

Waveguides:

Waveguides consists of metallic tube with rectangular or circular cross-section. The waves (EM waves) are guided through these structure and hence called guided waves ("the propagation is by means of guided waves" i.e. the waves are guided along the conducting or even ~~over~~ the dielectric surfaces).

- Wave guides are usually applied in H-wave frequency range (4 GHz to 18 GHz). Large guides are required to transmit RF Power at longer wavelengths.

- In wave guides, the electric and magnetic fields are confined to the space within the guide. There is no loss due to radiation. Dielectric losses are negligible since these waveguides are air filled. There will be power loss as heat in the walls which is very small.

- It is possible to propagate several modes of an energy (modes are nothing but solⁿ to Maxwell's field equations). A given waveguide has a definite cutoff frequency. For a given mode, if operating frequency is greater than cutoff frequency (f_c) then that particular mode is passed through the waveguides with out any attenuation. If $f < f_c$, then the particular mode will be attenuated even at shorter distance.

⇒ The principle of waveguides in which guiding of waves is accomplished by bouncing of waves abruptly within the guides.

⇒ wave guides field is in general a combination of both

(TE & TM).

TE (Transverse electric) $\rightarrow E_z = 0, H_z \neq 0$

TM (Transverse magnetic field) $\rightarrow H_z = 0, E_z \neq 0$

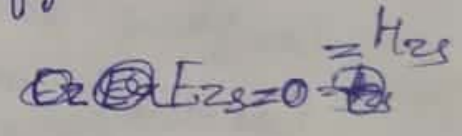
* Transmission line may operate from dc ($f=0$) to a very high frequency. Waveguide can operate only above a certain frequency called the cut off frequency and therefore acts as a high pass filter. Transmission lines become inefficient due to skin effect and dielectric losses; waveguides are used at that range of frequencies to obtain longer bandwidth and lower signal attenuation.

- Transmission line can support only a transverse electromagnetic (TEM) wave, whereas a waveguide can support many possible field configurations.

TEM (Transverse electromagnetic) \rightarrow

TE mode - $E_z = 0, H_z \neq 0$

TM " $\rightarrow E_z \neq 0, H_z = 0,$



$$E = (E_x, E_y, E_z)$$

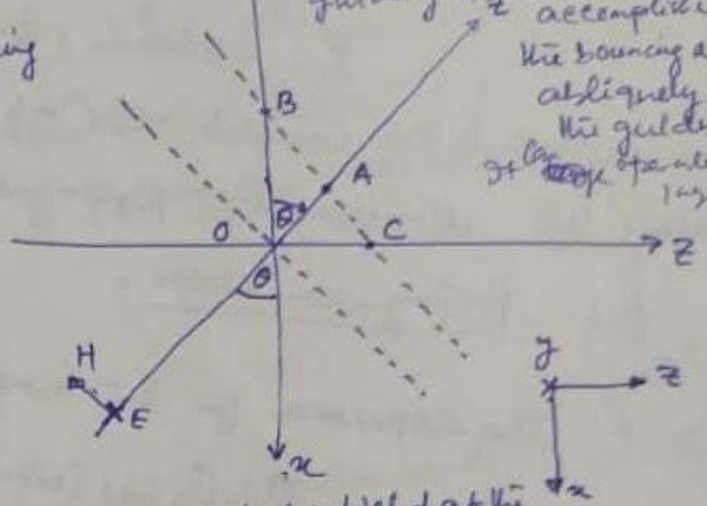
$$H = (H_x, H_y, H_z)$$

Uniform plane wave propagation in an arbitrary direction:

* In transmission lines made up of two or more parallel conductors, similarly the principle of waveguides in which guiding of waves is accomplished by the bouncing of waves obliquely within the guides. It can be thought of as large frequency

Let us consider a uniform plane wave propagating in the z' direction making an angle θ with the negative x axis.

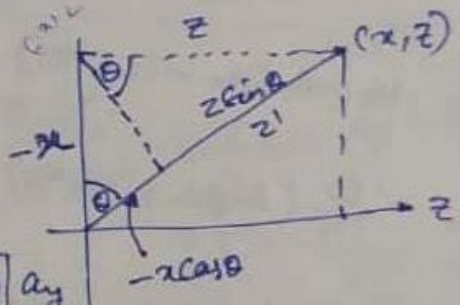
Let the electric field of the wave be entirely in the y direction.



Here we can write the expression for the electric field of the wave as: $\vec{E} = E_0 \cos(\omega t - \beta z') \hat{a}_y$ where $\beta = \omega \sqrt{\mu \epsilon}$ is the phase const.

we have

$$z' = -x \cos \theta + z \sin \theta$$



$$\vec{E} = E_0 \cos[\omega t - \beta(-x \cos \theta + z \sin \theta)] \hat{a}_y$$

$$\vec{E} = E_0 \cos[\omega t - (-\beta \cos \theta)x - (\beta \sin \theta)z] \hat{a}_y$$

$$\vec{E} = E_0 \cos[\omega t - \beta_x x - \beta_z z] \hat{a}_y$$

where $\beta_x = -\beta \cos \theta$ and $\beta_z = \beta \sin \theta$ are the phase constants in the positive x and positive z -direction respectively.

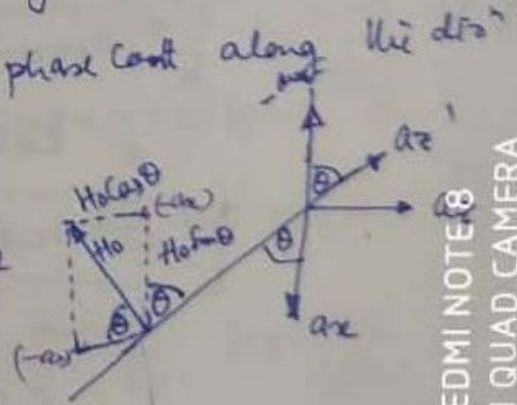
Here $|\beta_x|$ and $|\beta_z|$ are less than β , the phase const along the direction of propagation of the wave

$$\text{Here } \beta_x^2 + \beta_z^2 = (-\beta \cos \theta)^2 + (\beta \sin \theta)^2 = \beta^2$$

$$\text{and } \hat{a}_{z'} = (-\hat{a}_x) \cos \theta + \hat{a}_z \sin \theta$$

$$\hat{a}_{z'} = -\hat{a}_x \cos \theta + \hat{a}_z \sin \theta$$

$\hat{a}_{z'}$ is the unit vector directed along z' direction.



Then ~~is~~ the vector.

$$\vec{\beta} = \beta_x \hat{a}_x + \beta_z \hat{a}_z$$

$$\vec{\beta} = -\beta \cos \theta \hat{a}_x + \beta \sin \theta \hat{a}_z$$

which is the complete dirⁿ of propagation and the phase constant along the direction of propagation. Hence vector $\vec{\beta}$ is known as the propagation vector.

The expression for the magnetic field of the wave

$$\vec{H} = \vec{H}_0 \cos(\omega t - \beta z) \text{ where } |H_0| = \frac{E_0}{\eta}$$

From the fig, $\vec{H}_0 = +H_0 \sin \theta (\hat{a}_x) + H_0 \cos \theta (-\hat{a}_z)$

$$\vec{H}_0 = H_0 (-\sin \theta \hat{a}_x - \cos \theta \hat{a}_z)$$

$$\vec{H} = H_0 \sin(\omega t - \beta z) (-\hat{a}_x - \hat{a}_z)$$

$$H = H_0 (-\sin \theta \hat{a}_x - \cos \theta \hat{a}_z) \cos[\omega t - \beta(-x \cos \theta + z \sin \theta)]$$

$$= -\frac{E_0}{\eta} (\sin \theta \hat{a}_x + \cos \theta \hat{a}_z) \cos[\omega t - \beta x - \beta z]$$

Now generalizing the case of a uniform plane wave propagating in a completely arbitrary dirⁿ in three dimensions, and characterized by phase constants β_x, β_y and β_z in the x, y and z dir^s respectively.

So the expression for the electric field is

$$E = E_0 \cos(\omega t - \beta_x x - \beta_y y - \beta_z z + \phi_0)$$

$$= E_0 \cos[\omega t - (\beta_x \hat{a}_x + \beta_y \hat{a}_y + \beta_z \hat{a}_z) \cdot (x \hat{a}_x + y \hat{a}_y + z \hat{a}_z) + \phi_0]$$

$$\boxed{E = E_0 \cos[\omega t - \beta \cdot \vec{r} + \phi_0]}$$

where $\beta = \beta_x \hat{a}_x + \beta_y \hat{a}_y + \beta_z \hat{a}_z$

and propagating vector $\vec{r} = x \hat{a}_x + y \hat{a}_y + z \hat{a}_z$ is the position vector, ϕ_0 is the phase at the origin at $t=0$.

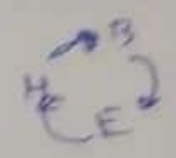
The expression for the magnetic field of the wave is then given by

$$H = H_0 \cos(\omega t - \beta \cdot r + \phi_0)$$

where $|H_0| = \frac{|E_0|}{\eta}$ and $E, H,$ and the dirⁿ of propagation are mutually perpendicular.

Since $E, H,$ and the dirⁿ of propagation are mutually perpendicular, so

$$\begin{aligned} \vec{E}_0 \cdot \vec{\beta} &= 0 \\ \vec{H}_0 \cdot \vec{\beta} &= 0 \\ \vec{E}_0 \cdot \vec{H}_0 &= 0 \end{aligned}$$



$E \times H$ should be directed along the propagation vector $\vec{\beta}$.

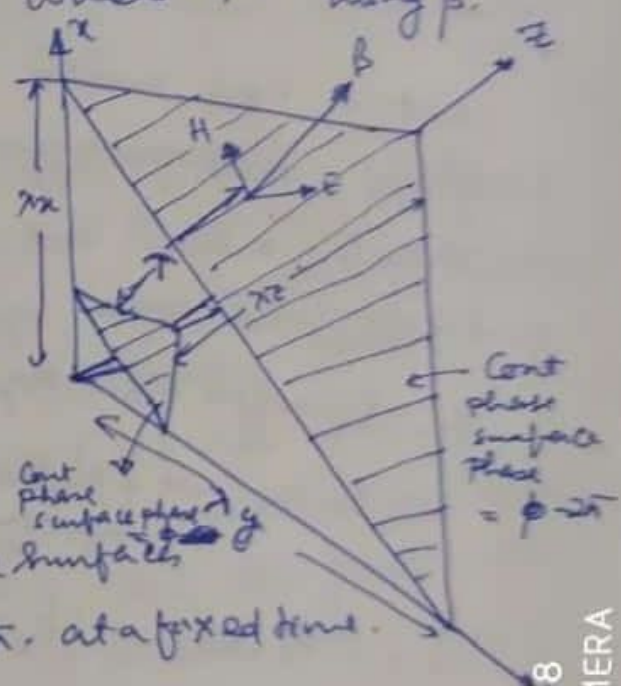
so that $\vec{\beta} \times \vec{E}_0$ is directed along \vec{H}_0 .

$$\begin{aligned} \vec{H}_0 &= \frac{\vec{a}_\beta \times \vec{E}_0}{\eta} = \frac{a_\beta \times E_0}{\sqrt{\mu\epsilon}} = \frac{\omega \sqrt{\mu\epsilon} a_\beta \times E_0}{\omega \mu} \\ &= \frac{\beta a_\beta \times E_0}{\omega \mu} = \frac{\vec{\beta} \times \vec{E}_0}{\omega \mu} \end{aligned}$$

where a_β is the unit vector along β .

$$\vec{H} = \frac{1}{\omega \mu} \vec{\beta} \times \vec{E}$$

Let the apparent wavelengths λ_x, λ_y and λ_z along the co-ordinates axes x, y and z respectively, are the distances measured along those respective axes betⁿ two consecutive constant phase surfaces betⁿ which the phase difference is 2π , at a fixed time.



$\beta_x, \beta_y,$ & β_z as being the phase constants along the x, y and z axes respectively.

$$\lambda_x = \frac{2\pi}{\beta_x}, \quad \lambda_y = \frac{2\pi}{\beta_y}, \quad \lambda_z = \frac{2\pi}{\beta_z}$$

$$\frac{1}{\lambda^2} = \frac{1}{(2\pi/\beta)^2} = \frac{\beta^2}{4\pi^2} = \frac{\beta_x^2 + \beta_y^2 + \beta_z^2}{4\pi^2} = \frac{1}{\lambda_x^2} + \frac{1}{\lambda_y^2} + \frac{1}{\lambda_z^2}$$

$$\frac{1}{\beta^2} = \frac{1}{\beta_x^2} + \frac{1}{\beta_y^2} + \frac{1}{\beta_z^2}$$

The apparent phase velocities v_{px} , v_{py} and v_{pz} along the x , y and z -axes respectively, and the velocities with which the phase of the wave progresses with time along the respective axes, thus

$$v_{px} = \frac{\omega}{\beta_x}$$

$$v_{py} = \frac{\omega}{\beta_y}$$

$$v_{pz} = \frac{\omega}{\beta_z}$$

$$\frac{1}{v_{pz}} = \left(\frac{\omega}{\beta}\right)^2 = \frac{\beta^2}{\omega^2} = \frac{\beta_x^2 + \beta_y^2 + \beta_z^2}{\omega^2}$$

$$\frac{1}{v_{pz}^2} = \frac{1}{v_{px}^2} + \frac{1}{v_{py}^2} + \frac{1}{v_{pz}^2}$$

Transverse Electric Waves in a Parallel-plate Waveguide

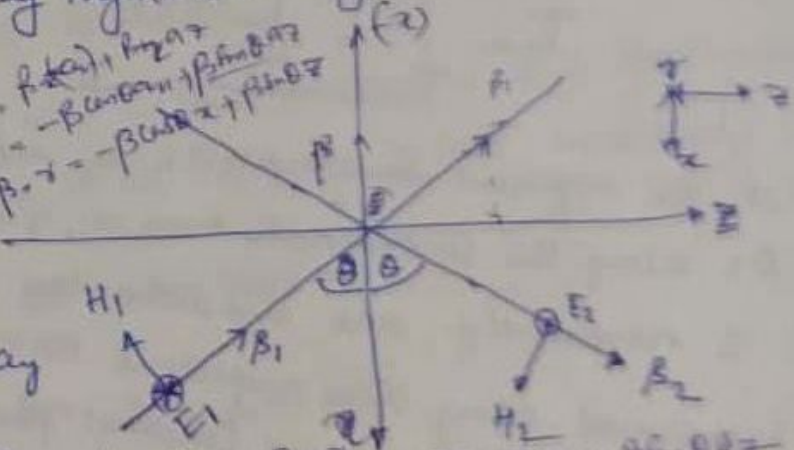
Let us now consider the superposition of two uniform plane waves propagating symmetrically with respect to the z -axis, as shown the electric fields are

$$E_1 = E_0 \cos(\omega t - \beta_1 \cdot r) a_y$$

$$= E_0 \cos(\omega t + \beta_x \cos \theta - \beta_z \sin \theta) a_y$$

$$E_2 = -E_0 \cos(\omega t - \beta_2 \cdot r) a_y$$

$$= -E_0 \cos[\omega t - \beta_x \cos \theta - \beta_z \sin \theta] a_y$$



The corresponding magnetic fields are given as H_1 and H_2 and the permittivities ϵ_1 and ϵ_2 respectively of the medium.

$$H_1 = \frac{E_0}{\eta} (-\sin \theta a_x - \cos \theta a_z) \cos(\omega t + \beta_x \cos \theta - \beta_z \sin \theta)$$

$$H_2 = \frac{E_0}{\eta} (\sin \theta a_x - \cos \theta a_z) \cos(\omega t - \beta_x \cos \theta - \beta_z \sin \theta)$$

where $\eta = \sqrt{\mu/\epsilon}$

$$H = H_1 + H_2$$

$$= -\frac{E_0}{\eta} (\sin \beta x_1 + \cos \beta x_1) \cos(\omega t + \beta z \cos \theta - \beta z \sin \theta)$$

$$+ \frac{E_0}{\eta} (\sin \beta x_1 - \cos \beta x_1) \cos(\omega t - \beta x \cos \theta - \beta z \sin \theta)$$

$$H_1 = -\frac{E_0}{\eta} \sin \beta x_1 \cos(\omega t + \beta x \cos \theta - \beta z \sin \theta)$$

$$+ \frac{E_0}{\eta} \sin \beta x_1 \cos(\omega t - \beta x \cos \theta - \beta z \sin \theta)$$

$$- \frac{E_0}{\eta} \cos \beta x_1 \cos(\omega t + \beta x \cos \theta - \beta z \sin \theta)$$

$$- \frac{E_0}{\eta} \cos \beta x_1 (\cos \omega t - \beta x \cos \theta - \beta z \sin \theta)$$

$$H = -\frac{E_0}{\eta} \sin \theta \left[\cos(\omega t + \beta x \cos \theta - \beta z \sin \theta) - \cos(\omega t - \beta x \cos \theta - \beta z \sin \theta) \right]$$

$$= \frac{E_0}{\eta} \sin \theta \left[\cos(\omega t + \beta x \cos \theta - \beta z \sin \theta) \right.$$

$$\left. + \cos(\omega t - \beta x \cos \theta - \beta z \sin \theta) \right] a_z$$

$$H = -\frac{E_0}{\eta} \sin \theta \left[-2 \sin(\beta x \cos \theta) \sin(\omega t - \beta z \sin \theta) \right] a_x$$

$$- \frac{E_0}{\eta} \sin \theta \left[2 \cos(\beta x \cos \theta) \cos(\omega t - \beta z \sin \theta) \right] a_z$$

$$H = \frac{2E_0}{\eta} \sin \theta \sin(\beta x \cos \theta) \sin(\omega t - \beta z \sin \theta) a_x$$

$$- \frac{2E_0}{\eta} \sin \theta \cos(\beta x \cos \theta) \cos(\omega t - \beta z \sin \theta) a_z \quad \text{--- (3)}$$

Here the factors $\sin(\beta x \cos \theta)$ and $\cos(\beta x \cos \theta)$ for the x -dependence and the factors $\sin(\omega t - \beta z \sin \theta)$ and $\cos(\omega t - \beta z \sin \theta)$ for the z -dependence, the composite fields have standing wave character in the x -dirⁿ and traveling wave character in the z -dirⁿ.

Thus we have standing waves in the x -dirⁿ moving bodily

$$P = -E_y H_z a z + E_y H_z a z$$

$$P = + 2 E_0 \sin(\beta x \cos \theta) \sin(\omega t - \beta z \sin \theta) \times \frac{2 E_0 \sin \theta \sin(\beta x \cos \theta)}{\eta} \sin(\omega t - \beta z \sin \theta)$$

$$+ \left\{ - 2 E_0 \sin(\beta x \cos \theta) \sin(\omega t - \beta z \sin \theta) \right. \\ \left. \times - \frac{2 E_0}{\eta} \cos \theta \cos(\beta x \cos \theta) \cos(\omega t - \beta z \sin \theta) \right\}$$

$$P = \frac{4 E_0^2}{\eta} \sin \theta \sin^2(\beta x \cos \theta) \sin^2(\omega t - \beta z \sin \theta) a z$$

$$+ \frac{4 E_0^2}{\eta} \cos \theta \sin(\beta x \cos \theta) \cos(\beta x \cos \theta) \sin(\omega t - \beta z \sin \theta) \cos(\omega t - \beta z \sin \theta) a z$$

$$P = \frac{4 E_0^2}{\eta} \sin \theta \sin^2(\beta x \cos \theta) \sin^2(\omega t - \beta z \sin \theta) a z$$

$$+ \frac{E_0^2}{\eta} \cos \theta \sin(\beta x \cos \theta) \sin 2(\omega t - \beta z \sin \theta) a z$$

The time-average Poynting vector is given by

$$\langle P \rangle = \frac{4 E_0^2}{\eta} \sin \theta \sin^2(\beta x \cos \theta) \langle \sin^2(\omega t - \beta z \sin \theta) \rangle a z$$

$$\langle P \rangle = \frac{1}{T} \int_0^T P dt = \frac{E_0^2}{\eta} \cos \theta \sin(2 \beta x \cos \theta) \langle \sin 2(\omega t - \beta z \sin \theta) \rangle a z$$

$$\langle P \rangle = \frac{2 E_0^2}{\eta} \sin \theta \sin^2(\beta x \cos \theta) a z$$

Then the time-average flow is entirely in the z-dirⁿ.

Since the composite electric field is directed entirely towards to the x-dirⁿ is the dirⁿ of time-average power flow whereas the composite magnetic field is not, the composite wave is known as Transverse electric or TE wave.

The TE wave for electric field in $\cos^n(\cdot)$ is zero when

$$\sin(\beta x \cos \theta) = 0 \quad \text{if } \beta x \cos \theta = \pm m \pi, \quad m = 0, 1, 2, \dots$$

$$a = \frac{\pm m \pi}{\beta \cos \theta} = \frac{\pm m \lambda}{2 \cos \theta}$$

Transverse Electric Waves in a Parallel Plate Waveguide

Let us consider that the superposition of two uniform plane waves propagating symmetrically with respect to the z-axis as shown.

The electric fields are

$$E_1 = E_0 \cos(\omega t - \beta_1 \cdot r) a_y$$

$$E_1 = E_0 \cos(\omega t + \beta_x \cos\theta - \beta_z \sin\theta) a_y$$

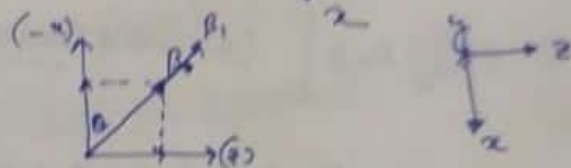
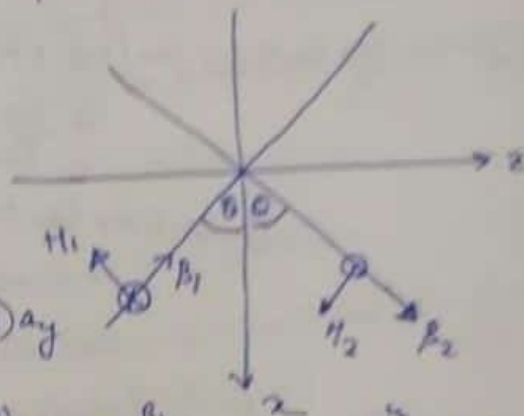
$$E_2 = E_0 \cos(\omega t - \beta_2 \cdot r) a_y$$

$$E_2 = -E_0 \cos(\omega t - \beta_x \cos\theta - \beta_z \sin\theta) a_y$$

$$E_2 = -E_0 \cos[\omega t - \beta_x \cos\theta - \beta_z \sin\theta] a_y$$

where $\beta = \omega \sqrt{\mu \epsilon}$ the wavenumber

ϵ, μ are permittivity and permeability of the medium.

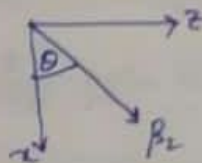


$$\beta_1 = +\beta \cos\theta a_x + \beta \sin\theta a_z$$

$$\beta_1 = -\beta \cos\theta a_x + \beta \sin\theta a_z$$

$$r = x a_x + y a_y + z a_z$$

$$\beta_1 \cdot r = -\beta \cos\theta x + \beta \sin\theta z$$



$$\beta_2 = \beta \cos\theta a_x + \beta \sin\theta a_z$$

$$\beta_2 \cdot r = \beta \cos\theta x + \beta \sin\theta z$$

The corresponding magnetic field given as

$$H_1 = \frac{E_0}{\eta} (-\sin\theta a_x - \cos\theta a_z) \cos[\omega t + \beta_x \cos\theta - \beta_z \sin\theta]$$

$$H_2 = \frac{E_0}{\eta} (\sin\theta a_x - \cos\theta a_z) \cos[\omega t - \beta_x \cos\theta - \beta_z \sin\theta]$$

$$\text{where } \eta = \sqrt{\frac{\mu}{\epsilon}}$$

The electric and magnetic fields of superposition of the waves are given by $E = E_1 + E_2$

$$E = E_0 \cos(\omega t + \beta_x \cos\theta - \beta_z \sin\theta) a_y - E_0 \cos[\omega t - \beta_x \cos\theta - \beta_z \sin\theta] a_y$$

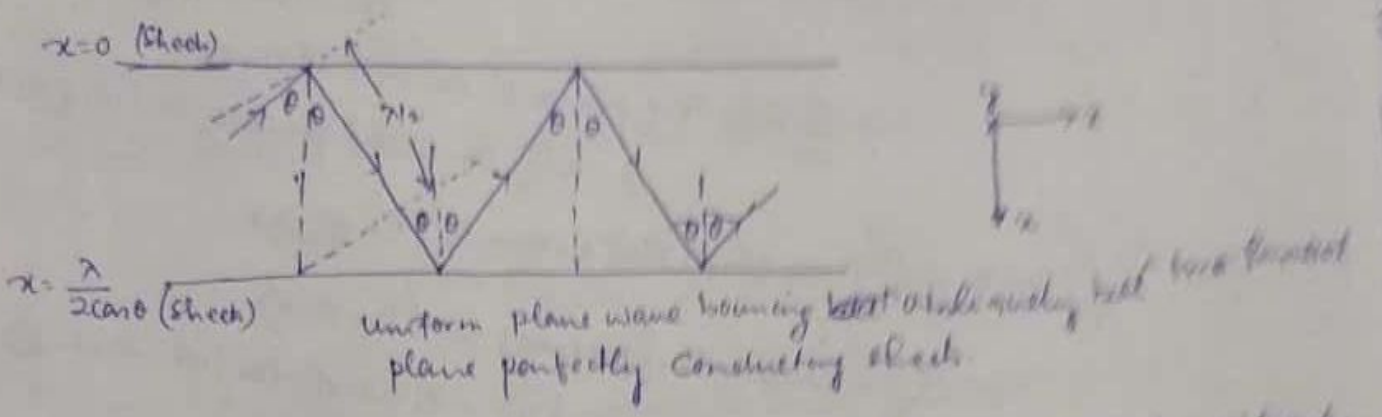
$$E = -2E_0 \sin(\beta_x \cos\theta) \sin(\omega t - \beta_z \sin\theta) a_y$$

$$\cos(A-B) - \cos(A+B) = 2 \sin A \sin B$$

$\beta_1 \cos \theta_0 - \beta_2 \sin \theta_0$

Let us consider that the superposition of

Thus if we place perfectly conducting sheets in this plane the wave will propagate undisturbed.



Let us consider that for uniform plane wave bouncing obliquely between two parallel perfectly conducting sheets, the above fig shows that the two sheets in the plane $x=0$ and $x = \frac{\lambda}{2 \cos \theta}$ thereby guiding the wave and hence the wave is in the z -dirⁿ, parallel to the plates then we have a parallel plate waveguide.

The above fig shows that constant phase surfaces of the obliquely bouncing wave that $\frac{\lambda}{2 \cos \theta}$ is simply one half of the apparent wavelength of that wave in the x -dirⁿ. Thus fields have one half apparent wavelength in the x -dirⁿ.

If ~~we~~ ~~place~~ we place the perfectly conducting sheets in the plane $x=0$ and $x = \frac{m\lambda}{2 \cos \theta}$, the fields will then have m number of one-half apparent wavelengths in the x -dirⁿ between the plates. The fields have no variation in the y -dirⁿ. Thus the fields are constant and corresponds to **TEM modes**. Here the subscript m is the mode number.

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the x dirⁿ, and m is denoted as the number of one half apparent wave lengths in that dirⁿ and the subscript '0' refers to the '0 dirⁿ' and '0' denoted as zero number of one half apparent wavelengths in that dirⁿ.

Case 1: \rightarrow Let us consider a parallel plate waveguide with perfectly conducting plates situated in the planes $x=0$ and $x=a$, and it's fixed space 'a' betⁿ them. Then for TEM waves guided by the plates,

$$a = \frac{m\lambda}{2 \cos \theta}$$

$$\cos \theta = \frac{m\lambda}{2a} = \frac{m}{2a} \frac{1}{f \mu \epsilon}$$

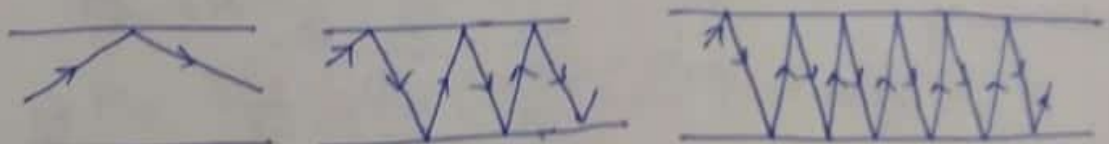
$x=0$ _____

$x=a$ _____

Case 2:

Since the wave of different wavelengths (or frequencies) bounces obliquely betⁿ the plates at different values of the angle θ , for very small wavelengths (very high frequencies), $\frac{m\lambda}{2a}$ is small, $\cos \theta \approx 0$, $\theta = 90^\circ$, and the wave simply slide betⁿ the plates as in the case of transmission line.

As λ increases (f decreases), $\frac{m\lambda}{2a}$ increases θ decreases and the wave bounce more and more obliquely.



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For $\lambda > 2a$, $\frac{m\lambda}{2a} > 1$, $\cos\theta > 1$, and θ has no real solution and this indicates that propagation does not occur for the wavelength in the waveguide modes. This condition is known as the cut off condition.

The cut off wavelength (λ_c) is given by

$$\lambda_c = \frac{2a}{m}$$

is simply the wavelength for which the spacing a is equal to a number of one half wavelengths.

Here the cut off frequency is given by

$$\lambda_c = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{\mu\epsilon}} = \frac{2\pi}{2\pi f\sqrt{\mu\epsilon}}$$

$$\frac{2a}{m} = \frac{1}{f\sqrt{\mu\epsilon}} \quad f_c = \frac{m}{2a\sqrt{\mu\epsilon}}$$

Propagation of a particular mode is possible only if f is greater than the values of f_c for that mode. Therefore, the wave of a given frequency f can propagate in all modes for which the cut off wavelengths are greater than wavelength or the cut off frequencies are less than the frequency.

$$\begin{aligned} \text{Again } \cos\theta &= \frac{m\lambda}{2a} = \frac{\lambda}{\lambda_c} = \frac{f_c}{f} \\ &= \frac{\lambda}{2a/m} = \frac{\lambda}{\lambda_c} = \frac{f_c}{f} \end{aligned}$$

$$\sin\theta = \sqrt{1 - \cos^2\theta} = \sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2} = \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$$\beta \cos\theta = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{\lambda_c} = \frac{2\pi}{\lambda_c} = \frac{2\pi}{\frac{2a}{m}} = \frac{m\pi}{a}$$

$$\beta \sin\theta = \frac{2\pi}{\lambda} \sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}$$

Since $\beta \sin \theta$ are the phase constant along the z -dirⁿ. So we can define the guide wavelength (λ_g), to be the wavelength in the z -dir along the guide.

$$\text{i.e. } \lambda_g = \frac{2\pi}{\beta \sin \theta} = \frac{2\pi}{\frac{2\pi}{\lambda} \sqrt{1 - (\lambda/\lambda_c)^2}} = \frac{\lambda}{\sqrt{1 - (\lambda/\lambda_c)^2}}$$

$$\boxed{\lambda_g = \frac{\lambda}{\sqrt{1 - (\lambda/\lambda_c)^2}}$$

Similarly the apparent wavelength in the z -dirⁿ, of the obliquely bounding uniform plane waves:

The phase velocity along the guide axis, is the apparent phase velocity in the z -dirⁿ, of the obliquely bounding uniform plane wave

$$\text{i.e. } v_{pz} = \frac{v_p}{\sin \theta} = \frac{v_p}{\sqrt{1 - (\lambda/\lambda_c)^2}} = \frac{v_p}{\sqrt{1 - (f_c/f)^2}}$$

$$\boxed{v_{pz} = \frac{v_p}{\sqrt{1 - (f_c/f)^2}}$$

we can obtain the expression for the electric field and magnetic field are

$$\vec{E} = -2E_0 \sin\left(\frac{m\pi x}{a}\right) \sin\left(\omega t - \frac{2\pi}{\lambda_g} z\right) \hat{y}$$

$$\vec{H} = \frac{2E_0}{\eta} \frac{\lambda}{\lambda_g} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\omega t - \frac{2\pi}{\lambda_g} z\right) \hat{z}$$

$$- \frac{2E_0}{\eta} \frac{\lambda}{\lambda_c} \cos\left(\frac{m\pi x}{a}\right) \cos\left(\omega t - \frac{2\pi}{\lambda_g} z\right) \hat{x}$$

the expression for the $TE_{m,0}$ mode fields in the parallel plate wave guides.

Dispersion:

As we know that for the propagating range of frequencies, the phase velocity and the wavelength along the axis of the parallel-plate waveguides are given by

$$v_{pz} = \frac{v_p}{\sqrt{1 - (f_c/f)^2}} \quad \text{and} \quad \lambda_g = \frac{\lambda}{\sqrt{1 - (f_c/f)^2}}$$

where $v_p = \frac{1}{\sqrt{\mu\epsilon}}$, $\lambda = \frac{v_p}{f}$ and f_c is the cut-off frequency.

As for a particular mode, the phase velocity of propagation along the guide axis varies with the frequency. As a consequence of this, the phase of the guided wave propagates

the field patterns of the different frequency components of a signal comprising a band of frequencies do not maintain the same phase relationships as they propagate down the guide.

This phenomenon is known as dispersion.