Engg Mechanics Module - III

Rectilinear Translation - Principle of Dynamics concept of inertial and non inertial frame of Reference, D'Alembert's principle.

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Rectilinear Motion

Dynamics is the branch of Mechanics which deals with analysis of moving bodies.

Dynamics is divided into types

→ Kinetics

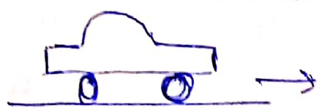
→ kinematics

Kinetics is study of bodies subject to force which is unbalanced.

kinematics is study of geometry and time dependens of motion without considering the force causing the motion.

Rectilinear motion

The simplest motion is along a st-line and motion is called as rectillinear motion.

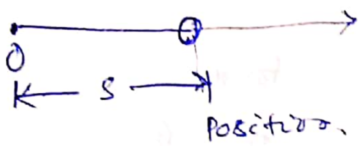


Position

The straight line path of a particle will be defined using a single co-ordinate ~~system~~. axis 'x'. The magnitude is discribe 's'.

it is (+)ve if they move (+)ve x-axis.

(−)ve for (−)ve x-axis



Displacement

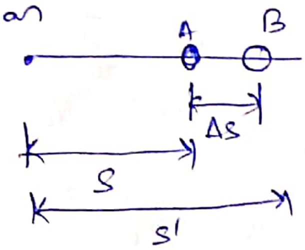
Displacement is defined as the change in position.

$\Delta s = s' - s$

Δs is (+)ve when it moves to right. $s' > s$

(−)ve when the final position is to be left.

(−)or



Velocity

~~It~~ If the particle moves through a displacement Δs during the time interval Δt the average velocity of the particle

$$V_{avg} = \frac{\Delta s}{\Delta t}$$

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If we take take small value time (Δt) the displacement is small (ΔS), the instantaneous velocity vector.

$$v = \lim_{\Delta t \to 0} \left(\frac{\Delta s}{\Delta t} \right)$$

$$\left(v = \frac{ds}{dt} \right) \quad (\xrightarrow{+}) \quad \text{--- (1)}$$

Acceleration

To give the velocity of the particle is known at two point. average accelⁿ of the particle during time interval.

$$a_{avg} = \frac{\Delta V}{\Delta t}$$

$$\Delta V = v' - v$$

Similarly the instaneous accelⁿ

$$a = \frac{dv}{dt} \text{ or } \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t} \quad \text{--- (2)}$$

$$\boxed{a = \frac{d^2 s}{dt}}$$

form eqⁿ (1) $\quad v = \frac{ds}{dt} \Rightarrow dt = \frac{v}{ds}$

form (2) $\quad a = \frac{dv}{dt} \Rightarrow dt = \frac{a}{dv}$

$$\frac{v}{ds} = \frac{a}{dv} \Rightarrow \boxed{v \cdot dv = a \cdot ds}$$

Constant accelⁿ ($a = a_c$)

When the accelⁿ is const the three eqⁿ

$$a_c = \frac{dv}{dt}, \quad v = \frac{ds}{dt} \quad \text{and} \quad a_c \cdot ds = v\, dv$$

velocity is a function of time

Integrati $\quad a_c = \frac{dv}{dt}$

Initial $v = v_0$, $t = 0$; final $v = v$, $t = t$

$$\int_{v_0}^{v} dv = \int_{t_0}^{t} a_c\, dt$$

$$\boxed{v = v_0 + a_c t}$$

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Position is function of time

$$v = \frac{ds}{dt} = v_0 + a_c t$$

$$\int_{s_0}^{s} ds = \int_0^t (v_0 + a_c t)\, dt$$

$$s = s_0 + v_0 t + \frac{1}{2} a_c t$$

(Const acceln)

velocity as a function position

$$v \cdot dv = a_c\, ds$$

Initially $v = v_0$ and $s = s_0$

$$\int_{v_0}^{v} v\,dv = \int_{s_0}^{s} a_c\, ds$$

$$\boxed{v^2 = v_0^2 + 2a_c(s - s_0)}$$

Ex-1

The car moves in a straight line such that for short time its velocity $v = (3t^2 + 2t)$ m/s where t in sec. Determine its position and accelerat. when t = 3s when t = 0, s = 0

Ans

$$v = f(t), \quad v = \frac{ds}{dt}, \quad s = 0 \quad t = 0$$

$$v = \frac{ds}{dt} = (3t^2 + 2t)$$

$$\int_0^s ds = \int (3t^2 + 2t)\, dt$$

$$s = t^3 + t^2$$

$$t = 3, \quad s = (3)^3 + (3)^2 = 36\ m.$$

Acceln

$$a = \frac{dv}{dt} = \frac{d}{dt}(3t^2 + 2t)$$

$$= 6t + 2$$

$$= 6 \times 3 + 2 = 20\ m/s$$

Ex-2

A small projectile is fored vertically downward into a fluid motion with initial velocity. Due to the drag resistance of fluid the projectile experience deceleration $a = (-0.4v^2)$ m/s² where v is m/s. Determine the projectile velocity and position 4s after it is fired.

Soln

Here $a = f(v)$

$$a = \frac{dv}{dt}$$

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(4)

$$a = \frac{dv}{dt} = -0.4v^3 \Rightarrow \frac{dv}{-0.4v^3} = dt$$

$$\int_{60}^{v} -\frac{dv}{0.4v^3} = \int_0^t dt$$

$$\Rightarrow \frac{1}{0.4}\left(-\frac{1}{2}\right)\frac{1}{v^2}\Big]_{60}^{v} = (t-0)$$

$$\cancel{= \frac{1}{0.4} \times -\frac{1}{2}}$$

$$= \frac{1}{0.8}\left[\frac{1}{v^2} - \frac{1}{(60)^2}\right] = t$$

$$v = \left\{\left[\frac{1}{(60)^2} + 0.8t\right]^{-1/2}\right\}$$

$t = 4s$

$v = 0.559 \, m/s \downarrow$

Position

$$v = \frac{ds}{dt} = \left[\frac{1}{(60)^2} + 0.8t\right]^{-1/2}$$

$$\int_0^s ds = \int_0^t \left[\frac{1}{(60)^2} + 0.8t\right]^{-1/2}$$

$$= s = \frac{2}{0.8}\left[\frac{1}{(60)^2} + 0.8t\right]_0^{1/2}\Big]_0^t$$

$$= \frac{1}{0.4}\left\{\left[\frac{1}{(60)^2} + 0.8t\right]^{1/2} - \frac{1}{60}\right\}$$

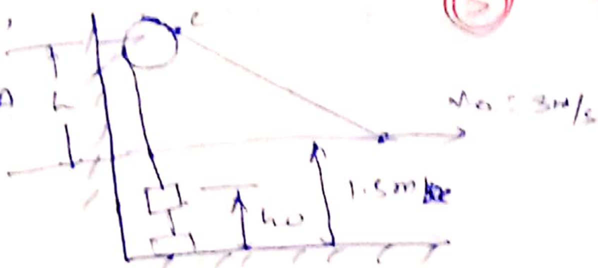
$t = 0.4$

$s = 4.43m.$

ex-3 (7.4 - (Timoschenko and young))

A Rope AB attach to a small block of negligible dimension at 'B' and passes over Pulley C so that ~~Pulley C~~ its free end A hangs 1.5m above ground when the block rests ~~from~~ on the Floor. The end 'A' of the rope is moved horizontally in a straight line by a man walking with a uniform velocity $v_0 = 3m/s$. Deduce the generalised expression for velocity. Also find the time reqd for the block to reach Puly if $h = 4.5m$ and pulley is negligibly small

A3 consider at time 't'
rope moves from A0 to A
block moves B at
height h0



$AoA = v_0 t$

let $AC = h + y$

$AC^2 = AA_0^2 + A_0C^2$

$(h+y)^2 = (v_0 t)^2 + h^2$

$y^2 + hy - v_0^2 t^2 = 0$

$\Rightarrow y = -h + \sqrt{h^2 + v_0^2 t^2}$

$$\frac{dy}{dt} = \frac{v_0^2 t}{\sqrt{h^2 + v_0^2 t^2}}$$

when block reaches 'C'

$h + 1.5 = -h + \sqrt{h^2 + v_0^2 t^2}$

$\Rightarrow (2h + 1.5)^2 = h^2 + v_0^2 t^2$

Put $h = 4.5$, $v_0 = 3$

$\boxed{t = 3.16s}$

Ex-4 (7.10) - Timoshenko and Youngs)

The Rectilinear motion of particle is defined by the displacement - time equation $x = x_0(2e^{-kt} - e^{-2kt})$, in which x_0 is the initial displacement k is the const and e is the natural logarithm base. find the max^m velocity of particle.

Sol^n

$x = x_0(2e^{-kt} - e^{-2kt})$

$\dot{x} = 2x_0(-k)e^{-kt} - x_0(-2k)e^{-2kt}$

$= -2kx_0(e^{-kt} - e^{-2kt})$

Max^m velocity, $\dot{x} = 0$, $e^{-kt} = e^{-2kt}$

$e^{kt} = 2 \Rightarrow t = \frac{1}{k}\ln 2$

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(6)

Therefore max^m velocity

$$\dot{x}\Big|_{max} = \dot{x}\Big|_{t=\frac{1}{k}\ln 2}$$

$$= -\frac{kx_0}{2}$$

Ex-5 (7.6 of Timoshenko and youngs)

A bullet leaves the muzzle of gun with velocity $v = 750\, m/s$, Assuming const acceleration from breech to muzzle, find the time t occupied by the bullet in travelling through the gun barrel, which is 750 mm long.

Solⁿ $u = 0$, $v = 750\, m/s$ $S = 750\, mm = 0.75\, m$

$$v^2 = u^2 + 2as$$

$$a = \frac{v^2 - u^2}{2s} = \frac{750^2 - 0}{2 \times 0.75}$$

$$= (375 \times 10^3)\, m/s^2$$

$$t = \frac{v - u}{a} = \frac{750 - 0}{375 \times 10^3} = 0.002\, s.$$

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Rectilinear motion: (variable motion)

When a ~~partic~~ particle has erratic or changing motion then its position, velocity and acceleration cannot be describe by single continuous mathematical function along entire path, for that type of motion we need s, v, a and t graph ~~for~~ can drawn. Then the graph can be construct of two variables, the differencial relation $v_0 = \frac{ds}{dt}$, $a = \frac{dv}{dt}$ and $a\,ds = v\,dv$ is used.

The s~t, v~t and a~t graph

Here first draw the v~t graph from s~t graph. Here in fig-1, the equation

$$\boxed{v = \frac{ds}{dt} \quad \text{slope of } s\sim t \text{ graph} = \text{velocity}}$$

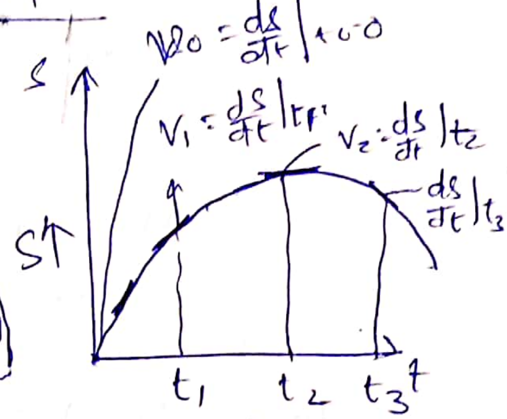


fig-1

from a~t graph ca

$$\boxed{\frac{dv}{dt} = a \quad \text{slope } v\sim t = \text{accel}^n}$$

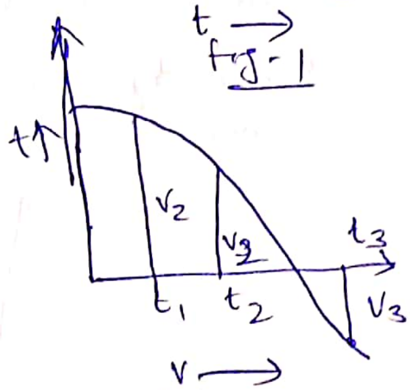
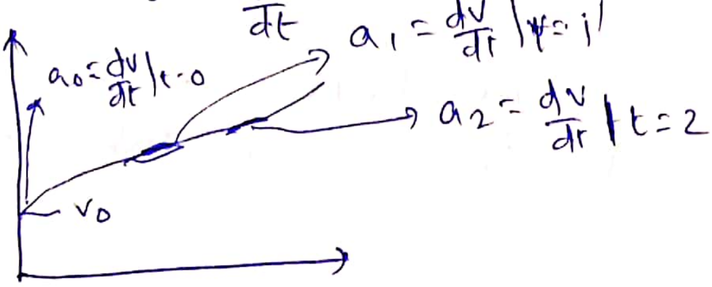
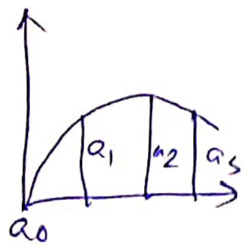


fig. 2

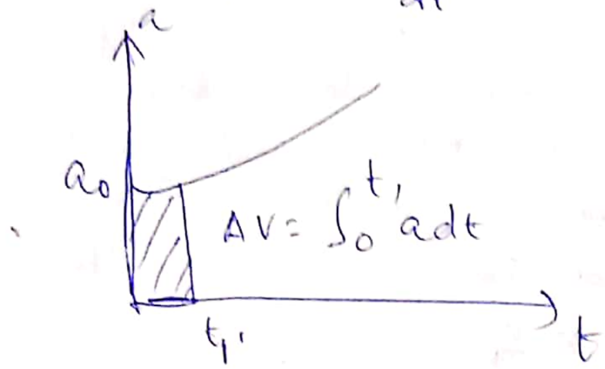
If s~t curve for each interval of time can be expressed $s = s(t)$, then eqn of v~t graph for same interval of time by differencial fuction $v = \frac{ds}{dt}$

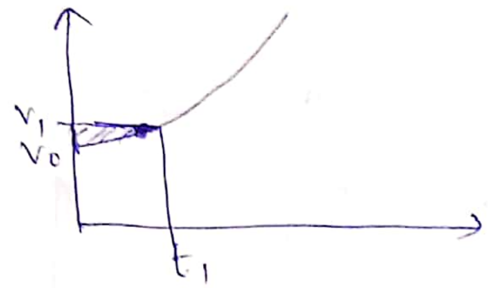




(8) $a = \frac{dv}{dt}$

If a-t graph is give then v-t graph is construct using $a = \frac{dv}{dt}$



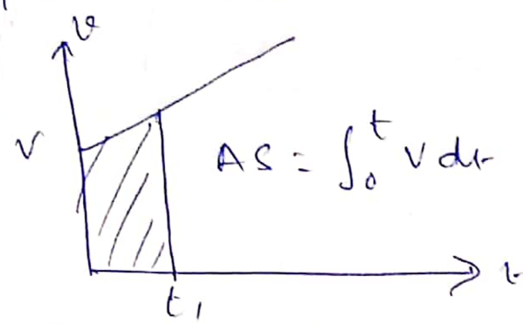


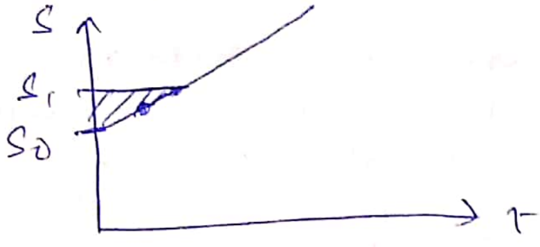
$\Delta v = \int a\,dt$

$\boxed{\text{Change velocity = area under a-t graph}}$

Here $v_1 = v_0 + \Delta v$

Similarly if v-t is given, we can draw s-t graph from this





$\Delta s = \int_0^t v\,dt$

displacement = area under v-t graph

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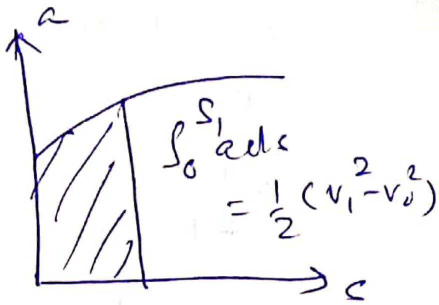
The v-s and a-s graph

If a-s graph can be constructed, the v-s graph can be determined

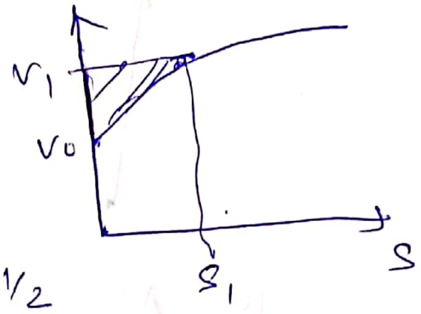
$$v\,dv = a\,ds$$

integrate $v = v_0$ at $s = s_0$

$v = v_1$ at $s = s_1$



$$\frac{1}{2}(v_1^2 - v_0^2) = \int_{s_0}^{s_1} a\,ds$$



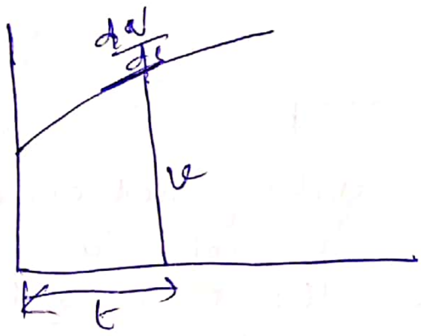
if the initial velocity is known v_0 at $s_0 = 0$, then

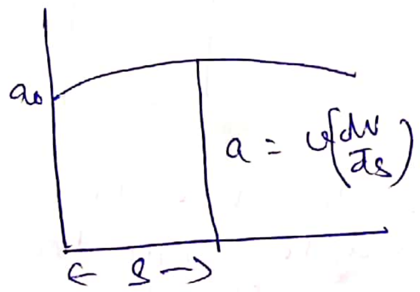
$$v_1 = \left(2\int_{s_0}^{s_1} a\,ds + v_0^2\right)^{1/2}$$

If v-s graph is known, accel^n 'a' at any point 's' can be determine by $a\,ds = v\,dv$

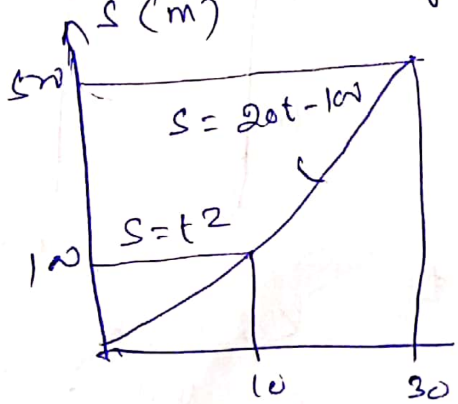
$$a = v\left(\frac{dv}{ds}\right)$$

$$\boxed{\text{accel}^n = \text{slope v-s graph}}$$





Ex-1 A car moves along a straight road such that its position is described by the graph in fig. Construct v-t and a-t graph for $0 \le t \le 30$ s



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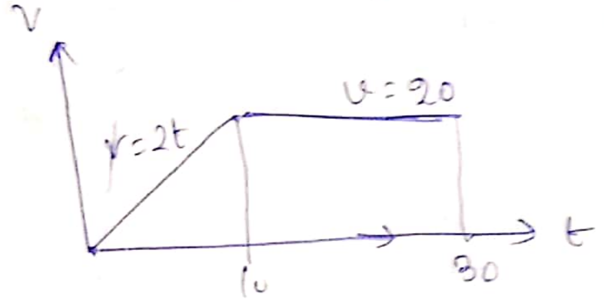
10 ~~s~~ s-t

v-t graph

$v = \frac{ds}{dt}$,

$0 \le t < 10s$, $s = t^2$ $v = \frac{ds}{dt} = 2t$ m/s

$10 \le t < 30$ $s = (20t - 100)$ $v = 20$ m/s

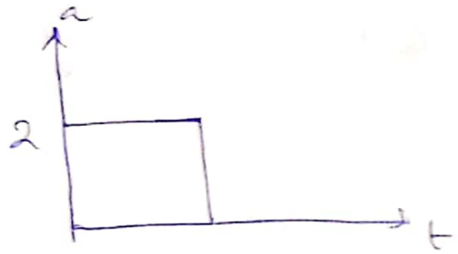


$v = \frac{\Delta s}{\Delta t} = \frac{500 - 100}{30 - 10} = 20$ ft/s.

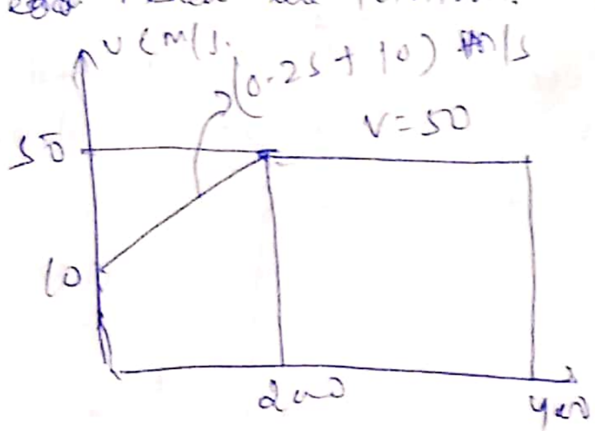
a-t graph

$0 \le t \le 10s$: $v = 2t$, $a = 2$ m/s^2

$10 < t < 30s$ $v = 20$ $a = 0$



Ex-2 The v-s graph describing motion of motorcycle, construct a-s graph and determine the time need the motorcycle to ~~eod~~ reach the position. $s = 400$ m



Ans

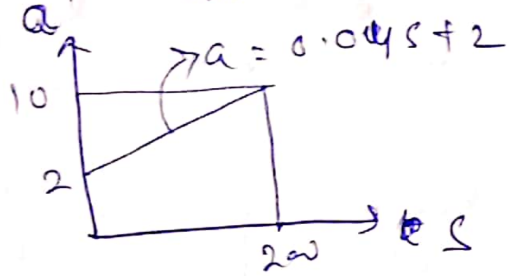
$0 < s < 200$ $v = (0.2s + 10)$ m/s

$a = v\frac{dv}{ds} = (0.2s + 10) \times 0.2$

$= 0.04s + 2$

$200\,ft < s < 400$ $\quad v = 50\,m/s$

$$a = v\frac{dv}{ds} = 0$$



Time

from v~s graph

$$v = \frac{ds}{dt}$$

$0 < s < 200$, $\quad v = (0.2s + 10)\,m/s$

$$dt = \frac{ds}{v} = \frac{ds}{0.2s+10}$$

$$\int_0^t dt = \int_0^s \frac{ds}{0.2s+10}$$

$$t = (5\ln(0.2s+10) - 5\ln 10)\,s$$

At $s = 200\,m$, $\quad t = 8.05\,s$

$200\,m < s < 400\,m$

$v = 50\,m/s \qquad dt = \frac{ds}{v} = \frac{ds}{50}$

$$\int_{8.05}^{t} dt = \int_{200}^{s} \frac{ds}{50}$$

$$\Rightarrow t - 8.05 = \frac{s}{50} - 4$$

$$\Rightarrow t = \left(\frac{s}{50} + 4.05\right)$$

$s = 400\,m$, $\quad t = \left(\frac{400}{50} + 4.05\right)$

$$= 12.05.$$

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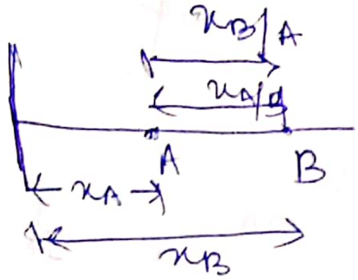
(12) Relative motion

In case of two or three particle in motion, when the motion of body is evaluate w.r.to another moving body, then it is called as relative motion.

Consider two bodies A and B moving along st. line, then relative motion characteristic describe



if $x_A > x_B$

Relative motion $(x_{B/A}) = x_B - x_A$

Relative velocity $= \dfrac{dx_{B/A}}{dt} = V_{B/A}$

$= V_B - V_A$

Relative acceln $= \dfrac{d^2 x_{B/A}}{dt} = \dfrac{dV_{B/A}}{dt} = a_{B/A}$

$= a_B - a_A$

$x_{B/A} = x_B - x_A$ $\quad x_{A/B} = x_A - x_B$

$x_{A/B} = -x_{B/A}$, $v_{A/B} = -v_{B/A}$

$a_{A/B} = -a_{B/A}$

if $v_B > v_A$

$\xrightarrow[V_A]{}$ $\quad \xrightarrow[V_B]{}$ $\quad \xrightarrow{V_{B/A}}$ $\quad \xleftarrow{V_{A/B}}$

$V_A > V_B$ $\quad \xrightarrow{V_A}$ $\quad \xrightarrow{V_B}$ $\quad \xleftarrow{V_{B/A}}$ $\quad \xrightarrow{V_{A/B}}$

$V_B > V_A$, $V_{B/A} = V_B - V_A$

$V_{A/B} = -(V_B - V_A) = V_A - V_B$

$V_A > V_B$

$V_{B/A} = V_B - V_A = -(V_A - V_B)$

$V_{A/B} = V_A - V_B$.

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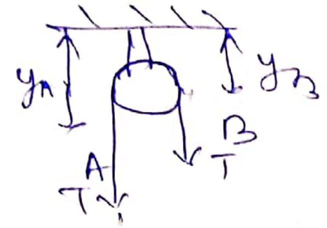
Dependent Relative motion

Here $y_A + y_B = \text{Cost}$

Since length of the string is const.



Here A and B are not independent. So it is dependent Relative motion.

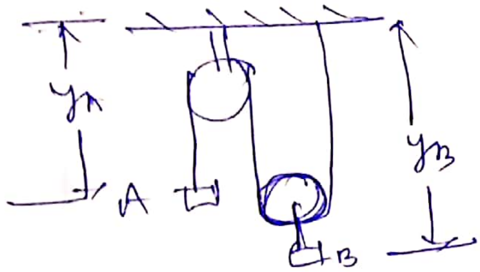
$y_A + y_B = \text{cost}$

$v_A + v_B = 0$

$a_A + a_B = 0$

Here

$(y_A + \Delta y_A) + 2(y_B + \Delta y_B)$
$= \text{const}$



$\Delta y_A + 2\Delta y_B = 0$

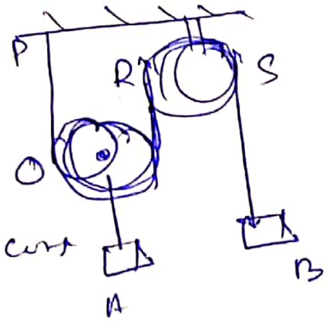
$v_A + 2v_B = 0$

$a_A + 2a_B = 0$

Ex Block A and B are initially rest when $y_A = 5m$ and $y_B = 4m$. Block B accelⁿ downward at const rate of $0.6\ m/s^2$. Determine relative vertical position and velocity of block 2s after the motion begins

Ans

If 'B' move downward A moves upward



$2y_A + y_B = \text{const}$

$2(y_A + \Delta y_A) + (y_B + \Delta y_B) = \text{const}$

$2\Delta y_A + \Delta y_B = 0$

$2v_A + v_B = 0$

$2a_A + a_B = 0$

of $a_B = 0.6\ m/s^2 (\downarrow)$ $\quad a_A = -0.3\ m/s^2$
$= 0.3\ m/s^2\ (\uparrow)$

$v_B = a_B t = 0.6 \times 2 = 1.2\ m/s \downarrow$

$v_A : 0.6\ m/s$

$y_B = y_B|_0 + v_B|_0 t + \frac{1}{2} a_B|_0 t^2$

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$= 4 + 0 + \frac{1}{2} 0.6 \times 4 = 5.2\ m$

$y_A = y_{A/0} + v_{A/0} t + \frac{1}{2} a_{A/0} t^2$

$= 5 + 0 + \frac{1}{2} \times 0.3 \times 4 = 4.4\ m$

$y_{B/A} = 5.2 - 4.4 = 0.8\ m$

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Principle of Dynamics

The relationship between the kind of motion of the particle and the forces producing, so it we take several axioms called Principle of Dynamics

Newton's law of motion

First law :- Every body continues in ~~the~~ its state of rest or uniform motion in a straight line except in so far it may be compelled by force to change that state.

Second law

The acceln of a given particle is proportional to the force applied to its and takes place in the direction of straight line in which force acts.

Third law :- To every action there is always an equal and contrary reaction or the mutual actions of any two bodies are alway equal and opposotely directed.

General equn of motion of Particle

$$f = ma \Rightarrow f = \frac{W}{g} a \qquad \left(\begin{array}{l} W = mg \\ m = \frac{W}{g} \end{array} \right)$$

Differential equn of Rectilinear motion

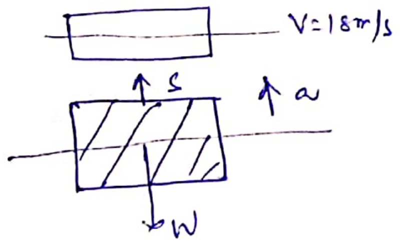
Since $a = \ddot{x}$

So $X = \frac{W}{g}\ddot{x}$

$\ddot{x}$ = acceln.

X = Resultant action of forces.

Ex-1 (7.15) Timosenko and youngs



$W = 4450N$

$v = 18m/s$

initial velocity : $u = 0$

distance travelled : $x = 1.8m$

$$S - W = \frac{W}{g} a$$

$$\Rightarrow S = W + \frac{W}{g}\cdot a \Rightarrow S = W\left(1 + \frac{a}{g}\right)$$

We know $v^2 - u^2 = 2as$

$$\Rightarrow 18^2 - 0^2 = 2a \times 1.8 \qquad \Rightarrow a = 90m/s^2$$

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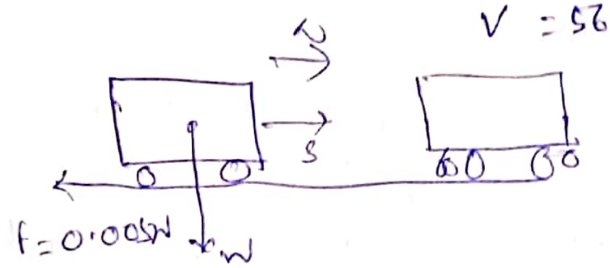
Put the value

$$S = 4450\left(1 + \frac{90}{9.81}\right) = 45275.7\,N$$

Ex. 2 (7.17) (Timoshenko and youngs)

$V = 56$ kmph $= 15.56$ m/s



$$S - f = \frac{W}{g}a$$

$$\Rightarrow S = f + \frac{W}{g}a$$

$$= 0.005W + \frac{Wa}{g}$$

$$v = u + at$$

$$\Rightarrow a = \left(\frac{15.56 - 0}{60}\right) = 0.26\,m/sec^2$$

$$S = W\left(0.005 + \frac{a}{g}\right)$$

$$= 1870\left(0.005 + \frac{0.26}{9.81}\right)$$

$$= 58.9\,kW$$

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D'Alembert's Principle

D'Alembert principle state that the system of external forces acting on a body in motion is in dynamic equilibrium with the inertia force of the body.

The differential equ^n of motion

$$X - m\ddot{x} = 0$$

X = Resultant of all applied force in direction motion.

m = mass of the particle.

This equ^n is known equ^n of dynamic equilibrium

The force is equal to the product of mass of the particle and its accel^o and direction opposite direction is called inertia force

$$-\Sigma m\ddot{x} = -\ddot{x}\Sigma m = -\frac{W}{g}\ddot{x}$$

equ^n of dynamic equilibrium can be expressed as

$$\Sigma x_i + \left(-\frac{W}{g}\ddot{x}\right) = 0$$

where $\left(-\frac{W}{g}\ddot{x}\right)$ is known as Inertia force.

$$F + F_E = 0$$

Initial and non Initial frame of Reference

To locate the position of object we need a frame of reference. The convenient way to set up a reference frame is to choose three mutually perpendicular axis x, y and z to the reference frame.

frames are divided into two.

→ Initial

→ non initial frame

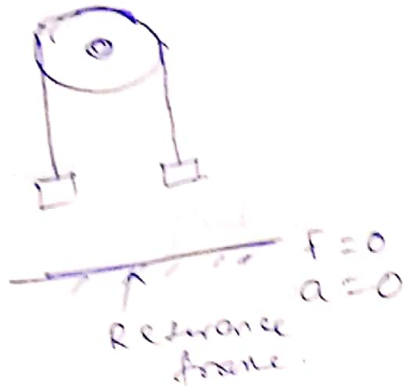
Initial frame

Initial frame of reference is one which is either fixed or moves with a const velocity is known Initial frame

Here f=0, and a=0

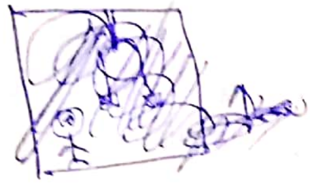
(f = force   a = acceleration)

Ex



Non Initial frame

Non Initial frame or accelarated frame moves with a accel^n a. ~~F=0~~, ~~a≠0~~ $F = 0$, $a \neq 0$



* How D'Alembert principle differ from Newton's second law of motion?

Ans According to Newton's law, for a moving body the resultant force is equal to product of mass and acceleration produced dir^n of force

$f = ma$

f = force, m = mass a = accel^n.

This equ^n of motion written in equilibrium simply by introducing ~~Interia~~ Inertia force in addition to the real force acting on the system.

$f - ma = 0$

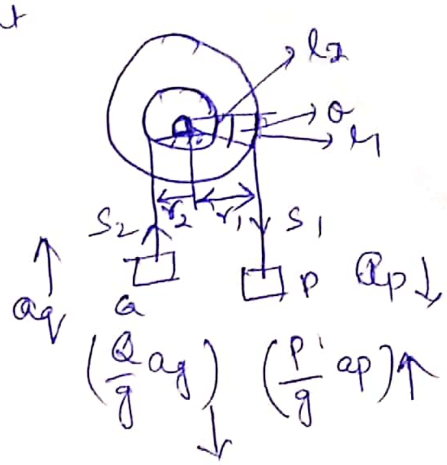
$f + (-ma) = 0$

$\boxed{f + F_I = 0}$

Ex Neglecting friction and inertia of two step pulley shown in the figure, find the acceleration a_p of the falling weight 'p'. Assuming $P = 35.6N$ and $Q = 53.4N$ and $r_1 = 2r_2$

Solⁿ Let accelⁿ of Q weight Q is upward dirⁿ a_q during any time interval t rotates @ θ.

displacement 'p' is downward



$$l_2 = r_2 \theta$$

$$l_1 = r_1 \theta$$

$$\frac{l_1}{l_2} = \frac{r_1 \theta}{r_2 \theta} = \frac{r_1}{r\theta}$$

$$\Rightarrow l_1 r_2 = l_2 r_1$$

Differential w.r.t two time

$$a_p r_2 = a_q r_1$$

$$a_q = \frac{a_p r_2}{r_1} = \frac{a_p}{2}$$

Now apply D'Alembert's principle. Moment about 'O'.

$$\Sigma M_O = 0$$

$$\left(\frac{P}{g} a_p - P\right) \times r_1 + \left(Q + \frac{Q}{g} a_q\right) r_2 = 0$$

$$= P\left(\frac{a_p}{9.81} - 1\right) 2r_2 + Q\left(1 + \frac{a_p}{2 \times 9.81}\right) r_2 = 0$$

$$= P\left(\frac{a_p}{9.81} - 1\right) \times 2 + Q\left(1 + \frac{a_p}{2 \times 9.81}\right) = 0$$

Put $P = 35.61N$, $Q = 53.4N$

$$\Rightarrow a_p = 1.784\ m/s$$

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