Two Dimensional Geometric Transformation

Transformation: Transformation is the process by which we can change the shape, size position and direction of any object w.r.t a co-ordinate system.

There are 2 types of transformation.

i) Geometric transformation

ii) Co-ordinate transformation.

Geometric transformation:

When an object is moving with respect to stationary(static) co-ordinate system then it is known as geometric transformation and it is applied to each and every point of the object.

Co-ordinate transformation:

When the co-ordinate system is moving w.r.t to the object and the object is static then it is called co-ordinate transformation

2-D transformation:

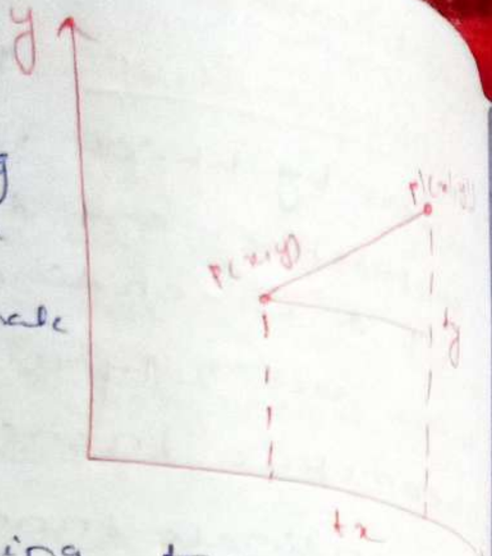
There are 3-basic two dimensional transformation..

a) Translation

b) Rotation

c) Scaling.

a) Translation:



Translation is applied to an object by responding it along a straight line path from one co-ordinate to another.

- So we translate on move 2-D point by adding translating distance tx and ty to the original co-ordinate position

$$x' = x + tx$$

$$y' = y + ty$$

where tx = translation distance along x-dirn

ty = translation distance along y-dirn

and translation distance pair(tx, ty) is known as translation vector or shift vector.

Matrix form:

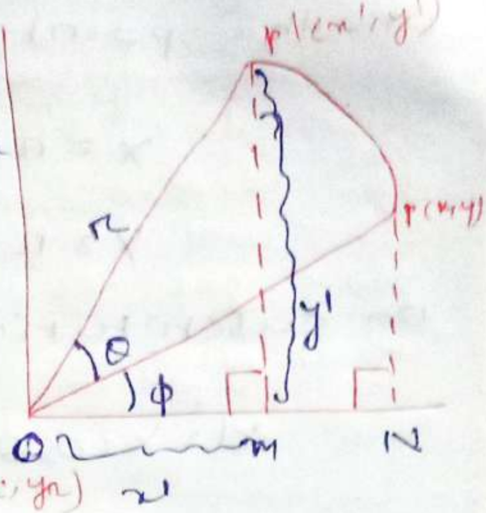
$$P = \begin{bmatrix} x \\ y \end{bmatrix} \quad T = \begin{bmatrix} tx \\ ty \end{bmatrix} \quad P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$\boxed{P' = P + T}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} tx \\ ty \end{bmatrix}$$

Rotation:

Rotation is the repositioning of an object along a circular path in a 2-D plane on x-y plane with certain angle



- The rotation angle is θ
- The object is to rotated about the rotation axis which is ⊥r to the x-y plane and passes through the rotation point i.e (x_r, y_r)
- Let us consider the rotation point P about the origin and the eqn will be

$$x' = OM$$

$$y' = p'M$$

consider the triangle $\Delta p'OM$

$$\cos(\theta+\phi) = \frac{OM}{Op'} = \frac{x'}{r}$$

$$\Rightarrow \cos\theta.\cos\phi - \sin\theta.\sin\phi = \frac{x'}{r}$$

$$\Rightarrow r\cos\theta.\cos\phi - r\sin\theta.\sin\phi = x' \quad \text{--- (1)}$$

Similarly $\sin(\theta+\phi) = \frac{p'M}{Op'} = \frac{y'}{r}$

$$\Rightarrow \sin\theta.\cos\phi + \cos\theta.\sin\phi = y'$$

$$\Rightarrow y' = r\sin\theta.\cos\phi + r\cos\theta.\sin\phi \quad \text{--- (2)}$$

But we know the original co-ordinate of the point in polar co-ordinate system

$$\left.\begin{aligned} x &= r\cos\phi \\ y &= r\sin\phi \end{aligned}\right\} \text{polar co-ordinate system}$$

on substituting eqn(1) and eqn(2)

$$x' = x\cos\theta - y\sin\theta \quad \text{———} (3)$$

$$y' = x\sin\theta + y\cos\theta \quad \text{———} (4)$$

Matrix form:

$$P' = \begin{bmatrix} x' \\ y' \end{bmatrix} \qquad P = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\boxed{P' = R \cdot P}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Q: Rotate the point P(2,-4), 30° about the origin by and find the corresponding (x,y) co-ordinate.

Ans

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad P = [x \ \ y]$$

$$P = [2, -4]$$

$$P = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$$

$$\boxed{P' = R \cdot P}$$

$$= \begin{bmatrix} \cos 30^\circ & -\sin 30^\circ \\ \sin 30^\circ & \cos 30^\circ \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3+\sqrt{2} \\ 1-2\sqrt{3} \end{bmatrix}$$

If $P = [x \ \ y]$

$$R^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \Rightarrow P' = R \cdot P$$

$$= [\sqrt{3}+2, 1-2\sqrt{3}]$$

(Ans)

Scaling: scaling transformation alters the size of an object.

- It is the process of expanding or compressing the dimension of an object.

- two get the transform co-ordinates multiply two scaling factor S_x and S_y with the original coordinate.

$$\boxed{\begin{aligned} x' &= x \cdot S_x \\ y' &= y \cdot S_y \end{aligned}}$$

- scaling factor S_x which scale the object in x-dir^n

- scaling factor S_y which scale the object in y-dir^n

- matrix form is

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- If $S_x = S_y$ scaling transformation is said to be homogeneous

- If scaling factor (S_x and S_y) greater than 1 implies expension otherwise compression

- when scaling transformation is performed the new object is to be located at a diff position w.r.t origin

- In scaling the point which remain fix is called origin
- unequal value S_x and S_y result in a differential scaling.
- The object transformed are both scaled and repositioned.
- Scaling factor with value less than 1 moves the object closer to the origin.
- We can also fixed the location of an object by a fix point method.
- For a vector with co-ordinate (x, y) and the scaled co-ordinate is (x', y')
- Fixed point of the object is (x_f, y_f)

$$x' = x_f + (x - x_f) S_x$$

$$\Rightarrow x' = x_f + x S_x - x_f S_x$$

$$\Rightarrow \boxed{x' = x S_x + x_f (1 - S_x)}$$

- Similarly

$$y' = y_f + (y - y_f) S_y$$

$$\Rightarrow y' = y_f + y S_y - y_f S_y$$

$$\Rightarrow \boxed{y' = y S_y + y_f (1 - S_y)}$$

Homogeneous co-ordinate system

Homogeneous co-ordinate is used to efficiently manipulate picture coordinates.

- The 2x2 matrix representation is converted to 3x3 matrix by this system and express any 2D transformation as matrix multiplication.
- We represent each cartesian position (x,y) with homogeneous co-ordinate

$$\boxed{x = \frac{x_h}{h} \qquad y = \frac{y_h}{h}}$$ (h=1 for simplicity)

- For 2D transformation h can be any non zero value.

For translation:

$x' = x + t_x$

$y' = y + t_y$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} x + 0 + t_x \\ 0 + \cancel{t_y}\, y + t_y \\ 0 + 0 + 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$

- Inverse of the transformation matrix is obtained by '-' ~~to~~ sign to the shifting vector

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} x - t_x \\ y - t_y \\ 1 \end{bmatrix}$$

For rotation! (anticlockwise)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\boxed{p' = R(\theta) \cdot p}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} x\cos\theta - y\sin\theta + 0 \\ x\sin\theta + y\cos\theta + 0 \\ 0 + 0 + 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \\ 1 \end{bmatrix}$$

$\Rightarrow x' = x\cos\theta - y\sin\theta$

$y' = x\sin\theta + y\cos\theta$

$h = 1$

Clockwise Rotation about origin

[θ becomes $(-\theta)$]

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) & 0 \\ \sin(-\theta) & \cos(-\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} x\cos\theta - y\sin\theta + 0 \\ -x\sin\theta + y\cos\theta + 0 \\ 0 + 0 + 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} x\cos\theta - y\sin\theta \\ -x\sin\theta + y\cos\theta \\ 1 \end{bmatrix}$$

$x' = x\cos\theta - y\sin\theta$

$y' = -x\sin\theta + y\cos\theta$

$h = 1$

Scaling! For 3x3 the scaling the 3x3 matrix in this homogeneous format.

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} x \cdot S_x \\ y \cdot S_y \\ 1 \end{bmatrix}$$

$\therefore x' = x S_x$

$y' = y \cdot S_y$

$h = 1$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1/S_x & 0 & 0 \\ 0 & 1/S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Composite transformation

We have to set up a sequence of transformation called composite transformation under matrix multiplication

- The multiplication of transformation matrices often is known as concatenation or composition of matrices.

- So we form composite transformation multiplying matrices in order from left to right.

Translation: If two successive translation vector (tx_1, ty_1) and (tx_2, ty_2) are applied to co-ordinate position P

so we have to calculate $P' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$

then matrix multiplication will be.

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & tx_1 \\ 0 & 1 & ty_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & tx_2 \\ 0 & 1 & ty_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & tx_1 + tx_2 \\ 0 & 1 & ty_1 + ty_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\boxed{T(tx_2, ty_2) \cdot T(tx_1, ty_1) = T(tx_1 + tx_2,\ ty_1 + ty_2)}$$

Rotation: If two rotation are applied to point p and the required eqn is

$$\boxed{P' = \{R(\theta_2) \cdot R(\theta_1)\} \cdot P}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_1 \cdot \cos\theta_2 - \sin\theta_1 \cdot \sin\theta_2 + 0 & -\cos\theta_2 \sin\theta_1 - \cos\theta_1 \sin\theta_2 + 0 & 0 \\ \cos\theta_1 \sin\theta_2 + \cos\theta_2 \cdot \sin\theta_1 + 0 & -\sin\theta_1 \cdot \sin\theta_2 + \cos\theta_1 \cdot \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0 \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$R(\theta_2) \cdot R(\theta_1) = R(\theta_1 + \theta_2)$$

$$\Rightarrow \boxed{R' = R(\theta_1 + \theta_2)}$$ Proved

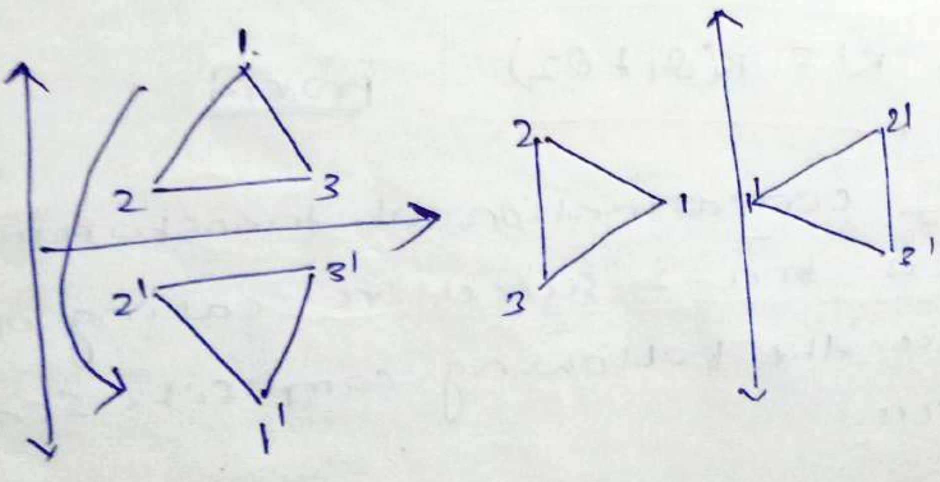
Scaling: concatenation of transformation matrices for 2 successive scaling operation produce the following composite scaling matrices.

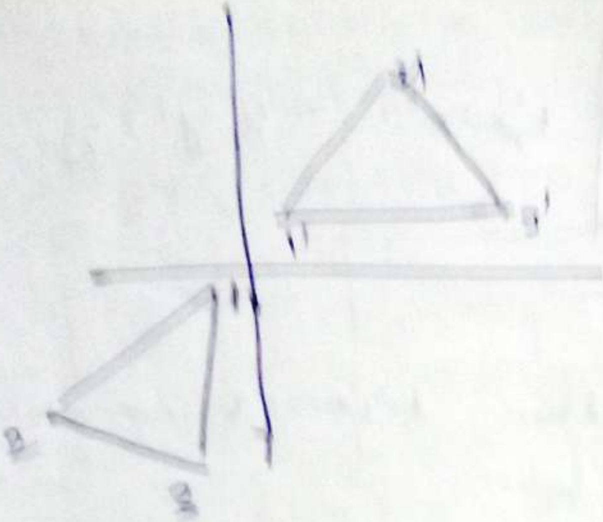
$$P' = \begin{bmatrix} Sx_1 & 0 & 0 \\ 0 & Sy_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Sx_2 & 0 & 0 \\ 0 & Sy_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} Sx_1 \cdot Sx_2 & 0 & 0 \\ 0 & Sy_1 \cdot Sy_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

So $\boxed{S(Sx_2 \cdot Sy_2) \cdot S(Sx_1 \cdot Sy_1) = S(Sx_1 \cdot Sx_2, Sy_1 \cdot Sy_2)}$

Reflection: Reflection is the transformation that produces mirror image of an object there must be reflected axis about which the object is to be reflected.





- to get reflected object the angle of rotation is 180°

- the path of reflection is ⊥ to the xy plane. so for rotating the object about the x-axis

- Hence required transformation matrix is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ Reflection about x-axis $\Rightarrow x' = x$ and $y' = -y$

Hence the path of reflection is ⊥ to the xy-plane. So for rotation the object about y-axis line $x = 0$. Hence the required transformation matrix is

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow x' = -x \ \& \ y' = y.$$

So for reflection the transformation matrix is

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

So as to ~~maintain~~ perform this the object is rotated about 180° in the x-y plane and the axis of rotation is ⊥ to xy plane

Shearing: A transformation that change the shape of an object such that the transformed object shape appears as if the object is composed of layers. that means transformation caused by shearing is nothing but sliding of layers.

$x' = x + Sh_x \cdot y$

$y' = y$

corresponding matrix is $\begin{bmatrix} 1 & sh_x \\ 0 & 1 \end{bmatrix}$

Homogeneous co-ordinate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & sh_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

suppose $sh_x = 2$ then the matrix will be

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The shearing parameter take any integer value. +ve value take co-ordinate to right and -ve value shift the co-ordinate value.

Transformation related to y-axis

$x' = x$

$y' = y + sh_y . x$

Homogeneous co-ordinate is

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We need to transfer x'y' axis onto xy axis

① Translate so that the origin (x_0, y_0) goto $(0,0)$ position

② Rotate the x' axis to x-axis.

<u>Translation:</u>

$$T(-x_0, -y_0) = \begin{bmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

clockwise rotation

$$R(-\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Window: A world co-ordinate area selected for display is known as window.

viewport: It defines where the object is to display on the o/p device.

- The window on viewport are rectangle in standard position.

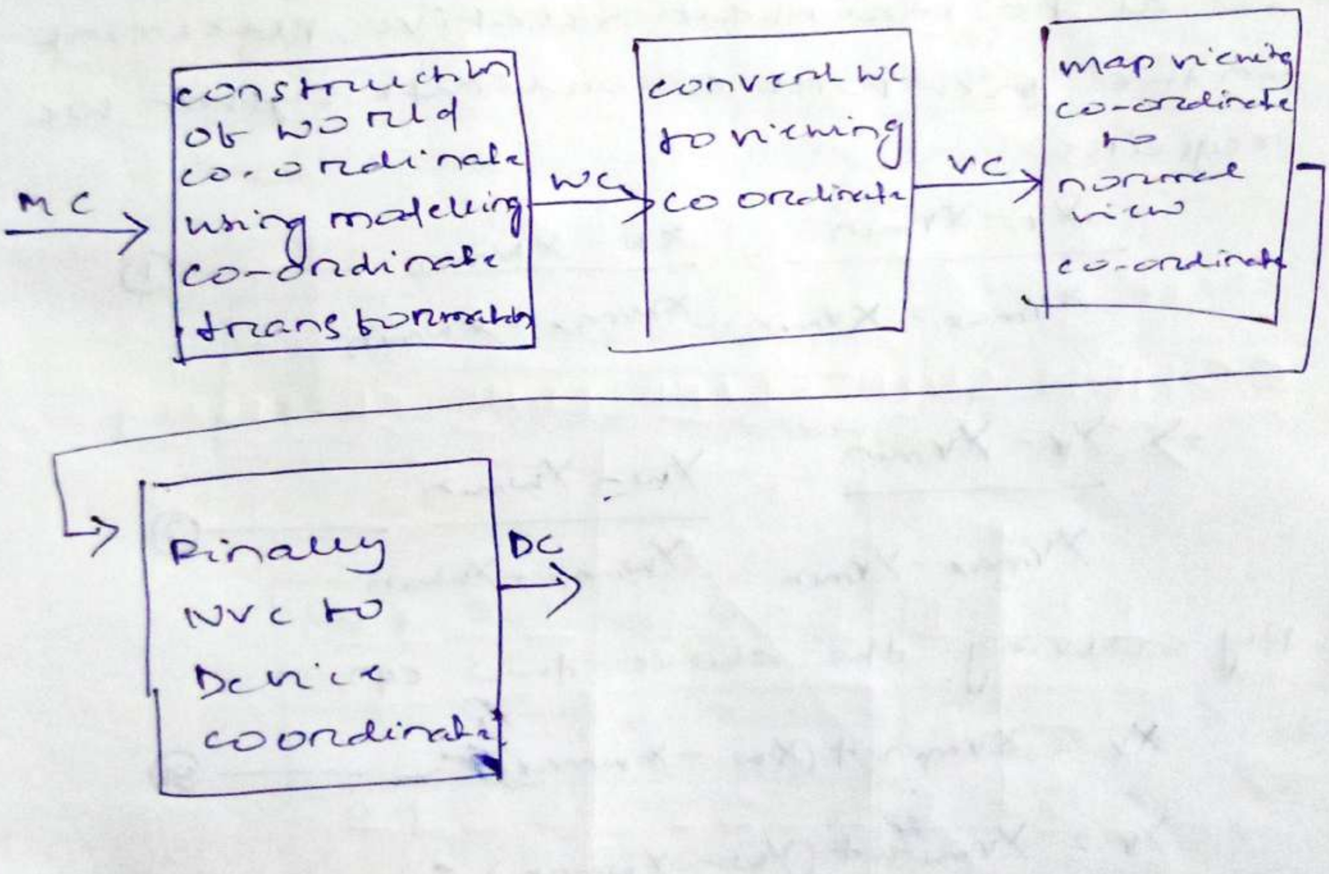
The 2D viewing is refered as mapping of a part of world co-ordinate screen to device co-ordinate screen and the mechanism behind this is known as viewing transformation. It is also known as windowing transformation.

- we have to establish the viewing reference frame
- then we transfer the world co-ordinate to viewing co-ordinate.
- then the view is defined in a normalised co-ordinate form i.e unit square where the viewport is to be displayed.
- so in this step the viewing co-ordinate system is mapped to normalised co-ordinate system.
- At last all part of the picture that lies outside the viewport are clipped.

- then the content of the viewport are transformed to the device co-ordinate system

- By changing the position of viewport we can view object at different position on the display of an o/p device.

- By varying the size of viewport we can change the size and the properties of display object (so we can achieve the zooming effect by this)



Window to viewport co-ordinate system

If co-ordinate position is at the center of the viewing window then it will displayed at the center of the view port.

$$\boxed{(V_w)_c = (V_p)_c}$$

- A point at position (x_w, y_w) in the window is mapped into the position (x_v, y_v) in association with viewport co-ordinate
- so as to maintain relative placement in the viewport co-ordinate system we require

$$\frac{x_v - x_{vmin}}{x_{vmax} - x_{vmin}} = \frac{x_w - x_{wmin}}{x_{wmax} - x_{wmin}} \quad \text{--- (1)}$$

$$\Rightarrow \frac{y_v - y_{vmin}}{y_{vmax} - y_{vmin}} = \frac{y_w - y_{wmin}}{y_{wmax} - x_{wmin}} \quad \text{--- (2)}$$

By solving the above two eqn

$$x_v = x_{vmin} + (x_w - x_{wmax}).S_x \quad \text{--- (3)}$$

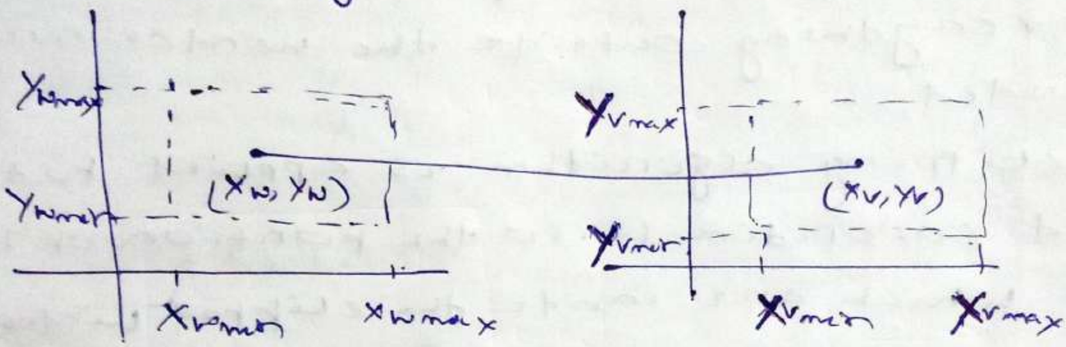
$$y_v = y_{vmin} + (y_w - y_{wmax}).S_y \quad \text{--- (4)}$$

where S_x and S_y are the scaling factor

$$S_x = \frac{X_{vmax} - X_{vmin}}{X_{wmax} - X_{wmin}} \quad \text{(5)}$$

$$S_y = \frac{Y_{vmax} - Y_{vmin}}{Y_{wmax} - Y_{wmin}} \quad \text{(6)}$$

To maintain the relative properties of the object $S_x = S_y$



Window Aspect ratio:

$$\boxed{A_w = \frac{X_{wmax} - X_{wmin}}{Y_{wmax} - Y_{wmin}}}$$

Viewport aspect ratio:

$$\boxed{A_v = \frac{X_{vmax} - X_{vmin}}{Y_{vmax} - X_{vmin}}}$$

Line clipping: The process of cutting off line which are outside the clip window is known as line clipping

Q: Find the normalization transformation window to viewport with window lower left corner at (1,1) and upper right corner at (3, 5) onto a viewport with left corner at (0, 0) and upper right corner at (1/2, 1/2)

Soln: Given that

$x_{wmin} = 1 \quad y_{wmin} = 1$

$x_{wmax} = 3 \quad y_{wmax} = 5$

$x_{vmin} = 0 \quad y_{vmin} = 0$

$x_{vmax} = 1/2 \quad y_{vmax} = 1/2$

$$N = \begin{bmatrix} 1 & 0 & x_{vmin} \\ 0 & 1 & y_{vmin} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_{wmin} \\ 0 & 1 & -y_{pmin} \\ 0 & 0 & 1 \end{bmatrix}$$

$$s_x = \frac{x_{vmax} - x_{vmin}}{x_{wmax} - x_{wmin}} \qquad s_y = \frac{y_{vmax} - y_{vmin}}{y_{wmax} - y_{wmin}}$$

$$= \frac{1/2 - 0}{3 - 1} \qquad = \frac{1/2 - 0}{5 - 1}$$

$$= 1/4 \qquad = 1/8$$

$$\therefore N = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 1/8 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

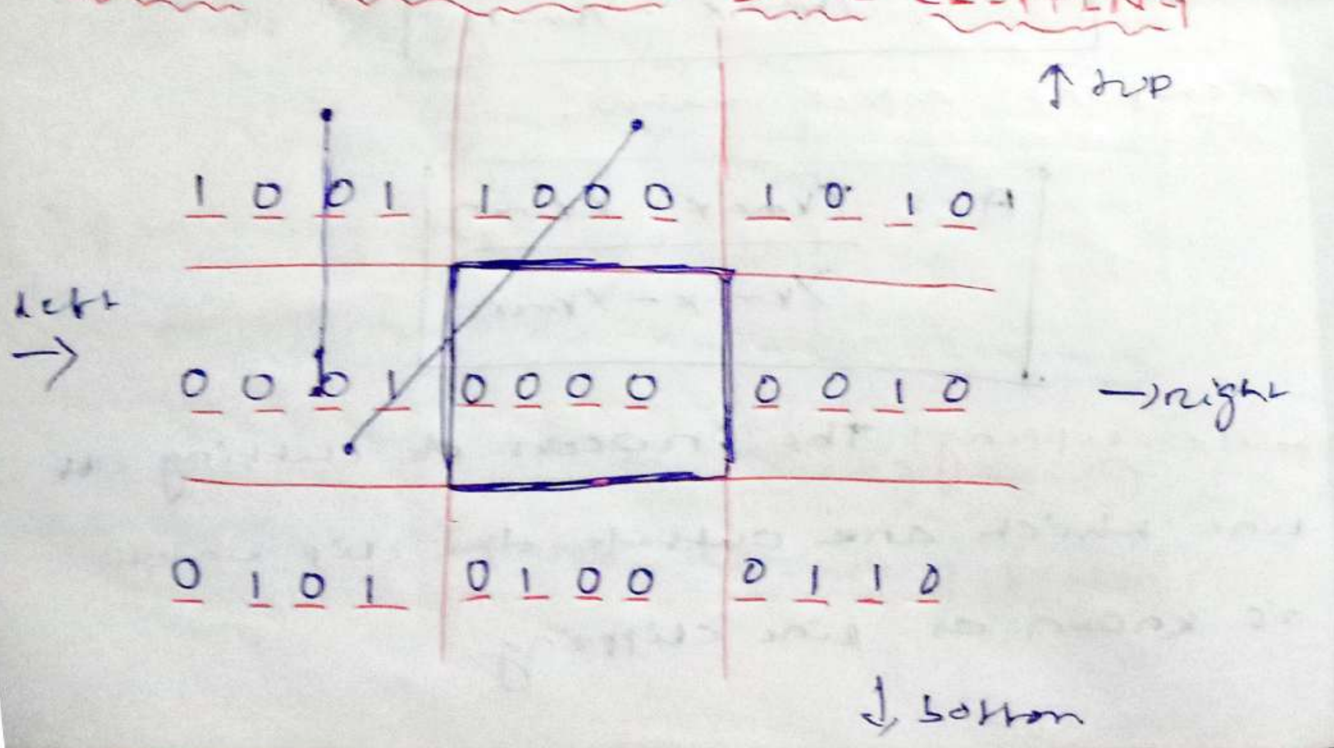
$$= \begin{bmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{8} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{4} & 0 & -\frac{1}{4} \\ 0 & \frac{1}{8} & -\frac{1}{8} \\ 0 & 0 & 1 \end{bmatrix}$$

Clipping window operation:

- The region against on which an object is to be clipped is called clipped window.
- The clipped window can be general polygon on a curve boundary.
- Hence we use the rectangular clipped region for the clipping purpose.
- So everything outside the window are discarded.
- The clipping algorithm is applied to the world co-ordinate. So the portion of the object which are inside the clipped window are mapped to device co-ordinate. and the clipping segement are not save for displaying.

COHEN SUTHERLAND LINE CLIPPING



this algorithm uses 4-bit binary code for checking (the line) the location of the points relative to the boundary of the clipping rectangle.

Let us divide the plane into 9 parts and assign a region code to each sub parts. The region code for the clipped window is

0 0 0 0

↓ ↓ ↓ ↓

4th 3rd 2nd 1st

bit bit bit bit.

bit 1 : left (X_{wmin})

bit 2 : right (X_{wmax})

bit 3 : below (Y_{wmin})

bit 4 : above (Y_{wmax}).

Algorithm:

1. If both the end points are in the clip window i.e 0000 then the entire line is inside the window (so the line is not required for clipping)
2. If not so then we will go for a test of both the end points by logical AND of outcode of the end points.

then that may give the results

a) a nonzero value.

b) A zero value.

(3) If we are getting the nonzero value then reject the entire segment.

(4) If we are getting a zero value for the nonzero end point then some portion of the line may inside the clip window.

Ex: 1st line

```
  1 0 0 1
  0 0 0 1
  -------
  0 0 0 1  (1st line rejected)
```

2nd line

```
  0 0 0 1
  1 0 0 0
  -------
  0 0 0 0  (the 2nd line is accepted and will be divided into segments)
```

(5) For the non zero end point, if we are getting the zero value, after logical ANDING we divide the entire line into diff. segment and check for each individual segment.

(6) compute the outcodes of the two end point of the segement.

(7) then enter onto a loop.

a) within the loop check to see if the outcode are zero. then accept the segement. Exit the loop.

b) If both the outcode are nonzero then take logical AND of both the outcodes and check for the non zero result, then reject the line/segement. Exit the loop.

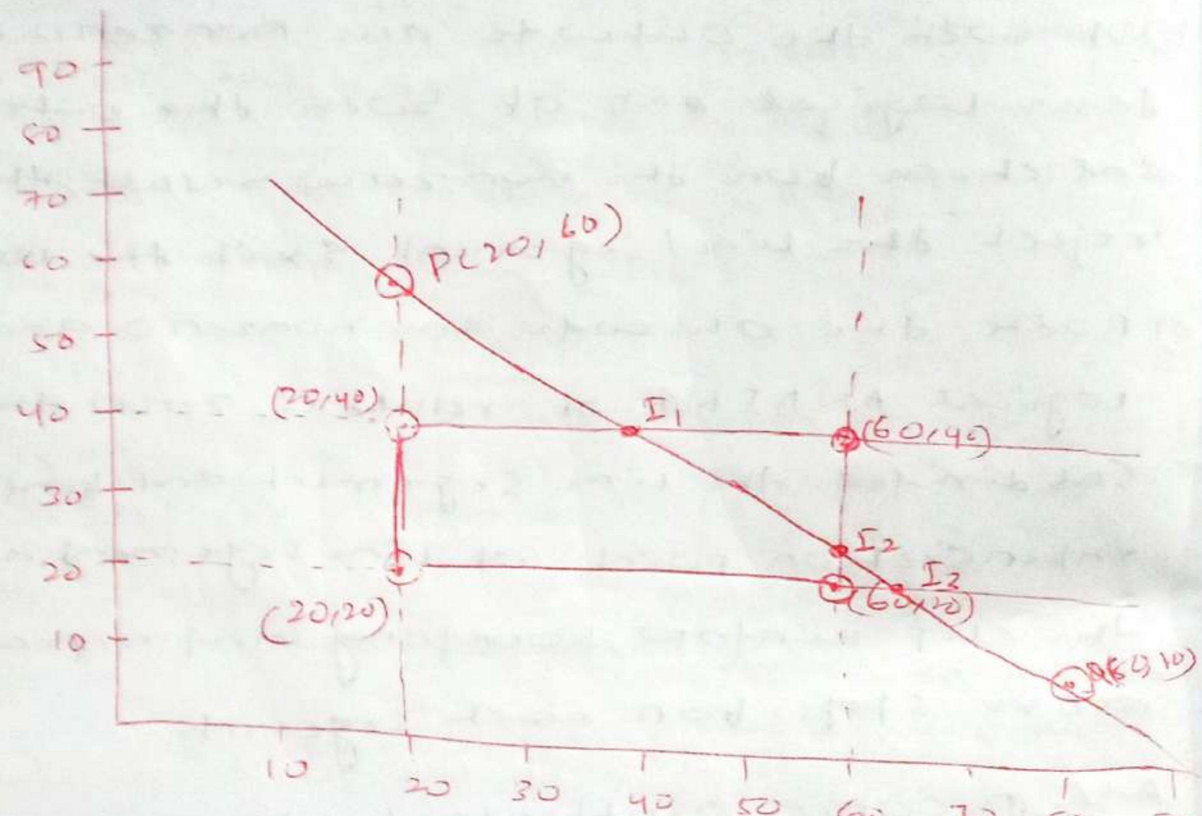
c) Both the outcode are nonzero and logical ANDING. If result is zero then subdivided the line segement and find out intersection point of line segement against the clip ~~window~~ boundary and repeat the above steps for each segement

Adv: (1) cohen sutherland line clipping algorithm is one of the oldest and most popular line clipping procedure

(2) This method speed up the processing of line segement by reperforming initial test that reduce the no. of intersection point that must be calculated.

Q1 Given a window A(20,20), B(60,20) C(60,40) D(20,40) using Cohen Sutherland line clipping algorithm find out the visible portion of the line P(20,60) to Q(80,40)

Ans:



How to find out the region code of the end point:

bit 4: sign$(y - y_{max})$

bit 3: sign$(y_{min} - y)$

bit 2: sign$(x - x_{max})$

bit 1: sign$(x_{min} - x)$

$$\text{sign } a = \begin{cases} 1 & \text{if } a > 0 \\ 0 & \text{otherwise.} \end{cases}$$

In this case y_{max} = above = 40

y_{min} = below = 20

x_{max} = left = 20

x_{min} = right = 60.

For $P(\overset{x}{20}, \overset{y}{60})$

bit 4: sign(60 − 40)

= sign(20) = 1

bit 3: sign(20 − 60)

= sign(−40) = 0.

bit 2: sign(20 − 60)

= 0.

bit 1: sign(20−20) = 0.

For Q(80, 10)

bit 4: sign(10−40) = 0.

bit 3: sign(20−10) = 1

bit 2: sign(80−60) = 1

bit 1: sign(20 − 80) = 0.

Logical ANDING of P and Q.

$$\begin{array}{cccc} 1 & 0 & 0 & 0 \\ \underline{0} & \underline{1} & \underline{1} & \underline{0} \\ 0 & 0 & 0 & 0 \end{array}$$

Let slope of PQ be m

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 60}{80 - 20} = -50/60 = -5/6$$

Let us push the point P towards the clipping window. So the '1' in the region code become 0. and to perform this we have to clip the line against the boundary line $y_{max} = 40$ (i.e. line (D))

Now we calculate the co-ordinate of I_1

$I_1(x, 40)$

$$y_2 - y_1 = m(x_2 - x_1)$$

$$\Rightarrow 40 - 10 = -5/6\,(x - 80)$$

$$\Rightarrow 30 = -5/6\,(x - 80)$$

$$\Rightarrow x - 80 = \frac{\overset{6}{\cancel{30}} \times 6}{-\cancel{5}} = 36$$

$$\Rightarrow x = -36 + 80 = 44.$$

The point Q is pushed the line towards the clipped window. So that 1's become 0. co-ordinate of I_3 is i.e. $I_3(x, 20)$

$y_{min} = 20$

$y_2 - y_1 = m(x_2 - x_1)$

$\Rightarrow 10 - 20 = -5/6\,(80 - x)$

$\Rightarrow -10 = -5/6\,(80 - x)$

$\Rightarrow 80 - x = -\frac{5}{6} \times \frac{1}{10_2}$

$\Rightarrow x = 80 - 12 = 68$

co-ordinate of I_2 is $I_2(60, y)$

$x_{max} = 60$

$y_2 - y_1 = m(x_2 - x_1)$

$\Rightarrow 20 - y = -\frac{5}{6}(68 - 60)$

$\Rightarrow y = 20 + \frac{5}{\underset{3}{6}} \times \overset{4}{8} \Rightarrow y = 20 + \frac{20}{3} = \frac{60+20}{3} = \frac{80}{3} = 26.66$

Q: Apply co-hen Sutherland line clipping algorithm to clip the line $P_1(10, 20)$ and $P_2(80, 90)$ against the window ABCD with $A(20, 20)$, $B(90, 20)$ $C(90, 70)$ and $D(20, 70)$.

Polygon clipping:

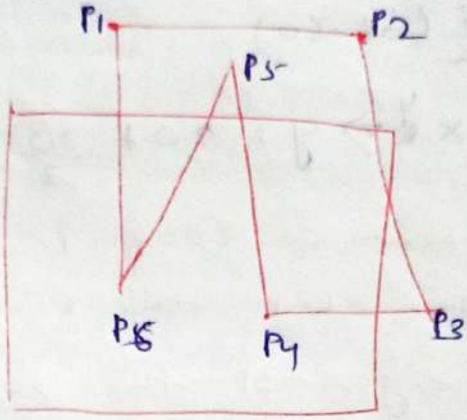
- A polygon boundary when processed with a line clipper may lead to a series of unconnected line segment.
- But in a polygon clipping we will get a boundary area after clipping.

- So output of a polygon clipper should be a sequence of vertices that defines the clipped polygon boundaries.

Sutherland Hodgeman polygon clipping algorithm

1) We can clip the polygon by processing the polygon boundary as a whole against each boundary edge.

2) So as to get a clipped polygon we have to process all polygon vertices against each clipped rectangle boundary one after other.



3) The clipping sequence first we take the initial set of polygon vertices. Let it be $P_1, P_2, P_3 \ldots P_n$ which implies a list of edge $\overline{P_1P_2}, \overline{P_2P_3} \ldots \overline{P_{n-1}P_n}, \overline{P_nP_1}$

4) a) Then we clip the polygon against the left rectangular boundary to produce a new sequence of vertices.

b) The new sequence of vertices could then be successfully passed to right boundary clipper

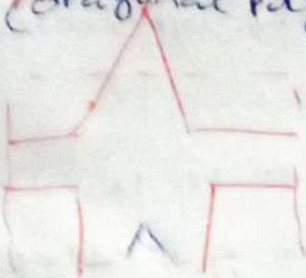
c) Then the resultant vertices are passed to bottom boundary clipper and top boundary clipper one after another.

4) So in each step a sequence of vertices o/p is generated and passed to the next window boundary clipper.


(original polygon)


(clip left)


(clip right)


(clip bottom)


(clip top)

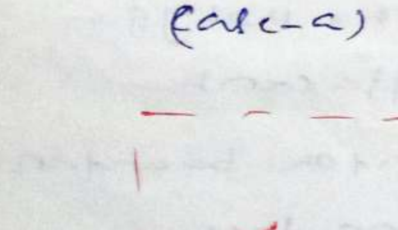
5) There are four possible cases arises when we process vertices in a sequence around the perimeter of the polygon and each vertices pair of adjacent polygon is passed to a window boundary clipper to make the following test

a) If the 1st vertex is in outside of the window boundary and 2nd vertex is inside then the intersection point of the of the polygon edge with window boundary and the 2nd vertex is added to the vertex list.

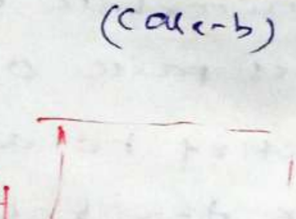
b) If both the i/p vertices are inside the window boundary only the 2nd vertex is added to the o/p vertex list.

c) If the first vertex is inside the window boundary and the 2nd vertex is outside, only the edge intersection point with window boundary is added to the o/p vertex list.

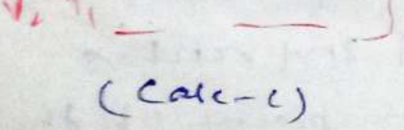
d) If both the i/p vertices are outside the window boundary then nothing is added to the o/p vertex list.



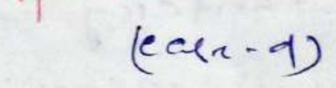
(case-a)



(case-b)



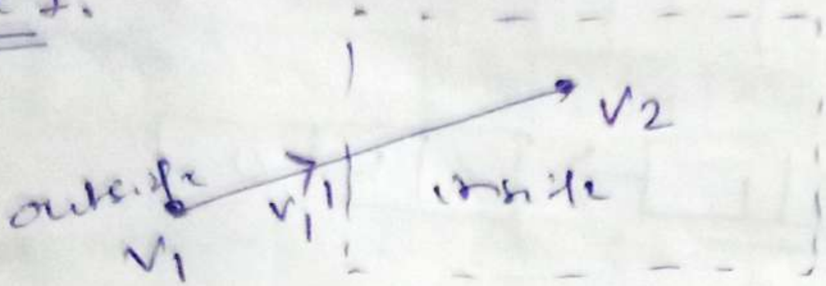
(case-c)



(case-d)

2 3 4 5 6

case 1:

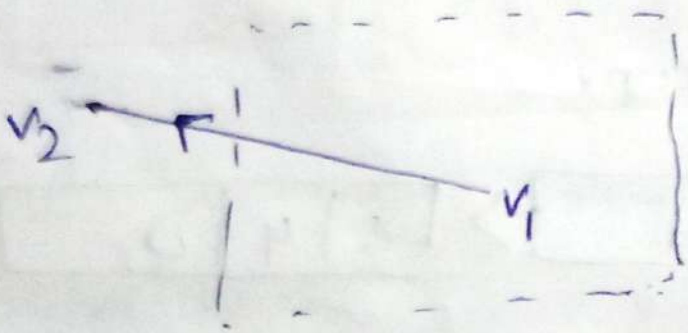


out → in.

O/P: intersection point + destination point

$V_1' V_2$

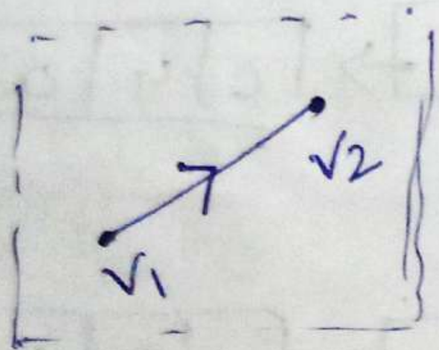
case-2:



in → out

O/P: intersection point. V_1'

(iii) case-3:



in → in

O/P: destination vertex V_2

case-4:

out -> out

output: null.

① perform $45°$ rotation of triangle $A(0,0)$, $B(1,1)$, $C(5,2)$

a) about the origin

b) about $P(-1,-1)$

Soln: $[A'B'C'] = R(45°)[ABC]$

$$= \begin{bmatrix} \cos 45° & -\sin 45° & 0 \\ \sin 45° & \cos 45° & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{3}{\sqrt{2}} \\ 0 & \frac{2}{\sqrt{2}} & \frac{7}{\sqrt{2}} \\ 1 & 1 & 1 \end{bmatrix}$$

b) $[A'B'C'] = R(-1,-1,45°)[ABC]$

$$= \begin{bmatrix} \cos 45° & -\sin 45° & -1(1-\cos 45°) + (-1)\sin 45° \\ \sin 45° & \cos 45° & -1(1-\cos 45°) - (-1)\sin 45° \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \sqrt{2}-1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 & \frac{3}{\sqrt{2}} - 1 \\ \sqrt{2}-1 & 2\sqrt{2}-1 & \frac{9}{\sqrt{2}} - 1 \\ 1 & 1 & 1 \end{bmatrix}$$

① magnify the triangle with vertices A(0,0), B(1,1), C(5,2) to twice its size while keeping C(5,2) is fixed

ans: magnifying the ΔABC keeping C fixed is same as magnifying about the fixed point C

$$[A'B'C'] = \begin{bmatrix} S_x & 0 & x_f(1-S_x) \\ 0 & S_y & y_f(1-S_y) \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 5(1-2) \\ 0 & 2 & 2(1-2) \\ 0 & 0 & 1 \end{bmatrix}$$

$$[A\ B\ C] = \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & -5 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -5 & -3 & 5 \\ -2 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

② Reflect the diamond shape polygon whose vertices are A(1,0) B(0,-2) C(1,0) D(0,2) about-

i) horizontal line $y=2$

ii) vertical line $x=2$

iii) the line $y=x+2$

i) the eqn of line $y=2$

$m=0,\ c=2$

$$[A'B'C'D'] = \begin{bmatrix} \frac{1-m^2}{m^2+1} & \frac{2m}{m^2+1} & \frac{-2cm}{m^2+1} \\ \frac{2m}{m^2+1} & \frac{m^2-1}{m^2+1} & \frac{2c}{m^2+1} \\ 0 & 0 & 1 \end{bmatrix} [A\ B\ C\ D]$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & -2 & 0 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 & 0 \\ 4 & 6 & 4 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

(b) The vertical line $x=2$ has no y-intercept and infinite slope, so we translate the line $x=2$ two unit vector case over to the y-axis then reflect about y-axis and finally translate over for two unit

$$[A'B'C'D'] = T^{-1}(2,0) \cdot T_{Refl\,y} \cdot T(2,0) \cdot [A,B,C,D]$$

$$= \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & -2 & 0 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & -4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & -2 & 0 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1-4 & -4 & -1-4 & -4 \\ 0 & -2 & 0 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -4 & -5 & -4 \\ 0 & -2 & 0 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}.$$

c) $y = x+2$ has slope $m=1$ and y-intercept $c=2$

$$[A'B'C'D'] = \begin{bmatrix} \frac{1-m^2}{m^2+1} & \frac{2m}{m^2+1} & \frac{-2cm}{m^2+1} \\ \frac{2m}{m^2+1} & \frac{m^2-1}{m^2+1} & \frac{2c}{m^2+1} \\ 0 & 0 & 1 \end{bmatrix} [A\,B\,C\,D]$$

$$= \begin{bmatrix} 0 & 1 & -2 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & -2 & 0 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -2 & -4 & -2 & 0 \\ 1 & 2 & 3 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$