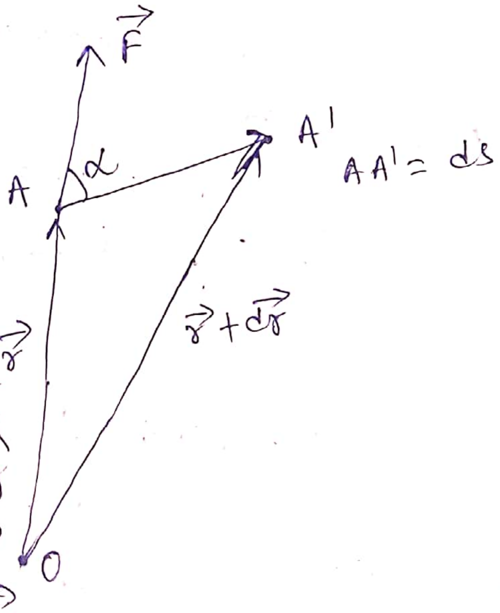
Work & Energy:-

→ Fundamental Problem in dynamics is to find the Posn of Particle w.r.t. to time when the acted force is known. By using Newton's 2nd law of motion $a = F/m$.

Then by applying Principles of Kinematics velocity & displacement can be found. & can only be used when acceleration is uniform, that means force acting on the particle is constant.

When forces vary with the Posn of Particle, the method of work & energy will be used. And relates force, mass, velocity & displacement & no need to determine acceleration.

Work of a force



A Particle moves from "A" to "A'". Posn vectors of A & A' are $\vec{r}$ & $(\vec{r}+\vec{dr})$

If the force acting on the particle is $\vec{F}$, then the work of the force $\vec{F}$ corresponding to the displacement $\vec{dr}$ is given by.

$$dU = \vec{F}.\vec{dr}$$

work is obtained by scalar Product i.e dot Product of $\vec{F}$ & $\vec{dr}$. By defination of scalar Product we get

$$dU = F\,ds\cos\alpha.$$

where ds is the magnitude of displacement

work done by a force = Component of force along the displacement × displacement

$= F\cos\alpha\ ds$

or = force × component of displacement along the line of force

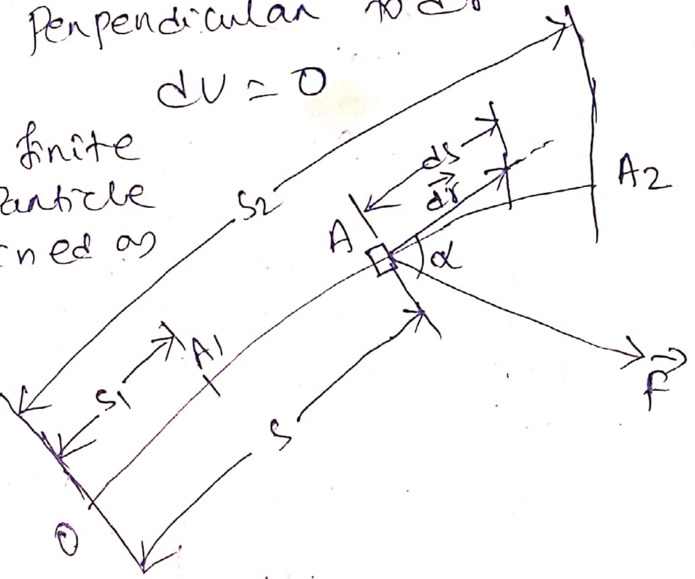
$= (F)ds\cos\alpha$

Unit of work = Newton metre (Nm) or Joule (J) 27

› Work done by force defined as the Product of Component of force in one direction of motion & the distance moved.

NOTE → If α is acute. dU is positive
→ If α is obtuse, dU is negative
→ If $\vec{F}$ & $d\vec{r}$ have same direction.
$dU = F\,ds$
→ If $\vec{F}$ & $d\vec{r}$ have opposite direction
$dU = -F\,ds$
→ If F is Perpendicular to $d\vec{r}$
$dU = 0$

The work $\vec{F}$ during a finite displacement of the particle from A_1 to A_2 is obtained as follows



$$\text{Work } U_{1\to 2} = \int_{A_1}^{A_2} \vec{F}\cdot d\vec{r}$$

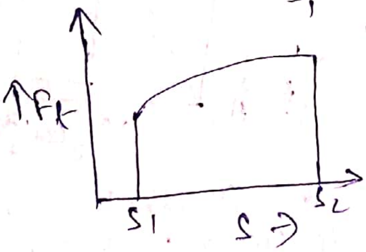
or

$$U_{1\to 2} = \int_{S_1}^{S_2} (F\cos\alpha)\,ds$$

$$= \int_{S_1}^{S_2} F_t\,ds$$

S_1, S_2 = distance travelled by the particle

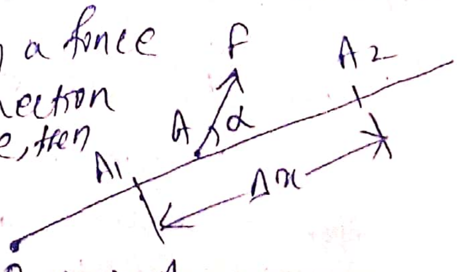
F_t = Tangential component of force.



$U_{1\to 2}$ is represented graphically in fig. A Curve F_t versus S is Plotted. Area under the curve is the work done.

Work of a constant force in Rectilinear Motion :-

A Particle is acted upon by a force of constant magnitude & direction is moving in a straight line, then



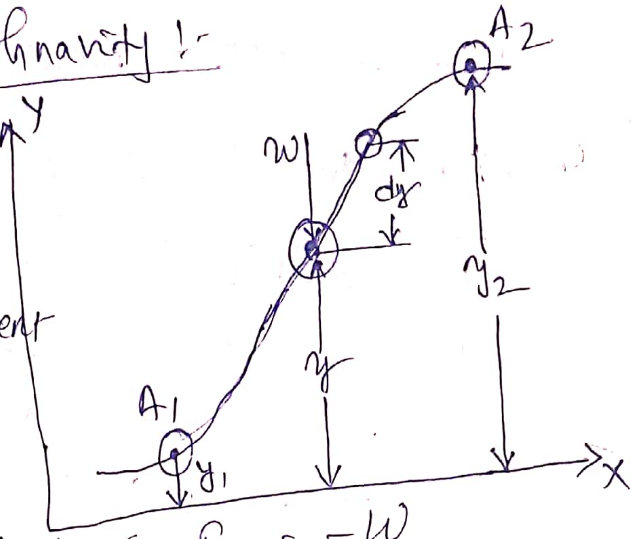
$U_{1\to 2} = (F\cos\alpha)\,\Delta x$

... from A_1 to A_2.

Work of force of gravity :-

A body of weight W is displaced from A_1 to A_2 in vertical plane.

Consider a small displacement dy at a distance y from the x axis.



Force acting on the body is $F_y = -W$

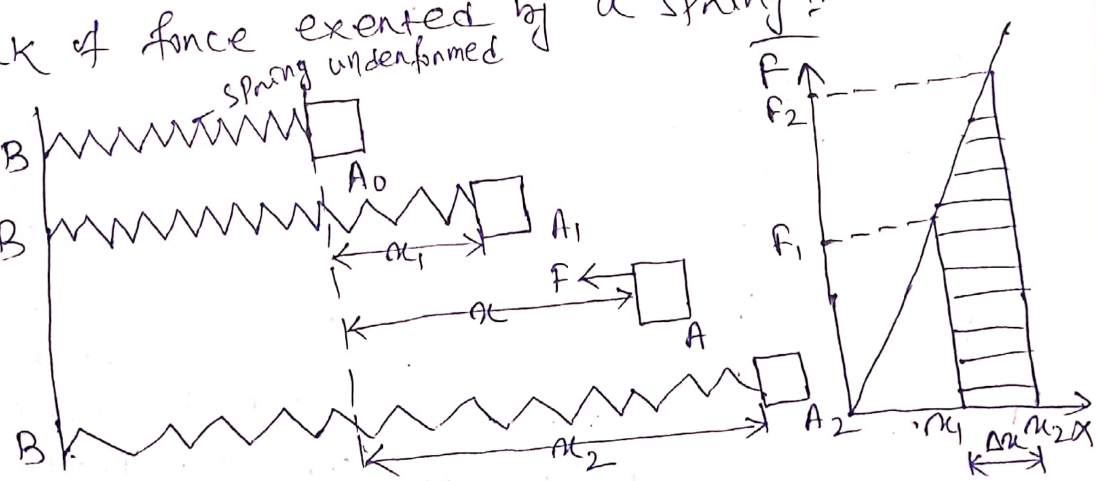
work done, $dU = F_y dy = -W dy$

$$U_{1\to 2} = -\int_{y_1}^{y_2} W\,dy = -W(y_2 - y_1) = -W\Delta y$$

Δy is the vertical displacement of the centre of gravity of the body.

In above case work done by weight is negative because weight acting downward while displacement upward. The work is positive when body moves down.

Work of force exerted by a spring :-



A body is attached to a tension spring whose other end is fixed at B. As body is displaced to right, spring undergoes deformation. & spring force F acts on the body from right to left, trying to pull the body to its original posn A_0.

$F = kx$, k = stiffness of the spring, constant on spring unit N/m.

For a small displacement dx from x.

work done, $dU = -F dx = -kx\,dx$

dU is -ve bcoz direction of force is opposite to that of displacement.

∴ for displacement x_1 to x_2

$$U_{1\to2} = -\int_{x_1}^{x_2} kx\,dx = -\frac{1}{2}k(x_2^2 - x_1^2)$$

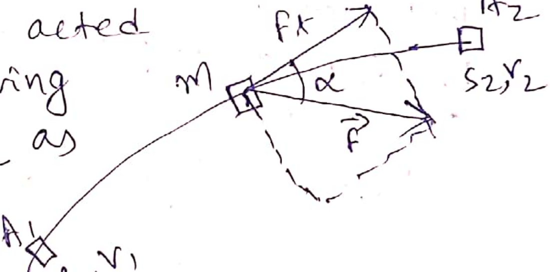
When spring is allowed to return its underformed posn both the spring force & displacement are in same direction. In this case work done will be positive.

Force vs displacement graph for spring is drawn. Work done by the spring force, for displacement ($x_2 - x_1$) is the shaded trapezodial area.

$$\boxed{U_{1\to2} = -\frac{1}{2}(F_1 + F_2)\Delta x.}$$

Kinetic Energy : Principle of Work & Energy :-

A particle of mass m is acted upon by a force $\vec{F}$ & is moving along a path from A_1 to A_2 as per figure.



Applying Newton's 2nd law in tangential direction using tangential components of force & of acceleration.

$F_t = ma_t$, $F_t = m \cdot \frac{dv}{dt}$ ∵ $v = \frac{ds}{dt}$

$$F_t = m \cdot \frac{dv}{ds} \cdot \frac{ds}{dt} = mv\frac{dv}{ds} \quad -(1)$$

$$F_t\,ds = mv\,dv$$

At A_1 , $v = v_1$ & $s = s_1$, At A_2 , $v = v_2$, $s = s_2$

Integrating (1) from A_1 to A_2

$$\int_{s_1}^{s_2} F_t\,ds = m\int_{v_1}^{v_2} v\,dv = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

LHS is the work of force $U_{1\to2}$ exerted on the particle for displacement from A_1 to A_2. RHS is change in KE.

$T_1 = \frac{1}{2}mv_1^2 =$ KE at A_1 , $T_2 = \frac{1}{2}mv_2^2 =$ KE at A_2

$U_{1\to 2} = T_2 - T_1$

Workdone by the force is equal to change in KE of the particle & known as Principle of work & energy.

$\therefore \; T_1 + U_{1\to 2} = T_2$

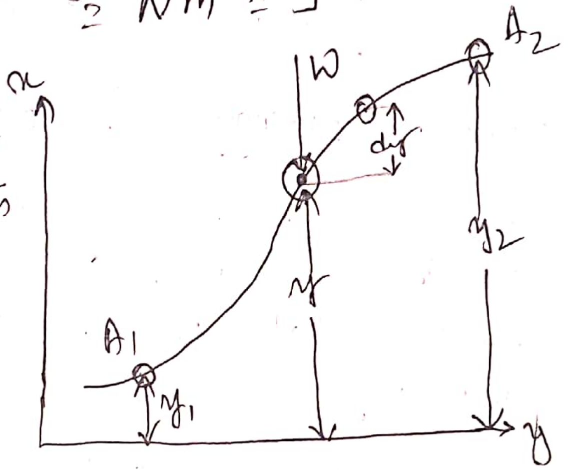
K.E. of the particle at A_2 is equal to sum of K.E. of the particle at A_1 & the work of force on the particle for displacement from A_1 to A_2.

Unit of K.E.

$T = \frac{1}{2} mv^2 = kg\,(m/s)^2 = (kg \cdot m/s^2)m$

$= Nm = J$

Potential Energy:-



Work of force of gravity W during displacement is obtained as.

$U_{1\to 2} = Wy_1 - Wy_2$

$Wy_1 = V_1 = P.E.$ at A_1

$Wy_2 = V_2 = P.E.$ at A_2

P.E. is defined as the energy possessed by a particle by virtue of its posn, If PE increases during displacement, the work $U_{1\to 2}$ is negative (as) in the case considered here.

As per Spring force

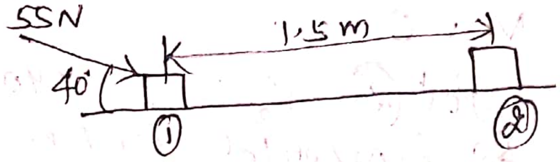
$U_{1\to 2} = \frac{1}{2} kx_1^2 - \frac{1}{2} kx_2^2 = V_1 - V_2$

$V_1 = \frac{1}{2} kx_1^2 =$ PE due to elastic force at A_1

$V_2 = \frac{1}{2} kx_2^2 =$ PE due to elastic force at A_2

If the PE due to elastic force increases, the workdone by the spring force $U_{1\to 2}$ is negative.

Ex-1 A block of mass 2.5 kg is resting at Posn ① on a smooth horizontal track as per fig. A 55N force at a constant inclination of 40° to the track is applied to the block. What is the speed of the block when it passes Posn ②.



$T_1 = 0$

$T_2 = \frac{1}{2} m v_2^2$

$= \frac{1}{2} \times 2.5 \times v_2^2$

$U_{1\to 2} = F\cos\alpha \times S = 55\cos 40° \times 1.5$

$\therefore U_{1\to 2} = T_2 - T_1$ & as $T_1 = 0$

$55\cos 40° \times 1.5 = \frac{1}{2}\, 2.5 \times v_2^2$

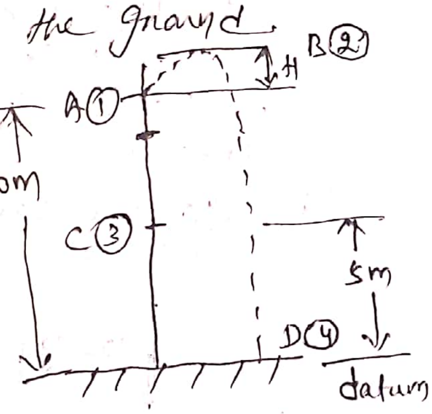
$\Rightarrow \boxed{v_2 = 7.11 \text{ m/s}}$

Ex-2 A Particle is projected upward from ~~bottom~~ location A with an initial velocity of 10 m/s with the max^m height reached above the ground.

a) what is the max^m height reached above the ground.

b) velocity with which particle passes point C.

c) velocity with which strikes the ground.



We can apply principle of conservation of energy.

When body is at A

$T_1 = \frac{1}{2} m v_1^2 = \frac{1}{2} m (10)^2$ (KE)

$V_1 = mg(10)$ (PE)

at B $\quad v_2 = 0 \Rightarrow T_2 = 0$

$V_2 = mg(10 + H)$

at D, $T_4 = \frac{1}{2} m v_4^2 = 0$

$v_4 = 0$

at C, $\quad T_3 = \frac{1}{2} m v_3^2$

$V_3 = mg(5)$

so, (a) $T_1 + V_1 = T_2 + V_2 \Rightarrow 50m + 10mg = 0 + mg(10 + H)$

$H = 5.09$ m

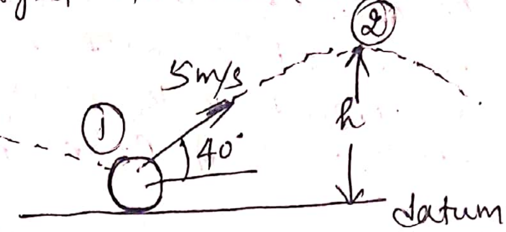
$\therefore h_{max} = 10 + 5.09 = 15.09$ m

(b) $T_1 + V_1 = T_3 + V_3 \Rightarrow 50m + 10mg = \frac{1}{2} m v_3^2 + 5mg$

$v_3 = 14.07$ m/s

(c) $T_1 + V_1 = T_4 + V_4 \quad 50m + 10mg = \frac{1}{2} m v_4^2 + 0 \Rightarrow v_4 = 17.21$ m/s

Ex-3 Figure shows a ball rebounding from the floor. Find the max^m height to which ball will rise.



Ans When ball is at Pos^n 1

$$T_1 = \frac{1}{2} m v_1^2 = \frac{1}{2} m (5)^2$$

$$V_1 = 0$$

Pos^n 2 $v_2 = 5\cos 40°$, at 2 Path is horizontal, So horizontal components alone considered

$$T_2 = \frac{1}{2} m v_2^2 = \frac{1}{2} m (5\cos 40°)^2$$

$$V_2 = mgh$$

By conservation of energy

$$T_1 + V_1 = T_2 + V_2$$

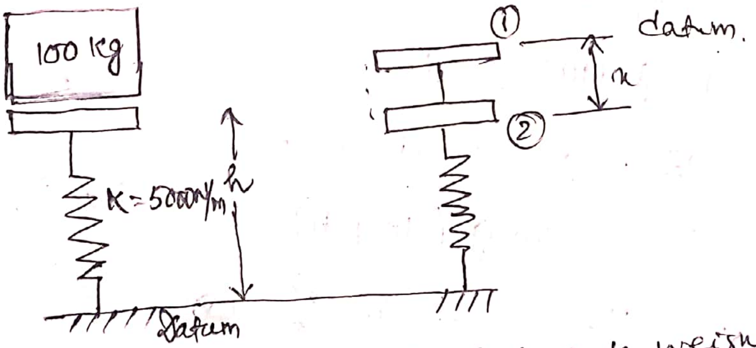
$$\frac{1}{2} m (5)^2 + 0 = \frac{1}{2} m (5\cos 40°)^2 + mgh$$

$$h = 0.5264\,m \text{ Ans.}$$

Ex-4 In figure, a block of mass 100 kg just contacts the free end of the spring.

(a) If the block is released instantaneously with zero initial velocity, what is the max^m direction of the spring? (The block is suddenly dropped onto the spring)

(b) Do the same part (a) if the block is lowered onto the spring in a very gradual manner.



At Pos^n 1 $T_1 = 0$, $V_1 = mgh$ (due to weight)

At Pos^n 2 $T_2 = 0$, $V_2 = \frac{1}{2} kx^2 + mg(h - x)$

conservation of energy

$$T_1 + V_1 = T_2 + V_2$$

$$mgh = \frac{1}{2} kx^2 + mg(h - x)$$

$$\frac{1}{2} kx^2 = 100(9.81)x, \quad x_{max} = 0.3924\,m.$$

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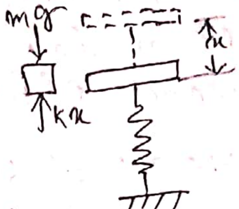
When block is gradually lowered, at the max^m deflection posn

For vertical equilibrium



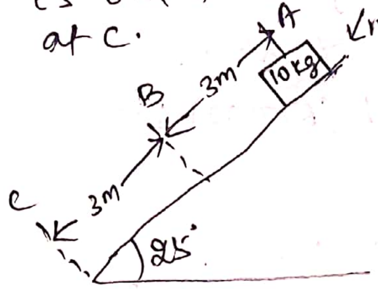
$$\Sigma F_y = 0$$

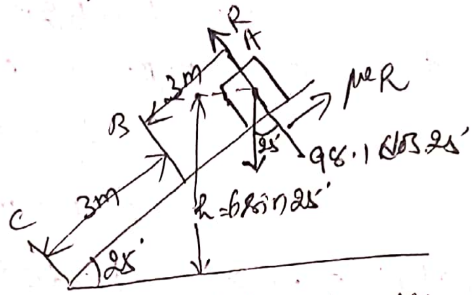
$$kx = mg$$

$$x = \frac{100 \times 9.81}{5000} = 0.1962\, m$$

NOTE:- deflection due to suddenly applied load is twice that due to gradually applied load.

Ex-5 A block is released from rest to slide down the incline of slope 25° as shown in the figure. The coefficient of friction is 0.2 for AB & is 0.15 for BC. What is the velocity of block at C.





Considering the equilibrium normal to the plane, for the FBD of the block shown in fig.

$$R = 98.1 \cos 25° = 88.9\, N$$

at A, $T_1 = 0$, $V_1 = mgh = 10 \times 9.81 \times 6 \sin 25° = 248.75\, J$

at C, $T_2 = \frac{1}{2} m v_2^2$; $V_2 = 0$

$$\text{work of friction} = -\mu_1 R\, AB - \mu_2 R\, BC$$

$$W_{nc} = -0.2(88.9)3 - 0.15(88.9)3$$

$$= -93.35\, J$$

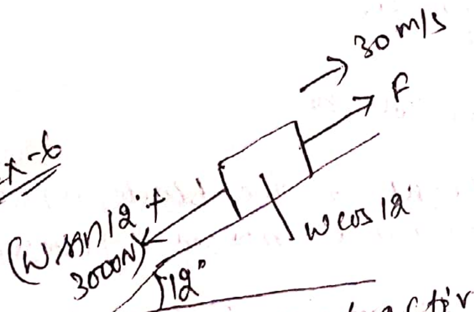
Conservation of energy:

$$W_{nc} = \Delta T + \Delta V$$

$$-93.35 = \frac{1}{2} 10\, v_2^2 - 248.75$$

$$v_2 = 5.57\, m/s$$

Ex-6



Calculate the o/p power of a car of mass 1000 kg when ascending a slope of 12° at a constant speed of 30 m/s against a resistance of 3 kN.

F is the tractive force.

∴ speed of the car is constant, $a = 0$ & $ma = 0$, using Newton's 2nd law in the x direction.

$\Sigma F_x = 0$, $F - W \sin 12° - 3000 = 0$, $W = 9810\, N$

$$F = 5039.61\, N$$

Power o/p, $F \times v = 5039.61 \times 30 = \underline{151.18\, kW}$

Ex-7. A Pump lifts 50 m³ of water to a tank at a height of 50 m. The velocity of delivery is 6 m/s. This work is done in half an hour. Find the i/p Power of the pump assuming an overall efficiency of 75%.

Weight of 50 m³ water $= 50 \times 1000 \times 9.81$

Mass density of water = 1000 kg/m³

$= 490500\ N$

Work done to lift water to a height of 50 m.

$= mgh = Wh = 490500 \times 50$

$W_1 = 24.525 \times 10^6\ J$

Work done to impart a velocity of 6 m/s.

$= \frac{1}{2} mv^2 = \frac{1}{2}(50000)\, 6^2$

$W_2 = 0.9 \times 10^6\ J$

Total energy expanded $= W_1 + W_2$

$= (24.525 + 0.9)\, 10^6\ J$

$= 25.425 \times 10^6\ J$

Output Power = energy expanded / second

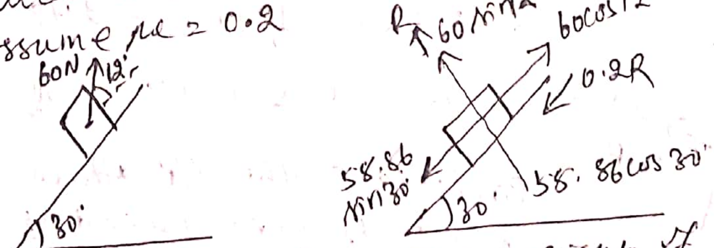
$= \frac{25.425 \times 10^6}{0.5 \times 60 \times 60} = 14125\ W$

Input Power = output Power / η

$= \frac{14125}{0.75} = 18833\ W = 18.83\ kW$

Ex-8. Find the total work done on a 6 kg block of. Pulled 5 m up an incline of slope 30° shown in fig.

Assume $\mu = 0.2$



The FBD is shown fig: The weight of the block as well as the applied force are resolved into x & y components. R is the normal reac^n.

weight of the block $= 6(9.81) = 58.86\ N$

$\Sigma F_y = 0$, $R + 60 \sin 12° - 58.86 \cos 30° = 0$, $R = 38.5\ N$

Friction force, $0.2R = 0.2(38.5) = 7.7\ N$

Work done by the force, $= (60 \cos 12° - 7.7 - 58.86 \sin 30°)\, 5$

$= 107.8\ Nm$ Ans.

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Principle of Impulse & Momentum:-

) Method can be used to solve problems involving: force, mass, velocity & time. Particularly useful for problems involving impulsive motion on impact.

Newton's 2nd law can be expressed as

$$\vec{F} = m\frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v})$$

$m\vec{v}$ is known as linear momentum.

multiplying both sides with dt

$$\vec{F}\,dt = d(m\vec{v})$$

integrating from time t_1 to t_2

$$\int_{t_1}^{t_2} \vec{F}\,dt = m\vec{v_2} - m\vec{v_1}$$

v_1 & v_2 are the velocities of particle at t_1 & t_2

Can be written as,

$$m\vec{v_1} + \int_{t_1}^{t_2} \vec{F}\,dt = m\vec{v_2} \quad -① $$

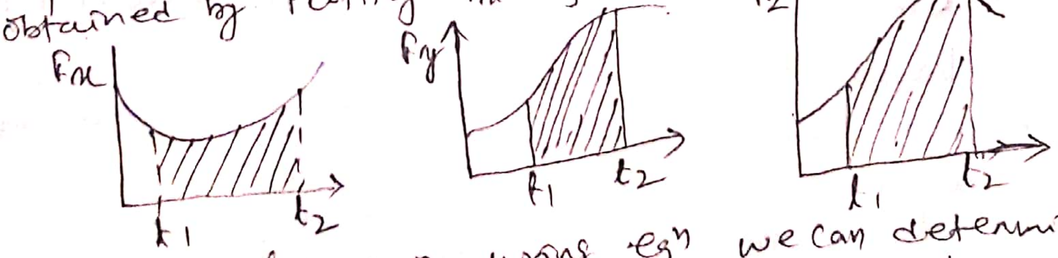
The integral in the above eqn called as impulse (linear impulse) of the force $\vec{F}$ during the time interval given & is a vector quantity.

$$Imp_{1\to2} = \int_{t_1}^{t_2} \vec{F}\,dt$$

Resolving $\vec{F}$ into rectangular components, we get

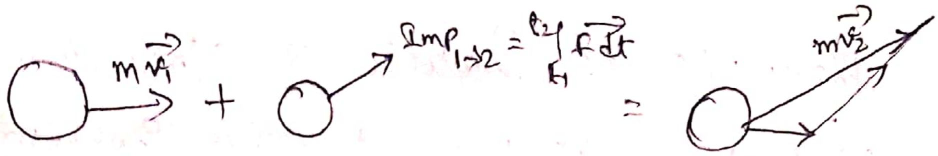
$$Imp_{1\to2} = \vec{i}\int_{t_1}^{t_2} F_x\,dt + \vec{j}\int_{t_1}^{t_2} F_y\,dt + \vec{k}\int_{t_1}^{t_2} F_z\,dt$$

Components of RHS represents the areas under the curves obtained by plotting F_x, F_y, F_z wrt time from t_1 to t_2.



Refer to figure & using eqn we can determine the final momentum $m\vec{v_2}$ of a particle by adding vectorically its initial momentum $m\vec{v_1}$ & impulse of the force $\vec{F}$ during the time interval considered.

WORK



eqⁿ (1) is written in equivalent components eqⁿ. in Scalar form!

$$(m v_x)_1 + \int_{t_1}^{t_2} F_x\,dt = (m v_x)_2$$

$$(m v_y)_1 + \int_{t_1}^{t_2} F_y\,dt = (m v_y)_2$$

$$(m v_z)_1 + \int_{t_1}^{t_2} F_z\,dt = (m v_z)_2$$

When several forces act on a particle the impulses of all forces should be considered & the below eqⁿ can be used to find the final momentum.

$$m\vec{v_1} + \Sigma Imp_{1\to2} = m\vec{v_2}$$

If the problem involves 2 particles or more, impulse momentum eqⁿ for each particle should be written. Momenta of all particle & the impulses of forces involved are vectorically added.

$$\Sigma m\vec{v_1} + \Sigma Imp_{1-2} = \Sigma m\vec{v_2}$$

If no external force is exerted on the system

$$\Sigma Imp_{1-2} = 0 \text{ \& hence}$$

$$\Sigma m\vec{v_1} = \Sigma m\vec{v_2}$$

It states that the momentum of the particles is conserved when the sum of the external forces on the systems is zero.

Impulsive force

When a constant force F is applied to a particle for a time t, then the product Ft is called the impulse.

When the force is large & the time interval is very short, near instantaneous, then the force is called an impulsive or impact force & the resulting motion is called an impulsive motion.

ex:- forces arise collision, explosion, driving of pile, hitting of cricket ball, sudden jamming of hoisting rope on the drum.

Impulse momentum eqn becomes.

$$m\vec{v_1} + \Sigma \vec{F}\,\Delta t = m\vec{v_2}$$

) Non impulsive forces like weight of the body, Spring force, & friction forces are negligible as compared to the large impulsive force.

NOTE:-

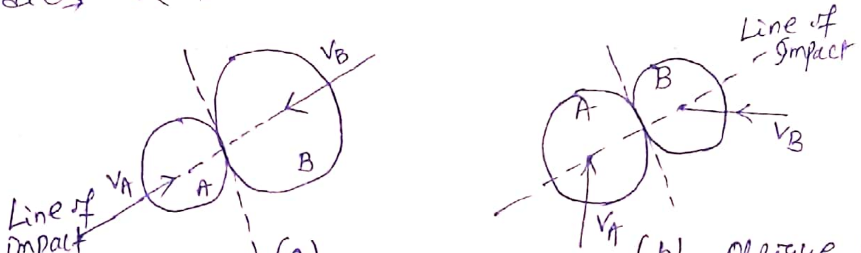
→ When "time" is the known quantity, momentum Principle is useful. In some situations only momentum Principle can give the Solution. But when "distance" is the known quantity work-energy Principle is most useful & convinient.

→ In impact problems where the impact force is not known, (not required) & where there is unavoidable loss of energy, momentum principle can directly be used & the law of conservation of energy can't directly be used.

Impact of Elastic Bodies:-

A collision of two moving bodies which occur in a very short interval of time & during which two bodies exert force on each other of very large magnitude is called an Impact.

And magnitude of impact depends on the shapes of bodies & their velocities & their elastic Properties.



(a) Direct Central impact

(b) Oblique Central impact

The common normal to the Surfaces in contact during the impact is called line of impact.

If the mass centre of two colliding bodies lie on the line of ~~contact~~ impact, then impact is called as <u>Central impact</u>.

Even if the mass centre of one of colliding bodies is not on line of impact, the impact said to be <u>eccentric impact</u>.

If the velocities of the two bodies are directed along the line of impact, then impact said to be <u>direct impact</u> (fig a)

If the velocity of one of the colliding bodies not directed along the line of impact, then impact said to be an <u>oblique impact</u> fig(b)

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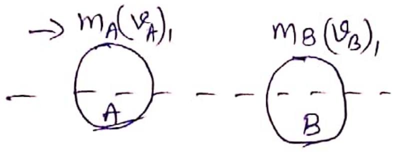
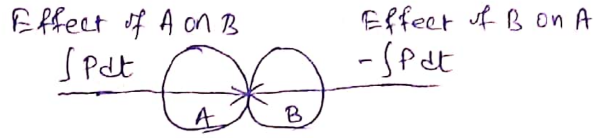
Direct Central Impact:-

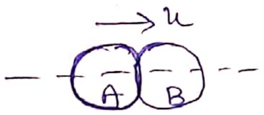
Two bodies A of mass "m_A" & B of mass "m_B" move in a straight line with velocities $(v_A)_1$ & $(v_B)_1$ respectively (fig a)



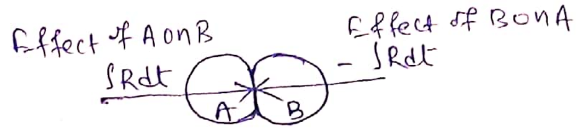
(a) Before impact



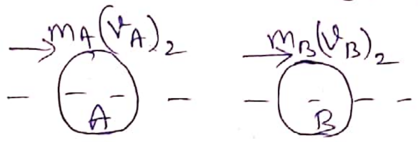
(b) deformation impulse.



(c) At max^m deformation



(d) Restitution impulse.



(e) After impact

It is assumed that $(v_A)_1 > (v_B)_1$. Hence A will eventually strike B. Under the impact the bodies will deform & at the end of "Period of deformation", both the bodies will have same velocity "u" (fig "c").

During the period of "restitution" the bodies will separate & have velocities $(v_A)_2$ & $(v_B)_2$ (fig "e")

Depending on the magnitude of impact forces & materials of the bodies involved the bodies may regain their original shapes or will have permanent deformation.

Taking the two bodies as a single system & there is no external force, then

$$m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

i.e. the total momentum of the two bodies is conserved.

Let "P" be the force acts on the two bodies during Period of deformation & varies from "zero" at the initial contact to the max^m value at the instant of max^m deformation.

Impulse during deformation $= \int P dt$

Impulse during restitution $= \int R dt$

where "R" the force on the two bodies during restitution & varies from max^m value at the instant of max^m deformation to zero at the instant of complete separation of bodies.

Now we apply; Impulse - momentum principle to body "A"

$$m_A(v_A)_1 - \int P\,dt = m_A u$$

$$m_A u - \int R\,dt = m_A(v_A)_2$$

in the above eqn ~~impulses~~ impulses have negative sign because they can be considered as reactions (fig 'b & d')

$$\int P\,dt = -m_A u + m_A(v_A)_1 \quad -(1)$$

$$\int R\,dt = -m_A(v_A)_2 + m_A u \quad -(2)$$

Divide (2) by (1)

$$e = \frac{\int R\,dt}{\int P\,dt} = \frac{u-(v_A)_2}{(v_A)_1 - u} \quad -(3)$$

Similarly for body B.

$$e = \frac{\int R\,dt}{\int P\,dt} = \frac{(v_B)_2 - u}{u-(v_B)_1} \quad -(4)$$

Since eqn (3) & (4) are equal, So they also equal to the sum of numerators by sum of denominators.

$$e = \frac{u-(v_A)_2 + (v_B)_2 - u}{(v_A)_1 - u + u - (v_B)_1} = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

e is known as the coefficient of restitution.

Can be defined as the ratio of relative velocity of separation to the relative velocity of approach.
Can also be defined as ratio of impulse during restitution period to the impulse during the deformation period.

Relative velocities are measured along line of impact.

e lies b/w 0 to 1.

0 - for perfectly plastic bodies (bodies cling together after impact & there will be no separation)

1 - for perfectly elastic bodies. (don't have any plastic deformation.

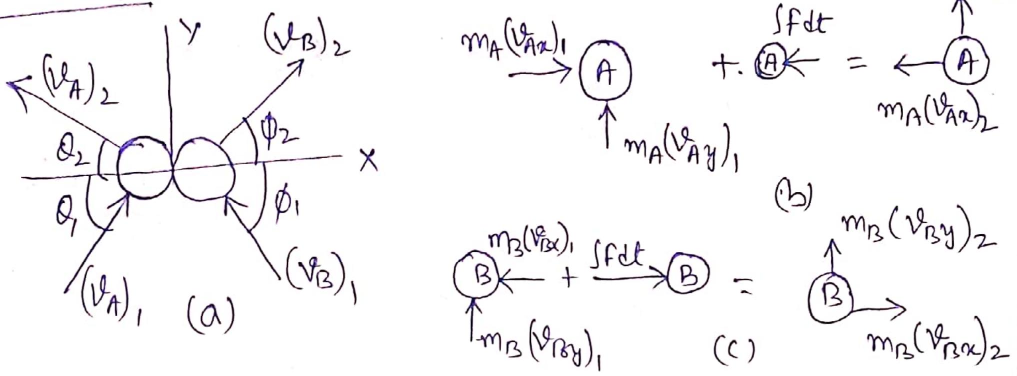
(1) Perfectly plastic ~~deformation~~ impact :- $e = 0$, Both particles stick together after impact.

② Perfectly Elastic impact :- $e = 1$, The total energy of two Particles as well as total momentum is conserved.

$$\frac{1}{2} m_A (v_A)_1^2 + \frac{1}{2} m_B (v_B)_1^2 = \frac{1}{2} m_A (v_A)_2^2 + \frac{1}{2} m_B (v_B)_2^2$$

This eqn expresses that K.E. of Particle is conserved.

Oblique Impact :-



Four unknowns are present in this problem; the bodies move away from each other with velocities of unknown magnitudes & unknown directions. In fig b & c, these are shown as the x & y components of final velocities.

Eqns required to find $(v_{Ax})_2$, $(v_{Ay})_2$, $(v_{Bx})_2$ & $(v_{By})_2$ are –

1. Momentum of the system is conserved along the line of impact (x axis), we use.

$$\Sigma m (v_x)_1 = \Sigma m (v_x)_2$$

2. $$e = \frac{(v_{Bx})_2 - (v_{Ax})_2}{(v_{Ax})_1 - (v_{Bx})_1}$$ The coefficient of restitution relates the relative velocity components of the bodies along the line of impact (x axis)

3. Momentum of body A is conserved along the common tangent to the surface in contact (y axis) since no impulse acts on A in this direction.

$$m_A (v_{Ay})_1 = m_A (v_{Ay})_2$$

4. Momentum of body B is conserved along the common tangent to the surfaces in contact (y axis) since no impulse acts on B in this direction

$$m_B (v_{By})_1 = m_B (v_{By})_2$$

Equation used for solving problem :-

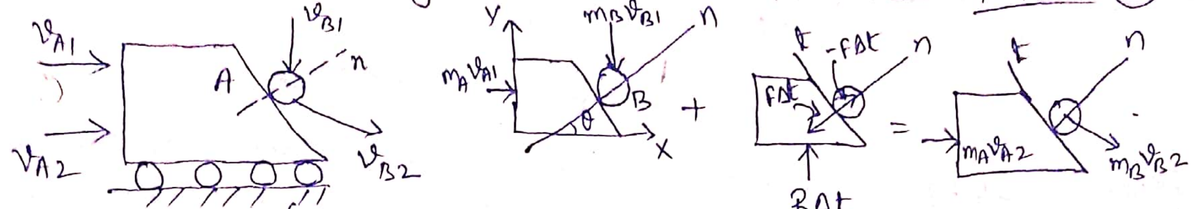
① Momentum of system is conserved along line of impact

$$\Sigma m v_1 = \Sigma m v_2$$

② The coefficient of restitution relates relative velocities of the bodies along the line of impact, just before & just after collision.

$$e = (v_B)_2 - (v_A)_2 / (v_A)_1 - (v_B)_1$$

One or both Colliding Bodies have Constrained Motion:- (41)



(a)

External Impulse (b)

In figure collision b/w block A & sphere B is shown. Block A is constrained to move in a horizontal direction.

In fig (b) FBD. The impulses of the internal forces F & −F are directed along the line of impact i.e. along the "n" axes. The external force is the floor reaction R in the vertical direction.

Due to the constraints, the initial & final velocities of A, v_{A1} & v_{A2} are horizontal.

The 3 eqns required for solving this type of Problem are.

$$m_B (v_{B1})_t = m_B (v_{B2})_t$$

Components of momentum of ball B along "t" axis is conserved.

$$m_A v_{A1} + m_B v_{Bx1} = m_A v_{A2} + m_B v_{Bx2}$$

x components of the total momentum of A & B is conserved.

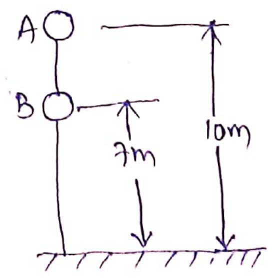
$$e = \frac{(v_{B2})_n - (v_{A2})_n}{(v_{A1})_n - (v_{B1})_n}$$

The coefficient of restitution relates the relative velocity components of the bodies along the line of impact (n axes)

−x−

Ex-1 A 50 gm glass marble dropped from a height of 10m rebounds to a height of 7 meters. Calculate the impulse & the average force b/w the floor & the marble taking the period of contact as 0.125 s. Reference Point is on the ground.

The velocity with which the marble hits the floor is obtained



$u = 0, \; x_0 = 10m., \; x = 0$

$u = v_1$ = velocity before impact

$v_1^2 = u^2 - 2g(x - x_0)$

$v_1 = \pm\sqrt{2g(10)} = -14 \text{ m/s} \downarrow$

velocity after impact

$x = v_2, \; u = 0, \; x_0 = 0, \; x = 7$

$u^2 = v_2^2 - 2g(x - x_0) \Rightarrow v_2 = \pm\sqrt{2g(7)} = \pm 11.71 \text{ m/s} \uparrow$

By impulse - momentum eqn; $m\vec{v_1} + \overrightarrow{Imp_{1\to 2}} = m\vec{v_2}$

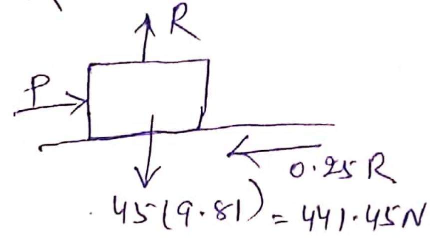
$Imp_{1-2} = mv_2 - mv_1 = 0.05\,(11.71 - (-14)) = 1.285 \text{ NS}$

If F is the average impulsive force b/w the marble & ground

$$F \Delta t = Imp_{1\to2}$$

$$\therefore F = \frac{1.285}{0.125} = 10.28\ N$$ Ans

Ex-2 A 45 kg body rests on a rough horizontal floor. It is acted upon by a horizontal force is given by the relation $P = 60t$ where P is in Newtons & t is in seconds. The starting value of the force is zero & force acts for 10s. If the friction coefficient b/w the floor & the body is 0.25, find the speed of the block at the end of 10 sec. Ans



Ans The FBD is shown in figure. For vertical equilibrium $R = 441.45 N$.

By impulse - momentum relation

$$\int_{t_1}^{t_2} \vec{F}\,dt = m(\vec{v_2} - \vec{v_1})$$

In horizontal direction.

$$F = 60t - 0.25R$$
$$= 60t - 0.25(441.45)$$
$$\boxed{F = 60t - 110.36}$$

As 'P' increases, friction force also increases to balance "P". When P equals to μR sliding will start. Time required for impending sliding motion can be obtained by equating $60t$ to μR $\Rightarrow$ $60t = 110.36 \Rightarrow t = 1.84$ sec

Impulse momentum eqn can be applied after 1.84 sec

$$\int_{t=1.84s}^{t=10s} (60t - 110.36)\,dt = 45(v_2 - 0)$$

$$\left[\frac{60t^2}{2} - 110.36t\right]_{1.84}^{10} = 45v_2$$

$$1998 = 45v_2$$

$$v_2 = 44.4\ m/s$$

If friction is neglected.

$$\int_{t=0}^{t=10} (60t)\,dt = 45(v_2 - 0)$$

$$\Rightarrow \quad v_2 = 66.7\ m/s$$

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Ex-3 A man weighing 700 N is in boat weighing 3000 N which (43) is floating in a still lake.

(a) if the man starts running along the length of the boat at a speed of 5 m/s wrt the boat, find the velocity of boat. Neglect the resistance to the motion of boat from the water.

(b) If he jumps off the boat with an absolute velocity of 8 m/s find the velocity of boat.

Ans There is no external impulsive force on the man-boat system. Hence by conservation of linear momentum.

$$m_m(v_{m1}) + m_b(v_{b1}) = m_m(v_{m2}) + m_b(v_{b2})$$

v_{m1}, v_{b1} = initial velocity of man, boat

v_{m2}, v_{b2} = final " " " "

m_m, m_b = masses of man & boat

$$v_{m1} = v_{b1} = 0$$

(a) Let the man run towards right as shown in fig.

Momentum eqn is written as

$$0 = \frac{700}{9.81} v_{m2} + \frac{3000}{9.81} v_{b2}$$

Relative velocity of man wrt boat

$$v_{mb} = v_{m2} - v_{b2} = 5 \text{ m/s}$$

$$\therefore v_{m2} = 5 + v_{b2}$$

Substitute the value of v_{m2}

$$\frac{700}{9.81}(5 + v_{b2}) + \frac{3000}{9.81} v_{b2} = 0$$

$$v_{b2} = -0.9459 \text{ m/s}$$

Negative sign indicates that boat moves to the left.

(b) Let the man jumps towards right.

By conservation of momentum.

$$0 = \frac{700}{9.81}(8) + \frac{3000}{9.81} v_{b2}$$

$$\therefore v_{b2} = -1.86 \text{ m/s}$$

boat moves towards left.

Ex-4 2 men, weighing 60 kg & 80 kg, dive off from the end of a boat of mass 450 kg. When they dive then relative velocity wrt boat is 10 m/s, the boat is rest in a steady lake. Determine the velocity of boat.

1. if 2 men dive simultaneously
2. if 60 kg man dives first followed by other man.

Ans Let suffix b means boat
" c means 60 kg man
" d means 80 kg man

By conservation of momentum.

$$m_b v_{b1} + m_c v_{c1} + m_d v_{d1} = m_b v_{b2} + m_c v_{c2} + m_d v_{d2}$$

Since system is initially at rest, LHS is zero.

$$m_b v_{b2} + m_c v_{c2} + m_d v_{d2} = 0$$

(1) Assume both men dive simultaneously towards right

Given $v_{cb} = v_{db} = 10\ m/s \rightarrow$

$$v_{c2} - v_{b2} = v_{d2} - v_{b2} = 10\,m/s$$

Substituting, we get

$$450\,(v_{b2}) + 60\,(10 + v_{b2}) + 80\,(10 + v_{b2}) = 0$$

$$v_{b2} = -2.37\ m/s \quad \text{towards left.}$$

(2) The 60 kg man dives followed by 80 kg man. The velocity of boat acquired by the jump of 60 kg man found from.

$$(m_b + m_d)(0) + m_c(0) = (m_b + m_d)v_{b2} + m_c v_{c2}$$

$$\because\ v_{c2} = 10 + v_{b2}$$

$$(450 + 80)\,v_{b2} + 60\,(10 + v_{b2}) = 0$$

$$v_{b2} = -1.016\ m/s$$

Now 80 kg man jumps off

$$v_{b1} = -1.016\ m/s$$

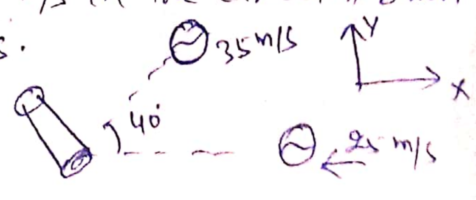
$$(m_b + m_d)v_{b1} = m_b v_{b2} + m_d v_{d2}$$

$$\because\ v_{d2} = 10 + v_{b2}$$

$$\Rightarrow (450 + 80)(-1.016) = 450\,v_{b2} + 80\,(10 + v_{b2})$$

$$\Rightarrow v_{b2} = -2.52\ m/s \quad \text{towards left}$$

Ex-5 A 105 gm base ball, pitched with a velocity of 25 m/s is hit by the bat & has a velocity of 35 m/s in the direction shown. The contact period b/w bat & ball is 0.02 s. Find the average impulsive force exerted by the ball.



Ans Initial velocity $\vec{v_1} = (-25\,\hat{i})$ m/s
final velocity $\vec{v_2} = (35\cos 40°\,\hat{i} + 35\sin 40°\,\hat{j})$ m/s

By impulse momentum eqn

$$m\vec{v_1} + Imp_{1\to 2} = m\vec{v_2}$$

$$0.105(-25\hat{i}) + \vec{F}\Delta t = 0.105(26.811\hat{i} + 22.497\hat{j})$$

$\because \Delta t = 0.02$ sec

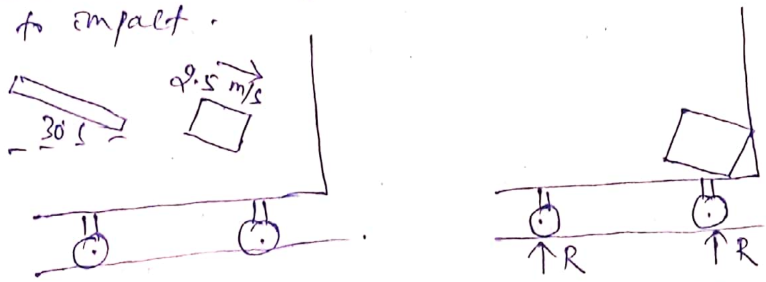
$$\therefore \vec{F} = (272\hat{i} + 118.1\hat{j})\,N.$$

$$\therefore F = \sqrt{272^2 + 118.1^2} = 296.5\,N.$$

$$\therefore \theta = \tan^{-1}\frac{118.1}{272} = 23.47° \text{ Ans}$$

Ex-6 A box weighing 8 kg is dropped from a height onto a slope with a velocity of 2.5 m/s. as shown in fig. The 25 kg cart is initially at rest. With the wheels fixed to base & can move freely. Find (a) final velocity of cart. (b) impulse exerted by the cart on the box (c) % of energy loss due to impact.

Ans



When box falls on the cart, there will be floor reach R in the vertical direction, which is an external force, but in the horizontal direction, there will be no impulsive force acting on the system, that means the momentum is conserved in horizontal direction.

Box-cart

$$m_B v_1 \cos 30° = (m_B + m_C) v_2$$

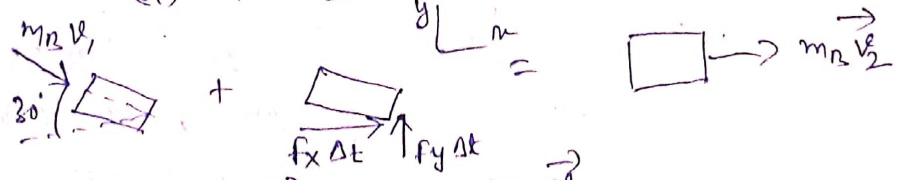
m_B, m_C = mass of box & cart

v_1, v_2 = initial & final velocity of cart & box on horizontal

$$8(2.5\cos 30°) = (8 + 25) v_2$$

$$(a) \Rightarrow v_2 = 0.5248 \text{ m/s}.$$

Box



$$m_B\vec{v_1} + Imp_{1\to 2} = m_B\vec{v_2}$$

in x direction we have.

$$8(2.5\cos 30°) + F_x\Delta t = 8(0.5248)$$

$F_x \Delta t = -13.12\ N/s$

In y-direction,

$$-8(2.5 \sin 30^\circ) + F_y \Delta t = 8(0)$$

$$F_y \Delta t = 10\ NS$$

$$\therefore F\Delta t = \sqrt{(F_x \Delta t)^2 + (F_y \Delta t)^2}$$

$$= \sqrt{(13.12)^2 + (10)^2} = \underline{16.49\ \text{sec.}}$$

$$\therefore F\Delta t = 16.49\ \text{sec}$$

(b) $\theta = \tan^{-1} \frac{10}{13.12} = 37.31^\circ$

K.E. of system before impact

$$T_1 = \frac{1}{2} m_B v_1^2 = \frac{1}{2}\, 8 (2.5)^2 = 25\ J$$

K.E. of system after impact

$$T_2 = \frac{1}{2}(m_B + m_C) v_2^2$$

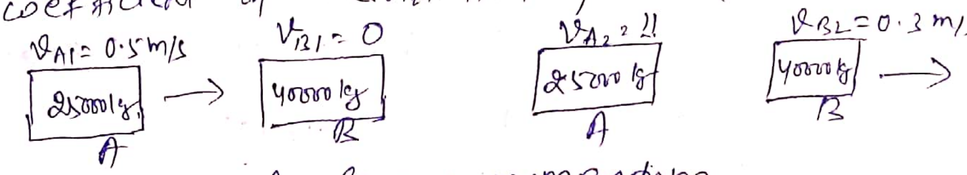
$$= \frac{1}{2}(8+25)(0.5248)^2$$

$$= 4.54\ J$$

% of energy lost $= \frac{T_1 - T_2}{T_1} \times 100$

(c) $= \frac{25 - 4.54}{25} \times 100 = \underline{81.8\%}$

Ex-7 Rail road car A, of mass 25000 kg, moving with a speed of 0.5 m/s collides with car B of mass 40000 kg which is at rest. After impact car B moves to right with a speed of 0.3 m/s. Find coefficient of restitution b/w A & B.



By conservation of linear momentum.

$$m_A v_{A1} + m_B v_{B1} = m_A v_{A2} + m_B v_{B2}$$

$$25000(0.5) + 0 = 25000\, v_{A2} + 40000(0.3)$$

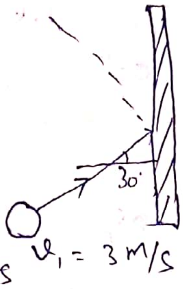
$$v_{A2} = 0.02\ m/s \rightarrow$$

$$e = \frac{v_{B2} - v_{A2}}{v_{A1} - v_{B1}} = \frac{0.3 - 0.02}{0.5 - 0} = \underline{0.56}$$

Ex-8 A ball is thrown against a frictionless wall. When it is on the verge of striking the wall, its velocity is 3m/s & the velocity forms an angle of 30' with the horizontal. Take e = 0.85 find the rebound velocity. (47)

$v_{x1} = 3\cos 30° = 2.598$ m/s

$v_{y1} = 3\sin 30° = 1.5$ m/s



Only the x component of the velocity changes during impact. v_y remains unchanged since no impulsive force acts on the ball in y direction during impact

$$e = \frac{-v_{x2}}{v_{x1}} \Rightarrow 0.85 = \frac{-v_{x2}}{2.598}$$

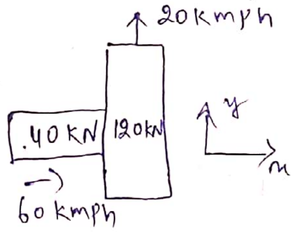
$v_{x2} = -2.208$ m/s ←

$v_{y2} = v_{y1} = 1.5$ m/s ↑

$$v_2 = \sqrt{(-2.208)^2 + (1.5)^2} = 2.669 \text{ m/s}$$

$$\theta = \tan^{-1}\frac{1.5}{2.208} = 34.19° \text{ Ans.}$$

Ex-9 A 40 kN car travelling at 60 kmph in a highway collides with a 120 kN truck which emerges with 20 kmph from a cross road at right angles to the highway. The two vehicles lock after collision. Find the velocity of the car & truck after impact, assuming that the vehicles undergo a perfectly inelastic collision (i.e. they stick together)



By using conservation of momentum principle along x-direction.

$$m_c v_{cx} + m_t v_{tx} = (m_c + m_t) v_x$$

$$\frac{40\times10^3}{9.81} \times \frac{60\times10^3}{60\times60} + 0 = \frac{(40+120)\times10^3}{9.81} v_x$$

$$v_x = 4.1667 \text{ m/s}$$

Conservation of momentum along y-axis.

$$m_c v_{cy} + m_t v_{ty} = (m_c + m_t) v_y$$

$$0 + \frac{120\times10^3}{9.81} \times \frac{20\times10^3}{60\times60} = \frac{(40+120)\times10^3}{9.81} \times v_y$$

$$v_y = 4.1677 \text{ m/s}$$

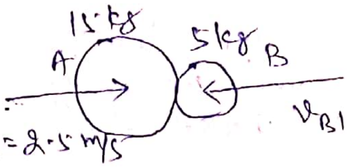
$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(4.1677)^2 \times 2}$$

$$v = 5.89 \text{ m/s}$$

$$\theta = \tan^{-1}\frac{v_y}{v_x} = 45° \text{ Ans}$$

Ex-10 Two bodies of masses 5 kg & 15 kg travelling along the same straight track, have respective velocities of 10 m/s right to left & 2.5 m/s left to right just before impact. Taking e = 0.75 find the velocities just after impact & calculate the energy loss due to impact.

Momentum eqn.



$v_{B1} = 10 \text{ m/s}$

$m_A v_{A1} + m_B v_{B1} = m_A v_{A2} + m_B v_{B2}$

$15 \times 2.5 + 5(-10) = 15 v_{A2} + 5 v_{B2}$ $\quad v_{A1} = 2.5 \text{ m/s}$

$3 v_{A2} + v_{B2} = -2.5 \quad -① $

Restitution eqn.

$$e = \frac{v_{B2} - v_{A2}}{v_{A1} - v_{B1}} \Rightarrow 0.75 = \frac{v_{B2} - v_{A2}}{2.5 - (-10)}$$

$v_{B2} - v_{A2} = 9.375 \quad -②$

① - ② gives $\quad 4 v_{A2} = -11.875 \Rightarrow v_{A2} = -2.97 \text{ m/s}$

$\therefore v_{B2} = 9.375 + v_{A2} = 9.375 - 2.97$

$v_{B2} = 6.4 \text{ m/s}$

Loss of K.E.

Initial K.E. $= \frac{1}{2} m_A v_{A1}^2 + \frac{1}{2} m_B v_{B1}^2$

$= \frac{1}{2} 15 (2.5)^2 + \frac{1}{2} 5 (10)^2$

$= 296.88 \text{ J}$

Final KE $= \frac{1}{2} m_A v_{A2}^2 + \frac{1}{2} m_B v_{B2}^2$

$= \frac{1}{2} 15 (2.97)^2 + \frac{1}{2} 5 (6.4)^2$

$= 168.55 \text{ J}$

Energy loss $= 296.88 - 168.55 = 128.3 \text{ J}$

Ans

Ex-11. A glass marble drops from a height of 3.5 m upon a horizontal floor. Taking $e = 0.85$, find the height to which the marble will rebound after the impact.

velocity of the glass marble when strikes the floor.

H = height of free fall (falling under influence of g)

$v_1 = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 3.5} = 8.29 \text{ m/s}$

velocity after impact $v_2 = e v_1 = 0.85 \times 8.29$

$\boxed{v_2 = 7.04 \text{ m/s}}$

If h is the height to which the sphere rises

$v_2 = \sqrt{2gh}$ (rising under deceleration)

$7.04 = \sqrt{2 \times 9.81 \times h}$

$\boxed{h = 2.526 \text{ m.}}$ Ans

Ex-12 An elastic ball drops from a height "h" & after rebounding twice from the floor & reaches a height $h/2$. Prove that $e = \left(\frac{1}{2}\right)^{1/4}$

Ans velocity of ball when it hits floor first time

$$v_1 = \sqrt{2gh}$$

Rebound velocity after first impact

$$v_2 = ev_1$$

v_3 = velocity when hitting 2nd time $= ev_1$

v_4 = Rebound velocity after 2nd impact

$$= e(ev_1) = e^2 v_1$$

height reached after 2nd rebound is related to rebound velocity.

$$v_4 = \sqrt{2g \times \text{height}}$$

$$e^2 v_1 = \sqrt{2g\frac{h}{2}}$$

$$\Rightarrow e^2\sqrt{2gh} = \sqrt{\frac{2gh}{2}}$$

$$\therefore e = \left(\frac{1}{2}\right)^{1/4}$$

Ex-13 A ball moving with a velocity of 10m/s impinges on a smooth fixed plane at an angle of 60° to the horizontal. Taking the coefficient of restitution as 0.9 find the velocity & direction of motion of ball after the impact.

$$\alpha = 90 - 60 = 30°$$

$$v_2 \sin\theta = v_1 \sin\alpha \quad -(1)$$

$$= 10 \sin 30° = 5 \quad -(1)$$

$$v_2 \cos\theta = ev_1 \cos\alpha$$

$$= 0.9 \times 10 \times \cos 30° \quad -(2)$$

$$= 7.794$$

(1) ÷ (2) yields $\tan\theta = \frac{5}{7.794}$

$$\Rightarrow \boxed{\theta = 32.7°}$$

$$v_2 = 5/\sin\theta = 5/\sin 32.7°$$

$$\boxed{v_2 = 9.26 \text{ m/s}}$$

The velocity after impact is inclined at angle $90 - 32.7 = \underline{57.3°}$