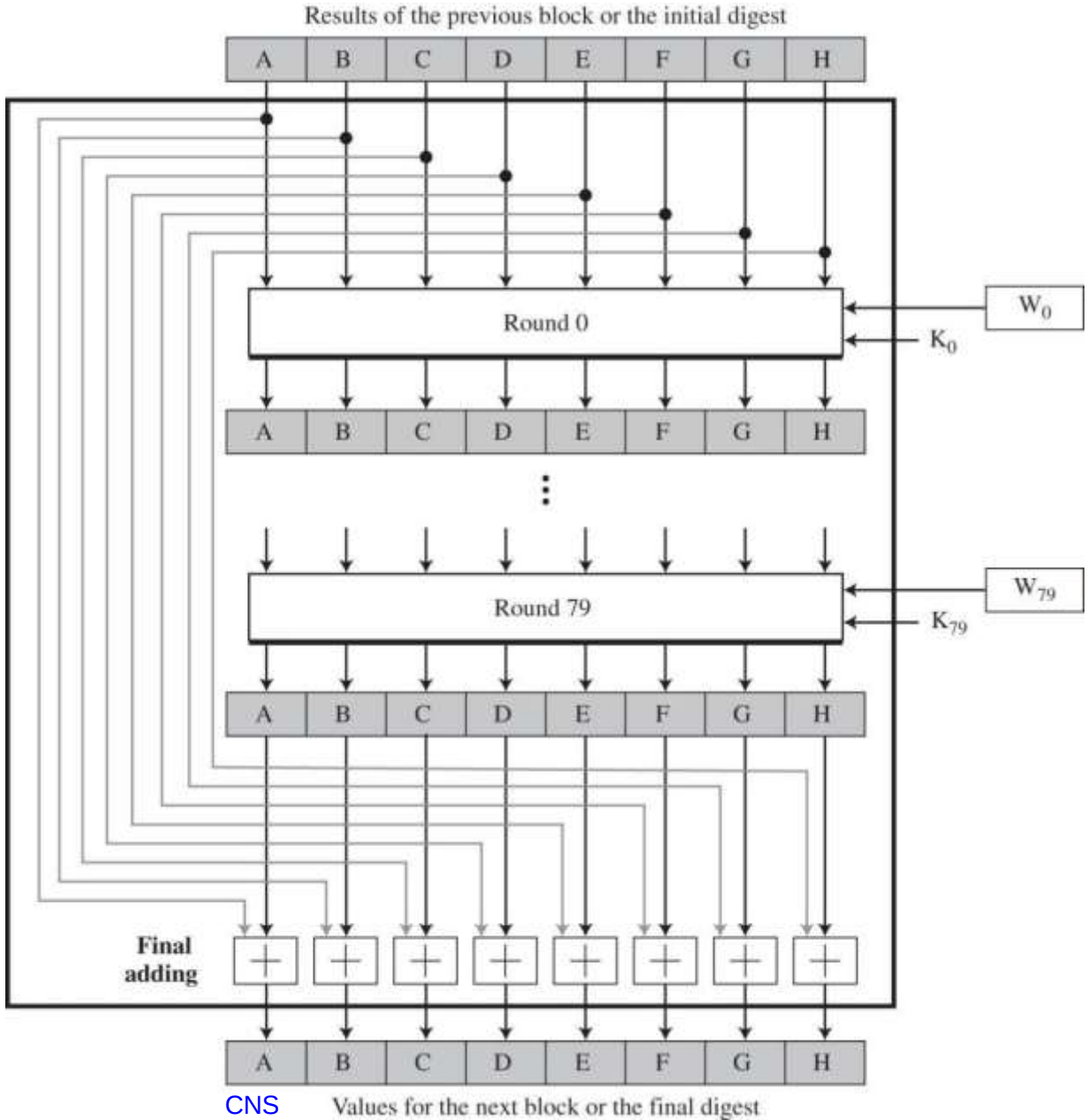
- Designed for appearance of ¯raŶdoŵŶess_
  - Created from first 8 primes (2, 3, 5, 7, 11, 13, 17, 19)
  - Take square root
  - Take first 64 digits of fractional part



A	6A09E667F3BCC908
B	BB67AE8584CAA73B
C	3C6EF372EF94F828
D	A54FE53A5F1D36F1
E	510E527FADE682D1
F	9B05688C2B3E6C1F
G	1F83D9ABFB41BD6B
H	5BE0CD19137E2179

Compress
SHA-512 Coion Function

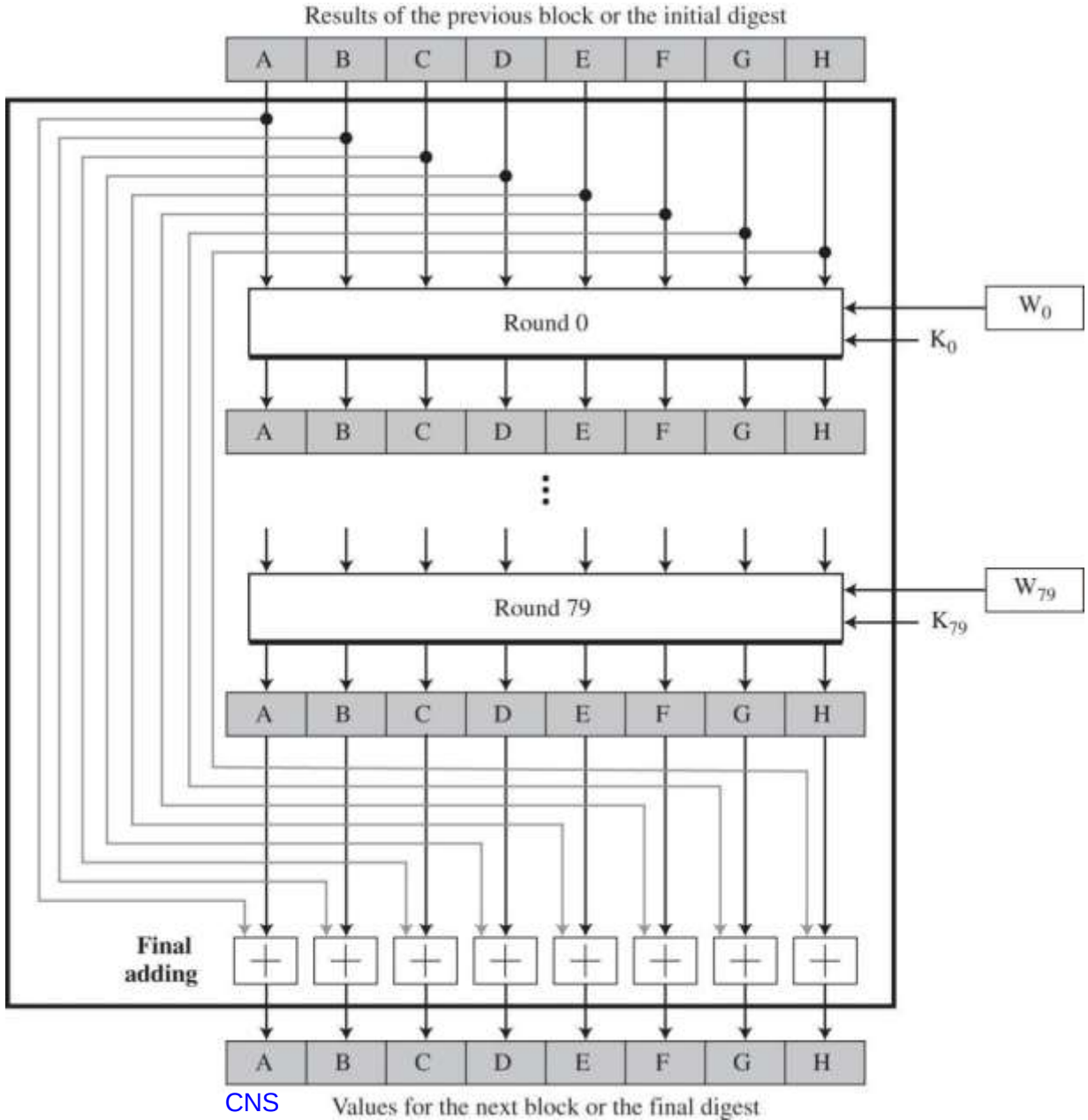
- 80 rounds
  - Each creates new intermediate message digest
- Final stage is sum (mod 2^{64}) of:
  - Initial round digest
  - Final round digest



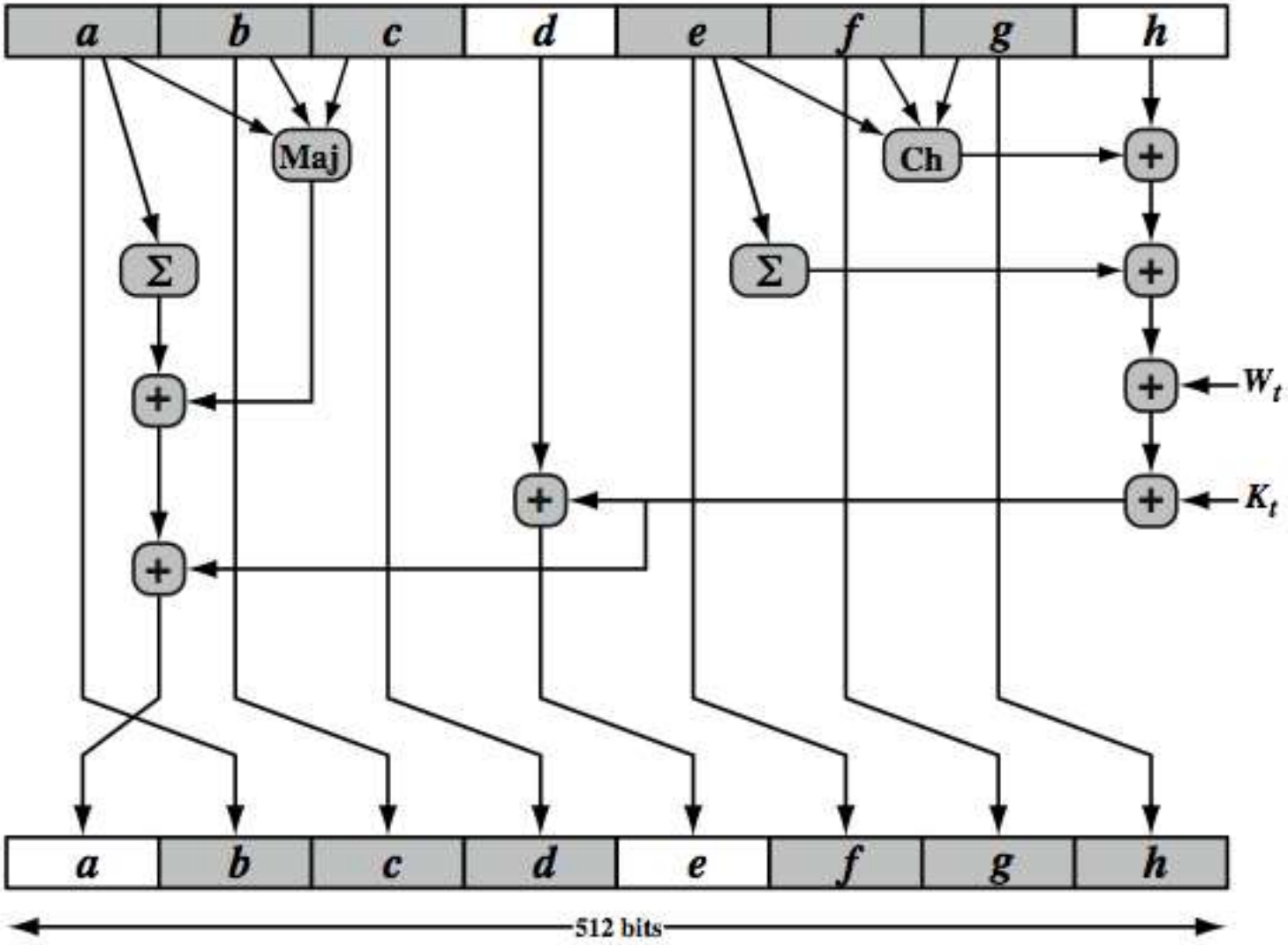
SHA-512 Compression Function

Each round ***i*** function of:

- Previous message digest
- Word $\mathbf{W}_i$
- RouŶd ̄ keLJ $\mathbf{K}_i$ created from fractional parts of square root of first 80 prime numbers (like initial message digest values)



SHA-512 Round Function



SHA-512 Round Function

Ch(e,f,g) = (e AND f) XOR (NOT e AND g)

Maj(a,b,c) = (a AND b) XOR (a AND c) XOR (b AND c)

∑(a) = ROTR(a,28) XOR ROTR(a,34) XOR ROTR(a,39)

∑(e) = ROTR(e,14) XOR ROTR(e,18) XOR ROTR(e,41)

+ = addition modulo 2^64

Kt = a 64-bit additive constant

Wt = a 64-bit word derived from the current 512-bit input block.

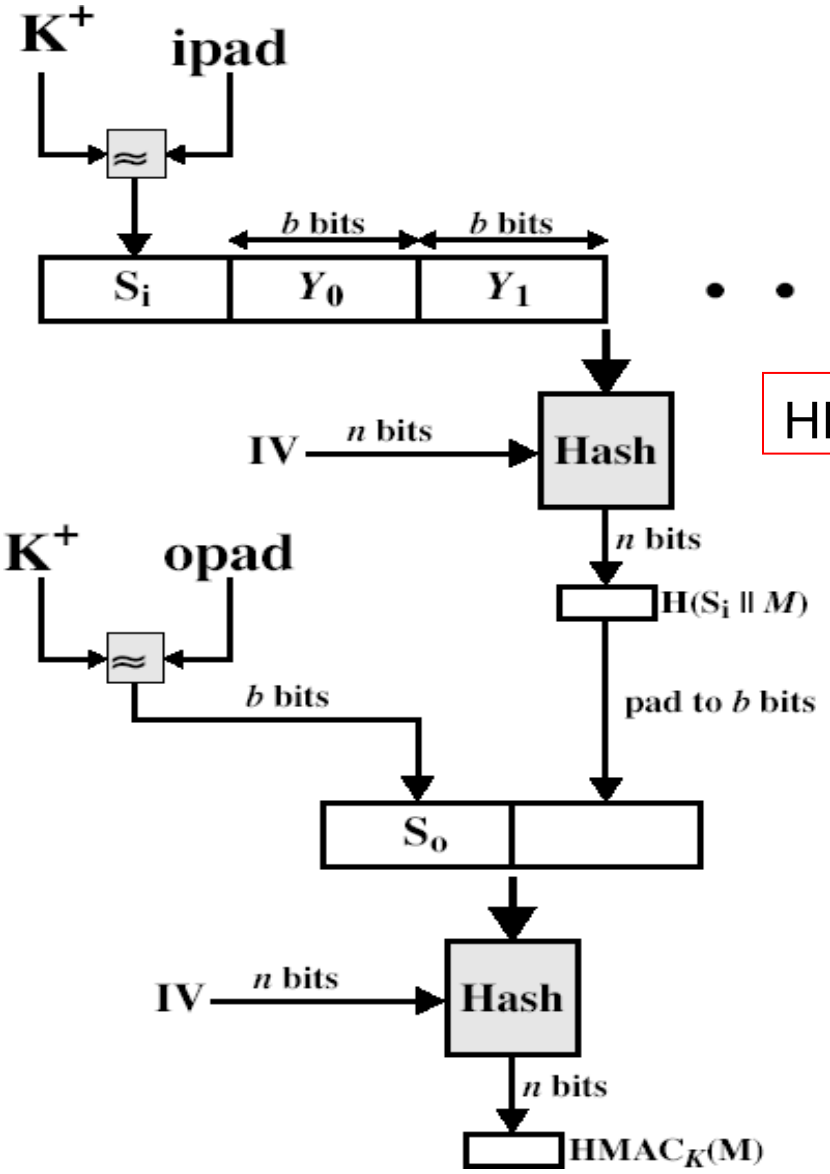
Other Secure HASH functions

	SHA-1	SHA-512	MD5	RIPEMD-160
Digest length	160 bits	512 bits	128 bits	160 bits
Basic unit of processing	512 bits	1024 bits	512 bits	512 bits
Number of steps	80 (4 rounds of 20)	80	64 (4 rnds of 16)	160 (5 paired rnds of 16)
Maximum message size	$2^{64}-1$ bits	$2^{128}-1$ bits		

HMAC

- Uses a MAC derived from a cryptographic hash code, such as SHA-1.
- **Motivations:**
  - Cryptographic hash functions executes faster in software than encryptoin algorithms such as DES
  - Library code for cryptographic hash functions is widely available
  - No export restrictions from the US

HMAC Overview



$HMAC_K = H[(K^+ \oplus opad)||H[(K^+ \oplus ipad)||M]]$

H = hash function
M = Message
Y_i = ith block of M, $0 \le i \le L-1$
L = no. of blocks in M
b = no. of bits in a block (based on chosen hash fn)
n = length of hash code
K = secret key
K^+ = K padded with zeros on the left so that the length is b bits
ipad = 00110110(0x36) repeated b/8 times
opad = 01011010(0x5C) repeated b/8 times

HMAC Advantages

- Existing hash function can be implemented in HMAC
- Easy to replace with more secure or updated hash algorithm
- HMAC is proven more secure than hash algorithms

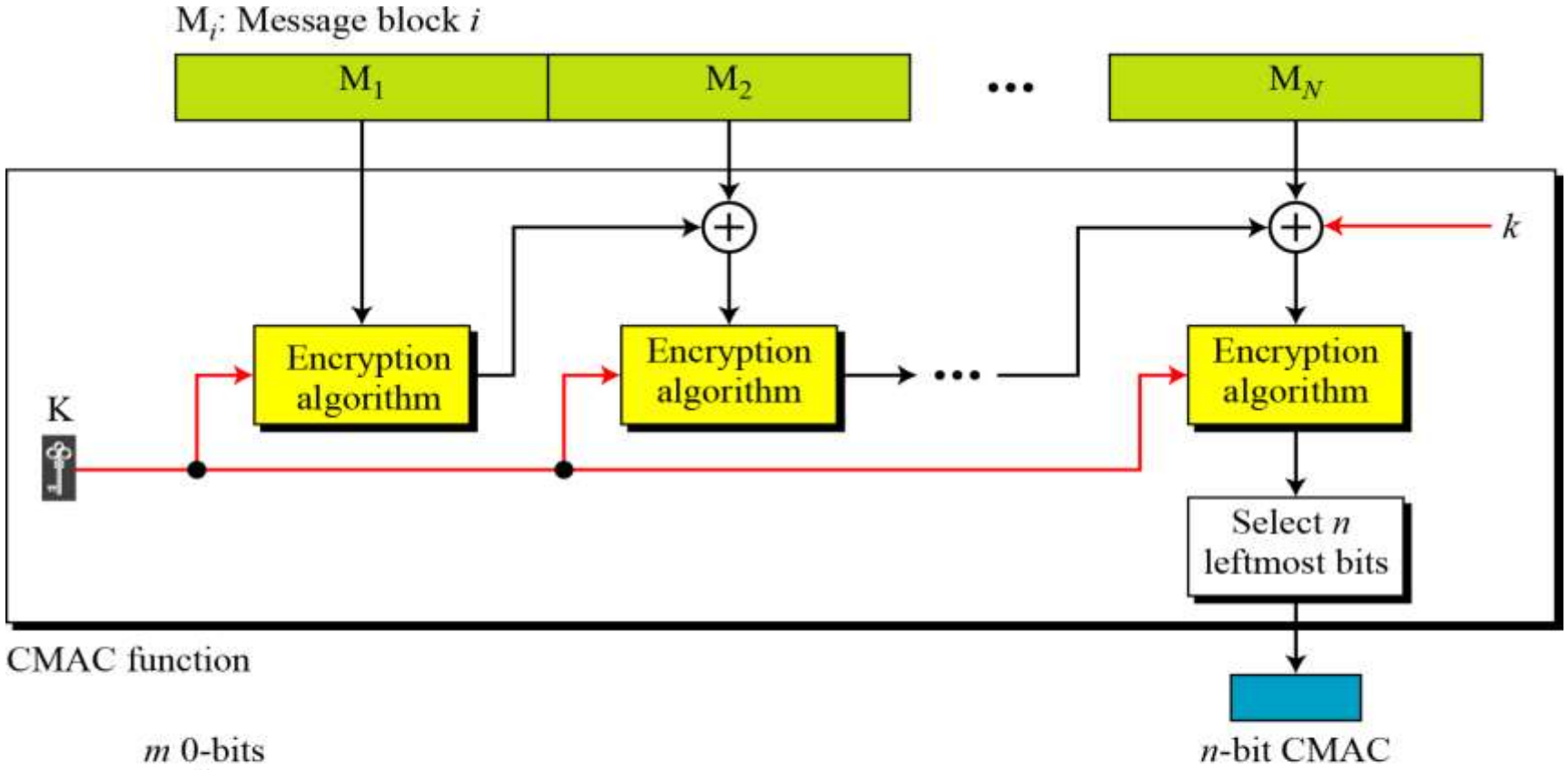
HMAC Security

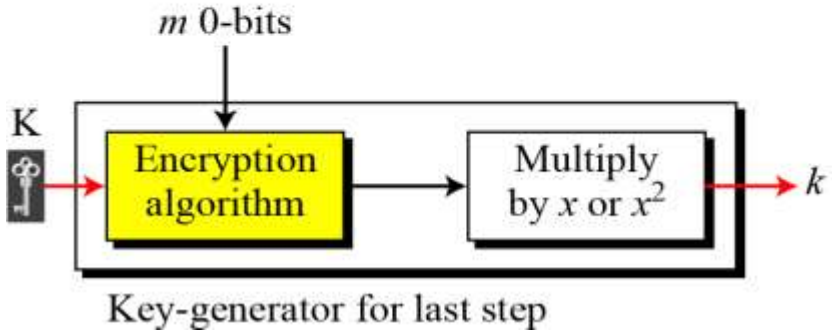
- Proved security of HMAC relates to that of the underlying hash algorithm
- Attacking HMAC requires either:
  - brute force attack on key used
  - birthday attack (but since keyed would need to observe a very large number of messages)
- Choose hash function used based on speed verses security constraints

CMAC (Cipher-based MAC)

- Hashless MAC
  - Uses an encryption algorithm (DES, AES, etc.) to generate MAC
  - Based on same idea as cipher block chaining
- Compresses result to size of single block (unlike encryption

CMAC Overview





- Message broken into N blocks
- Each block fed into an encryption algorithm with key
- Result XOR'd with Ŷedžt ďloÐk ďefore encryption to make final MAC

CMAC Facts

- **Advantages**:
  - Can use existing encryption functions
  - Encryption functions have properties that resist pre-image and collision attacks
    - Cipher text designed to appear like random noise – good approximation of random oracle model
    - Most exhibit strong avalanche effect – minor change in message gives great change in resulting MAC
- **Disadvantage**:
  - Encryption algorithms (particularly when chained) can be much slower than hash algorithms

Unit 3 – Outline

- Authentication requirement
- Authentication functions
- MAC
- Hash function
- Security of hash function and MAC
- MD5 and SHA
- HMAC and CMAC
- Digital signature and authentication protocols
- DSS – EI Gamal – Schnorr

Authentication Protocols

- Used to convince parties of each others identity and to exchange session keys
- May be **one-way authentication** or **mutual authentication**
- Key issues are
  - **confidentiality** – to protect session keys
  - **timeliness** – to prevent replay attacks

Replay Attacks

- **Examples of replay attacks:**
  - **simple replay** (copies message and replays)
  - **repetition that can be logged** (within time frame)
  - **repetition that cannot be detected** (Actual msg suppressed)
  - **backward replay without modification** (to sender)
- **Countermeasures** include
  - use of sequence numbers (generally impractical)
  - timestamps (needs synchronized clocks)
  - challenge/response (using unique nonce)

Mutual Authentication Using Symmetric Encryption

- A two-level hierarchy of keys are used.
- Usually with a trusted Key Distribution Center (KDC)
  - each party shares own master key with KDC
  - KDC generates session keys used for connections between parties
  - master keys used to distribute the session keys to them

Needham-Schroeder Protocol

- Original third-party key distribution protocol
- For session between A and B mediated by KDC
- Protocol overview is:

1. A→KDC : $ID_A \,||\, ID_B \,||\, N_1$

2. KDC→A : $E_{Ka}[Ks \,||\, ID_B \,||\, N_1 \,||\, E_{Kb}[Ks||ID_A]]$

3. A→B : $E_{Kb}[Ks||ID_A]$

4. B→A : $E_{Ks}[N_2]$

5. A→B : $E_{Ks}[f(N_2)]$

Needham-Schroeder Protocol

- Used to securely distribute a new session key for communications between A & B
- But is vulnerable to a replay attack if an old session key has been compromised
  - then message 3 can be resent convincing B that is communicating with A
- Modifications to address this require:
  - timestamps
  - using an extra nonce

Mutual Authentication Using Public-Key Encryption

- Have a range of approaches based on the use of public-key encryption
- Need to ensure that they have correct public keys for other parties
- Using a central Authentication Server (AS)
- Various protocols exist using timestamps or nonces

Denning AS Protocol

- Denning presented the following:

  1. $A \rightarrow AS : ID_A \,||\, ID_B$

  2. $AS \rightarrow A : E_{KRas}[ID_A||KU_a||T] \,||\, E_{KRas}[ID_B||KU_b||T]$

  3. $A \rightarrow B : E_{KRas}[ID_A||KU_a||T] \,||\, E_{KRas}[ID_B||KU_b||T] \,||\, E_{KUb}[E_{KRa}[K_s||T]]$

- Note session key is chosen by A, hence AS need not be trusted to protect it
- Timestamps prevent replay but require synchronized clocks

One-Way Authentication

- Required when sender & receiver are not in communications at same time (eg. email)
- Have header in clear so can be delivered by email system
- May want contents of body protected & sender authenticated

Using Symmetric Encryption

- Can refine use of KDC ďut Đan't have final exchange of nonces, vis:

  1. A→KDC : $ID_A \,||\, ID_B \,||\, N_1$

  2. KDC→A : $E_{Ka}[Ks \,||\, ID_B \,||\, N_1 \,||\, E_{Kb}[Ks||ID_A]\,]$

  3. A→B : $E_{Kb}[Ks||ID_A] \,||\, E_{Ks}[M]$

- Does not protect against replays
  - could rely on timestamp in message, though email delays make this problematic

Public-Key Approaches

- Have seen some public-key approaches
- If confidentiality is major concern, can use:

  $A \rightarrow B$: $E_{KUb}[Ks] \,||\, E_{Ks}[M]$

  – has encrypted session key, encrypted message
- If authentication needed use a digital signature with a digital certificate:

  $A \rightarrow B$: $M \,||\, E_{KRa}[H(M)] \,||\, E_{KRas}[T||ID_A||KU_a]$

  – with message, signature, certificate

Digital Signatures

Inclusion: A conventional signature is included in the document; it is part of the document.
But when we sign a document digitally, we send the signature as a separate document.
Verification: For a CS, when the recipient receives a document, she compares the signature on the document with the signature on file.
For a DS, the recipient receives the message and the signature. The recipient needs to apply a verification technique to the combination of the message and the signature to verify the authenticity.

Digital Signatures

Relationship: For a CS, there is normally a one-to-many relationship between a signature and documents.

For a DS, there is a one-to-one relationship between a signature and a message.

Duplicity: In CS, a copy of the signed document can be distinguished from the original one on file.

In DS, there is no such distinction unless there is a factor of time on the document.

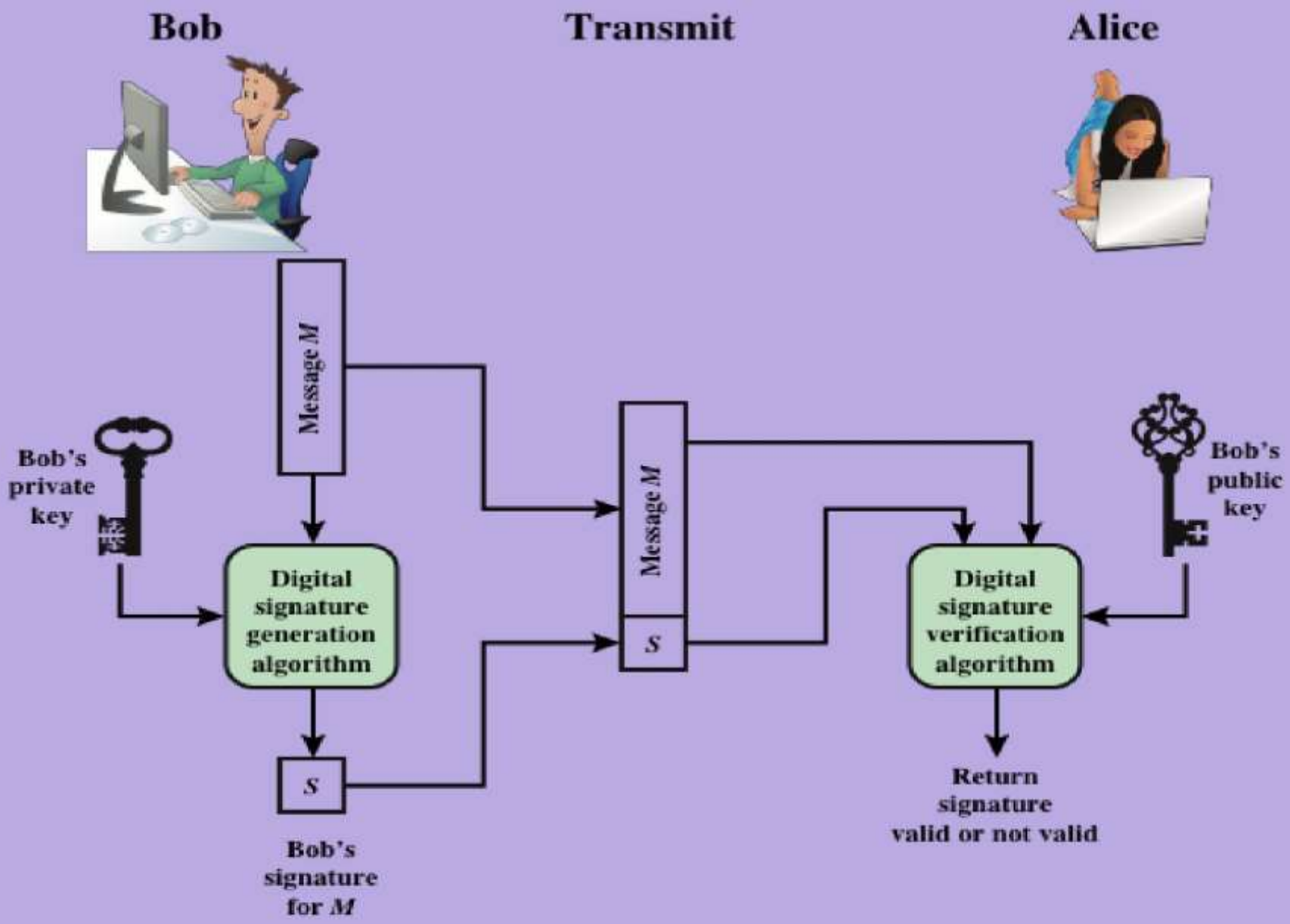


Figure 13.1 Generic Model of Digital Signature Process

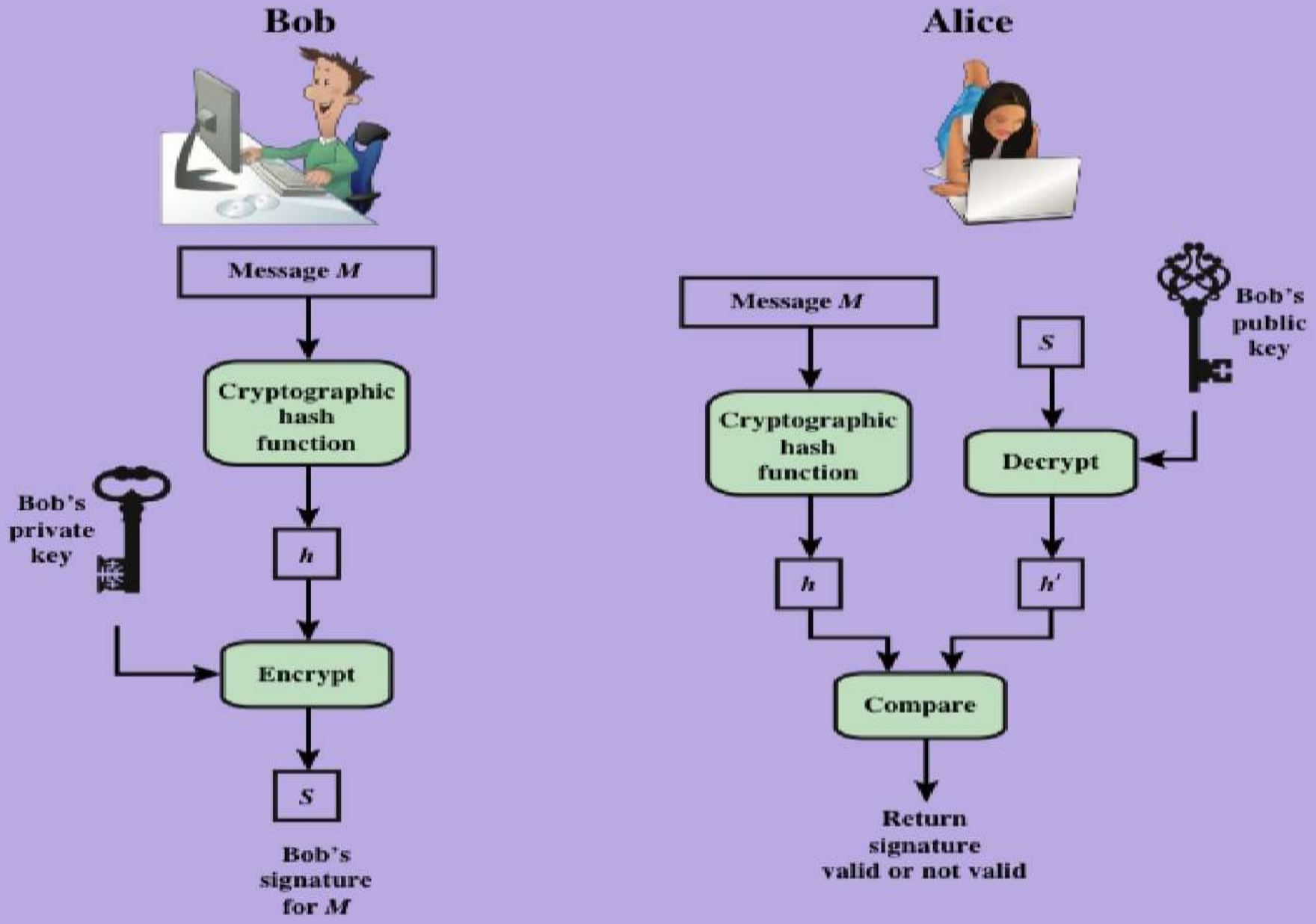


Figure 13.2 Simplified Depiction of Essential Elements of Digital Signature Process

Digital Signature Properties

It must verify the author and the date and time of the signature

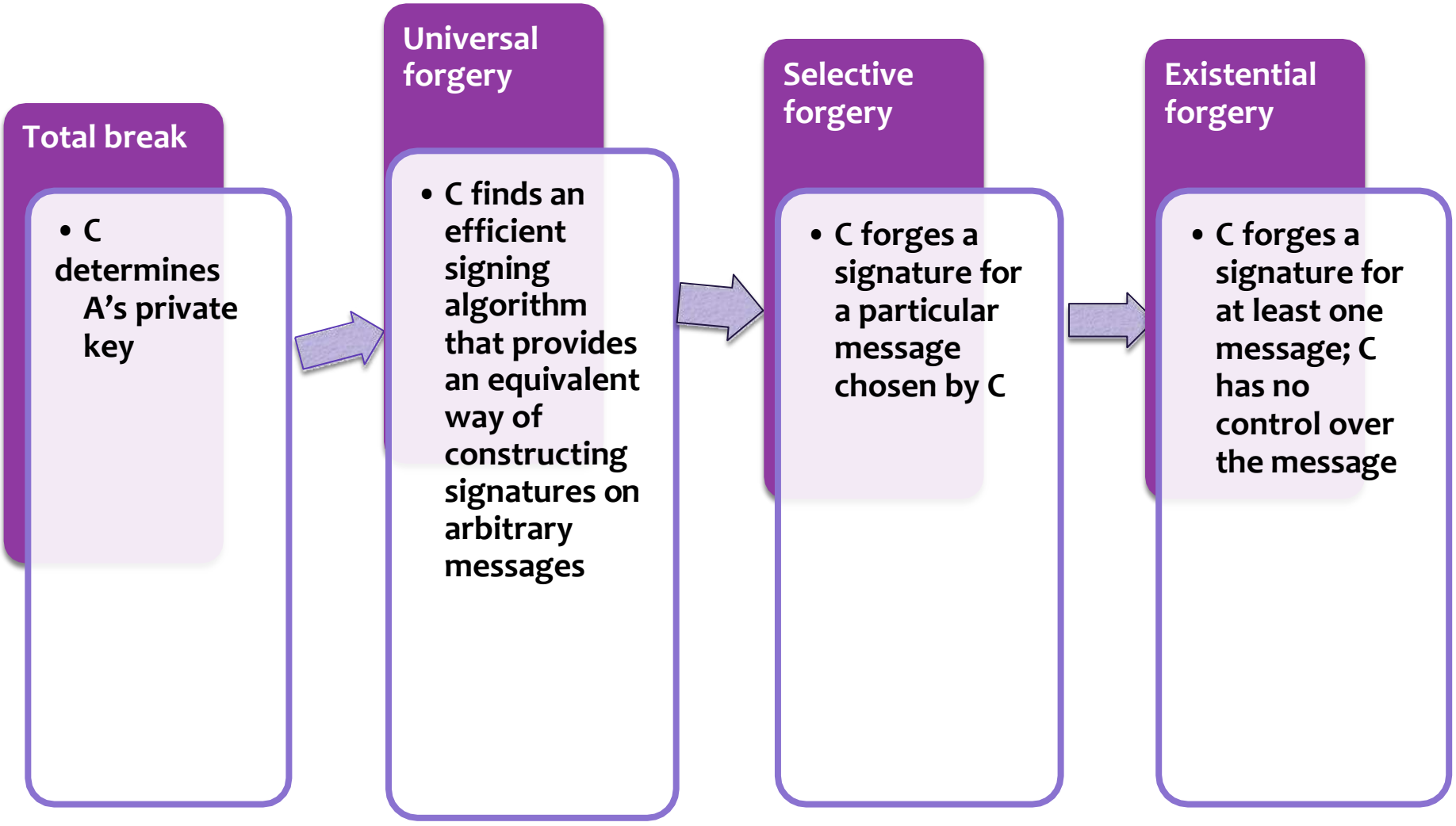
It must authenticate the contents at the time of the signature

It must be verifiable by third parties, to resolve disputes

Attacks

Key-only attack	C only knows A's public key
Known message attack	C is given access to a set of messages and their signatures
Generic chosen message attack	C chooses a list of messages before attempting to break A's signature scheme, independent of A's public key; C then obtains from A valid signatures for the chosen messages
Directed chosen message attack	Similar to the generic attack, except that the list of messages to be signed is chosen after C knows A's public key but before any signatures are seen
Adaptive chosen message attack	C may request from A signatures of messages that depend on previously obtained message- signature pairs

Forgeries


Total break
• C determines A's private key
Universal forgery
• C finds an efficient signing algorithm that provides an equivalent way of constructing signatures on arbitrary messages
Selective forgery
• C forges a signature for a particular message chosen by C
Existential forgery
• C forges a signature for at least one message; C has no control over the message

Digital Signature Requirements

- The signature must be a bit pattern that depends on the message being signed
- The signature must use some information unique to the sender to prevent both forgery and denial
- It must be relatively easy to produce the digital signature
- It must be relatively easy to recognize and verify the digital signature
- It must be computationally infeasible to forge a digital signature, either by constructing a new message for an existing digital signature or by constructing a fraudulent digital signature for a given message
- It must be practical to retain a copy of the digital signature in storage

ELGamal Cryptosystem

- The ElGamal Algorithm provides an alternative to the RSA for public key encryption.
- Security of the RSA depends on the **difficulty of factoring large integers.**
- Security of the ElGamal depends on the **difficulty of computing discrete logs** in a large prime modulus.
- ElGamal has the **disadvantage** that the cipher text is twice as long as the plaintext.
- It has the **advantage** the same plaintext gives a different cipher text (with near certainty) each time it is encrypted.

ELGamal Cryptosystem

Alice chooses

i) A large prime p_A (say 200 to 300 digits).

ii) A primitive element α_A modulo p_A.

iii) A (possibly random) integer d_A with $2 \le d_A \le p_A - 2$.

Alice computes

iv) $\beta_A \equiv \alpha_A^{d_A} \pmod{p_A}$.

Alice's *public key* is (p_A, α_A, β_A). Her *private key* is d_A.

ELGamal Cryptosystem

Bob encrypts a short message M ($M < p_A$) and sends it to Alice like this:

i) Bob chooses a random integer k (which he keeps secret).

ii) Bob computes $r \equiv \alpha_A^{\,k} \pmod{p_A}$ and $t \equiv \beta_A^{\,k} M \pmod{p_A}$, and then discards k.

Bob sends his encrypted message (r, t) to Alice.

ELGamal Cryptosystem

When Alice receives the encrypted message (r, t), she decrypts (using her private key d_A) by computing tr^{-d_A}.

Note
$$tr^{-d_A} \equiv \beta_A^{\,k} M (\alpha_A^{\,k})^{-d_A} \pmod{p_A}$$
$$\equiv (\alpha_A^{\,d_A})^{k} M (\alpha_A^{\,k})^{-d_A} \pmod{p_A}$$
$$\equiv M \pmod{p_A}$$

ELGamal Cryptosystem

Even if Eve intercepts the ciphertext (r, t), she cannot perform the calculation above because she doesn't know d_A.

$$\beta_A \equiv \alpha_A^{\,d_A} \pmod{p_A}, \text{ so } \quad d_A \equiv L_{\alpha_A}(\beta_A)$$

Eve can find d_A if she can compute a discrete log in the large prime modulus p_A, presumably a computation that is too difficult to be practical.

Caution: Bob should choose a different random integer k for each message he sends to Alice.

ElGamal Example

- $p_A=11$ and $\alpha_A=2$
- Alice computes her key:
  - chooses $d_A=5$ & computes $\beta_A=2^5 \bmod 11 = 10$
  - Public Key = (11, 2, 10)
  - Private Key = (5)
- Bob send message $m=1$ as $(9,1)$ by
  - chosing random $k=6$
  - computing $r = \alpha_A^k \bmod p_A = 2^6 \bmod 11 = 9$
  - computing $t = \beta_A^k.m \bmod p_A = 10^6.1 \bmod 11 = 1$
- Alice recovers original message by computing:
  - $m = tr^{-d_A} \bmod p_A = 1.9^{10-5} \bmod 11 = 1$

ElGamal Example

- $p_A=19$ and $\alpha A=10$
- Alice computes her key:
  - A chooses $d_A=5$ & computes $\beta_A=10^5 \bmod 19 = 3$
  - Public Key = (19, 10, 3)
  - Private Key = (5)
- Bob send message $m=17$ as $(11,5)$ by
  - chosing random $k=6$
  - computing $r = \alpha_A{}^k \bmod p_A = 10^6 \bmod 19 = 11$
  - computing $t = \beta_A{}^k.m \bmod p_A = 3^6 \,.\, 17 \bmod 19 = 5$
- Alice recovers original message by computing:
  - $m = tr^{-d_A} \bmod p_A = 5.11^{18-5} \bmod 19 = 17$

Schnorr Digital Signature

- Scheme is based on discrete logarithms
- Minimizes the message-dependent amount of computation required to generate a signature
- Multiplying a $2n$-bit integer with an n-bit integer
- Main work can be done during the idle time of the processor
- Based on using a prime modulus p, with $p - 1$ having a prime factor q of appropriate size
- Typically p is a 1024-bit number, and q is a 160-bit number

Schnorr Digital Signature

Generation of Private-Public Key Pair:

1. Choose **prime p and q**, such that q is a prime factor of p-1.
2. Choose an integer a, such that $a^q=1 \bmod p$. (**a, p and q are the global public keys** and common to all users of a group)
3. Choose a random integer s with $0<s<q$. (**This 's' is user's private key**)
4. Calculate $v=a^{-s} \bmod p$. (**This 'v' is user's public key**)

Schnorr Digital Signature

Generation of signature with Private-Public Key Pair (s-v):

1. Choose a random integer r with $0<r<q$ and compute $x = a^r \bmod p$.
2. Concatenate the message M with x and hash the result to compute the value $e = H(M||x)$
3. Compute $y = (r+se) \bmod q$.

The signature consists of the pair (e, y)

Schnorr Digital Signature

Signature verification by any other user:

1. Compute $x' = a^y v^e \mod p$.
2. Verify that $e = H(M||x')$

To Prove this:

$$x' = a^y v^e = a^y a^{-se} = a^{y-se} = a^r = x \pmod p$$

Hence **$H(M||x') = H(M||x)$**

Unit 4 – Outline

- Authentication applications:
  - Kerberos
  - X.509
- Authentication services
- Internet Firewalls for Trusted System:
  - Roles of Firewalls
  - Firewall related terminology
  - Types of Firewalls
  - Firewall designs
  - Firewalls design principles.
- SET for E-Commerce Transactions
- Intruder
  - Intrusion detection system
- Virus and related threats
  - Countermeasures
- Trusted systems
- Practical implementation of cryptography and security.

Security Concerns

- Key concerns are **confidentiality** and **timeliness**
- To provide confidentiality must encrypt identification and session key info
- Which requires the use of previously shared private or public keys
- Need timeliness to prevent **replay attacks**
- Provided by using sequence numbers or timestamps or challenge/response

KERBEROS

- Users wish to access services on servers.
- Three threats exist:
  - User pretends to be another user.
  - User alter the network address of a workstation.
  - User eavesdrop on exchanges and use a replay attack.

Kerboros

- Provides a centralized authentication server to authenticate users to servers and servers to users.
- Relies on conventional encryption, making no use of public-key encryption
- Two versions: version 4 and 5
- Version 4 makes use of DES

Kerberos

- **Motivation**: 3 Approaches
  1. Rely on each individual client to assure the identity of the user to enforce security policy based on user identification.
  2. Require that client systems authenticate themselves to servers
  3. Require that the user to prove his or her identity for each service invoked

Kerberos

- **Motivation**: Requirements
  - **Secure:** Eavesdropper should not be able to obtain the necessary info. to impersonate a user.
  - **Reliable:** Should be highly reliable and should employ a distributed server architecture with one system able to back up another.
  - **Transparent:** User should not be aware that authentication is taking place beyond the requirement to enter the password.
  - **Scalable:** System should be capable of supporting large numbers of clients and servers.

Kerberos Version 4

- Terms:
  - C = Client
  - AS = authentication server
  - V = server
  - ID_C = identifier of user on C
  - ID_V = identifier of V
  - P_c = password of user on C
  - ADc = network address of C
  - K_V = secret encryption key shared by AS an V
  - TS = timestamp
  - || = concatenation

A Simple Authentication Dialogue

(1) C → AS: $ID_c \| P_c \| ID_v$

(2) AS → C: Ticket

(3) C → V: $ID_c \| Ticket$

$Ticket = E_{Kv}[ID_c \| P_c \| ID_v]$

Version 4 Authentication Dialogue

- **Problems:**
  - User would need a new ticket for every different service.
  - Since the plaintext transmission of the password is involved, an eavesdropper could capture the pw and use any accessible service.

  Lifetime associated with the ticket-granting ticket

  If too short → repeatedly asked for password

  If too long → greater opportunity to replay

Ticket Granting Server Scenario

Once Per User Logon Session

1. C → AS: $ID_C \| ID_{tgs}$
2. AS → C: $E(K_C, Ticket_{tgs})$

Once Per Type of Service

3. C → TGS: $ID_C \| ID_V \| Ticket_{tgs}$
4. TGS → C: $Ticket_V$

Once Per Service Session

5. C → V: $ID_C \| Ticket_V$

$Ticket_{tgs} = E(K_{tgs} [IDc \| AD_c \| ID_{tgs} \| TS_1 \| Lifetime_1])$

$Ticket_V = E(K_V , [IDc \| AD_c \| ID_V \| TS_2 \| Lifetime_2])$

Ticket Granting Server Scenario

- **Problems:**
  - Lifetime associated with the ticket-granting ticket
    - If too short → repeatedly asked for password
    - If too long → greater opportunity to replay
  - There may be a requirement for servers to authenticate themselves to users.

Version 4 Authentication Dialogue

Authentication Service Exhange: To obtain Ticket-Granting Ticket

(1) C → AS: IDc || IDtgs ||TS1

(2) AS → C: $E_{Kc}[K_{c,tgs}|| IDtgs || TS_2 || Lifetime_2 || Ticket_{tgs}]$

Ticket-Granting Service Echange: To obtain Service-Granting Ticket

(3) C → TGS: IDv ||$Ticket_{tgs}$ ||Authenticatorc

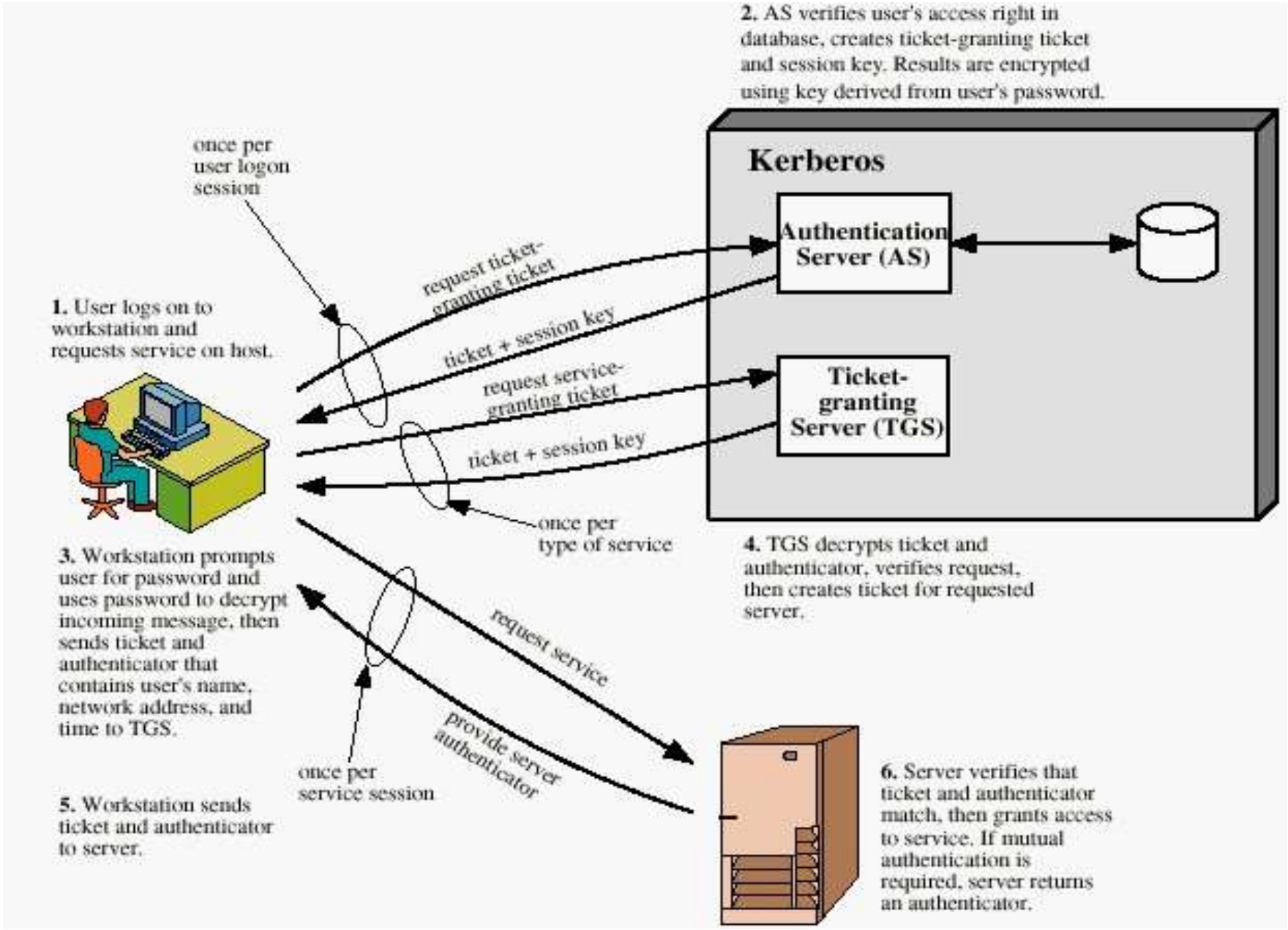
(4) TGS → C: $E_{Kc}[K_{c,v}|| IDv || TS_4 || Ticket_v]$

Client/Server Authentication Exhange: To Obtain Service

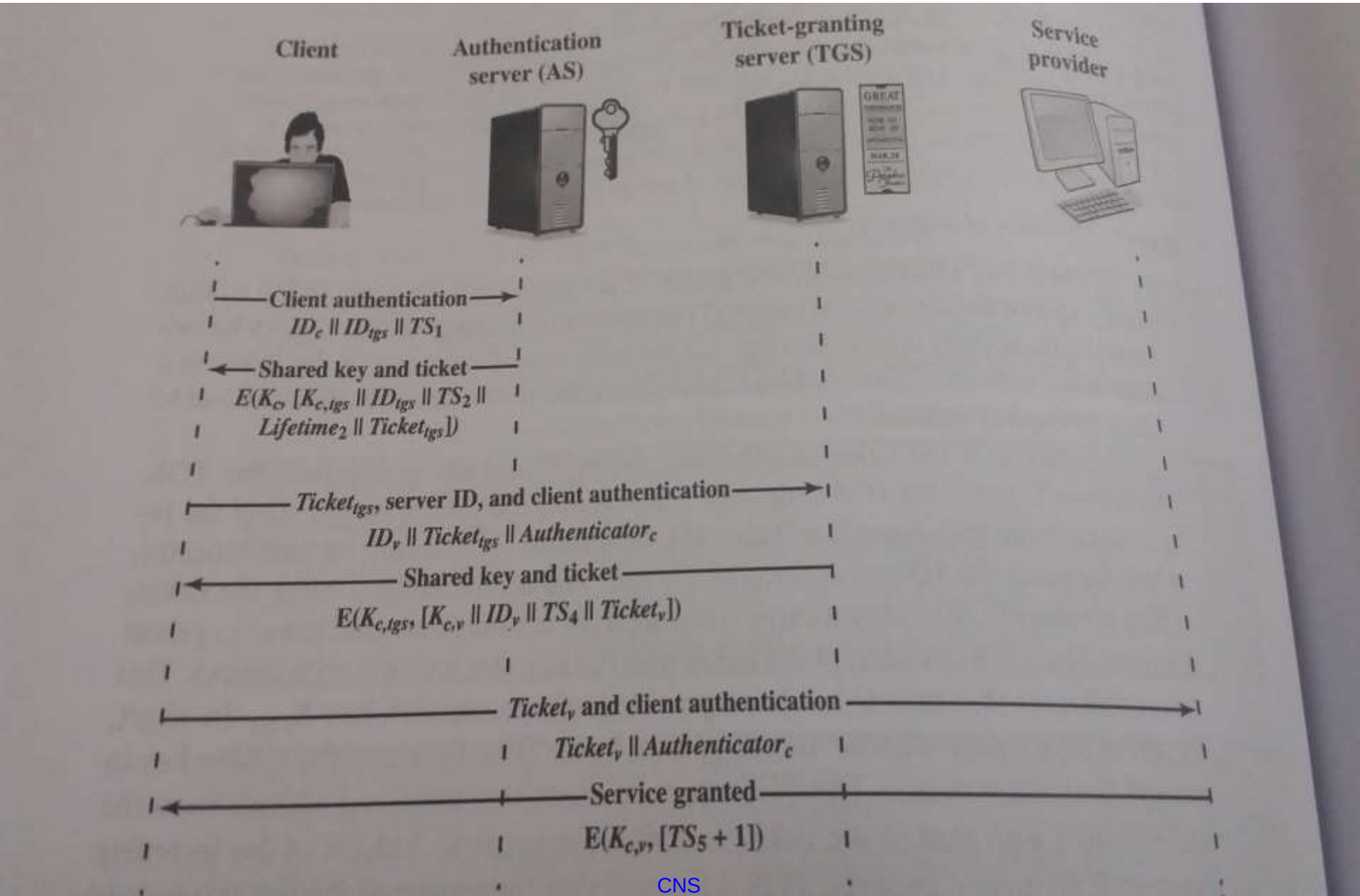
(5) C → V: $Ticket_v$ || $Authenticator_c$

(6) V → C: EKc,v[TS5 +1]

Overview of Kerberos



Kerberos Exchanges



Kerberos Realms & Multiple Kerbe

A Full-Service **Kerberos Environment** called **Kerberos Realm** consists:

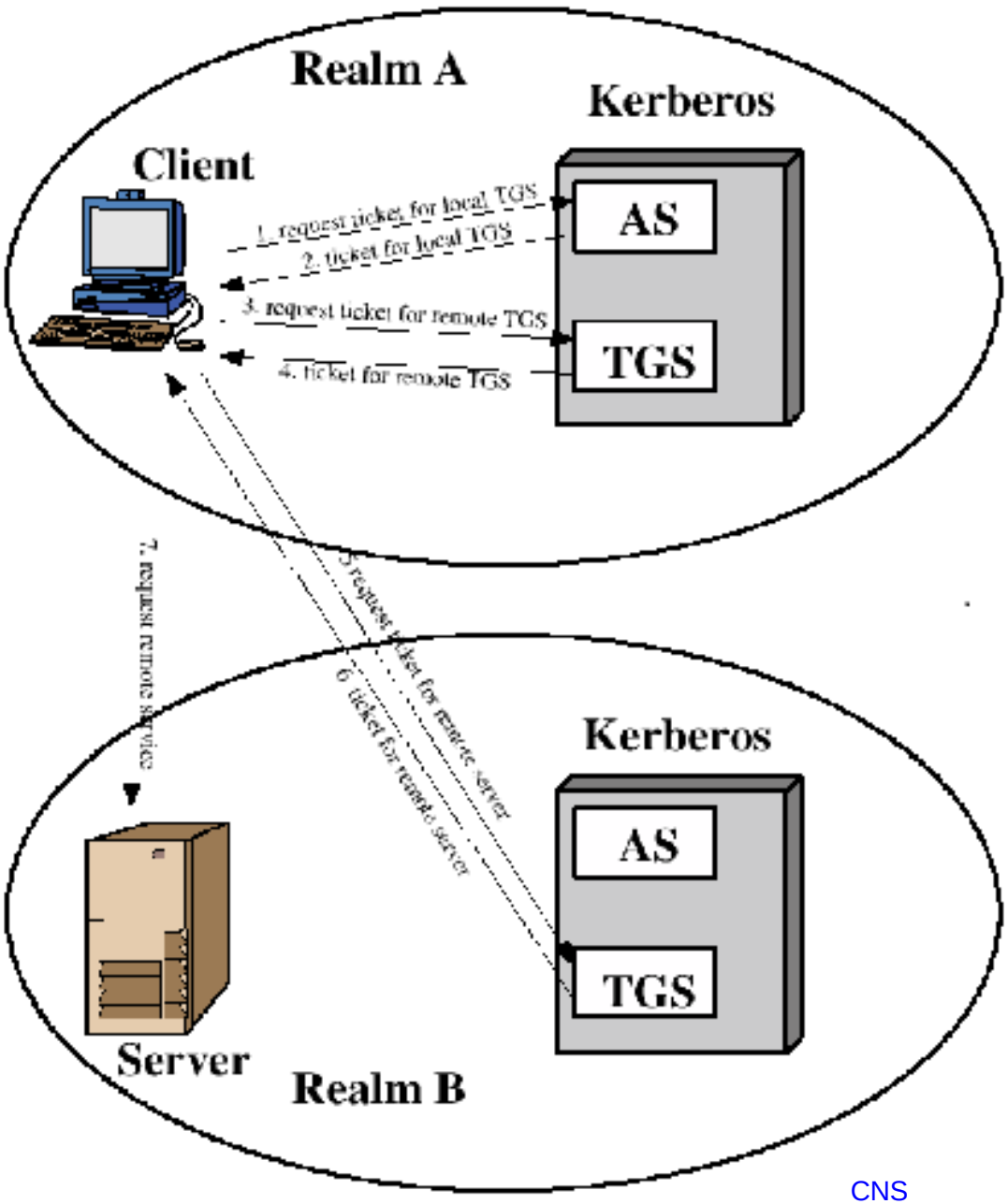
1. Kerberos Server
2. No. of Clients
3. No. of Application Servers

Kerberos Realms & Multiple Kerber

Application Servers Requirements:

1. Kerberos Server must have the user ID and hashed passwords of all participating users in its database. All users are registered with the Kerberos Server.
2. Kerberos Server must share a secret key with each server. All servers are registered with the Kerberos server.
3. The Kerberos Server in each interoperating realm shares a secret key with the server in other realm. 2 Kerberos servers are registered with each other.

Request for Service in Another Realm



1. Request ticker for local TGS
2. Ticket for local TGS
3. Request ticker for remote TGS
4. Ticket for remote TGS
5. Request ticker for remote server
6. Ticket for remote server
7. Request for remote service

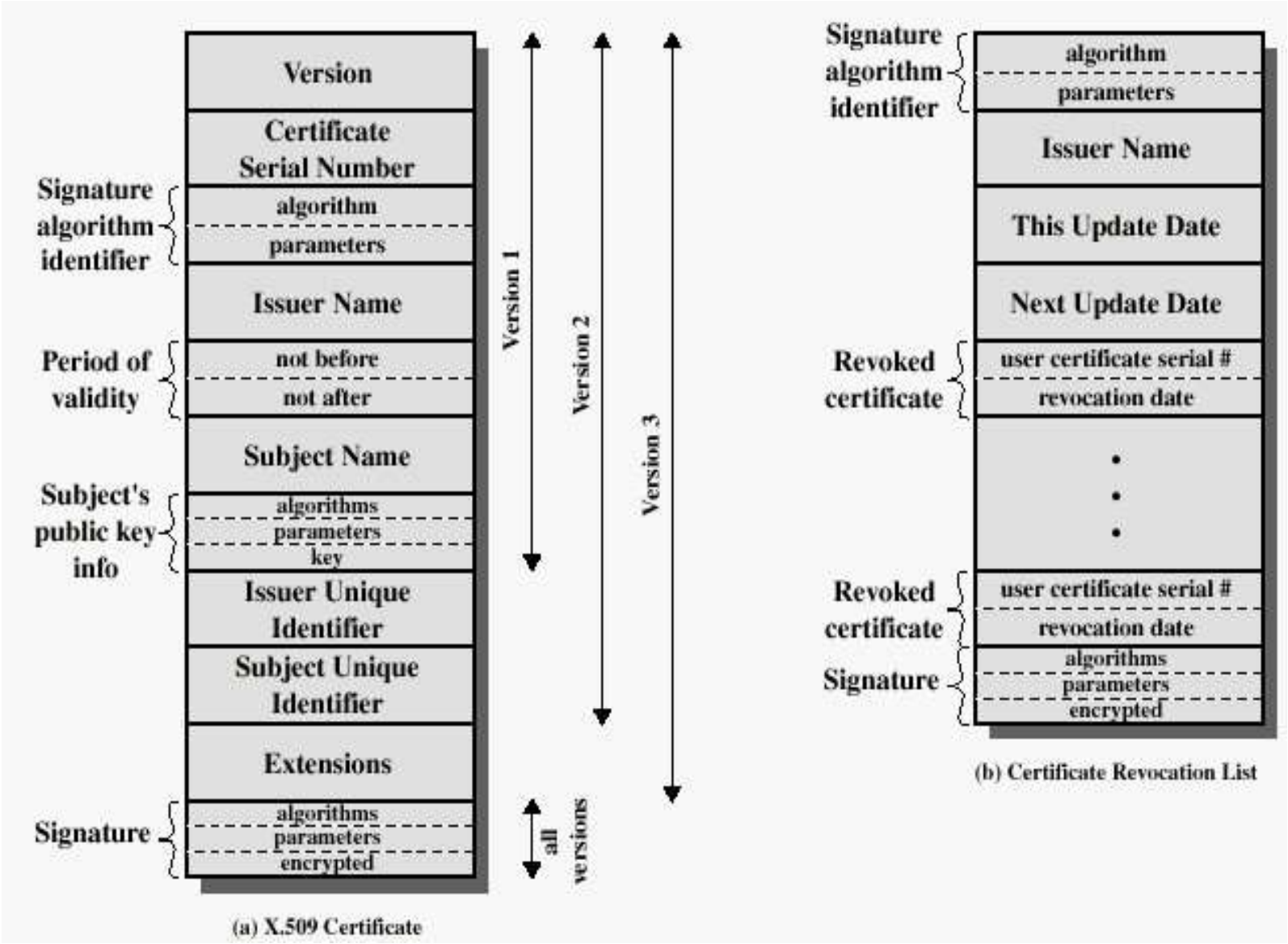
Difference Between Version 4 and 5

- Encryption system dependence (V.4 DES)
- Internet protocol dependence
- Message byte ordering
- Ticket lifetime
- Authentication forwarding
- Interrealm authentication

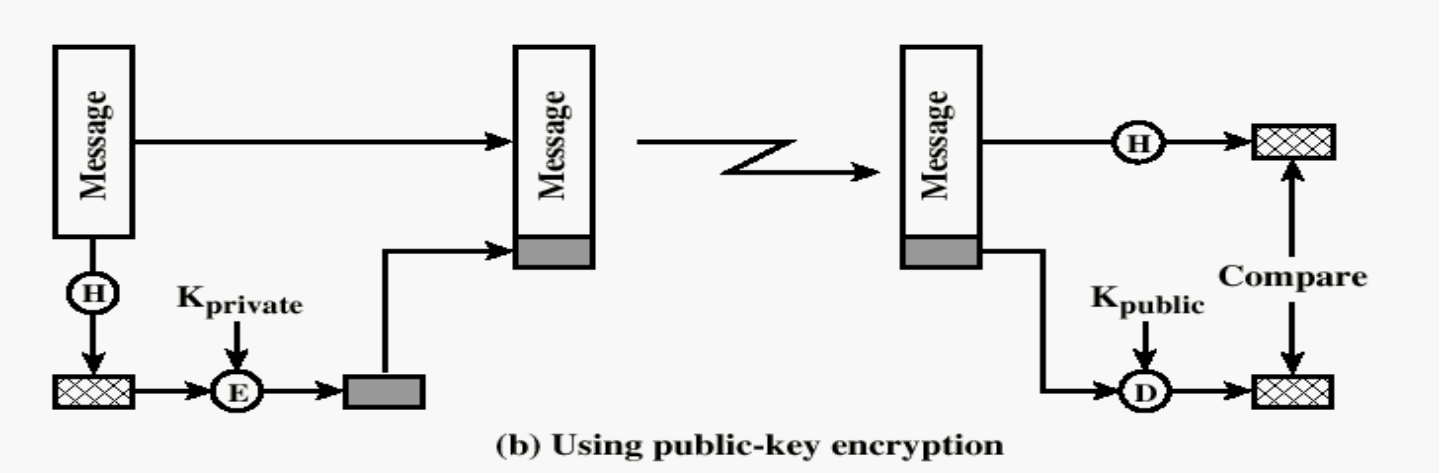
X.509 Authentication Service

- Distributed set of servers that maintains a database about users.
- Each certificate contains the public key of a user and is signed with the private key of a CA.
- Is used in S/MIME, IP Security, SSL/TLS and SET.
- RSA is recommended to use.

X.509 Formats



Typical Digital Signature Approach

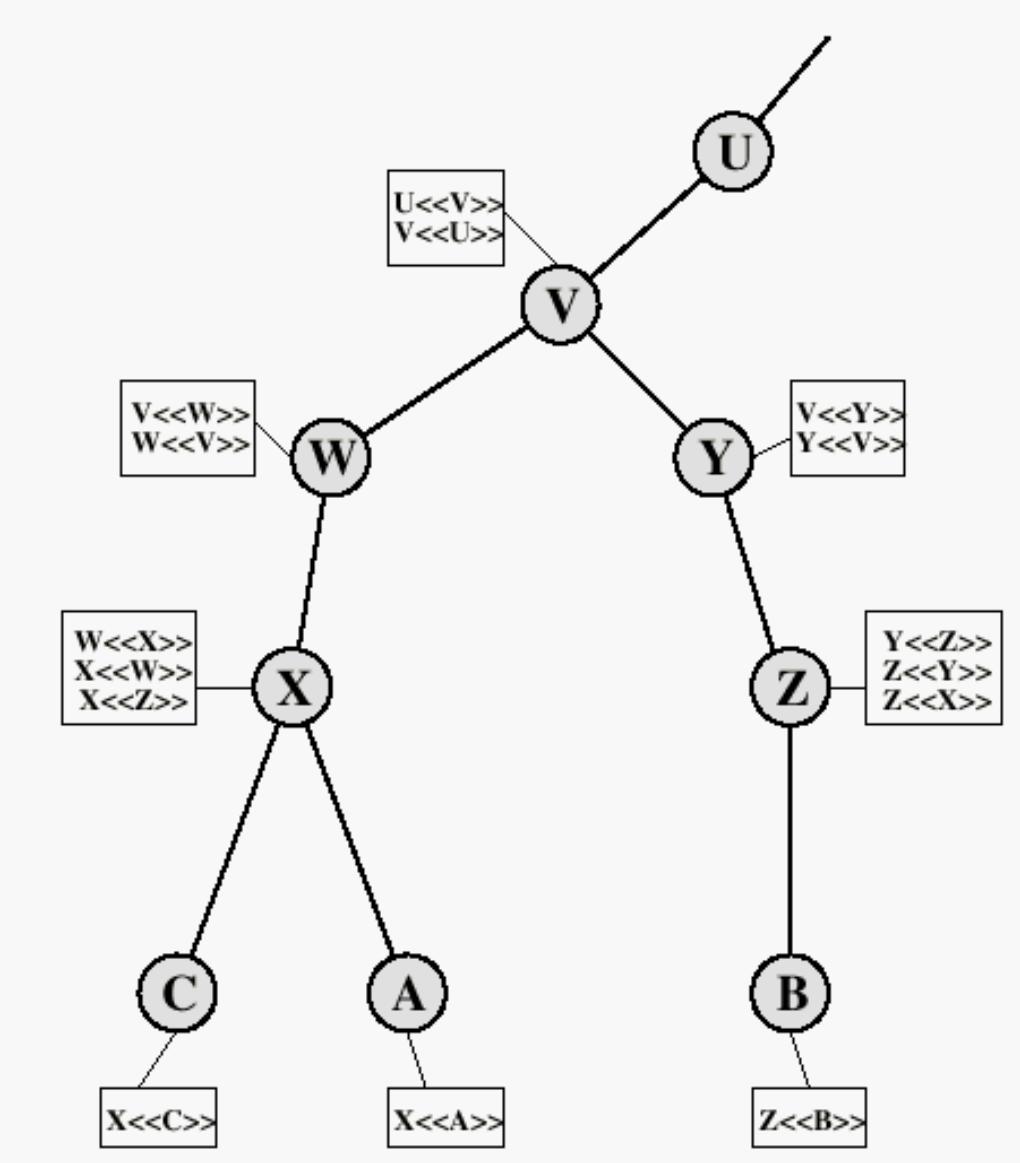


(b) Using public-key encryption

Obtaining a User's Certificate

- Characteristics of certificates generated by CA:
  - Any user with access to the public key of the CA can recover the user public key that was certified.
  - No part other than the CA can modify the certificate without this being detected.

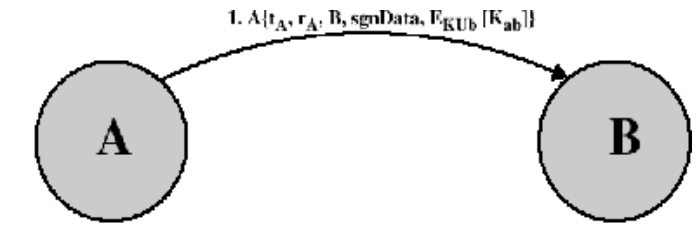
X.509 CA Hierarchy



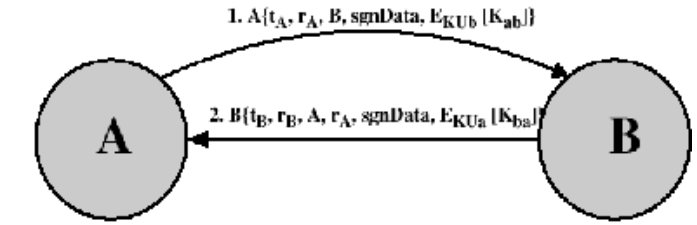
Revocation of Certificates

- Reasons for revocation:
  - The users secret key is assumed to be compromised.
  - The user is no longer certified by this CA.
  - The CA's certificate is assumed to be compromised.

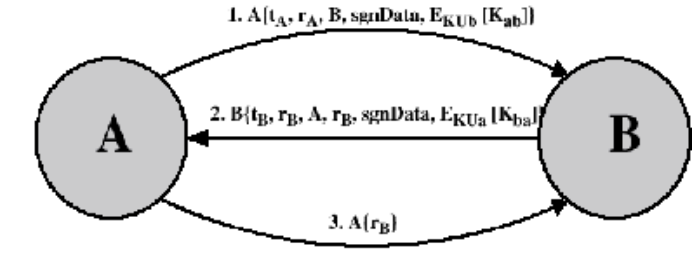
Authentication Procedures



(a) One-way authentication



(b) Two-way authentication

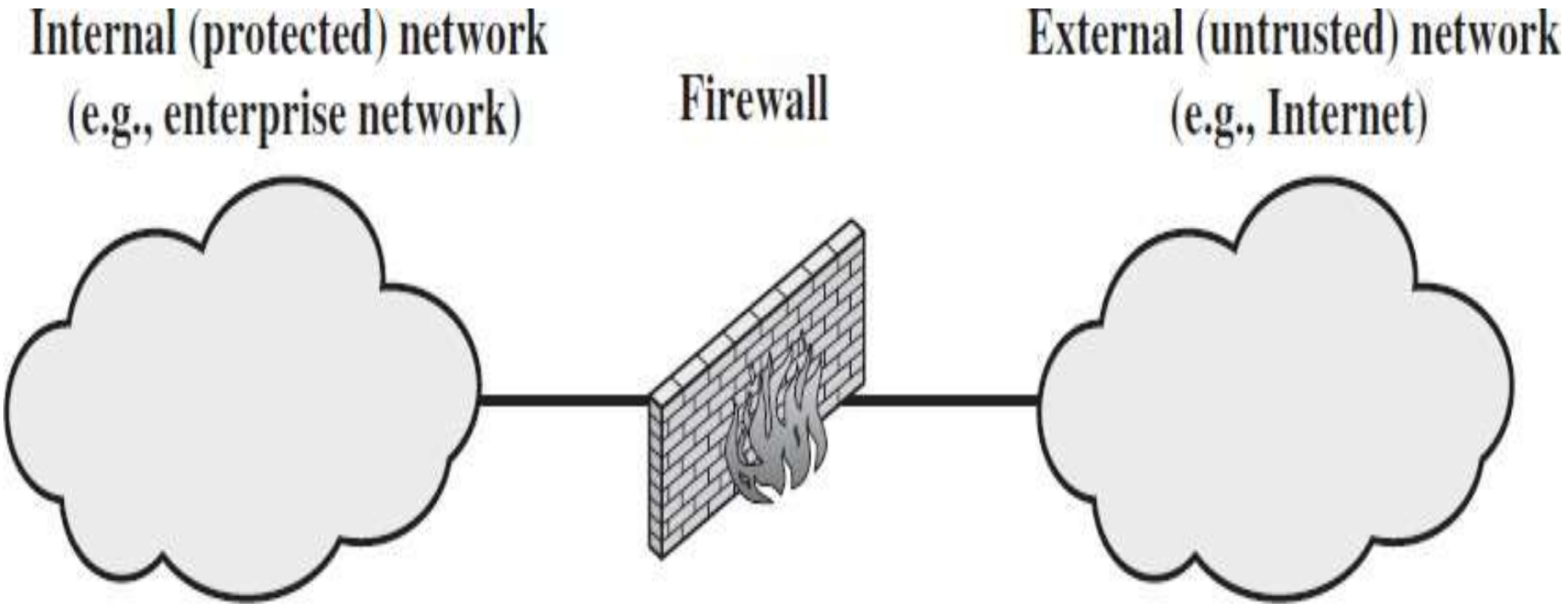


(c) Three-way authentication

Figure 4.5 X.509 Strong Authentication Procedures

Firewalls

Firewall


Internal (protected) network
(e.g., enterprise network)
Firewall
External (untrusted) network
(e.g., Internet)

Need for Firewalls

- **Centralized data processing system**, with a central mainframe supporting a no. of directly connected terminals
- **Local area networks** (LANs) interconnecting PCs and terminals to each other and the mainframe
- **Premises network**, consisting of a number of LANs, interconnecting PCs, servers, and a mainframe or two
- **Enterprise-wide network**, consisting of multiple, geographically distributed premises networks interconnected by a private wide area network (WAN)
- **Internet connectivity**, in which the various premises networks all hook into the Internet and may or may not also be connected by a private WAN

Design Goals of Firewalls

1. **All traffic from inside to outside, and vice versa, must pass through the firewall.**
   - This is achieved by physically blocking all access to the local network except via the firewall.
2. **Only authorized traffic, as defined by the local security policy, will be allowed to pass.**
   - Various types of firewalls are used, which implement various types of security policies.
3. **The firewall itself is immune to penetration.**
   - This implies the use of a hardened system with a secured operating system.

Characteristics of Firewalls

- **Service control:** Determines the types of Internet services that can be accessed, inbound or outbound. The firewall may filter traffic on the basis of IP address, protocol, or port number.
- **Direction control:** Determines the direction in which particular service requests may be initiated and allowed to flow through the firewall.
- **User control:** Controls access to a service according to which user is attempting to access it.
- **Behavior control:** Controls how particular services are used. For example, the firewall may filter e-mail to eliminate spam, or it may enable external access to only a portion of the information on a local Web server.

Capabilities of Firewalls

1. **A firewall defines a single choke point** that keeps unauthorized users out of the protected network, prohibits potentially vulnerable services from entering or leaving the network, and provides protection from various kinds of IP spoofing and routing attacks.
2. **A firewall provides a location for monitoring security-related events** like auditing and alarms.
3. **A firewall is a convenient platform for several Internet functions that are not security related** like network address translator and network mgmt. function.
4. **A firewall can serve as the platform for IPSec and**

Limitations of Firewalls

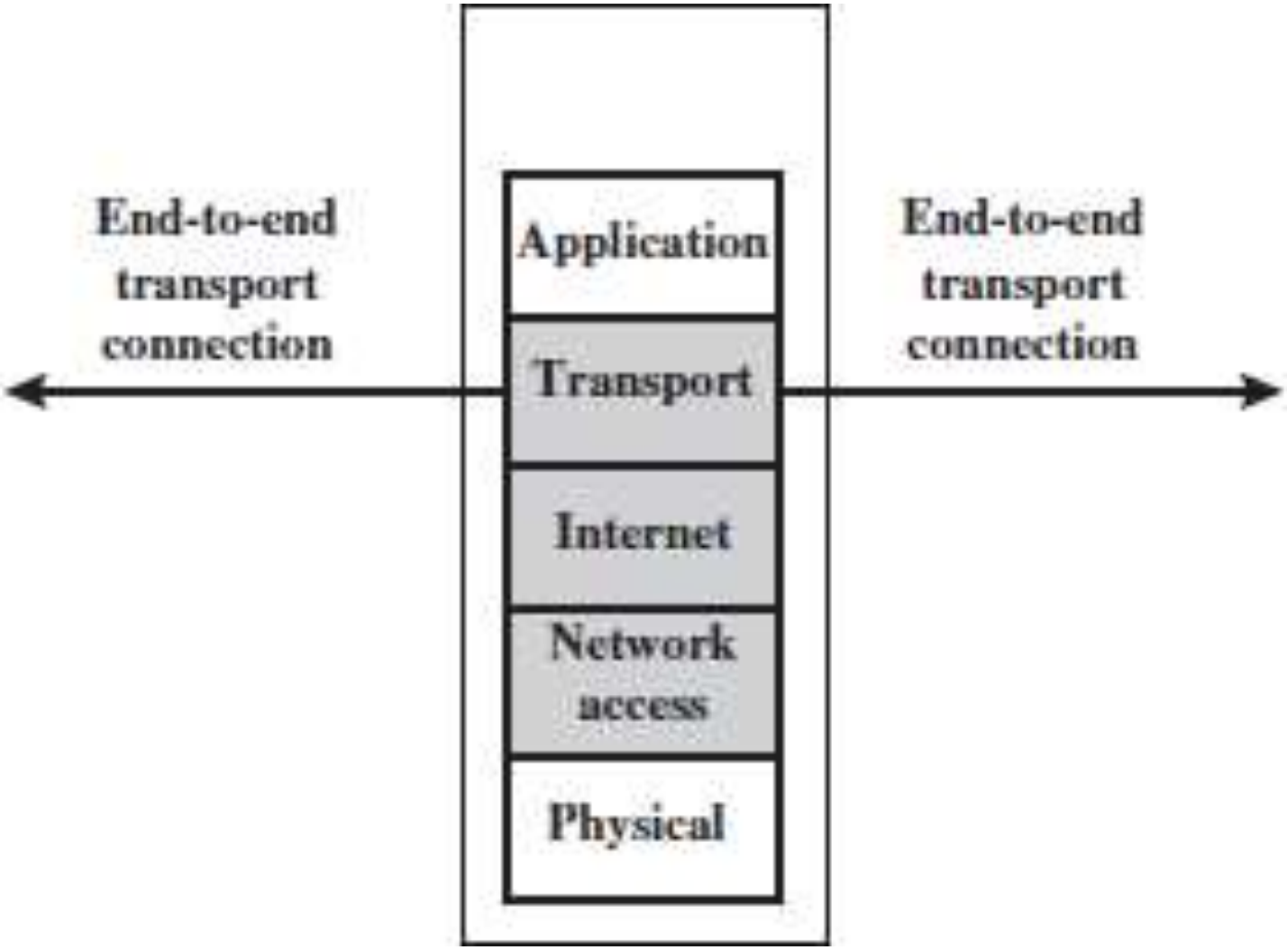
1. **The firewall cannot protect against attacks that bypass the firewall.**
2. **The firewall may not protect fully against internal threats,** such as a disgruntled employee or an employee who unwittingly cooperates with an external attacker.
3. **An improperly secured wireless LAN may be accessed from outside the organization.**
4. **A laptop, PDA, or portable storage device may be used and infected outside the corporate network, and then attached and used internally.**

Types of Firewalls

1. Packet Filtering Firewall

- A firewall may act as a **packet filter**.
- It can operate as a **positive filter**, allowing to pass only packets that meet specific criteria, or as a **negative filter**, rejecting any packet that meets certain criteria.
- A packet filtering firewall applies a **set of rules** to each incoming and outgoing IP packet and then **forwards or discards** the packet.
- The firewall is typically configured to filter packets

1. Packet Filtering Firewall



1. Packet Filtering Firewall

- Filtering rules are based on info. in network packet:
  - **Source IP address:** The IP address of the system that originated the IP packet (e.g., 192.178.1.1)
  - **Destination IP address:** The IP address of the system the IP packet is trying to reach (e.g., 192.168.1.2)
  - **Source and destination transport-level address:** The transport-level (e.g., TCP or UDP) port number
  - **IP protocol field:** Defines the transport protocol
  - **Interface:** For a firewall with three or more ports, which interface of the firewall the packet came from or which interface of the firewall the packet is destined for

1. Packet Filtering Firewall

- The packet filter is typically set up as a **list of rules** based on matches to fields in the IP or TCP header.
- If there is a **match** to one of the rules, that rule is invoked to **forward or discard** the packet.
- If there is **no match** to any rule, then a **default action** is taken.
- Two default policies are possible:
  - **Default = discard:** That which is not expressly permitted is prohibited.
  - **Default = forward:** That which is not expressly prohibited is permitted.

1. Packet Filtering Firewall

- Following table gives some examples of packet filtering rule sets.
- In each set, the rules are applied top to bottom.
- The “*” in a field is a wildcard designator that matches everything.
- We assume that the default = discard policy is in force.

action	ourhost	port	theirhost	port	comment
block	*	*	SPIGOT	*	we don't trust these people
allow	OUR-GW	25	*	*	connection to our SMTP port

Rule Set B

action	ourhost	port	theirhost	port	comment
block	*	*	*	*	default

Rule Set C

action	ourhost	port	theirhost	port	comment
allow	*	*	*	25	connection to their SMTP port

Rule Set D

action	src	port	dest	port	flags	comment
allow	{our hosts}	*	*	25		our packets to their SMTP port
allow	*	25	*	*	ACK	their replies

Rule Set E

action	src	port	dest	port	flags	comment
allow	{our hosts}	*	*	*		our outgoing calls
allow	*	*	*	*	ACK	replies to our calls
allow	*	*	*	>1024		traffic to nonservers

Packet Filtering Firewall-Rule Set

A. **Inbound mail is allowed, but only to a gateway host,** however, packets from SPIGOT, are blocked because that host has a history of sending massive files in e-mail messages.

B. This is an explicit statement of the default policy.

C. This rule set is intended to specify that any inside host can send mail to the outside.

D. This rule set achieves the intended result that was not achieved in C.

E. This rule set is one approach to handling FTP connections.

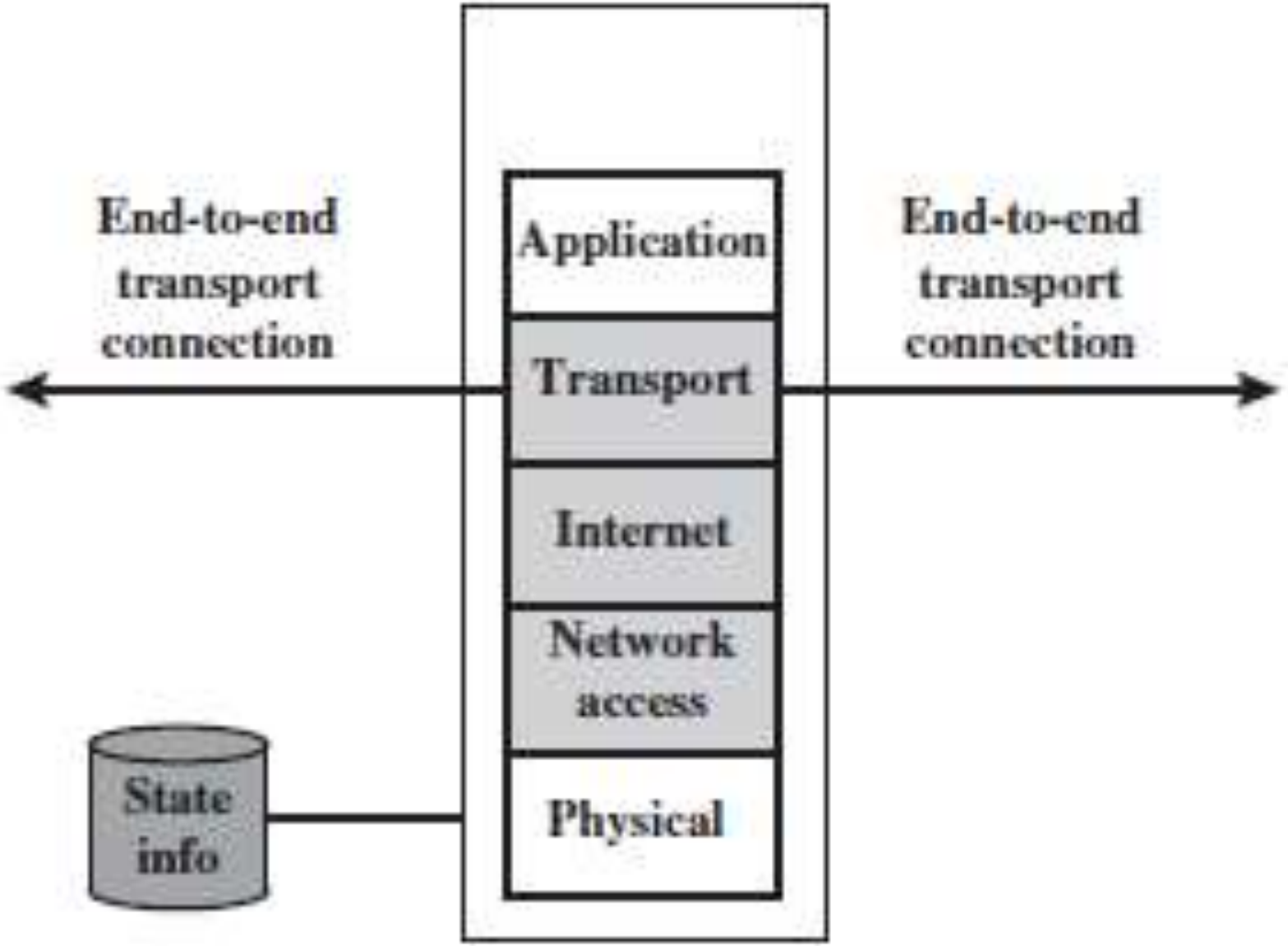
Attacks on Packet Filtering Firewall

- **IP address spoofing:** The intruder transmits packets from the outside with a source IP address field containing an address of an internal host.
  - The **countermeasure** is to discard packets with an inside source address if the packet arrives on an external interface.
- **Source routing attacks:** The source station specifies the route that a packet should take as it crosses the Internet, that this will bypass security measures that do not analyze the source routing information.
  - The **countermeasure** is to discard all packets that use this option.

Attacks on Packet Filtering Firewall

- **Tiny fragment attacks:** The intruder uses the IP fragmentation option to create extremely small fragments and force the TCP header information into a separate packet fragment.
    - **Countermeasure:** A tiny fragment attack can be defeated by enforcing a rule that the first fragment of a packet must contain a predefined minimum amount of the transport header. If the first fragment is rejected, the filter can remember the packet and discard all subsequent fragments.

2. Stateful Inspection Firewall



2. Stateful Inspection Firewall

- A **packet filtering firewall** permit inbound network traffic on all high-numbered ports for TCP-based traffic to occur.
- This creates a **vulnerability** that can be exploited by unauthorized users.
- A **stateful inspection packet firewall** tightens up the rules for TCP traffic by creating a **directory** of currently established outbound TCP connections
- The packet filter will now allow incoming traffic to high-numbered ports only for those packets that fit the profile of one of the entries in this directory.

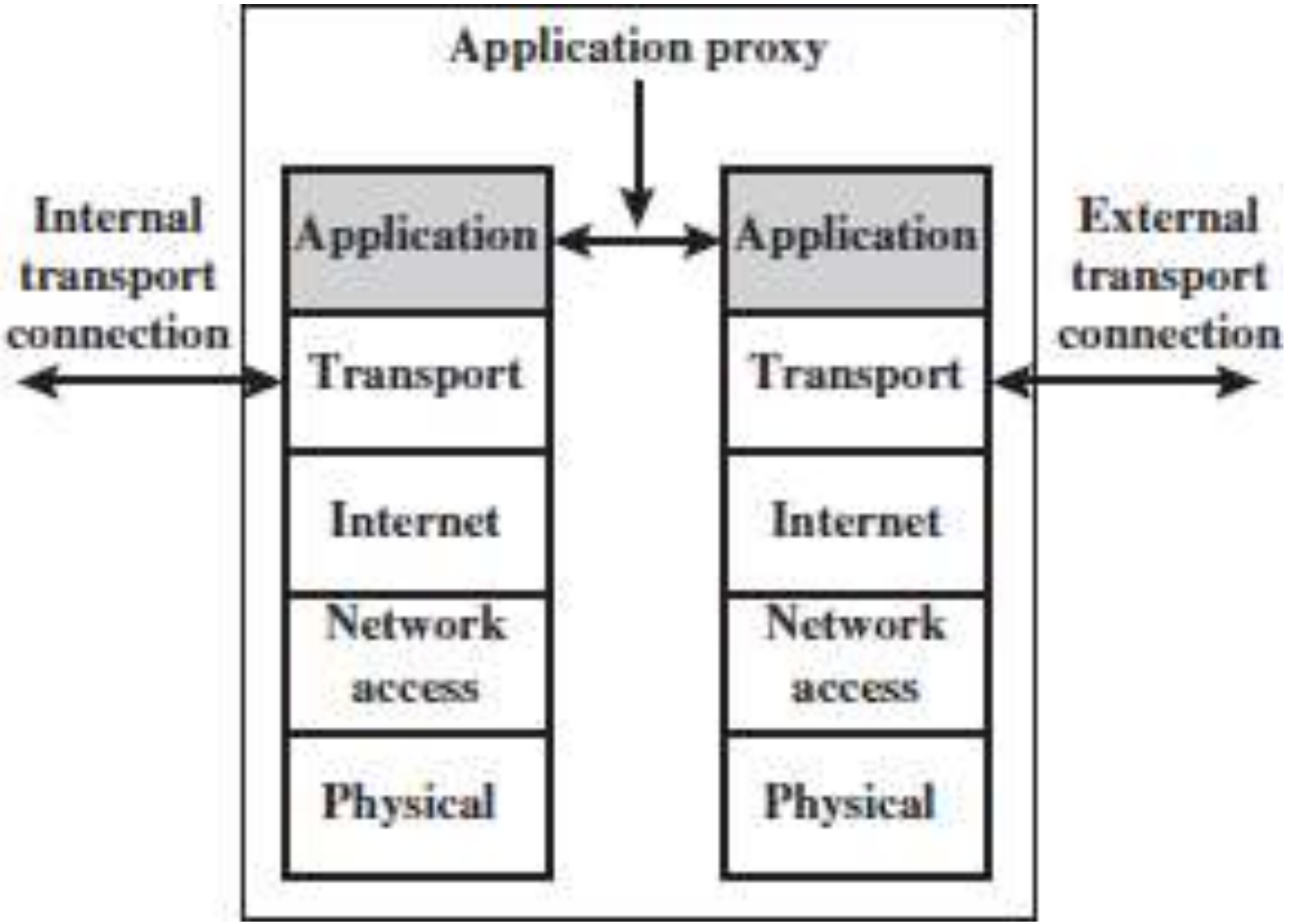
2. Stateful Inspection Firewall

Source Address	Source Port	Destination Address	Destination Port	Connection State
192.168.1.100	1030	210.22.88.29	80	Established
192.168.1.102	1031	216.32.42.123	80	Established
192.168.1.101	1033	173.66.32.122	25	Established
192.168.1.106	1035	177.231.32.12	79	Established
223.43.21.231	1990	192.168.1.6	80	Established
2122.22.123.32	2112	192.168.1.6	80	Established
210.922.212.18	3321	192.168.1.6	80	Established
24.102.32.23	1025	192.168.1.6	80	Established
223.21.22.12	1046	192.168.1.6	80	Established

2. Stateful Inspection Firewall

- A stateful packet inspection firewall **reviews** the same packet information as a packet filtering firewall, but also **records** information about TCP connections.
- Some stateful firewalls also **keep track of TCP sequence numbers** to prevent attacks that depend on the sequence number, such as **session hijacking**.
- Some even **inspect** limited amounts of application data for some well-known protocols like **FTP, IM and SIP commands,** in order to identify and track related connections.

3. Application Level Gateway



3. Application Level Gateway

- An **application-level gateway**, also called an **application proxy**, acts as a relay of application-level traffic.
- The user contacts the gateway using a **TCP/IP application**, such as **Telnet or FTP**, and the gateway asks the user for the name of the remote host to be accessed.
- When the user responds and provides a **valid user ID** and **authentication information**, the gateway contacts the application on the remote host and relays TCP segments containing the application data between the two endpoints.

3. Application Level Gateway

- If the **gateway does not implement the proxy code for a specific application**, the service is not supported and cannot be forwarded across the firewall.
- Further, the gateway can be **configured to support only specific features of an application** that the network administrator considers acceptable while denying all other features.

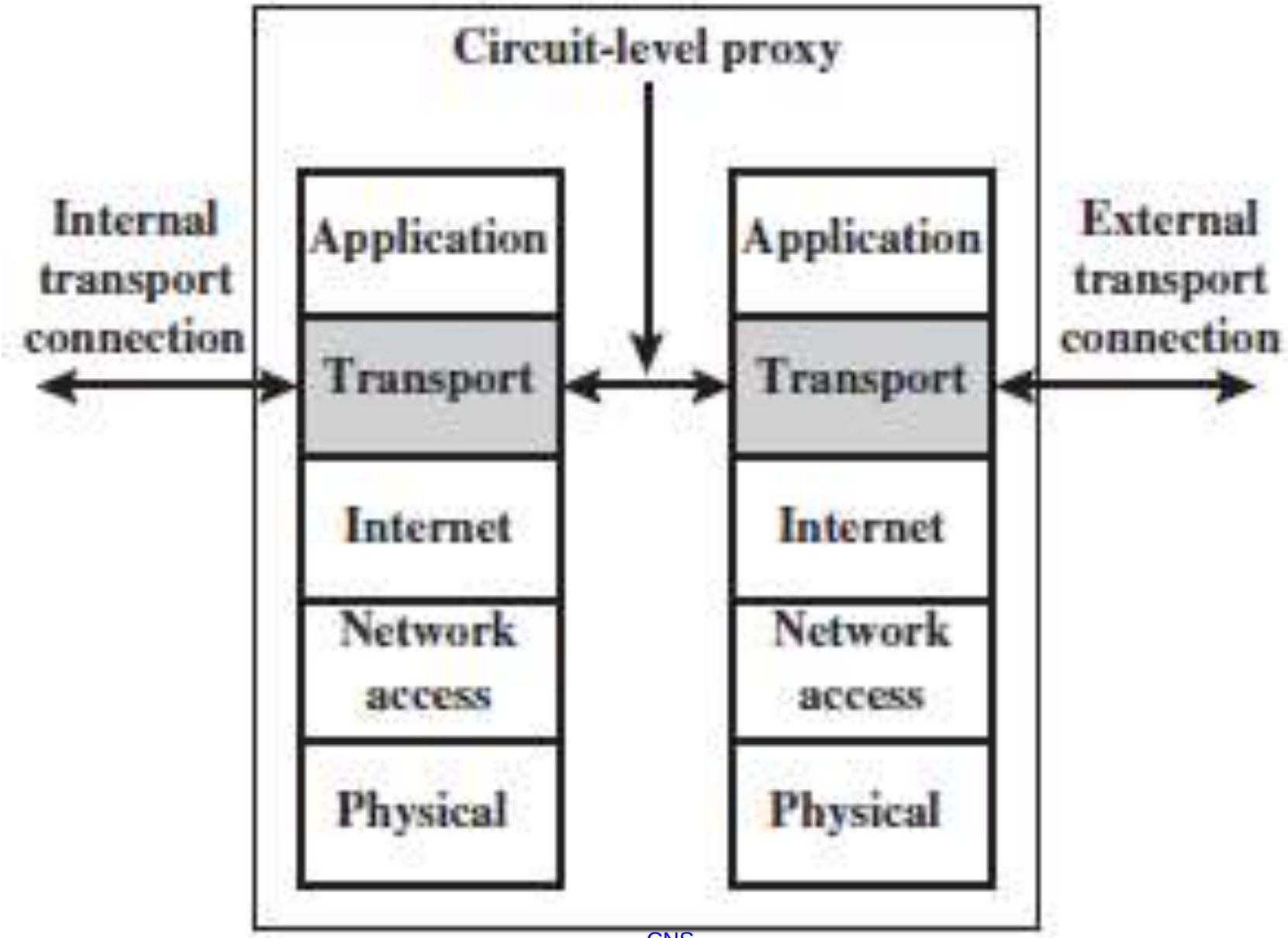
3. Application Level Gateway

- Application-level gateways tend to be more **secure than packet filters.**
- Rather than trying to deal with the numerous possible combinations that are to be allowed and forbidden at the TCP and IP level, the application-level gateway need only **scrutinize a few allowable applications**.
- In addition, it is **easy to log and audit** all incoming traffic at the application level.

3. Application Level Gateway

- A prime disadvantage of this type of gateway is the **additional processing overhead** on each connection.
- In effect, there are **two spliced connections** between the end users, with the gateway at the splice point, and the gateway must examine and forward all traffic in both directions.

4. Circuit Level Gateway



4. Circuit Level Gateway

- A fourth type of firewall is the circuit-level gateway or **circuit-level proxy.**
- This can be a **stand-alone system** or it can be a **specialized function performed by an application-level gateway.**
- As with an application gateway, a circuit-level gateway **does not permit an end-to-end TCP** connection.
- Rather, the gateway sets up **two TCP connections**, one between itself and a TCP user on an inner host and one between itself and a TCP user on an outside host.

4. Circuit Level Gateway

- Once the two connections are established, the gateway typically **relays TCP segments** from one connection to the other without examining the contents.
- The **security function** consists of determining which connections will be allowed.
- A typical use of circuit-level gateways is a situation in which the **system administrator trusts the internal users**.

4. Circuit Level Gateway

- The gateway can be configured to support **application-level** or proxy service on **inbound connections** and **circuit-level functions** for **outbound connections**.
- In this configuration, the gateway can incur the **processing overhead of examining incoming application data** for forbidden functions but does not incur that overhead on outgoing data.