ENGINEERING MECHANICS

Civil engineering (2nd Sem.)

Class notes with assignments (Module 2 and module 3)

Reference book:

Engineering Mechanics by S Timoshenko, D.H Young and J.V.Rao, McGraw Hill.

STRUCTURE:

MODULE 2

Rectilinear Translation- Kinematics- Principles of Dynamics- Concept of Inertial and Non-inertial frame of reference, D'Alemberts Principles.

MODULE 3

Momentum and impulse, Work and Energy- impact

Curvilinear translation- Kinematics- equation of motion- projectile- D'Alemberts Principle in curvilinear motion, Moment of momentum, Work- Energy in curvilinear motion.

Kinetics of Rotation of rigid body

- : Rectilinear Translation : -

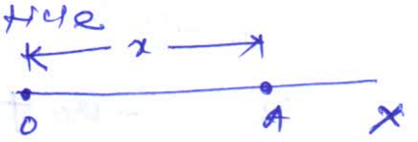
In statics, it was considered that the rigid bodies are at rest. In dynamics, it's considered that they are in motion. Dynamics is commonly divided into two branches. Kinematics and kinetics.

In, kinematics we are concerned with space time relationship of a given motion of a body and not at all with the forces that cause the motion.

- In kinetics we are concerned with finding the kind of motion that a given body or system of bodies will have under the action of given forces or with what forces must be applied to produce a desired motion.

Displacement

from the fig: displacement of a particle can be defined by it's x-coordinate, measured from the fixed reference point O.



- When the particle is to the right of fixed point O, this displacement can be considered positive and when it's towards the ~~sig~~ left hand side it's considered as negative.

General displacement time equation

$$x = f(t) \quad -(1)$$

where $f(t)$ = function of time

for example $\boxed{x = c + bt}$

In the above equation c, represents the initial displacement at $t=0$, while the constant b shows the rate at which displacement increases. It is called uniform rectilinear motion.

Second example is $\boxed{x = \frac{1}{2} g t^2}$
where x is proportional to the square of time.

Velocity

Acceleration

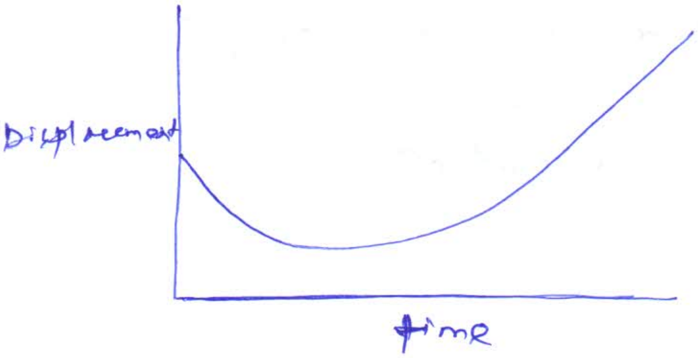
Example The rectilinear motion of a particle is defined by the displacement - time equation $x = x_0 - v_0 t + \frac{1}{2} a t^2$
construct displacement - time and velocity diagram for this motion and find the displacement and velocity at time $t = 2s$. $x_0 = 750$ mm, $v_0 = 500$ mm/s
$a = 0.125$ m/s^2
The equation of motion is

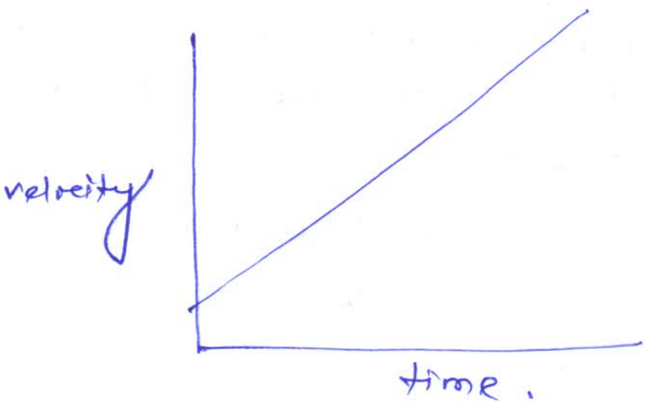
$$x = x_0 - v_0 t + \frac{1}{2} a t^2 \quad \text{—(1)}$$

$$v = \frac{dx}{dt} = -v_0 + at \quad \text{—(2)}$$

substituting x_0, v_0 and a in equation (1)

~~$x = 75 - 500$~~





Q-1 A bullet leaves the muzzle of a gun with velocity $v = 750\ m/s$. Assuming constant acceleration from breech to muzzle find time t occupied by the bullet in travelling through gun barrel which is 750 mm long.

initial velocity of bullet $u = 0$
final velocity of bullet $v = 750\ m/s$.
total distance $s = 0.75\ m$.
$t = ?$

We have $v^2 - u^2 = 2as$,

$\Rightarrow v^2 = 2as \Rightarrow a = \frac{v^2}{2s} = \frac{750^2}{2 \times 0.75}$
$= 375000\ m/sec^2$

Again $v = u + at$

$\Rightarrow 750 = 375000 \times t$
$\Rightarrow t = \frac{750}{375000} = \boxed{0.002\ sec.}$

Q-2 A stone is dropped into well and falls vertically with constant acceleration $g = 9.81\ m/sec^2$. The sound of impact of stone on the bottom of well is heared after 6.5 sec. If velocity of sound is 336 m/s. How deep is the well?

$V = 336\ m/sec$
let s = depth of well
t_1 = time taken by the stone into the well
t_2 = time taken by the sound to be heared.
total time $t = (t_1 + t_2) = 6.5\ sec$.

Now $s = ut + \frac{1}{2}gt^2$

$\Rightarrow s = 0 + \frac{1}{2}gt^2$
$\Rightarrow t_1 = \sqrt{\frac{2s}{g}}$

When the sound travels with uniform velocity
$s = Vt_2$ or $t_2 = \frac{s}{V}$

$$\sqrt{\frac{2s}{g}} + \frac{s}{V} = 6.5$$

$$\Rightarrow \frac{2s}{g} = \left(6.5 - \frac{s}{336}\right)^2$$

$$\Rightarrow 2s = 9.81\left(6.5 - \frac{s}{336}\right)^2$$

$$= 9.81\left(\frac{2184 - s}{336}\right)^2$$

$$= 0.0291\,(2184 - s)^2$$

$$= 0.0291\,(4769856 + s^2 - 4368s)$$

$$= 138802.809 + 0.0291s^2 - 127.10888$$

$$\Rightarrow 0.0291s^2 - 129.10888 + 138802.809 = 0$$

$$\Rightarrow s =$$

$$0.2038s = 42.25 + 0.0000088 5s^2 - 0.0386s$$

~~s = 174~~

$$0.00000885s^2 - 0.1658s + 42.25 = 0$$

$$\boxed{s = 17.31\text{ m.}}$$

A2 A rope AB is attached at B to a small block of negligible dimensions and passes over a pulley C so that it's free end A hangs 1.5 m above ground when the block rests on the floor. The end A of the rope is moved horizontally in a str. line by a man walking with a uniform velocity $v_0 = 3$ m/s. Plot the velocity - time diagram

(b) find the time t required for the block to reach the pulley if h = 4.5 m, pully dimension are negligible.

A3 A particle starts from rest and moves along a str. line with constant acceleration a. If it acquires a velocity $v = 3$ m/s. after having travelled a distance s = 7.5 m. find magnitude of acceleration.

Principles of Dynamics:

Newton's law of motion:

First law: Everybody continues in it's state of rest or of uniform motion in a straight line except in so far as it may be compelled by force to change that state.

Second Law: The acceleration of a given particle is proportional to the force applied to it and takes place in the direction of the straight line in which the force acts.

Third law To every action there is always an equal and contrary reaction or the mutual action's of any two bodies are always equal and oppositely directed.

General Equation of Motion of a Particle:

$$\boxed{ma = f}$$

Differential equation of Rectilinear motion:

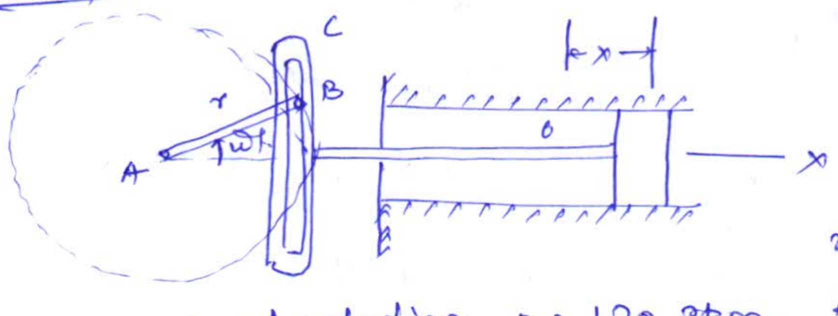
Differential form of equation for rectilinear motion can be expressed as

$$\boxed{\frac{W}{g}\ddot{x} = X}$$

where $\ddot{x}$ = acceleration

X = Resultant acting force.

Example



For the engine shown in fig., the combined wt. of piston and piston rod $W = 450N$, crank radius $r = 250mm$ and uniform speed of rotation $n = 120$ rpm. Determine the magnitude of resultant force acting in piston (a) at extreme position and (b) at the middle position

piston has a simple harmonic motion represent(ing)
displacement-time equation

$$x = r\cos\omega t \quad \text{---}(1)$$

$$\omega = \frac{2\pi n}{60} = \frac{2\pi\times120}{60} = 4\pi \text{ rad/s.}$$

$$\dot{x} = -r\omega\sin\omega t$$

$$\ddot{x} = -r\omega^2\cos\omega t \quad -(2)$$

Differential equation of motion

$$\frac{W}{g}\ddot{x} = X$$

$$\Rightarrow -\frac{W}{g}r\omega^2\cos\omega t = X$$

$$\Rightarrow X = -\frac{450}{9.81}\times0.25\,(4\pi)^2\cos(4\pi t)$$

for extreme position

$\cos\omega t = -1$

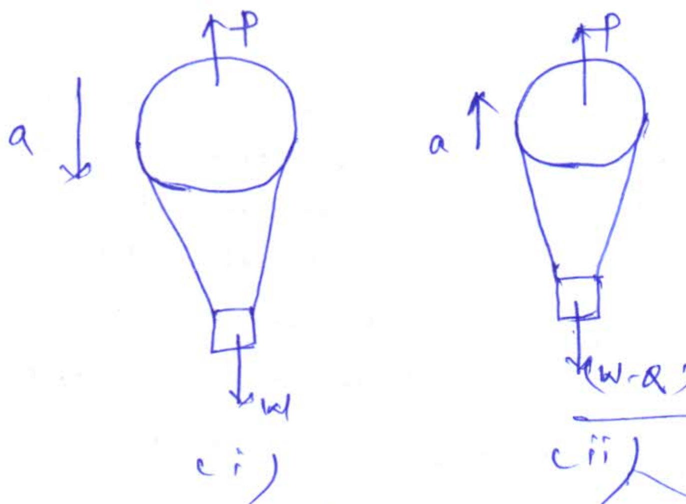
so $\boxed{X = 1810N.}$

~~for extre~~ middle position $\cos\omega t = 0$.

so Resultant force $= 0$.

E-2 A ballon of gross wt W is falling vertically down ward with constant acceleration a. What amount of ballast Q must be thrown out in order to give balloon an equal upward acceleration a.

P = buoyant force.



(i) considering 1st case when ballon is falling,

$$\frac{W}{g}a = W - P \quad -(1)$$

(ii) $$\frac{W-Q}{g}a = P - (W-Q) - (2)$$

Eq(1) + Eq(2)

$$\frac{Q}{g}\cdot a = W + W - Q = 2W + Q$$

$$\Rightarrow Q\left(\frac{a}{g} - 1\right) = 2W$$

$$\Rightarrow Q = \frac{2Wg}{(a-g)}$$

$$\frac{Wa}{g} = (W-P)$$

$$\frac{(W-Q)a}{g} = P-(W-Q)$$

$$\frac{Wa+(W-Q)a}{g} = W-\cancel{P}+\cancel{P}-(W-Q) = Q$$

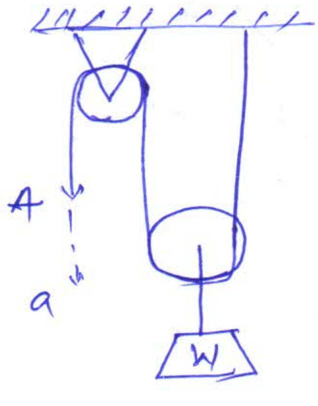
$$\Rightarrow \frac{Wa+Wa-Qa}{g} = Q$$

$$\Rightarrow 2Wa = Qg+Qa$$

$$\Rightarrow \boxed{Q = \frac{2Wa}{(g+a)}}$$

Q.1 A wt. W = 4450N is supported in a vertical plane by string and pulleys arranged shown in fig. If the free end A of ~~the~~ the string is pulled vertically downward with constant acceleration a = 18 m/s² find tension S in the string.

Differential equation of motion for the system is



$$2S - W = \frac{W}{g} \times \frac{a}{2}$$

$$\Rightarrow 2S = W + \frac{Wa}{2g}$$

$$= \cancel{\frac{W}{2}\left(2+\frac{a}{2g}\right)}$$

$$= W\left(1+\frac{a}{2g}\right)$$

$$\Rightarrow S = \frac{W}{2}\left(1+\frac{a}{2g}\right)$$

$$= \frac{4450}{2}\left(1+\frac{18}{2\times 9.81}\right) = \boxed{4266.28\ N.}$$

$$\frac{Wa}{g} = (W-P)$$

$$\frac{(W-Q)a}{g} = P-(W-Q)$$

$$\frac{Wa+(W-Q)a}{g} = W-\cancel{P}+\cancel{P}-(W-Q) = Q$$

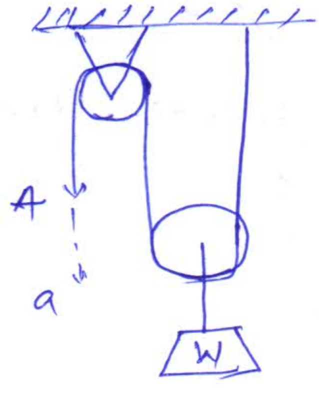
$$\Rightarrow \frac{Wa+Wa-Qa}{g} = Q$$

$$\Rightarrow 2Wa = Qg+Qa$$

$$\Rightarrow \boxed{Q = \frac{2Wa}{(g+a)}}$$

Q.1 A wt. $W = 4450N$ is supported in a vertical plane by string and pulleys arranged shown in fig. If the free end A of ~~oth~~ the string is pulled vertically downward with constant acceleration $a = 18 m/s^2$ find tension S in the string.

Differential equation of motion for the system is



$$2S - W = \frac{W}{g} \times \frac{a}{2}$$

$$\Rightarrow 2S = W + \frac{Wa}{2g}$$

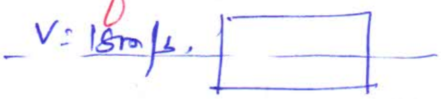
$$= \cancel{\frac{W}{2}\left(2+\frac{a}{2g}\right)}$$

$$= W\left(1+\frac{a}{2g}\right)$$

$$\Rightarrow S = \frac{W}{2}\left(1+\frac{a}{2g}\right)$$

$$= \frac{4450}{2}\left(1+\frac{18}{2\times 9.81}\right) = \boxed{4266.28\ N.}$$

Q.2 An elevator of gross wt W = 4450N starts to move upward direction with a constant acceleration and acquires a velocity $v = 18 m/s$, after travelling a distance = 1.8 m. find tensile force S in the cable during it's motion.

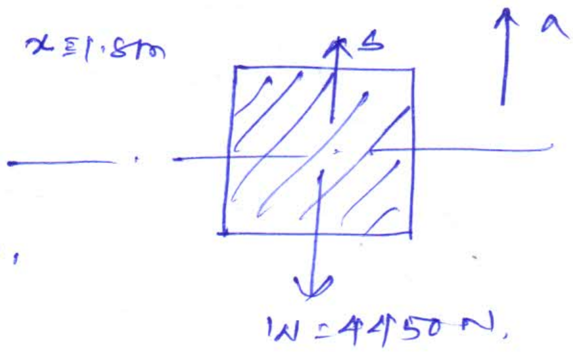


W = 4450N.

V = 18 m/s.

initial velocity u = 0

distance travelled x = 1.8 m,



$$S - W = \frac{W}{g} \cdot a$$

$$\Rightarrow S = W + \frac{W}{g} a = W\left(1 + \frac{a}{g}\right) \quad \text{---(1)}$$

Now applying equation of kinematics

$$v^2 - u^2 = 2as$$

$$\Rightarrow 18^2 - 0 = 2a \times 1.8$$

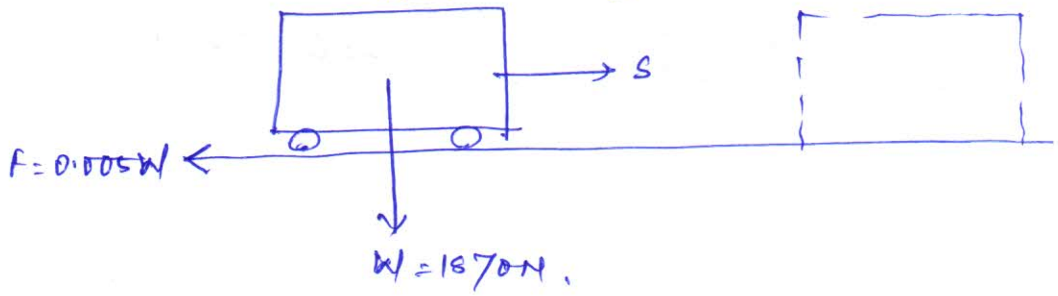
$$\Rightarrow a = \frac{18^2}{2 \times 1.8} = \boxed{90 m/s^2}$$

substituting the value of a in eq. (1)

$$S = 4450\left(1 + \frac{90}{9.81}\right) = \boxed{45275.7 \text{ N.}}$$

A-1

Q.3 A train weighing 1870N without the locomotive starts to move with constant acceleration along a straight track and in first 60s acquires a velocity of 56 Kmph. Determine the tension S in draw bar both locomotive and train if the air resistance is 0.005 times the wt. of the train.



$S - F = \frac{W}{g} \cdot a$

$\Rightarrow S = 0.005W + \frac{Wa}{g}$ —— (1)

from eq. of kinematics,

$v = u + at$

$\Rightarrow a = \left(\frac{15.56 - 0}{60}\right) = 0.26\ m/sec^2$

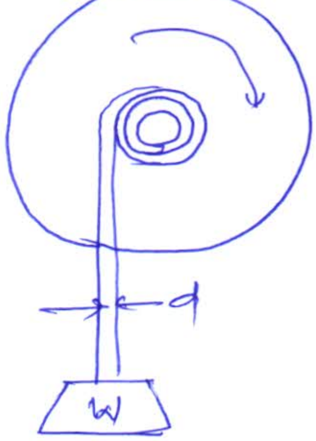
substituting the value of a in eq. (1)

$S = W\left(0.005 + \frac{a}{g}\right)$

$= 1870\left(0.005 + \frac{0.26}{9.81}\right) = \boxed{58.9\ kN.}$

A-2

Q. 4 A wt. W is attached to the end of a small flexible rope of dia. d = 6.25mm. and is raised vertically by winding the rope on a reel. if the reel is turned uniformly at a rate of 2 rps. what will be the tension in rope.



dia of rope d = 6.25mm = 0.00625m.

No of revolutions N = 2rps.

let x = initial radius of reel.

t = time taken for N revolutions.

Net radius after t sec.

$R = [x + (Ntd)]$

Now mass velocity $v = R\omega$

$\omega = 2\pi N.$

$\therefore v = (x + Ntd)\, 2\pi N$

acceleration of rope = $a = \frac{dv}{dt}$

$a = \frac{d}{dt}\left[2\pi N x + 2\pi N^2 td\right] = 2\pi N^2 d$

$S - W = \frac{W}{g} \cdot a \Rightarrow S = W + \frac{Wa}{g} = W\left(1 + \frac{a}{g}\right)$

$\Rightarrow S = W\left(1 + \frac{2\pi N^2 d}{g}\right)$ ~~W(1+~~

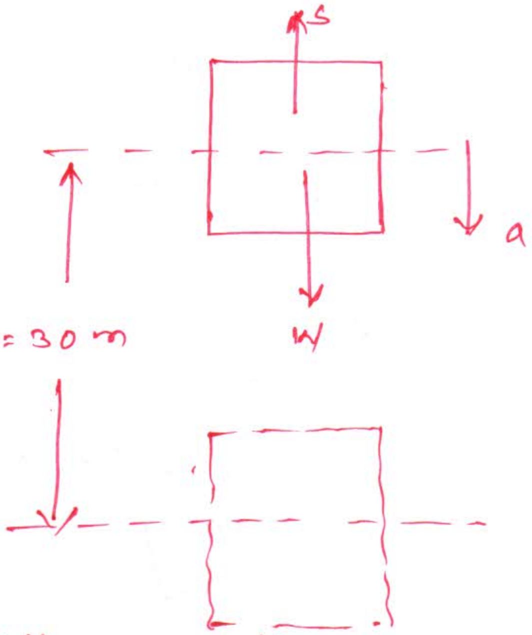
$\Rightarrow S = W\left(1 + \frac{2\pi \times 2^2 \times 0.00625}{9.81}\right)$

$=$

Ass-3

Q. 5

A mine case of wt $W = 8.9$ kN starts from rest and moves downward with constant acceleration travelling a distance $s = 30$ m in 10 sec. find the tensile force in the cable.



Wt. of case $W = 8.9$ kN.

initial velocity $u = 0$.

distance travelled $s = 30$ m

time $t = 10$ sec.

$s = \cancel{ut}^{0} + \frac{1}{2} a t^2$

$\Rightarrow 30 = \frac{1}{2} a \times 10^2$

$\Rightarrow t = \frac{60}{10^2} = \boxed{0.6 \text{ m/sec}^2}$

Differential equation of rectilinear motion

$W - S = \frac{W}{g} \cdot a$

$\Rightarrow S = W - \frac{W}{g} a = W\left(1 - \frac{a}{g}\right)$

$= 8.9\left(1 - \frac{0.6}{9.81}\right)$

$\Rightarrow \boxed{S = 8.35 \text{ kN.}}$ (Ans)

D' Alembert's Principle

Differential equation of motion (rectilinear) can be written as

$$X - m\ddot{x} = 0 \quad \text{—— (1)}$$

Where X = Resultant of all applied force in the direction of motion

m = mass of the particle

The above equation may be treated as equation of dynamic equilibrium. To represent this equation, in addition to the real forces acting on the particle a fictitious force $m\ddot{x}$ is required to be considered. This force is equal to the product of mass of the particle and it's acceleration and directed in opposit direction, and is called the inertia force of the particle.

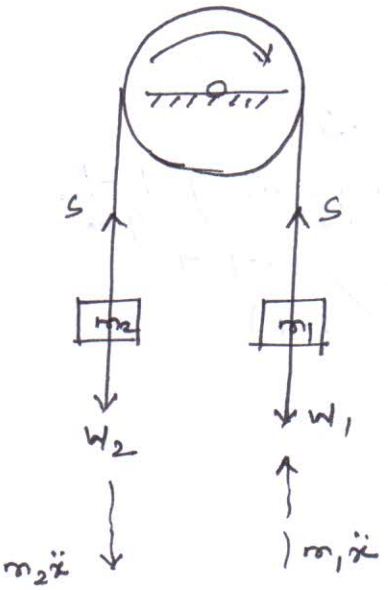
$$\boxed{-\Sigma m\ddot{x} = -\ddot{x}\,\Sigma m = -\frac{W}{g}\ddot{x}}$$

Where W = total weight of the body

So the equation of dynamic equilibrium can be expressed as:

$$\boxed{\Sigma X_i + \left(-\frac{W}{g}\ddot{x}\right) = 0} \quad \text{—— (2)}$$

Example 1



for the example shown considering the motion of pulley as shown by the arrow mark, where upward acceleration $\ddot{x}_2$ for W_2 and downward acceleration $\ddot{x}_1$ for W_1

- corresponding inertia forces and their direction are indicated by dotted line.
- By adding inertia forces to the real forces (such as W_1, W_2 and tension in strings S) we obtain, for each particle, a system of forces in equilibrium.

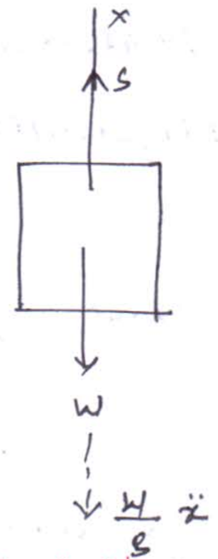
The equilibrium equation for the entire system without S

$$W_2 + m_2\ddot{x} = W_1 - m_1\ddot{x}$$

$$\Rightarrow (m_1 + m_2)\ddot{x} = (W_1 - W_2) \Rightarrow \boxed{\ddot{x} = \frac{W_1 - W_2}{(W_1 + W_2)} \cdot g}$$

Example 2 A body is moving in upward direction by a rope.

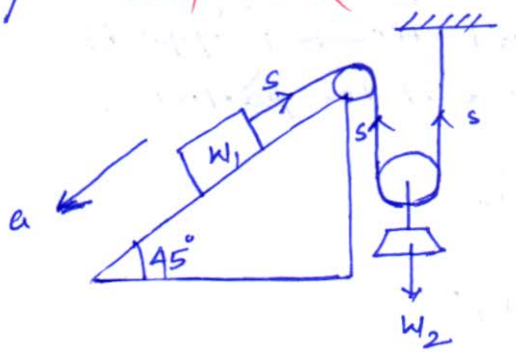
So the equation of dynamic equilibrium considering the real and inertia forces.

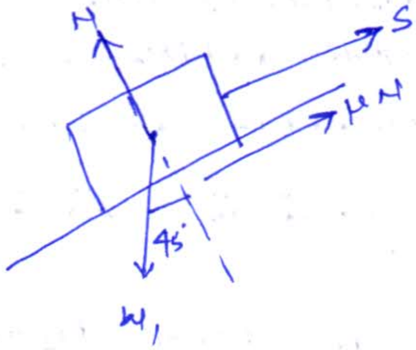


$S - W - \frac{W}{g}a = 0$, so tensile force in rope

$\Rightarrow \boxed{S = W\left(1+\frac{a}{g}\right)}$

Q.1 Find tension S in the string during motion of the system if $W_1 = 900N$, $W_2 = 450N$. The μ betn the inclined plane and block $W_1 = 0.2$





When W_1 moves downward in the inclined plane with an acceleration a, then acceleration of $W_2 = \frac{a}{2}$

Considering dynamic equilibrium of W_1, from D' Alembert's principle

$$\left(W_1 \sin 45° - \mu N - S\right) - \frac{W_1}{g}a = 0$$

$$\Rightarrow \frac{W_1}{g}a = W_1 \sin 45° - \mu N - S$$

$$= W_1 \sin 45 - \mu W_1 \cos 45 - S$$

$$\Rightarrow a = \left(900 \times \frac{1}{\sqrt{2}} - 0.2 \times 900 \times \frac{1}{\sqrt{2}} - S\right)\frac{9.81}{900}$$

$$= \left(636.4 - 127.28 - S\right) 0.0109$$

$$6.93676 - 1.387352$$

$$\Rightarrow a = \cancel{5.549} - 0.0109S \quad \text{---(1)}$$

Similarly ~~from~~ for weight W_2

$$2S - W_2 - \frac{W_2}{g}\frac{a}{2} = 0$$

$$\Rightarrow \cancel{\frac{W_2 a}{2g}} = W_2\left(1 + \frac{a}{2g}\right) = 2S$$

$$\Rightarrow 2S = \frac{450}{2}\left(1 + \frac{a}{19.62}\right) = 225 + 11.46a \quad \text{---(2)}$$

substituting the value of S in eq. (1)

~~a = 5.549 − 2.4525 − 0.125a~~ $\Rightarrow a =$

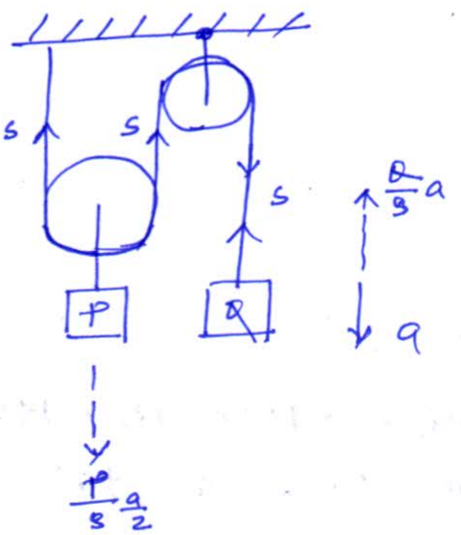
$$a = 6.93676 - 1.387352 - 0.0109(225 + 11.46a)$$

$$= \cancel{6.93}\ 5.549408 - 2.4525 - 0.124914a$$

$$= 3.096908 - 0.124914a$$

$$\Rightarrow \boxed{a = 2.75\ m/s^2}$$

Q.2 Two weights P and Q are connected by the arrangement shown in fig. Neglecting friction and inertia of pulley and cord find the acceleration a of wt. Q
Assume $P = 178\ N$, $Q = 133.5\ N$.



Applying D'Alembert's principle for Q

$$Q - S - \frac{Q}{g}a = 0$$

$$\Rightarrow S = Q\left(1 - \frac{a}{g}\right) \quad \text{---(1)}$$

$$= 133.5\left(1 - \frac{a}{9.81}\right)$$

Applying D'Alembert's principle to P

$$\cancel{P - 2S = P}$$

$$2S - P - \frac{Pa}{2g} = 0$$

$$\Rightarrow 2S = P\left(1 + \frac{a}{2g}\right)$$

$$\Rightarrow S = \frac{P}{2}\left(1 + \frac{a}{2g}\right) \quad \text{---(2)}$$

$$= \frac{178}{2}\left(1 + \frac{a}{19.62}\right)$$

$$133.5\left(1 - \frac{a}{9.81}\right) = 89\left(1 + \frac{a}{19.62}\right)$$

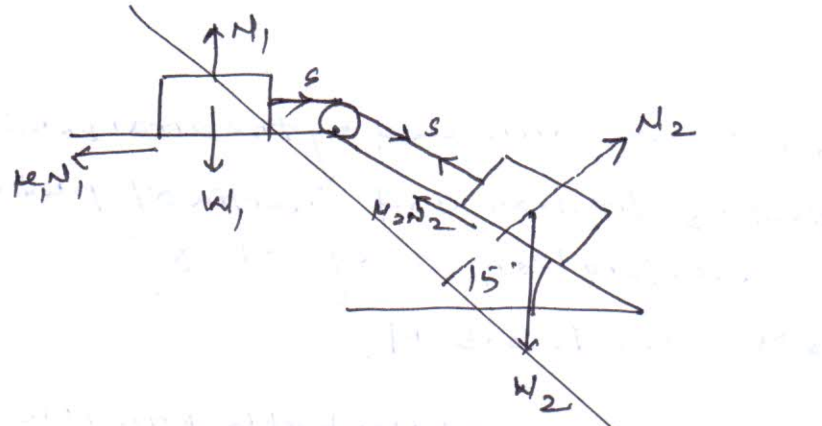
$$\Rightarrow 133.5 - 13.608a = 89 + 4.536a$$

$$\Rightarrow 18.144a = 44.5$$

$$\Rightarrow \boxed{a = 2.45\ m/s^2} \quad \text{(Ans)}$$

Q.3 ~~Assuming the car in the fig. to have a velocity of 6 m/s find shortest distance s in which it can be stopped with constant decelaration without disturbing the block.~~ Data: $c = 0.6\ m$, $h = 0.9\ m$
$\mu = 0.5$

Q.3 Two blocks of wt $W_1 = 150N$ and $W_2 = 500N$ are connected by an inextensible string. Find the acceler of the blocks and tension in the string. $\mu_1 = 0.1$, $\mu_2 = 0.$



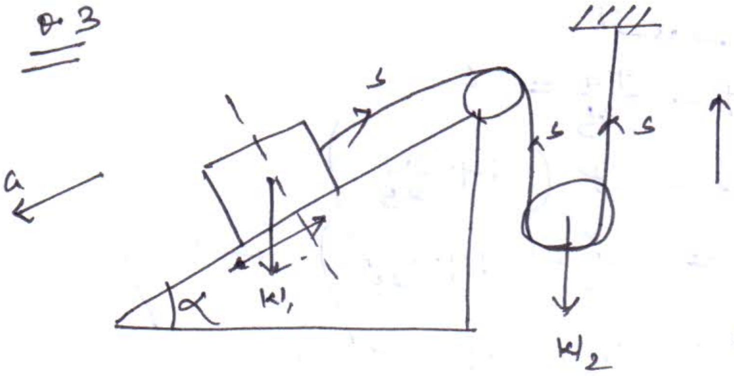
for block 1

$S - \mu_1 N_1 = 0$

$\Rightarrow S = \mu_1 W_1 = 0.1 \times 150 = 15N.$

for block 2

Q.3



$W_1 = 890N$ $W_2 = 445N.$

$\mu = 0.2$ $\alpha = 45°$

find S.

considering equilibrium of W_1 and applying D'Alembert's principle

$$W_1 \sin 45° - \mu N_1 - S - \frac{W_1}{g} a = 0$$

$$\Rightarrow S = W_1 \sin 45 - \mu N - \frac{W_1}{g} a$$

$$= \frac{890}{\sqrt{2}} - 0.2 \times 890 \times \frac{1}{\sqrt{2}} - \frac{890}{9.81} a$$

$$= 629.32 - 125.865 - 90.72a$$

$$\boxed{S = 503.455 - 90.72a} \quad \text{---(1)}$$

Applying D'Alembert's principle for W_2

$$2S - W_2 - \frac{W_2}{g}\frac{a}{2} = 0$$

$$\Rightarrow 2S = W_2\left(1 + \frac{a}{2g}\right)$$

$$\Rightarrow S = \frac{W_2}{2}\left(1 + \frac{a}{2g}\right) = \frac{445}{2}\left(1 + \frac{a}{19.62}\right) = 222.5 + 11.34a \quad \text{---(2)}$$

equating (1) and (2)

$503.455 - 90.72a = 222.5 + 11.34a$

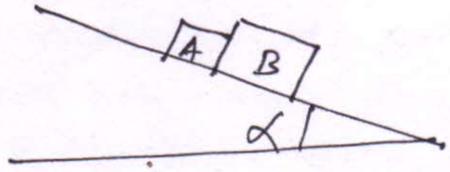
$\Rightarrow 102.0604\, a = 280.955$

$\Rightarrow \boxed{a = 2.75\ m/s^2}$

so $S = 222.5 + 11.34 \times 2.75$

$= \boxed{253.71\ N.}$

Q.4

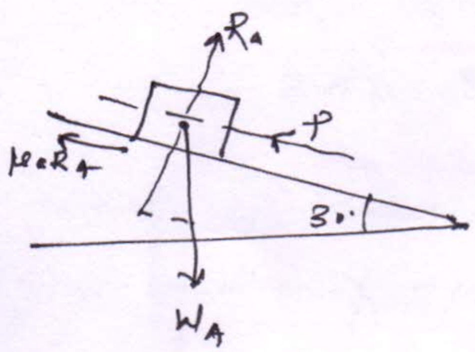


$W_A = 44.5\,N \quad W_B = 89\,N$

$\alpha = 30° \quad \mu_a = 0.15$

$\mu_B = 0.3$

find pressure P betn blocks.



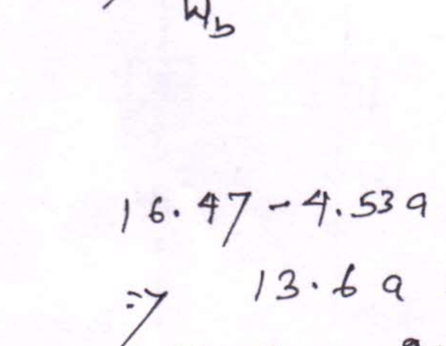
$W_a \sin 30 - P - \mu_a R_a - \frac{W_a}{g} a = 0$

$\Rightarrow P = W_a \sin 30 - \mu_a R_a - \frac{W_a}{g} a$

$= 44.5 \times \frac{1}{2} - 0.15 \times 44.5 \times \cos 30 - \frac{44.5}{9.81} a$

$= 22.25 - 5.78 - 4.53a$

$= 16.47 - 4.53a \quad \text{--- (1)}$



$P + W_b \sin 30 - \mu_b R_b - \frac{W_b}{g} a = 0$

$\Rightarrow P = -\frac{W_b}{2} + 0.3 \times 89 \cos 30 + \frac{89}{9.81} a$

$= -\frac{89}{2} + 23.122 + 9.07a$

$= -21.378 + 9.07a \quad \text{--- (2)}$

$16.47 - 4.53a = -21.378 + 9.07a$

$\Rightarrow 13.6a = 37.848$

$\Rightarrow a = 2.78\ m/s^2$

$P = 3.87\ N.$

Momentum and Impulse

We have the differential equation of rectilinear motion of a particle

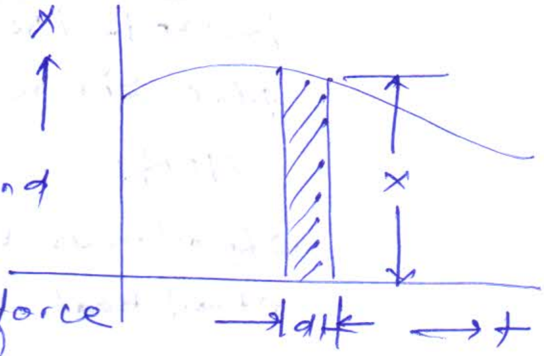
$$\frac{W}{g}\ddot{x} = X$$

Above equation may be written as

$$\frac{W}{g}\frac{d\dot{x}}{dt} = X$$

or $$\boxed{d\left(\frac{W}{g}\dot{x}\right) = X\,dt} \quad \text{—— (1)}$$

In the above equation we will assume force X as a function of time represented by a force time diagram.



The right hand side of eq. (1) is then represented by the area of shaded elemental strip of ht X and width dt. This quantity i.e $(X\,dt)$ is called impulse of the force X in time dt. The expression on the left hand side of the expression $\left(\frac{W}{g}\dot{x}\right)$ is called momentum of particle.

So the eq. (1) represents the differential change in momentum of a particle in time dt.

Integrating eq. (1) we have

$$\boxed{\frac{W}{g}\dot{x} + C = \int_0^t X\,dt} \quad \text{—— (2)}$$

where C is a constant of integration

Now assuming an initial moment, $t=0$, the particle has an initial velocity $\dot{x}_0$

so $$\boxed{C = -\frac{W}{g}\dot{x}_0} \quad \text{—— (3)}$$

So equation (2) becomes

$$\boxed{\frac{W}{g}\dot{x} - \frac{W}{g}\dot{x}_0 = \int_0^t X\,dt} \quad \text{—— (4)}$$

from equation (41) it is clear that the total change of momentum of a particle during a finite interval of time is equal to the impulse of acting force.

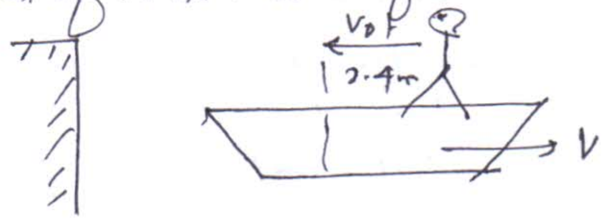
~~Example~~ in other words

$$\boxed{F \cdot dt = d(mv)}$$

where $m \times v =$ momentum

~~Recoil of gun~~

Q-1 A man of wt 712 N stands in a boat so that he is 4.5 m from a pier on the shore. He walks 2.4 m in the boat towards the pier and then stops. How far from the pier will he be at the end of time. Wt of boat is 890 N.



wt of man $W_1 = 712$ N

wt of boat $W_2 = 890$ N

Let v_0 is the initial velocity of man and t is time

then $v_0 t = x$

$\Rightarrow v_0 t = 2.4$ m

$\Rightarrow v_0 = \left(\frac{2.4}{t}\right)$ m/s.

let V = velocity of boat towards right

according to conservation of momentum

$$W_1 v_0 = (W_1 + W_2) V$$

$$\Rightarrow V = \frac{W_1 v_0}{(W_1 + W_2)}$$

distance covered by boat

$$s = V \cdot t = \frac{W_1 v_0}{(W_1 + W_2)} \cdot t$$

$$\Rightarrow s = \frac{712 \times 2.4}{\cancel{t}\,(712 + 890)} \cdot \cancel{t} = \boxed{1.067 \text{ m}}$$

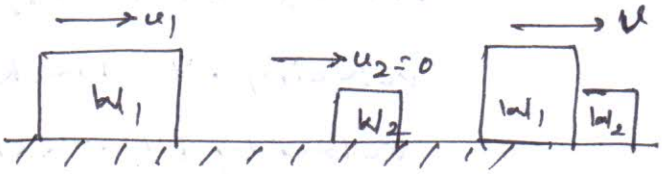
position of man from pier

$= 4.5 + s - x$

$= 4.5 + 1.667 - 2.4 = \boxed{3.167\ m}$ (Ans).

Q.2 A locomotive wt 534 kN has a velocity of 16 kmph and backs into a frieght car of wt 86 kN that is at rest on a track. after coupling at what velocity v the entire system continues to move. Neglect friction.

conservation of momentume



$W_1 u_1 + W_2 u_2 = (W_1 + W_2) V$

$\Rightarrow V = \dfrac{534 \times 4.45}{(534 + 86)} = \boxed{3.82\ m/s.}$

Q.3 A 667.5 man sits in a 333.75 N canoe and fire a rifle bullet horizontally. ~~directed over~~ find velocity v with which the canoe will move after the shot. the rifle has a muzzle velocity 660 m/s and wt. of bullet is 0.28 N.

Wt. of man $W_1 = 667.5$ N.

Wt. of canoe $W_2 = 333.75$ N.

Wt. of bullet $W_3 = 0.28$ N.

velocity of muzzle $u = 660$ m/s.

$V =$ final velocity of canoe.

According to conservation of momentum

$\cancel{W_3 u}$ $W_3 u = (W_1 + W_2) V$

$\Rightarrow V = \dfrac{0.28 \times 660}{(667.5 + 333.75)} = \boxed{0.182\ m/s.}$

Q.4 A wood block wt 22.25 N rests on a smooth horizontal surface. A revolver bullet weighing 0.14 N is shot horizontally into the side of block. If the block attains a velocity of 3 m/s what is muzzle velocity.

Wt. of wood block $W_1 = 22.25$ N.

Wt. of bullet $W_2 = 0.14$ N.

velocity of block $V = 3$ m/s.

velocity of muzzle $= u$

According to conservation of momentum

$$\cancel{W_1 v} = W_2 u = (W_1 + W_2) V$$

$$\Rightarrow u = \frac{(22.25 + 0.14)\,3}{0.14}$$

$$= \boxed{479.98 \text{ m/s.}}$$

Conservation of momentum

When the sum of impulses due to external force is zero the momentum of the system remain conserved

When $\sum \int_0^t X\,dt = 0$

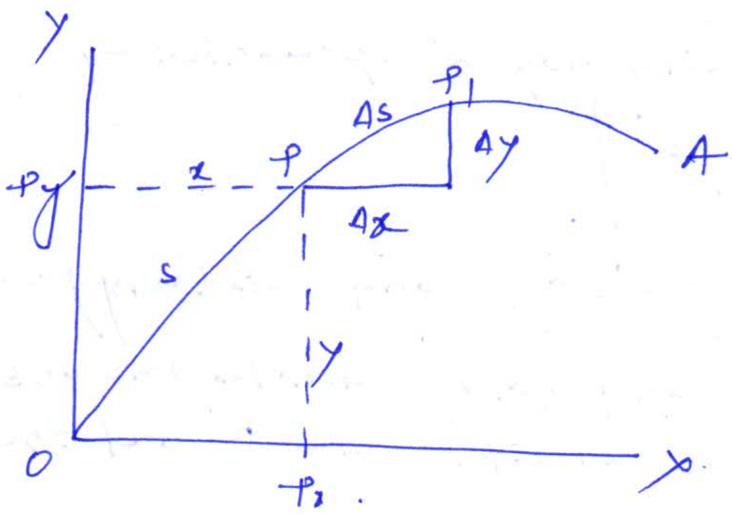
$$\boxed{\sum \left(\frac{W}{g}\right) \dot{x}_2 = \sum \left(\frac{W}{g}\right) \dot{x}_1}$$

$\because$ final momentum = initial momentum.

Curvilinear Translation

When moving particle describes a curved path it is said to have curvilinear motion.

Displacement



consider a particle P in a plane on a curved path. To define the particle we need two coordinate x and y as the particle moves, these coordinates ~~move~~ change with time and the displacement time equations are

$$x = f_1(t) \qquad y = f_2(t) \qquad \text{---} (1)$$

The motion of particle can also be expressed as

$$y = f(x) \qquad s = f_1(t)$$

where $y = f(x)$ represents the equation of path OA

and $s = f_1(t)$ gives displacement s measured along the path as a function of time.

velocity :-

Considering an infinitesimal time difference from t to $t+\Delta t$ during which the particle move from P to P_1 along it's path.

then velocity of particle may be expressed as

$$\boxed{\bar{v}_{av} = \frac{\Delta s}{\Delta t}}$$

$$\boxed{\begin{aligned} (v_{av})_x &= \frac{\Delta x}{\Delta t} \\ (v_{av})_y &= \frac{\Delta y}{\Delta t} \end{aligned}}$$

(average velocity along x and y coordinates)

It can also be expressed as

$$v_x = \frac{dx}{dt} = \dot{x}$$

$$v_y = \frac{dy}{dt} = \dot{y}$$

so the total velocity may be represented by

$$\boxed{v = \sqrt{\dot{x}^2 + \dot{y}^2}}$$

and $\cos(v, x) = \frac{\dot{x}}{v}$ and $\cos(v, y) = \frac{\dot{y}}{v}$

where (v, x) and (v, y) denotes the angles betn the direction of velocity vector $\vec{v}$ and the coordinate axes.

Acceleration :-

The acceleration particles may be described as

$$\boxed{\begin{aligned} a_x &= \frac{d\dot{x}}{dt} = \ddot{x} \\ a_y &= \frac{d\dot{y}}{dt} = \ddot{y} \end{aligned}}$$

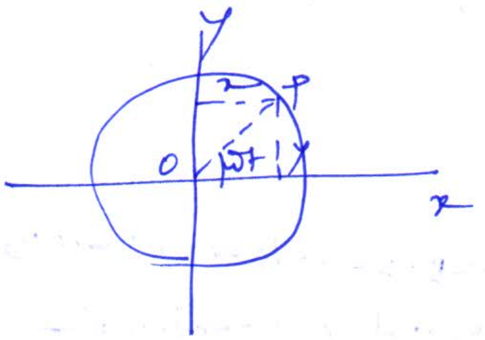
It is also known as instantaneous acceleration

Total acceleration $a = \sqrt{\ddot{x}^2 + \ddot{y}^2}$

Considering particular path for above case.

$x = r\cos\omega t \qquad y = r\sin\omega t.$

$x^2 + y^2 = r^2$



$\dot{x} = -r\omega\sin\omega t \qquad \dot{y} = r\omega\cos\omega t$

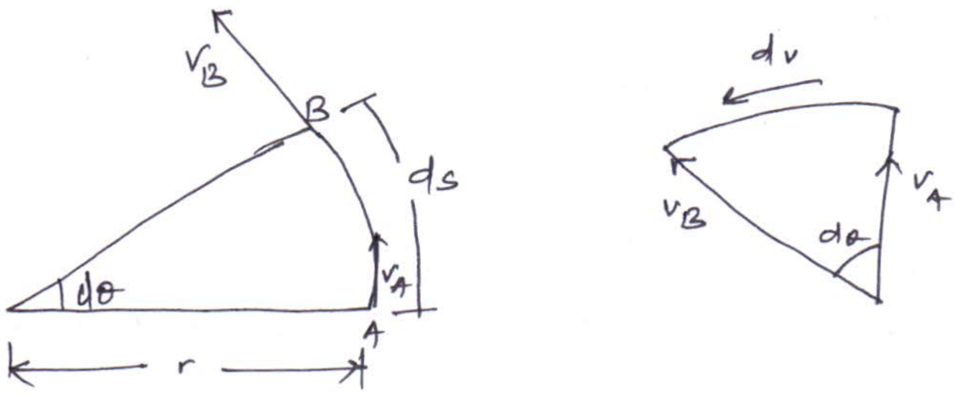
$v = \sqrt{\dot{x}^2 + \dot{y}^2}$

$\ddot{x} = -r\omega^2\cos\omega t \qquad \ddot{y} = -r\omega^2\sin\omega t$

$a = \sqrt{\ddot{x}^2 + \ddot{y}^2}$

D'Alembert's Principle in Curvilinear Motion

Acceleration during circular motion



V_A = tangential velocity at A
= tangential velocity at B
$= V_B = V$

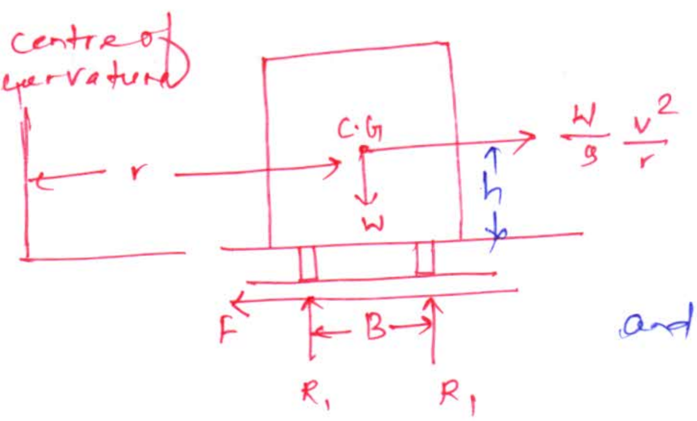
Now $dv = v\,d\theta = v\frac{ds}{r} = \frac{v}{r}ds$

acceleration $= \frac{dv}{dt} = \boxed{\frac{v^2}{r}}$

so when a body moves with uniform velocity v along a curved path of radius r, it has a radial inward acceleration of magnitude $\frac{v^2}{r}$

Applying D'Alembert's principle to get equilibrium condition an inertia force of magnitude $\frac{W}{g}a = \frac{W}{g}\frac{v^2}{r}$ must be applied in outward direction it is known as centrifugal force.

Motion on a level road



Consider a body is moving with uniform velocity on a curvilinear curve of radius r. Let the road is flat.

Let W = wt. of the body

and inertia force is given by

$\frac{W}{g}a = \frac{W}{g}\frac{v^2}{r}$

Condition for skidding :-

Let W = wt. of vehicle

R_1, R_2 = reactions at wheel

F = frictional force.

$\frac{W}{g} \cdot \frac{v^2}{r}$ = inertia force

Skidding takes place when the frictional forces reaches limiting value i.e

$F = \mu W$

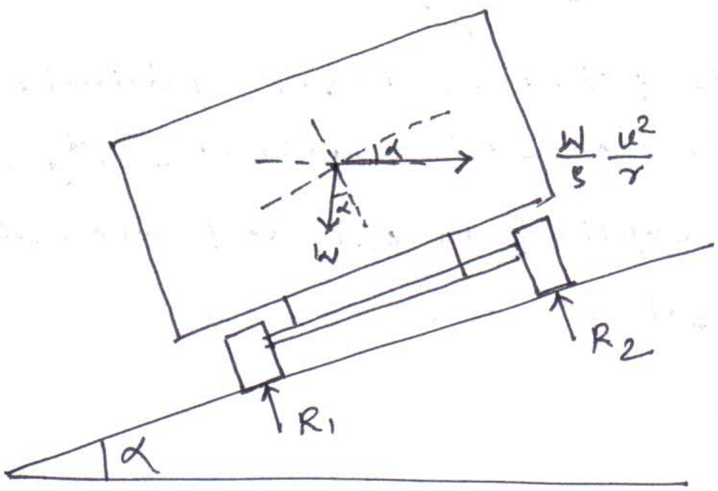
Then maxm permissible speed to avoid skidding

$$\boxed{v = \sqrt{\frac{gr}{2} \frac{B}{h}}}$$

The distance betn inner and outer wheel is equal to the gauge of railway track and expressed as G.

so $$\boxed{v = \sqrt{\frac{gr}{2} \frac{G}{h}}}$$

Designed speed and angle of Braking



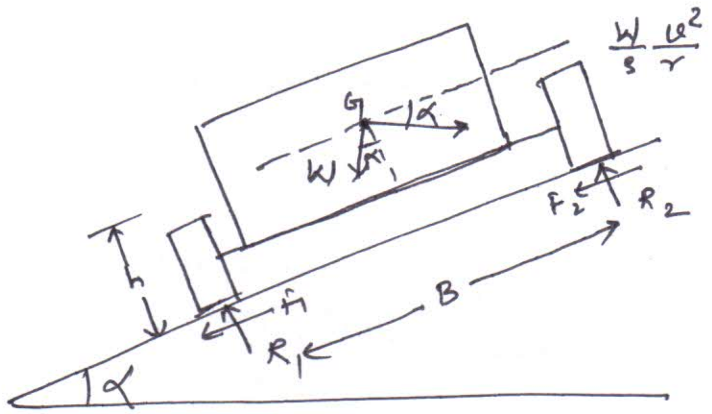
Σ of all the forces in the inclined plane

$\frac{W}{g} \frac{v^2}{r} \cos\alpha - W \sin\alpha = 0$

$\Rightarrow \boxed{\tan\alpha = \frac{v^2}{gr}}$

Relation betn the angle of braking and designed speed is $\boxed{\tan\alpha = \frac{v^2}{gr}}$

Condition for skidding and overturning:-



(a) condition for skidding

$$\boxed{v = \sqrt{\tan(\alpha + \phi) \times g r}}$$

where α = angle of inclination

$\tan\phi = \mu$

g = ~~coeffic~~ gravitational acceleration

r = radius of curve

~~then the~~ vehicle will skid if the velocity is more than this value.

(b) condition for overturning:

Limiting speed from consideration of overturning

$$\boxed{v = \sqrt{g r \frac{G + (2he/G)}{2h - e}}}$$

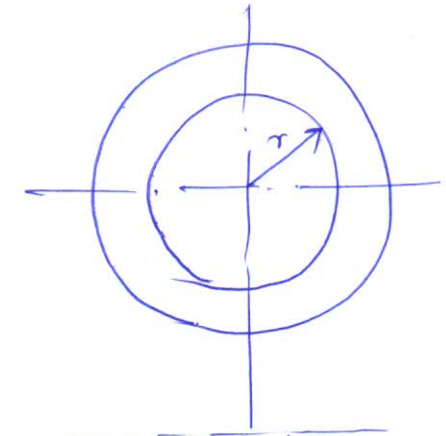
Q.1 A circular ring has a mean radius r = 500 mm and is made of steel for which w = 77.12 kN/m³ and for which ultimate strength in tension is 413.85 MPa. Find the uniform speed of rotation about its geometrical axis perpendicular to the plane of the ring at which it will burst ?

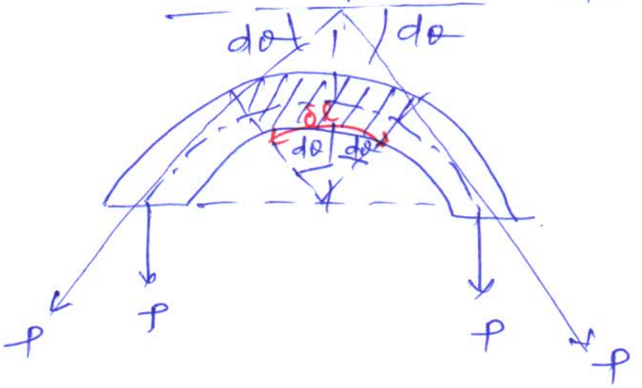
mean radius $r = 500\,mm = 0.5\,m$.

density of the wheel $w = 77.12\,kN/m$

σ_t = ultimate strength $= 413.85 \times 10^6\,Pa$.

Now considering an infinitesimal small elementary ring extruded at an angle of $2d\theta$





centrifugal force acting

$$\delta F_c = \frac{dW}{g} \cdot \frac{v^2}{r}$$

Let P = tension on the ring

A = cross-sectional area of ring.

dW = wt. of the element

$= w \times$ volume

$= w \times A \times \delta l$

$= w \times A \times r\,2d\theta$

Now centrifugal force

$$\frac{w}{g}(A\,\delta l) \times \frac{v^2}{r} = \frac{w}{g} \times A\,\cancel{r}\,2d\theta \times \frac{v^2}{\cancel{r}} = \frac{2wA\,d\theta\,v^2}{g}$$

Balancing forces along the radius $= 2P \sin d\theta$

$$= \frac{2wA\,d\theta\,v^2}{g} \quad \text{---(1)}$$

as $d\theta$ is very small $\sin d\theta \simeq d\theta$

Eq. (1) may be written as

$$\cancel{2}P\,\cancel{d\theta} = \frac{\cancel{2}wA\,\cancel{d\theta} \cdot v^2}{g}$$

$$\Rightarrow \boxed{P = \frac{wAv^2}{g}} \quad \text{---(2)}$$

Tensile stress on the ring $\sigma_t = \frac{P}{A} = \frac{wv^2}{g}$

Now substituting the values

$$413.85 \times 10^6 = \frac{77.12 \times 10^3 \times v^2}{9.81} \Rightarrow v = 229.45\,m/s.$$

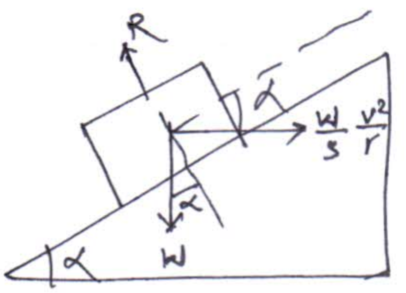
Now $v = \frac{\pi D N}{60} \Rightarrow N = \frac{60 \times 229.45}{\pi \times 1} = \boxed{4382.\ rpm.}$

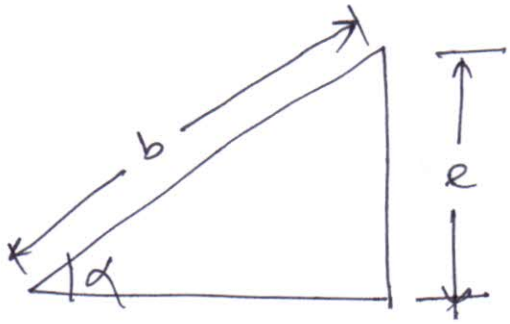
D' Alembert's Principle in Curvilinear Motion

Equation of motion of a particle may be written as

$$\left.\begin{aligned} X - m\ddot{x} &= 0 \\ Y - m\ddot{y} &= 0 \end{aligned}\right\} \quad \text{— (1)}$$

Q.2 Find the proper super elevation 'e' for a 7.2 m highway curve of radius r = 600m in order that a car travelling with a speed of 80 Kmph will have no tendency to skid sidewise.





$b = 7.2\,m \qquad r = 600\,m \qquad v = 80\,Kmph = 22.23\ m/s.$

Resolving along the inclined plane'

$$W \sin\alpha = \frac{W}{g} \cdot \frac{v^2}{r} \cos\alpha$$

$$\Rightarrow \boxed{\tan\alpha = \frac{v^2}{rg}}$$

from the geometry $\sin\alpha = \frac{e}{b}$, since α is very small

let $\sin\alpha \simeq \tan\alpha$

$$\frac{v^2}{rg} = \frac{e}{b} \Rightarrow e = \frac{bv^2}{rg} = \frac{7.2 \times 22.23^2}{600 \times 9.81}$$

$$= 0.604\,m \quad \text{(Ans)}$$

Q.3 A racing car travels around a circular track of 300m radius with a speed of 384 kmph. What angle α should the floor of the track make with horizontal in order to safeguard against skidding.

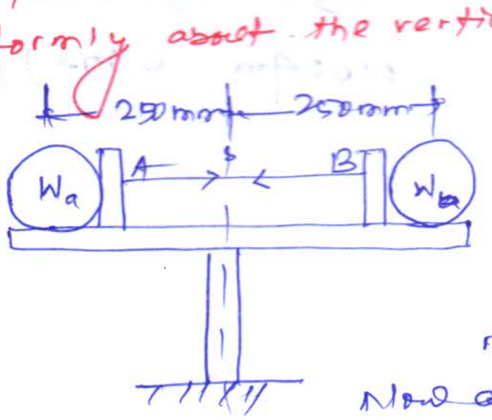
velocity $v = 384$ kmph $\quad r = 300m$

$= 106.67$ m/s.

We have angle of braking $\tan\alpha = \frac{v^2}{rg}$

$\Rightarrow \alpha = \tan^{-1}\left(\frac{106.67^2}{300 \times 9.81}\right) = \boxed{75.5^\circ}$ (Ans)

Q.4 Two balls of wt $W_a = 44.5N$ and $W_b = 66.75N$ are connected by an elastic string and supported on a turntable as shown. When the turntable is at rest, the tension in the string is $S = 222.5N$ and the balls exert this same force on each of the stops A and B. What forces will they exert on the stops when the turntable is rotating uniformly about the vertical axis CD at 60 rpm?



We have;

$W_a = 44.5N \quad W_b = 66.75N$

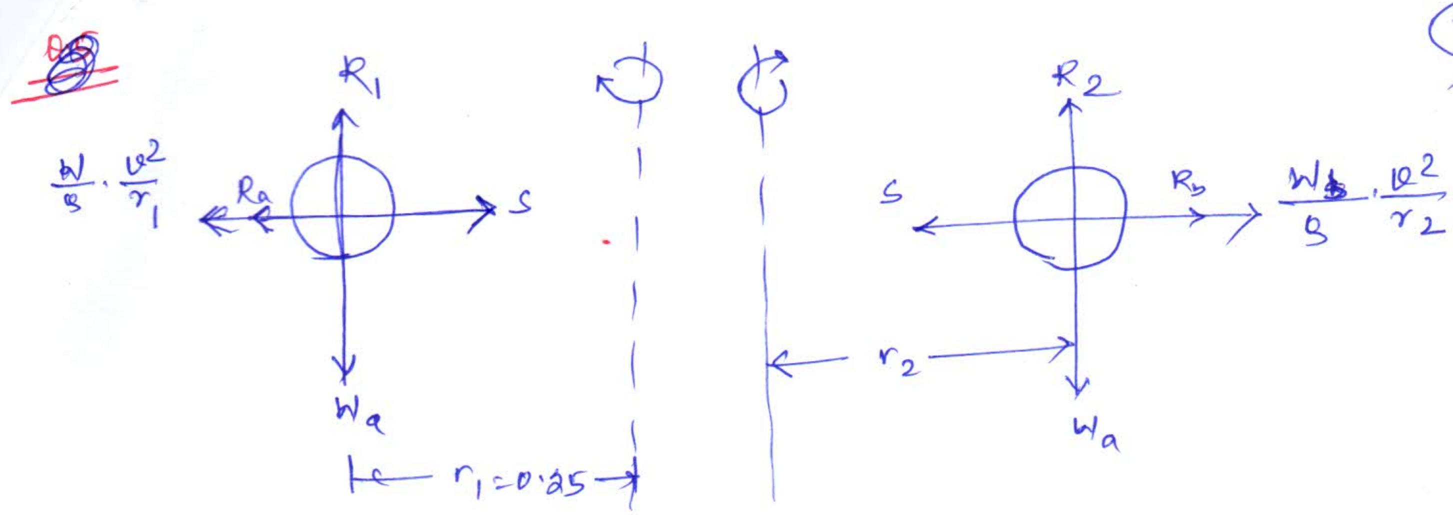
$S = 222.5N$

$N = 60 rpm$,

radius of rotation $r_1, r_2 = 0.25m$

Now angular velocity

$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 60}{60} = 2\pi$ rad/s.



~~consider~~ Considering the left hand side ball

$$R_a + \frac{W_a}{g} \cdot r_1 \omega^2 = S$$

$$\Rightarrow R_a = 222.5 - \frac{44.5}{9.81} \times 0.25 \times (2\pi)^2$$

$$= \boxed{177.72\ N.}$$

Considering the ball on right hand side

$$R_b + \frac{W_b}{g} \times r_2 \times \omega^2 = S$$

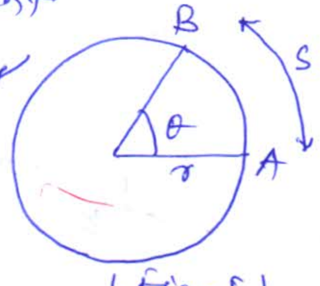
$$\Rightarrow R_b = 222.5 - \frac{66.75}{9.81} \times 0.25 \times (2\pi)^2$$

$$= \boxed{155.34\ N.}$$

Rotation of Rigid Bodies:-

Angular motion:-

The rate of change of angular displacement with time is called angular velocity and denoted by ω.



(fig-1)

$$\boxed{\omega = \frac{d\theta}{dt}} \quad —(1)$$

- The rate of change of angular velocity with time is called angular acceleration and denoted by α

$$\boxed{\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}} \quad —(2)$$

Angular acceleration may also be expressed as:

$$\alpha = \frac{d\omega}{dt} = \frac{d\omega}{d\theta} \cdot \frac{d\theta}{dt}$$

$$\Rightarrow \boxed{\alpha = \omega \cdot \frac{d\omega}{d\theta}} \quad —(3) \quad \left(\because \frac{d\theta}{dt} = \omega\right)$$

Relationship between angular motion and linear motion

from fig-1 $s = r\theta$

tangential velocity (linear) of the particle p.

$$\boxed{v = \frac{ds}{dt} = r \cdot \frac{d\theta}{dt}} \quad —(4)$$

linear acceleration $\boxed{a_t = \frac{dv}{dt} = r\frac{d^2\theta}{dt^2}}$ —(5)

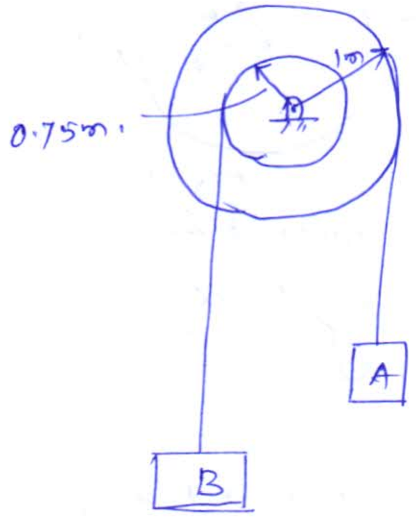
If $\frac{v^2}{r}$ = radial acceleration

Then $\boxed{a_n = \frac{v^2}{r} = r\omega^2}$ —(6) (where a_n = radial acceleration)

uniform angular velocity (ω)

$$\boxed{\omega = \frac{2\pi N}{60} = \text{rad/sec}} \quad —(7)$$

Q. The step pulley starts from rest and accelerates at 2 rad/s^2. How much time is required for block A to move 20 m. Find also the velocity of A and B at that time.



When A moves by 20 m, the angular displacement of pulley θ is given by

$r\theta = s$

$\Rightarrow 1 \times \theta = 20$

$\Rightarrow \boxed{\theta = 20 \text{ rad}}$

$\alpha = 2 \text{ rad/s}^2$ and $\omega_0 = 0$

from kinematic relation

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\Rightarrow 20 = 0 \times t + \frac{1}{\cancel{2}} \times \cancel{2} t^2$

$\Rightarrow \boxed{t = 4.472 \text{ sec.}}$

Velocity of pulley at this time

$\omega = \omega_0 + \alpha t$

$= 0 + 2 \times 4.472$

$= \boxed{8.944 \text{ rad/s}}$

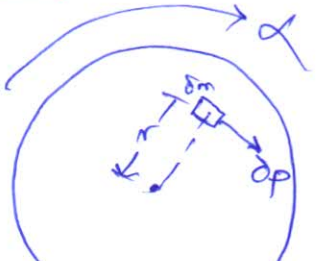
Velocity of block A $v_A = 1 \times 8.944$

$= \boxed{8.944 \text{ m/s}}$

Velocity of block B $v_B = 0.75 \times 8.944$

$= \boxed{6.708 \text{ m/s.}}$

Kinematics of rigid body for rotation:-



Consider a wheel rotating about it's axis in clockwise direction with an acceleration α. Let δm be mass of an element at a distance r from the axis of rotation, δp be the resulting force on this element

$\delta p = \delta m \times a$ (a = tangential acceleration)

but $a = r \times \alpha$ (α = angular acceleration)

$\therefore \boxed{\delta p = \delta m \, r \, \alpha}$

Rotational moment $\delta M_t = \delta p \times r$

$= \delta m \, r^2 \alpha$

$M_t = \Sigma \, \delta M_t = \Sigma \, \delta m \, r^2 \alpha$

$= \alpha \, \Sigma \, \delta m \, r^2$

$= \alpha I$

$\Rightarrow \boxed{M_t = \alpha I}$ (I = mass moment of inertia)

Product of mass moment of inertia and angular velocity of rotating body is called angular momentum

so $\boxed{\text{Angular momentum} = I \omega}$

Kinetic energy of rotating bodies

$\boxed{K.E = \frac{1}{2} I \omega^2}$

Q.2 A flywheel weighing 50 kN and having radius of gyration 1 m losses its speed from 400 rpm to 280 rpm in 2 min. calculate

(a) retarding torque, (b) change in KE during the period, (c) change in angular momentum.

we have $\omega_0 = 400 \, rpm = \frac{2\pi \times 400}{60} = 41.89 \, rad/s$

$\omega = 280 \, rpm = \frac{2\pi \times 280}{60} = 29.32 \, rad/s.$

$t = 2 \, min = 120 \, sec.$

$\omega = \omega_0 + \alpha t$

$\Rightarrow \alpha = \frac{\omega - \omega_0}{t} = \boxed{-.1047 \, rad/s^2}$

Wt of flywheel = 50000 N

mass of " $= \dfrac{50000}{9.81} = 5096.84$ kg.

Radius of gyration $k = 1$ m.

$I = mk^2$

$= 5096.84 \times 1 = 5096.84$

(a) Retarding torque

$I\alpha = 5096.84 \times 0.1047$

$= \boxed{533.64 \text{ Nm.}}$

(b) change in KE

= initial KE − final KE

$= \frac{1}{2} I\omega_0^2 - \frac{1}{2} I\omega^2$

$= \frac{1}{2} \times 5096.84 \left(41.89^2 - 29.32^2\right)$

$= \boxed{\cancel{2280442.9} \text{ Nm}}$ $\boxed{2281115.462 \text{ Nm}}$

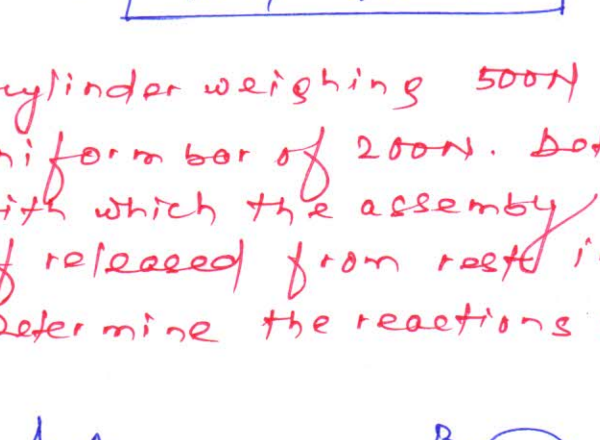
(c) change in angular momentum

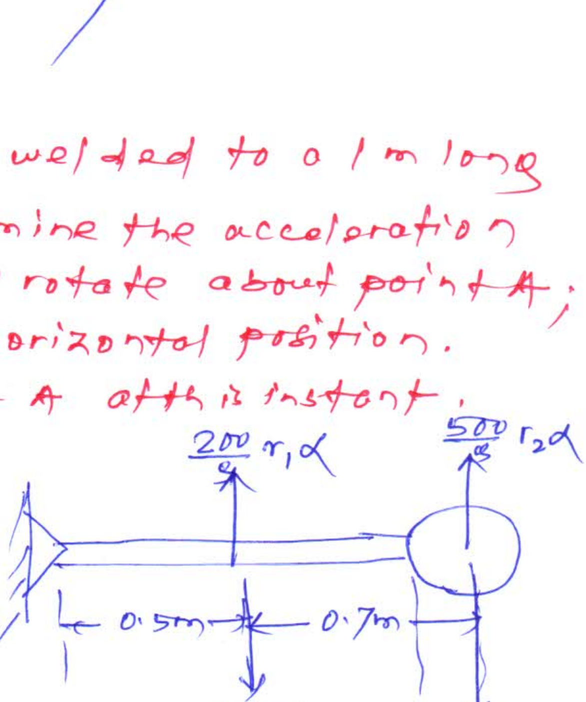
$I\omega_0 - I\omega$

$= 5096.84 (41.89 - 29.32)$

$= \boxed{64067.278 \text{ Nm.}}$

Q.3 A cylinder weighing 500N is welded to a 1 m long uniform bar of 200N. Determine the acceleration with which the assemby will rotate about point A, if released from rest in horizontal position. Determine the reactions at A at this instant.





③

Let α = angular acceleration of the assembly

I = mass moment of inertia of the assembly

$I = I_G + Md^2$ (transfer formula)

mass MI of bar about A $= \frac{1}{2} \times \frac{200}{9.81} \times 1^2 + \frac{200}{9.81} \times (0.5)^2$

$= 6.7968$

mass MI of cylinder about A

$= \frac{1}{2} \frac{500}{9.81} \times 0.2^2 + \frac{500}{9.81} \times 1.2^2$

$= 74.4$

MI of the system $= 6.7968 + 74.4 = 81.2097$

Rotational moment about A

$M_t = 200 \times 0.5 + 500 \times 1.2 = 700 \text{ Nm}$.

$M_t = I\alpha$

or $\alpha = \frac{700}{81.2097} = \boxed{8.6197}$ rad/sec

Instantaneous acceleration of rod AB is vertical and $= r_1 \alpha = 0.5 \times 8.6197$

$= 4.31$ m/s.

Similarly instantaneous acceleration of cylinder

$= r_2 \alpha = 1.2 \times 8.6197$

$= 10.34$ m/s.

Applying D'Alembert's dynamic equilibrium

$R_A = 200 + 500 - \frac{200}{9.81} \times 4.31 - \frac{500}{9.81} \times 10.34$

$\Rightarrow \boxed{R_A = 84.93 \text{ N}.}$ (Ans)

Engineering Mechanics

Assignment problem: Module 2 and 3

Rectilinear Translation: assignment 1

FIG. B

5. A particle starts from rest and moves along a straight line with constant acceleration a. If it acquires a velocity $v = 10$ fps after having traveled a distance $s = 25$ ft, find the magnitude of the acceleration. *Ans.* $a = 2$ ft/sec.^{2}

6. A bullet leaves the muzzle of a gun with velocity $v = 2{,}500$ fps. Assuming constant acceleration from breech to muzzle, find the time t occupied by the bullet in traveling through the gun barrel, which is 30 in. long. *Ans.* $t = 0.002$ sec.

7. A ship while being launched slips down the skids with uniform acceleration. If 10 sec is required to traverse the first 16 ft, what time will be required to slide the total distance of 400 ft? With what velocity v will the ship strike the water? *Ans.* $t = 50$ sec; $v = 16$ fps.

8. Water drips from a faucet at the uniform rate of n drops per second. Find the distance x between any two adjacent drops as a function of the time t that the trailing drop has been in motion. Neglect air resistance and assume constant acceleration $g = 32.2$ ft/sec^2. *Ans.* $x = gt/n + g/2n^2$.

9. A stone is dropped into a well and falls vertically with constant acceleration $g = 32.2$ ft/sec.^{2}. The sound of impact of the stone on the bottom of the well is heard 6.5 sec after it is dropped. If the velocity of sound is 1,120 fps, how deep is the well? *Ans.* 577 ft.

10. The rectilinear motion of a particle is defined by the displacement-time equation $x = x_0(2e^{-kt} - e^{-2kt})$, in which x_0 is the initial displacement, k is a constant, and e is the natural logarithmic base. Sketch the displacement-time and velocity-time curves for this motion and find the maximum velocity of the particle. *Ans.* $\dot{x}_{max} = -kx_0/2$, when $t = \ln 2/k$.

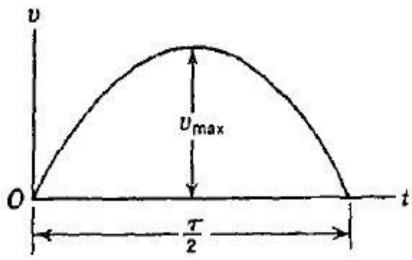


FIG. C

11. If the velocity-time diagram for the rectilinear motion of a particle is the half wave of a sine curve as shown in Fig. C, find the total distance x that the particle travels during the half-period time interval $\tau/2$. *Ans.* $x = \tau v_{max}/\pi$.

12. If the velocity-time curve shown in Fig. C is a parabola with vertical axis, find the distance traveled by the particle during the time interval $\tau/2$. *Ans.* $x = \tau v_{max}/3$.

13. An automobile starting from rest increases its speed from 0 to v with a constant acceleration a_1, runs at this speed for a time, and finally comes to rest with constant deceleration a_2. If the total distance traveled is s, find the total time t required. *Ans.* $t = s/v + (v/2)\,(1/a_1 + 1/a_2)$.

14. The greatest possible acceleration or deceleration that a train may have is a, and its maximum speed is v. Find the minimum time in which the train can get from one station to the next if the total distance is s. *Ans.* $t_{min} = s/v + v/a$.

Assignment 2

1. An elevator of gross weight $W = 1{,}000$ lb starts to move upward with constant acceleration and acquires a velocity $v = 6$ fps, after traveling a distance $s = 6$ ft. Find the tensile force S in the cable during this accelerated motion. Neglect friction. *Ans.* $S = 1{,}093$ lb.

2. The elevator of Prob. 1, when stopping, moves with constant deceleration and from the constant velocity $v = 6$ fps comes to rest in 2 sec. Determine the pressure P transmitted during stopping to the floor of the elevator by the feet of a man weighing 170 lb. *Ans.* $P = 154$ lb.

3. A train weighing 200 tons without the locomotive starts to move with constant acceleration along a straight horizontal track and in the first 60 sec acquires a velocity of 35 mph. Determine the tension S in the draw-bar between the locomotive and train if the total resistance to motion due to friction and air resistance is constant and equal to 0.005 times the weight of the train. *Ans.* $S = 12{,}620$ lb.

4. The driver of an automobile, traveling along a straight level highway, suddenly applies the brakes so that the car slides for 2 sec, covering a distance of 32.2 ft before coming to a stop. Assuming that during this time the car moved with constant deceleration, find the coefficient of friction between the tires and the pavement. *Ans.* $\mu = 0.5$.

5. A mine cage of weight $W = 1$ ton starts from rest and moves downward with constant acceleration, traveling a distance $s = 100$ ft in 10 sec. Find the tensile force in the cable during this time. *Ans.* $S = 1{,}876$ lb.

6. A particle of weight W is dropped vertically into a medium that offers a resistance proportional to the square of the velocity of the particle. The buoyancy of the medium is negligible, and the resisting force is f when the velocity is 1 fps. What uniform velocity will the particle finally attain? *Ans.* $v = \sqrt{W/f}$.

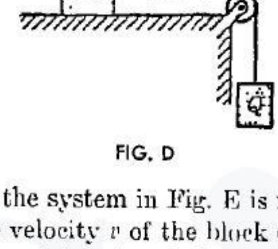


FIG. D

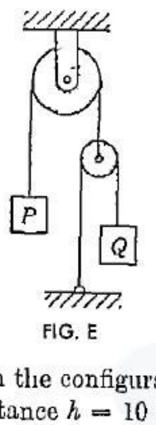


FIG. E

7. If the system in Fig. E is released from rest in the configuration shown, find the velocity v of the block Q after it falls a distance $h = 10$ ft. Neglect friction and inertia of the pulleys and assume that $P = Q = 10$ lb. *Ans.* $v = 16.05$ fps.

8. A length l of smooth straight pipe held with its axis inclined to the horizontal by an angle of 30° contains a flexible chain also of length l. Neglecting friction and assuming that, after release, the chain falls vertically as it emerges from the open end of the pipe, find the velocity v with which it leaves the pipe. *Ans.* $v = \sqrt{3gl/2}$.

Assignment 3

1. A body starts to move vertically upward under the influence of gravity with an initial velocity $v_0 = 20$ fps. Find (*a*) the maximum height to which it will rise and (*b*) the time required for it to return to its initial position. Take the starting point as the origin so that $x_0 = 0$ and neglect air resistance. *Ans.* (*a*) $x_{max} = 6.2$ ft; (*b*) $t = 1.24$ sec.

2. A train is moving down a slope of 0.008 with a velocity of 30 mph. At a certain instant the engineer applies the brakes and produces a total resistance to motion equal to one-tenth of the weight of the train. What distance x will the train travel before stopping? *Ans.* $x = 327$ ft.

3. An elevator weighing 1,000 lb is moving upward with a uniform velocity of 12 fps. In what distance x will it stop after the power is shut off if the friction force opposing motion is 20 lb? *Ans.* $x = 2.19$ ft.

4. To determine experimentally the coefficient of friction between two materials, a small block of weight $W = 10$ lb is projected with initial velocity $v_0 = 30$ fps along a horizontal plane covered with the same material. If the block travels a total distance $x = 45$ ft before coming to rest, what is the coefficient of friction? *Ans.* $\mu = 0.31$.

5. Referring to Fig. A, find the acceleration a of the falling weight P if the coefficient of friction between the block Q and the horizontal plane on which it slides is μ. Neglect inertia of the pulley and friction on its axle. The following numerical data are given: $P = 10$ lb, $Q = 12$ lb, $\mu = \frac{1}{3}$. *Ans.* $a = 3g/11$.

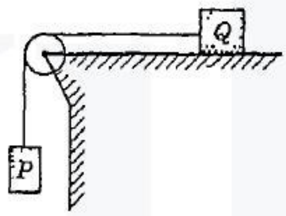


FIG. A

6. A train moves with a uniform speed of 36 mph along a straight level track. At a certain instant the engineer moves the throttle so as to increase the traction by 20 per cent. What distance x will the train cover before acquiring a speed of 45 mph if the resistance to motion is constant and equal to $\frac{1}{200}$ of the weight of the train? *Ans.* 4.61 miles.

7. A police investigation of tire marks shows that a car traveling along a straight level street had skidded for a total distance of 145 ft after the brakes were applied. The coefficient of friction between tires and pavement is estimated to be $\mu = 0.6$. What was the probable speed of the car when the brakes were applied? *Ans.* 51 mph.

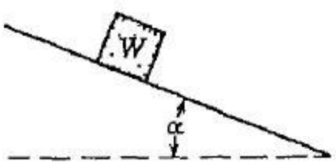


FIG. B

8. A small block of weight W rests on an adjustable inclined plane as shown in Fig. B. Friction is such that sliding of the block impends when $\alpha = 30°$. What acceleration will the block have when $\alpha = 45°$? Neglect any difference between static and kinetic friction. *Ans.* $a = 0.3g$.

Assignment 4

1. Two weights P and Q are connected by the arrangement shown in Fig. A. Neglecting friction and the inertia of the pulleys and cord, find the acceleration a of the weight Q. Assume that $P = 40$ lb and $Q = 30$ lb. *Ans.* $a = 8.05$ ft/sec^2.

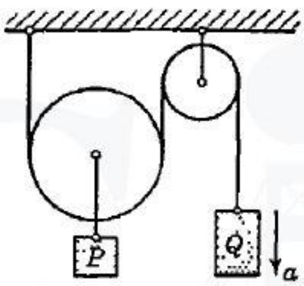


FIG. A

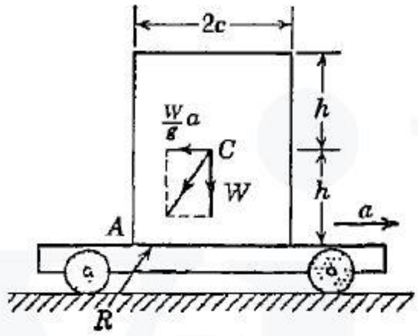


FIG. B

2. A block of weight W, height $2h$, and width $2c$ rests on a flat car which moves horizontally with constant acceleration a (Fig. B). Determine (a) the value of the acceleration a at which slipping of the block on the car will impend if the coefficient of friction is μ and (b) the value of the acceleration at which tipping of the block about the edge A will impend, assuming sufficient friction to prevent slipping. *Ans.* $a_1 = \mu g$; $a_2 = cg/h$.

friction to prevent slipping. *Ans.* $a_1 = \mu g$; $a_2 = cg/h$.

3. Assuming the car in Fig. B to have a velocity of 20 fps, find the shortest distance s in which it can be stopped with constant deceleration without disturbing the block. The following data are given: $c = 2$ ft, $h = 3$ ft, $\mu = 0.5$. *Ans.* $s = 12.4$ ft.

4. Neglecting friction and the inertia of the two-step pulley shown in Fig. C, find the acceleration a of the falling weight P. Assume $P = 8$ lb, $Q = 12$ lb, and $r_2 = 2r_1$. *Ans.* $a = 2g/11$.

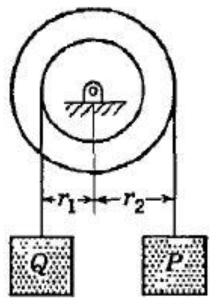


FIG. C

5. A mathematical pendulum hanging from the ceiling of a railway car inclines to the vertical by an angle α during starting of the train. What is the corresponding acceleration of the train? *Ans.* $a = g \tan \alpha$.

6. A spring-suspended mass hangs from the ceiling of an elevator cage. How will its natural period of free vertical vibration be affected by acceleration of the cage?

9. Two blocks of weights P and Q are connected by a flexible but inextensible cord and supported as shown in Fig. F. If the coefficient of friction between the block P and the horizontal surface is μ and all other friction is negligible, find (a) the acceleration of the system and (b) the tensile force S in the cord. The following numerical data are given: $P = 12$ lb, $Q = 6$ lb, $\mu = \frac{1}{3}$. *Ans.* $a = 3.58$ ft/sec²; $S = 5.33$ lb.

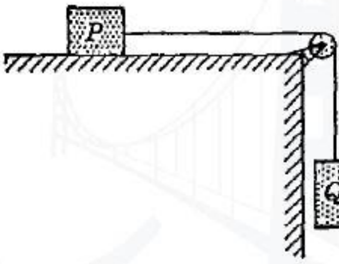


FIG. F

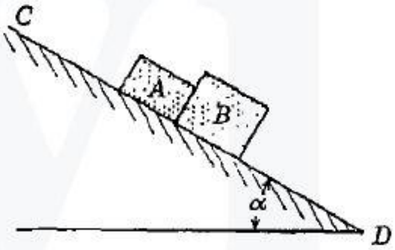


FIG. G

10. Two blocks A and B under the action of gravity slide down the inclined plane CD that makes with the horizontal the angle $\alpha = 30°$ (Fig. G). If the weights of the blocks are $W_a = 10$ lb and $W_b = 20$ lb and the coefficients of friction between them and the inclined plane are $\mu_a = 0.15$ and $\mu_b = 0.30$, find the pressure P existing between the blocks during the motion. *Ans.* $P = 0.87$ lb.

12. Find the tension S in the string during motion of the system shown in Fig. I if $W_1 = 200$ lb, $W_2 = 100$ lb. The system is in a vertical plane, and the coefficient of friction between the inclined plane and the block W_1 is $\mu = 0.2$. Assume the pulleys to be without mass. *Ans.* $S = 57$ lb.

13. A rectangular block of weight $Q = 200$ lb rests on a flatcar of weight $P = 100$ lb which may roll along the horizontal plane AB without friction (Fig. J). The car and block together are to be accelerated by the weight W arranged as shown in the figure. Assuming that there is sufficient friction between the block and the car to prevent sliding, find the maximum value of the weight W by which the car can be accelerated. What will this acceleration be? *Ans.* $W_{max} = 100$ lb; $a = 8.05$ ft/sec².

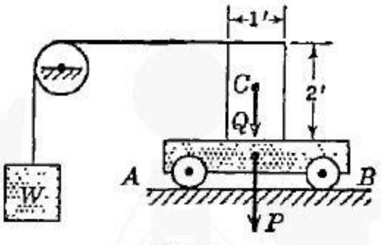


FIG. J

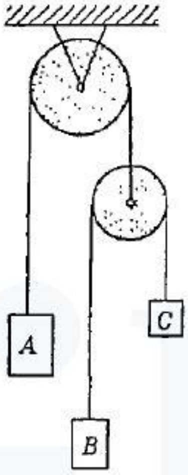


FIG. K

***14.** A system of weights and pulleys is arranged in a vertical plane as shown in Fig. K. Neglecting friction and the inertia of the pulleys, find the acceleration of each weight if their magnitudes are in the ratio $W_a : W_b : W_c = 3:2:1$. *Ans.* $\ddot{x}_a = g/17$; $\ddot{x}_b = 5g/17$; $\ddot{x}_c = -7g/17$.

Assignment 5

1. For the ideal system shown in Fig. A, the weight W_1 hangs at the height $x_1 = h$ above the floor in the equilibrium configuration. Calculate the potential energy of the system with reference to this configuration if W_1 is pulled down to the floor ($x_1 = 0$). Neglect the mass of the spring and cord and rotational inertia of the pulleys. The following numerical data are given: $W_1 = W_2 = 10$ lb, $k = 2$ lb/in., $h = 4$ in. *Ans.* $V = 4$ in.-lb.

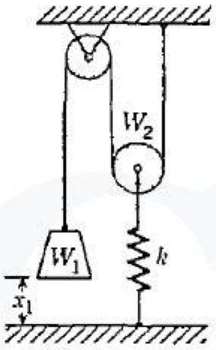


FIG. A

2. If the system in Fig. A is released from rest in the configuration defined by $x_1 = 0$, what maximum height above the floor will the block W_1 attain after release? *Ans.* $(x_1)_{max} = 8$ in.

***3.** Calculate the period of free vibration of the system in Fig. A if the weight W_1 performs small oscillations $x_1' = a \cos pt$ about its position of equilibrium. Use the same numerical data as in Probs. 1 and 2. *Ans.* $\tau = 1.60$ sec.

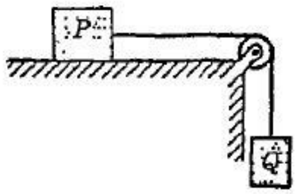


FIG. D

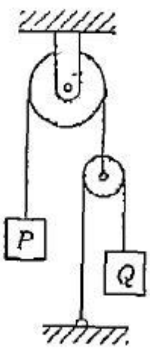


FIG. E

7. If the system in Fig. E is released from rest in the configuration shown, find the velocity v of the block Q after it falls a distance $h = 10$ ft. Neglect friction and inertia of the pulleys and assume that $P = Q = 10$ lb. *Ans.* $v = 16.05$ fps.

8. A length l of smooth straight pipe held with its axis inclined to the horizontal by an angle of 30° contains a flexible chain also of length l. Neglecting friction and assuming that, after release, the chain falls vertically as it emerges from the open end of the pipe, find the velocity v with which it leaves the pipe. *Ans.* $v = \sqrt{3gl/2}$.

Activ
Go to

Assignment 6

3. Several identical blocks, each of mass m, rest in a row on a perfectly smooth horizontal plane so that their centers of gravity lie on a straight line. Another block, also of mass m, is moving along this line with velocity v and squarely strikes one end of the row. Discuss what will happen if the blocks are all perfectly elastic.

4. A wood block weighing 9.95 lb rests on a rough horizontal plane, the coefficient of friction between the two being $\mu = 0.4$. If a bullet weighing 0.05 lb is fired horizontally into the block with muzzle velocity $v = 2{,}000$ fps, how far will the block be displaced from its initial position? Assume that the bullet remains inside the block. *Ans.* 3.88 ft.

5. A golf ball dropped from rest onto a cement sidewalk rebounds eight-tenths of the height through which it fell. Neglecting air resistance, determine the coefficient of restitution. *Ans.* $e = 0.9$.

6. For the two balls in Fig. A find the velocities v_1' and v_2' after an elastic impact if, before impact, $v_1 = v$, $v_2 = 0$, and $W_2 = 2W_1$. *Ans.* $v_1' = -v/3$; $v_2' = +2v/3$.

7. For the two balls in Fig. A, find the velocities v_1' and v_2' after impact if $v_1 = v$, $v_2 = 0$, $W_2 = 3W_1$, and the coefficient of restitution $e = \frac{1}{2}$. *Ans.* $v_1' = -v/8$; $v_2' = +3v/8$.

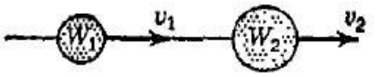


FIG A

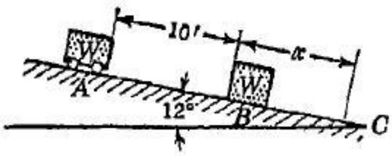


FIG. B

Assignment 7

PROBLEM SET 7.3

1. A mortar fires a projectile across a level field so that the range r is a maximum and equal to 1,000 yd. Find the time of flight. *Ans.* $t = 13.7$ sec.

2. In Fig. A, a projectile is fired horizontally from point A with initial velocity $v_0 = 360$ fps. Find the range r to the target B. *Ans.* $r = 2{,}840$ ft.

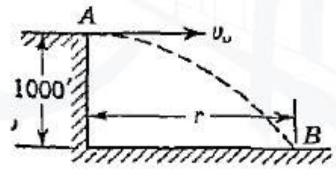


FIG. A

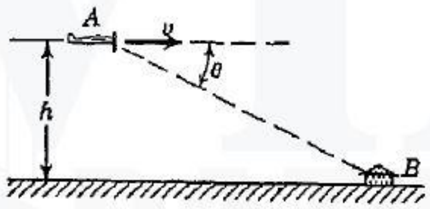


FIG. B

3. In Fig. B the pilot of an airplane flying horizontally with constant speed $v = 300$ mph at an elevation $h = 2{,}000$ ft above a level plain wishes to bomb a target B on the ground. At what angle θ below the horizontal should he see the target at the instant of releasing the bomb in order to score a hit? Neglect air resistance. *Ans.* $\theta = 22°12'$.

4. A mortar having muzzle velocity $v_0 = 707$ fps fires for maximum range across a level plain. Neglecting air resistance, calculate the time of flight of the shell. *Ans.* $t = 31.1$ sec.

Assignment 8

1. A circular ring has a mean radius $r = 20$ in. and is made of steel for which $w = 0.284$ lb/in.^{3} and for which the ultimate strength in tension is 60,000 psi. Find the uniform speed of rotation about its geometric axis perpendicular to the plane of the ring at which it will burst. *Ans.* 4,300 rpm.

2. Find the proper superelevation e for a 24-ft highway curve of radius $r = 2{,}000$ ft in order that a car traveling with a speed of 50 mph will have no tendency to skid sidewise. *Ans.* $e = 2.00$ ft.

3. Racing cars travel around a circular track of 1,000-ft radius with a speed of 240 mph. What angle α should the floor of the track make with the horizontal in order to safeguard against skidding? *Ans.* $\alpha = 75°27'$.

4. A particle A of weight W is suspended in a vertical plane by two strings as shown in Fig. A. Determine the tension S in the inclined string OA (*a*) an instant before the horizontal string AB is cut and (*b*) an instant after this string is cut. Assume the string OA inextensible. *Ans.* $S_1 = W \sec \alpha$; $S_2 = W \cos \alpha$.

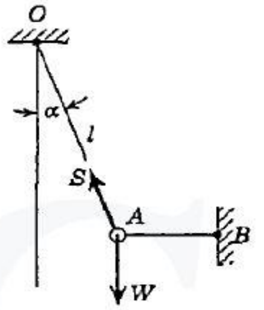


FIG. A

5. Two balls of weights $W_a = 10$ lb and $W_b = 15$ lb

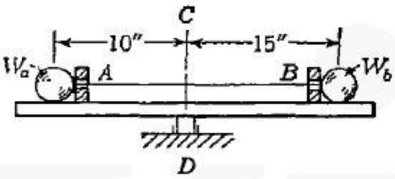


FIG. B

Assignment 9

1. A small ball of weight W, attached to the end of a string, is supported by a smooth horizontal plane and travels with uniform speed v_0 in a circular path of radius r (Fig. A). By pulling the string at the lower end, the radius of the path is reduced to $r/2$. Determine the new velocity of the ball and the tension S in the string. *Ans.* $v_1 = 2v_0$; $S = 8Wv_0^2/gr$.

2. A conical pendulum of length $l = 20$ in. rotates with constant speed v in a horizontal circular path of radius $r = 10$ in. as shown in

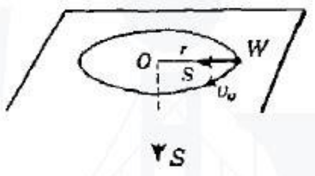


FIG. A

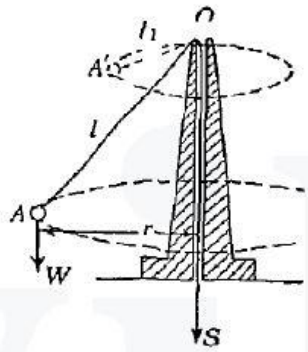


FIG. B

the speed of the ball? *Ans.* 14.9 in.

3. If the ball shown in Fig. A is given an initial velocity v_0 in the circular path of radius r and the coefficient of friction between it and the horizontal plane is μ, determine, by using Eq. (60), the time interval t required for the ball to come to rest. *Ans.* $t = v_0/\mu g$.

4. A heavy particle suspended vertically by a long string so that it can swing freely under the influence of gravity is allowed to describe a small horizontal elliptical path centered about the position of equilibrium O. If the major and minor semiaxes of the ellipse are a and b, respectively, find the ratio of the velocity v_x with which the particle crosses the y axis to the velocity v_y with which it crosses the x axis. *Ans.* $v_x/v_y = a/b$.

5. The motion of a particle of mass m in the xy plane is defined by the equations

$$x = a \cos pt \qquad y = b \sin pt$$

where a, b, and p, are constants. Calculate the moment of momentum of the particle with respect to the origin O. *Ans.* $H_0 = abpm$.

6. The motion of a particle of mass ...

Assignment 10

1. A simple pendulum of weight W and length l as shown in Fig. A is released from rest at A ($\alpha = 60°$), swings downward under the influence of

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gravity, and strikes a spring of stiffness k at B. Neglecting the mass of the spring, determine the compression that it will suffer. *Ans.* $\delta = \sqrt{Wl/k}$.

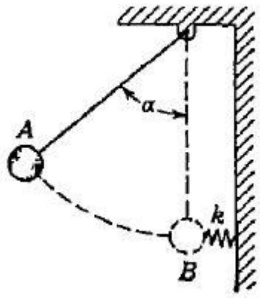

FIG. A

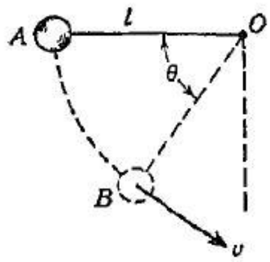

FIG. B

2. The simple pendulum in Fig. B is released from rest at A with the string horizontal and swings downward under the influence of gravity. Express the velocity v of the bob as a function of the angle θ. *Ans.* $v = \sqrt{2gl \sin \theta}$.

3. If the simple pendulum of weight W in Fig. B is released from rest in the position A, find the tension T in the string OB as a function of the angle θ. *Ans.* $T = 3W \sin \theta$.

4. If the pendulum in Fig. C is released from rest in its position of unstable equilibrium as shown, find the value of the angle φ defining the position in its downward fall at which the axial force in the rod changes from compression to tension. *Ans.* $\varphi = \arccos \frac{2}{3} = 48°11'$.

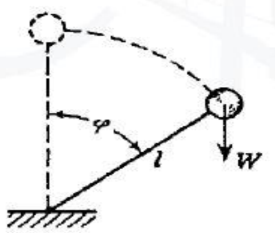

FIG. C

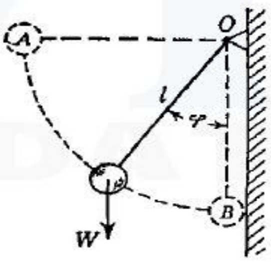

FIG. D

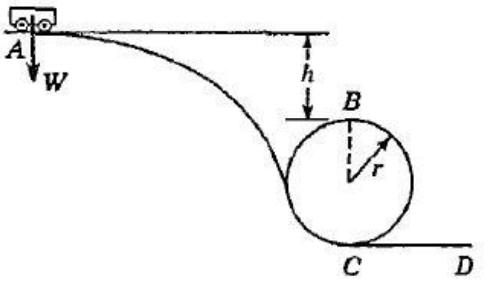

FIG. E

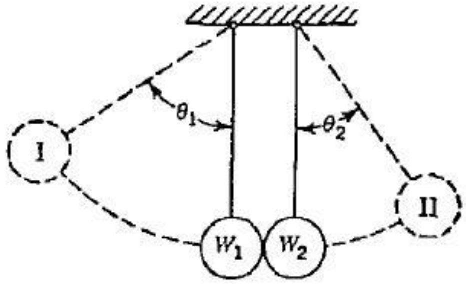

FIG. F

7. Referring to Fig. F, assume that the ball I of weight W is released from rest in the position $\theta_1 = 60°$ and swings downward to where it strikes the ball II of weight $3W$. Assuming an elastic impact, calculate the angle θ_2 through which the larger pendulum will swing after the impact. *Ans.* $\theta_2 = 28°57'$.

8. In the system shown in Fig. F, the ball I is allowed to swing downward from rest in the position defined by the angle $\theta_1 = 45°$ and to strike the ball II, which, after impact, swings upward to the position defined by the angle $\theta_2 = 30°$. If the weights of the balls are equal, find the coefficient of restitution e for the materials. *Ans.* $e = 0.35$.

9. In Fig. G a small ball of weight $W = 5$ lb starts from rest at O and rolls down the smooth track OCD under the influence of gravity. Find the reaction R exerted on the ball at C if the curve OCD is defined by the equation $y = h \sin (\pi x/l)$ and $h = l/3 = 3$ ft. *Ans.* $R = 15.95$ lb.