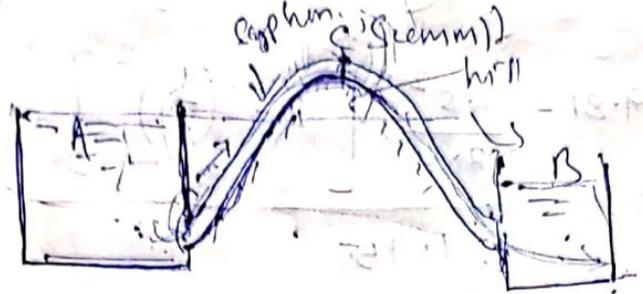


Syphon

Syphon is a long bend pipe which is used to transfer liquid from a reservoir at a higher level ground



The ~~point~~ highest point of the syphon is called as Summit. As the pt. summit 'C' is above the free surface of the water in the tank A, the pressure at 'C' will be less than atmospheric pressure. Thus

Theoretically the pressure is reduced -10.3 m of water, but in actual practice the pressure is only -7.6 m of water or $10.3 - 7.6 = 2.7$ m of water absolute.

If the pressure at 'C' becomes less than 2.7 m of water absolute, the dissolved air and other gases would come out from water and collect at the Summit, the flow of the water will be ~~ob~~ obstructed.

Use of Syphon in following cases

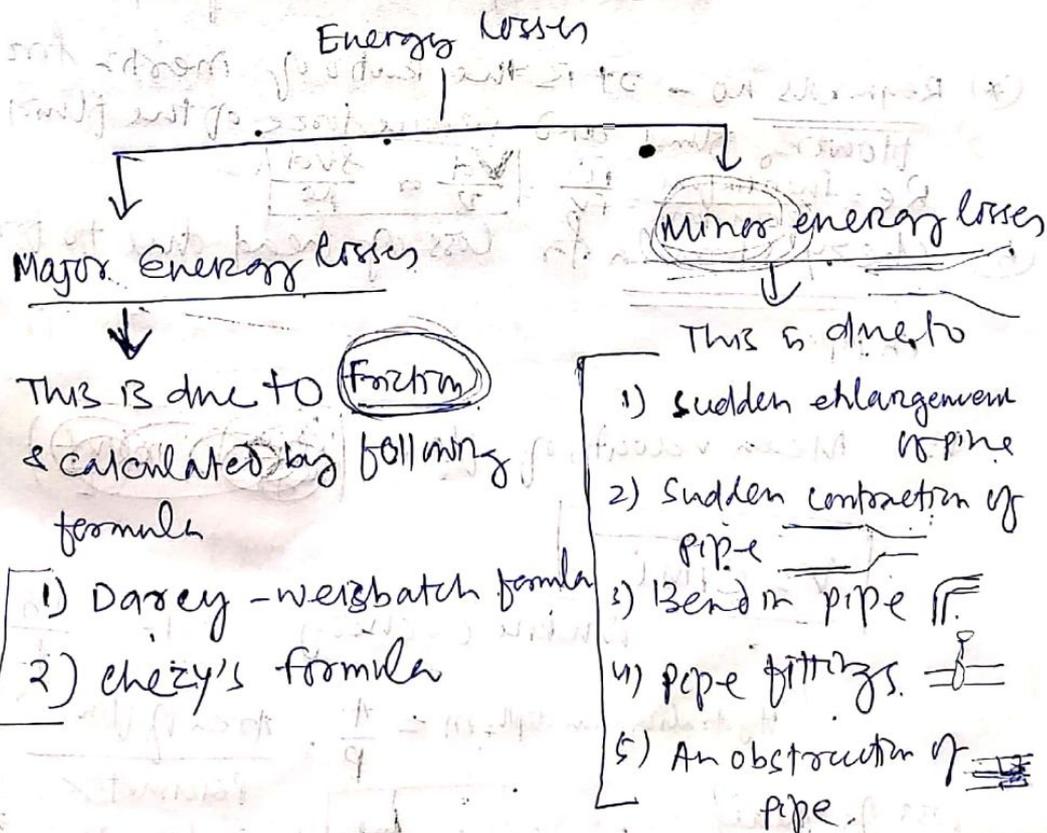
- i) To carry water from one reservoir to another reservoir separated by hill or ridge.
- ii) To take the water liquid of the tank which is not having outlet.
- iii) To empty a channel not provided with outlet sluice.

Module III Flow through pipes

A pipe is a closed conduit (circular section) which is used for carrying fluid under pressure. The flow in a pipe is termed pipe flow only when the fluid completely fill the cross-section and there is no free surface of fluid.

Loss of energy in pipes

When a fluid flows through a pipe, the fluid experiences some resistance due to which some of energy of the fluid is lost. The loss of energy is classified as



Loss of energy (or head) due to friction

- a) Poiseuille - Weisbach formula: The loss of head (or energy) in pipes due to friction is calculated from Poiseuille - Weisbach equation

$$h_f = \frac{4 f L v^2}{d \times 2g}$$

where h_f = loss of head due to friction.

$$f = \begin{cases} \frac{16}{Re} & \text{for } Re < 2000 \text{ (viscous flow)} \\ \frac{0.079}{Re^{0.25}} & \text{for } Re \text{ varying from } 4000 \text{ to } 10^6 \end{cases}$$

L = Length of pipe

v = mean velocity of flow

d = diameter of pipe.

- (*) Reynolds no. :- It is the ratio of inertia force of flowing fluid and viscous force of the fluid.
- $$Re = \frac{\text{Inertia force}}{\text{viscous force}} = \frac{F_i}{F_v} = \frac{Vd}{\nu} \text{ or } \frac{\rho Vd}{\mu}$$

- (b) Chezy's formula for loss of head due to friction in pipe.

v = Mean velocity of flow = ~~$\sqrt{\frac{4fL}{d \times 2g}}$~~

$$v = C \sqrt{mi}$$

where C = Chezy's const. = $\frac{8.5}{f}$

Hydraulic mean depth $m = \frac{A}{P} = \frac{\text{Area of flow}}{\text{Perimeter}}$

Loss of head cent feet (slope) $i = \frac{h_f}{L}$ = head loss due to friction

where h_f = head loss due to friction.

L = Length of pipe

The value of $m = d/4$, d = dia of pipe.

Problem =

In a pipe of diameter 350 mm and length 75 m water is flowing at a velocity of 2.8 m/s. Find the head loss due to friction using 1) Darcy-Weisbach formula 2) Chezy's formula, $C = 95$.

Assume kinematic viscosity of water as 0.012 Stokes

Solⁿ Given Dia of pipe - $D = 350 \text{ mm} = 0.35 \text{ m}$.

Length of pipe $L = 75 \text{ m}$.

vel. = $V = 2.8 \text{ m/s}$

Chezy's constant $C = 95$

kinematic viscosity of water = $\nu = 0.012 \text{ Stokes}$
 $= 0.012 \times 10^{-4} \text{ m}^2/\text{s}$

Head lost due to friction

1) Darcy-Weisbach formula

$$h_f = \frac{4fLV^2}{d \cdot 2g}$$

$$Re = \frac{V \times D}{\nu} = \frac{2.8 \times 0.35}{0.012 \times 10^{-4}} = 8.167 \times 10^5$$

$$f = \frac{0.0791}{(Re)^{1/4}} = \frac{0.0791}{(8.167 \times 10^5)^{1/4}} = 0.00263$$

Head lost

$$h_f = \frac{4 \times 0.00263 \times 75 \times (2.8)^2}{0.35 \times 2 \times 9.81} = 0.9 \text{ m}$$

① Chezy's formula

$$v = C \sqrt{mi}, \quad C = 55$$

$$m = A/P = \frac{\pi d^2}{4} \div \pi d = \frac{d}{4} = \frac{0.34}{4} = 0.0875 \text{ m.}$$

$$2.8 = 55 \sqrt{0.0875 \times i}$$

$$0.0875 \times i = \left(\frac{2.8}{55}\right)^2 = 0.0259$$

$$\text{or } i = 0.0296$$

$$\text{BWL } i = \frac{hf}{L} = 0.0296$$

$$\frac{hf}{75} = 0.0296$$

$$\text{or } hf = 75 \times 0.0296 = 2.22 \text{ m}$$

Q-4 - An oil of sp. gravity 0.7 is flowing through a pipe of diameter 300 mm at a rate of 500 lit/s. find the head lost due to friction and power required to maintain the flow for the length of 1000 m, take $\nu = 0.29$ Stokes.

5/2 Given - Sp. gravity of oil $S = 0.7$
 Dia of pipe $d = 300 \text{ mm} = 0.3 \text{ m}$
 Discharge $Q = 500 \text{ lit/s} = 0.5 \text{ m}^3/\text{s}$

Pipe length $L = 1000 \text{ m}$.

$$\text{velocity } (v) = \frac{Q}{\text{Area}} = \frac{0.5}{\frac{\pi}{4} d^2} = \frac{0.5 \times 4}{\pi \times 0.3^2} = 7.073 \text{ m/s}$$

$$\therefore \text{ Reynolds no. } Re = \frac{vd}{\nu} = \frac{7.073 \times 0.3}{0.29 \times 10^{-4}} = 7.316 \times 10^4$$

coefficient of friction

$$f = \frac{0.079}{Re^{1/4}} = \frac{0.079}{(7.316 \times 10^4)^{1/4}} = 0.0048$$

Head lost due to friction

$$h_f = \frac{4 \times 0.0048 \times 1000 \times 7.073^2}{0.3 \times 2 \times 9.81} = 163.18 \text{ m}$$

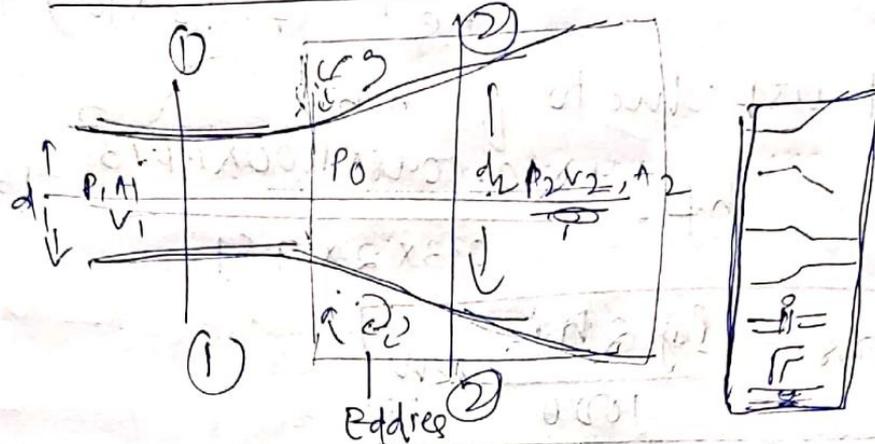
$$\text{Power} = \frac{\rho g Q h_f}{1000} \text{ kW}$$

$$\rho = \text{density of the oil} = \frac{0.7 \times 1000}{0.85} = 760 \text{ kg/m}^3$$

$$\therefore \text{power required} = \frac{760 \times 9.81 \times 0.5 \times 163.18}{1000} \\ = 560.28 \text{ kW}$$

Minor energy losses

1) Loss of head due to sudden enlargement



The fig shows a liquid flowing through a pipe which has suddenly enlargement. Due to sudden enlargement, the flow is decelerated abruptly and eddies are developed resulting in loss of energy.

Applying Bernoulli's eqn to secⁿ 1-1 & 2-2

we have

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + \text{head loss due to enlargement}$$

But $z_1 = z_2$, as horizontal.

hence the above eqn reduces to

$$h_e = \left(\frac{P_1}{\rho g} - \frac{P_2}{\rho g} \right) + \frac{v_1^2}{2g} - \frac{v_2^2}{2g} \quad \text{--- (1)}$$

Now, the force acting on liquid on the control volume in the secⁿ 1-1 & 2-2 in the flow direction given by

$$F_x = P_1 A_1 + P_0 (A_2 - A_1) - P_2 A_2$$

Assuming $P_0 = P_1$, we have

$$F_x = (P_1 - P_2) A_2 \quad \text{--- (2)}$$

Consider momentum of liquid at section 1-1 and 2-2.

are mass & velocity i.e. $\rho A_1 v_1$ & $\rho A_2 v_2$ respectively.

Consider momentum of fluid ~~as shown below~~
 $\frac{\rho A v \times v}{\text{mass} \times \text{velocity}} = \rho A_2 v_2^2 - \rho A_1 v_1^2$

But from continuity equation, we have

$$A_1 v_1 = A_2 v_2$$

$$\therefore \boxed{A_1 = \frac{A_2 v_2}{v_1}}$$

\therefore change of momentum/sec.

$$= \rho A_2 v_2^2 - \rho \frac{A_2 v_2}{v_1} \times v_1^2$$

$$= \rho A_2 (v_2^2 - v_1 v_2) \quad \text{--- (3)}$$

Now net force = change in momentum.

$$(P_1 - P_2) A_2 = \rho A_2 (v_2^2 - v_1 v_2)$$

$$\therefore \frac{P_1 - P_2}{\rho} = v_2^2 - v_1 v_2$$

Dividing both side by ρ , we get

$$\frac{P_1 - P_2}{\rho g} = \frac{v_2^2 - v_1 v_2}{g}$$

$$\therefore \boxed{\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{v_2^2 - v_1 v_2}{g}}$$

Substituting the value of $\left(\frac{P_1}{\rho g} - \frac{P_2}{\rho g}\right)$ in eqn (1)

we get

$$h_e = \frac{v_2^2 - v_2 v_1}{g} + \frac{v_1^2}{2g} - \frac{v_2^2}{2g}$$

Rearranging we get

$$h_e = \frac{(v_1 - v_2)^2}{2g}$$

Q: At a sudden enlargement of a water main from 240mm and 480mm, diameter the hydraulic gradient line rise by 10mm. Calculate the rate of flow. (2)

SW
 D1 = smaller side $D_1 = 240\text{mm} = 0.24\text{m}$
 " larger side $D_2 = 480\text{mm} = 0.48\text{m}$
 Rise of hydraulic gradient line = 10mm

$$\left[\left(\frac{P_2}{\rho g} + z_2 \right) - \left(\frac{P_1}{\rho g} + z_1 \right) \right] = 10\text{mm} = 0.01\text{m}$$

Rate of flow

Applying Bernoulli's eqn to small & large pipe
 section 1-1 & 2-2

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_e \quad \text{--- (1)}$$

$$\text{But } h_e = \frac{(v_1^2 - v_2^2)}{2g} \quad \text{--- (2)}$$

From continuity eqn

$$A_1 v_1 = A_2 v_2$$

$$v_1 = \frac{A_2 v_2}{A_1} = \frac{\pi D_2^2 v_2}{\pi D_1^2} = \left(\frac{D_2}{D_1} \right)^2 v_2$$

$$v_1 = \left(\frac{0.48}{0.24} \right)^2 v_2 = 4v_2$$

Substituting this value of v_1 in eqn (1), we get

$$h_e = \frac{(4v_2^2 - v_2^2)}{2g} = \frac{3v_2^2}{2g}$$

Now substituting the values of h_e and v_1 in eqn (1), we get

$$\frac{P_1}{\rho g} + \left(\frac{4v_2^2}{2g}\right) + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + \left(\frac{9v_2^2}{2g}\right) h_c$$

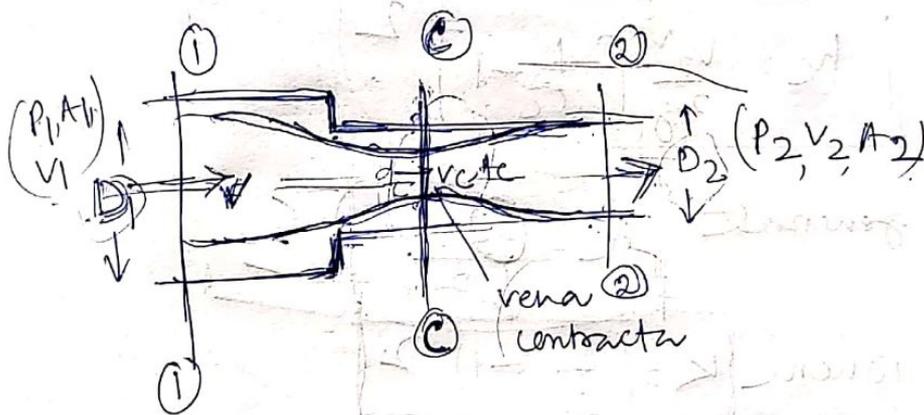
$$\therefore \left(\frac{P_2}{\rho g} + z_2\right) - \left(\frac{P_1}{\rho g} + z_1\right) = \frac{16v_2^2}{2g} - \frac{v_2^2}{2g} - \frac{9v_2^2}{2g}$$

$$\therefore 0.01 = \frac{6v_2^2}{2g}$$

$$\therefore v_2 = \left(\frac{0.01 \times 2 \times 9.8}{6}\right)^{1/2} = 0.181 \text{ m/s}$$

Rate of flow $Q = A_2 v_2 = \frac{\pi}{4} \times 0.08^2 \times 0.181$
 $= 0.03275 \text{ m}^3/\text{s}$

Head loss due to Sudden contraction



In above figure due to sudden contraction the streamlines converge to minimum cross-section called the vena-contracta and then expand to fill the downstream pipe.

Hence the loss of head due to sudden contraction = ~~loss of head~~ Loss up to vena-contracta + loss due to sudden enlargement due to vena contracta.

$$\therefore h_c = \text{negligible due to small} + \left(\frac{v_c^2 - v_2^2}{2g}\right)$$

From continuity equation we have

$$A_1 v_1 = A_2 v_2$$

$$\frac{v_1}{v_2} = \left(\frac{A_2}{A_1} \right) = \frac{1}{\left(\frac{A_1}{A_2} \right)} = \frac{1}{C_c}$$

$$C_c = \frac{A_c}{A_2}$$

$$v_1 = \frac{v_2}{C_c}$$

$$C_c = \frac{A_c}{A_2}$$

Substituting the value of v_1 in eqn (1) we get

$$h_c = \left(\frac{v_1}{C_c} - v_2 \right)^2 = \frac{v_2^2}{2g} \left(\frac{1}{C_c} - 1 \right)^2$$

$$h_c = \frac{v_2^2}{2g} \left[\frac{1}{C_c} - 1 \right]^2$$

In general

$$h_c = \frac{K v_2^2}{2g}$$

$$\text{where } K = \left(\frac{1}{C_c} - 1 \right)^2$$

From experiment $C_c = 0.62 \neq 0.38 \left(\frac{A_2}{A_1} \right)^2$

and this is due to loss coefficient K is a function of ratio

$$\frac{A_1}{A_2} \text{ or } \frac{D_1}{D_2}$$

and $K = 0.375$ & $C_c = 0.62$

For gradual contraction (conical nozzle)

K is a function of cone angle $\alpha \approx 0.1$

Note if the value C_c is not given the loss of head due to contraction may be taken as

$$h_c = \left(0.5 \frac{v_2^2}{2g} \right) \text{ etc}$$

$Q = A$ horizontal pipe carries water
 at the rate $0.04 \text{ m}^3/\text{s}$. Its diameter,
 which is 300 mm reduces abruptly to 150 mm.
 Calculate the pressure across the contraction.
 Take coefficient of contraction = 0.62.

Solⁿ Given $Q = 0.04 \text{ m}^3/\text{s}$

$$D_1 = 300 \text{ mm} = 0.3 \text{ m}, \quad A_1 = \frac{\pi}{4} (0.3)^2 = 0.0707 \text{ m}^2$$

$$D_2 = 150 \text{ mm} = 0.15 \text{ m}, \quad A_2 = \frac{\pi}{4} \times 0.15^2 = 0.01767 \text{ m}^2$$

$$C_c = 0.62$$

Press loss across the contraction ($P_1 - P_2$)

From continuity equation, we have

$$A_1 v_1 = A_2 v_2 = Q$$

$$v_1 = \frac{Q}{A_1} = \frac{0.04}{0.0707} = 0.566 \text{ m/s}$$

$$\therefore v_2 = \frac{Q}{A_2} = \frac{0.04}{0.01767} = 2.26 \text{ m/s}$$

Applying Bernoulli's eqn before and after

contraction we get

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + hc \quad (\text{as } z_1 = z_2) \quad \text{--- (1)}$$

$$hc = \left(\frac{1}{C_c} - 1 \right)^2 \frac{v_2^2}{2g} = \left[\frac{1}{0.62} - 1 \right]^2 \times \frac{2.26^2}{2 \times 9.81} = 0.0978$$

Substituting these values in equation (1)

we get

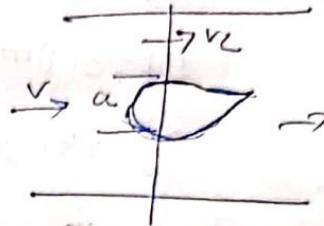
$$\frac{p_1}{\rho g} - \frac{p_2}{\rho g} = \frac{2 \cdot 26^2}{2 \times 9.81} + 0.0978 - \frac{0.566^2}{2 \times 9.81}$$

$$= 0.3418$$

Hence $p_1 - p_2 = \rho g (0.3418) = 9.81 \times 0.3418 = 3.35 \frac{\text{N}}{\text{m}^2}$

Loss of head due to obstruction in pipe

$$h_{obs} = \left[\frac{A}{C_c (A-a)} \right]^2$$



where $A =$ Area of pipe

$a =$ Area of obstruction

$v_2 =$ velocity of liquid in pipe

Loss of head at the entrance to pipe

Loss of head at the entrance to pipe h_i given by:

$$h_i = 0.5 \frac{v^2}{2g}, \quad \text{where } v = \text{vel of liquid in pipe}$$

Loss of head at the exit of pipe (h_o)

$$h_o = \frac{v^2}{2g}, \quad \text{where } v = \text{velocity of fluid at outlet of the pipe}$$

Loss of head due to bend in pipe (h_b)

$$h_b = K \frac{v^2}{2g}, \quad \text{where } v = \text{mean velocity of flow of fluid}$$

$K =$ coefficient of bend, g depends on radius of curvature of bend and dia of pipe.

Loss of head in various pipe fittings

The loss of head in the various pipe fittings (such as valve, constriction etc) may also be represented as.

$$h_{\text{fittings}} = K \frac{v^2}{2g}$$

where v = mean velocity flow in the pipe.
& K = value of coefficient depend on pipe fitting.

Hydraulic gradient ^{Line} and total energy lines



Total energy line (TEL or EGL)

The total head (total energy per unit weight) w.r.t any arbitrary datum is

$$\text{Total head} = \left(\frac{p}{\rho g} + z \right) + \frac{v^2}{2g} \quad \text{or}$$

the sum of pressure head, datum head & velocity head.

TEL: If the total energy at various points along the axis of the pipe is plotted and joined by a line, the line so obtained is called the Total energy line or Energy gradient line. (EGL)



Hydraulic gradient line (HGL)

The sum of potential head and pressure head $\left(\frac{p}{\rho g} + z \right)$ at any point is called the piezometric head.

If a line is drawn joining the perimeter levels at various points, the line so obtained is called Hydraulic gradient line. (HGL)

Pts. to remember

1. TEL always drops in the dirⁿ of flow because of loss of head.
2. HGL may rise or fall depending on the pressure change.
3. HGL - always below TEL and the vertical intercept both the two is equal to the velocity head $\left(\frac{v^2}{2g}\right)$.
4. For a pipe uniform cross-section the slope of HGL is equal to TEL.

Q. A horizontal pipeline uniformly is connected to a water tank at one end and discharges freely into the atmosphere at the other end. For 1st 25 m of its length from the tank, the pipe is 150mm diameter and its diameter is suddenly enlarged to 300mm. The height of water level in the tank is 8m above the centre of the pipe. considering all losses of head which occur.

- (1) Determine the rate of flow.
- (2) Draw the HGL & TEL. Take $f = 0.01$ for both secⁿ of pipe.

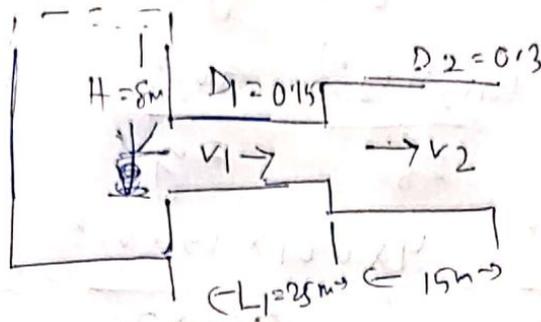
50m

Given $L = 40 \text{ m} = L_1 + L_2$

$L_1 = 25 \text{ m}$ $D_1 = 150 \text{ mm} = 0.15 \text{ m}$

$L_2 = 15 \text{ m}$ $D_2 = 300 \text{ mm} = 0.3 \text{ m}$

$H = 8 \text{ m}$ $f = 0.01$



① Ratio of flow

Applying Bernoulli's eqn to free surface of water in the tank and outlet of the pipe

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + \text{All losses}$$

$$0 + 0 + 8 = 0 + \frac{v_2^2}{2g} + 0 + \underbrace{h_i}_{\text{entrance}} + \underbrace{h_{f1}}_{\text{friction}} + \underbrace{h_e}_{\text{enlargement}} + h_{f2}$$

where $h_i = \text{loss of head in entrance} = 0.5 \frac{v_1^2}{2g}$

$h_{f1} = \text{Head loss due to friction} = \frac{4fL_1v_1^2}{D_1 \times 2g}$ (pipe-1)

$h_{f2} = \text{Head loss due to friction in pipe-2} = \frac{4fL_2v_2^2}{D_2 \times 2g}$

$h_e = \text{head loss due to sudden enlargement}$

$$= \frac{(v_1 - v_2)^2}{2g}$$

Again from continuity equation, we have

$$A_1 v_1 = A_2 v_2$$

$$v_1 = \frac{A_2}{A_1} v_2 = \left(\frac{D_2}{D_1}\right)^2 v_2 = \left(\frac{0.3}{0.15}\right)^2 v_2 = 4v_2$$

Substituting the value of v_1 (in terms of v_2) in different head losses, we have

$$h_{f1} = \frac{0.5 v_1^2}{2g} = \frac{0.5 \times 4 v_2^2}{2g} = \frac{8 v_2^2}{2g}$$

$$h_{f1} = \frac{4fL_1 v_1^2}{D_1 \times 2g} = \frac{4 \times 0.01 \times 20 \times (4v_2)^2}{0.15 \times 2g} = 106.6 \frac{v_2^2}{2g}$$

$$h_e = \frac{(v_1 - v_2)^2}{2g} = \frac{(4v_2 - v_2)^2}{2g} = \frac{9v_2^2}{2g}$$

$$h_{f2} = \frac{4fL_2 v_2^2}{D_2 \times 2g} = \frac{4 \times 0.01 \times 15 \times v_2^2}{0.3 \times 2g} = \frac{2v_2^2}{2g}$$

Substituting these values in Bernoulli's eqn. (1)

$$8 = \frac{v_2^2}{2g} + \frac{8v_2^2}{2g} + \frac{106.6v_2^2}{2g} + \frac{9v_2^2}{2g} + \frac{2v_2^2}{2g}$$

$$= \frac{126.6v_2^2}{2g}$$

$$\therefore v_2 = \sqrt{\frac{8 \times 2 \times g}{126.6}} = 1.11 \text{ m/s}$$

$$\text{Hence } Q = \text{Rate of flow} = A_2 v_2 = \frac{\pi}{4} (0.3)^2 \times 1.11 = 0.078 \text{ m}^3/\text{s}$$

(ii) TEL & HGL

The corresponding losses

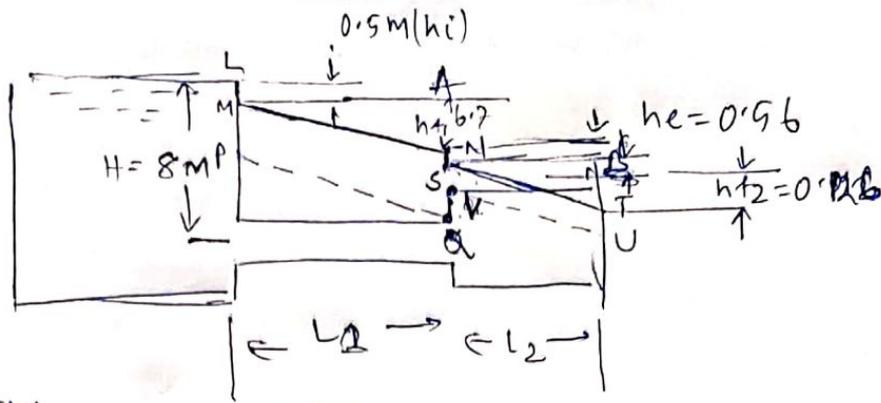
$$h_i = \frac{8v_2^2}{2g} = \frac{8 \times (1.11)^2}{2 \times 9.81} = 0.5 \text{ m}$$

$$h_{f1} = \frac{106.6 v_2^2}{2g} = \frac{106.6 \times (1.11)^2}{2 \times 9.81} = 6.7 \text{ m}$$

$$h_e = \frac{9v_2^2}{2g} = \frac{9 \times (1.11)^2}{2 \times 9.81} = 0.56 \text{ m}$$

$$h_{f2} = \frac{2v_2^2}{2g} = \frac{2 \times (1.11)^2}{2 \times 9.81} = 0.12 \text{ m}$$

To draw TEL & HGL



TEL $L = \text{free surface.}$

→ $L_M = h_i = 0.5\text{m}$

→ From M draw horizontal line. Taking MP horizontal line equal to length of pipe \$L\$. Draw vertical line down from point A. $AM - AN = h_1 = 6.7\text{m}$

→ Join MN

→ From N, draw line NS vertically down from

$h_e = 0.56\text{m}$

→ From S, draw SB horizontal line from point and from point U draw vertical line in upward direction, meeting at B.

from $BT = h_2 = 0.26$

→ Join.

→ The line LMNST represents the TEL.

HGL (Hydraulic gradient line)

③ Form M, take $MP = \frac{V^2}{2g} = \frac{(4 \times 0.11)^2}{2 \times 9.81} = 1\text{m}$

→ Draw the line PQ || to MN

→ From the point U, draw line UV || to TS

→ Join QV.

→ The line PQVU represents the (HGL)