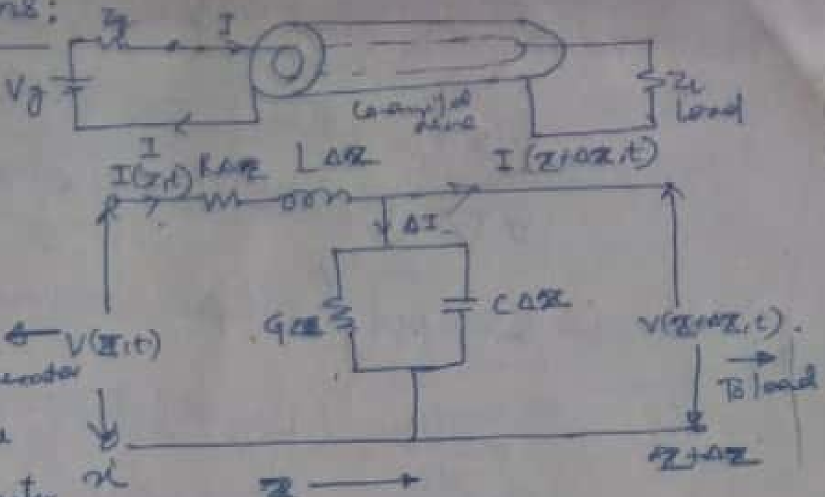
...mission Line Equations:



...n Consider a two cut ...mission line having ...tributed parameters, ...ncremental length (To generator) ...ch a line Δz will have ...nted parameters, representin... ...type Equivalent cut.

... voltages and currents at different points are shown.

...pplying Kirchhoff's law to the Equivalent Cut, we have the voltage Equation,

$$V(z,t) = R\Delta z\, I(z,t) + L\Delta z \frac{\partial I(z,t)}{\partial t} + V(z+\Delta z,t)$$

$$-\left\{\frac{V(z+\Delta z,t) - V(z,t)}{\Delta z}\right\} = R I(z,t) + L\frac{\partial I(z,t)}{\partial t}$$

taking $\Delta z \to 0$

$$\boxed{-\frac{\partial V}{\partial z} = RI + L\frac{\partial I}{\partial t}} \quad \text{--- (1)}$$

...e current Equation is

$$I(z,t) = I(z+\Delta z,t) + \Delta I$$

$$= I(z+\Delta z,t) + G\Delta z\, V(z+\Delta z,t) + C\Delta z \frac{\partial V(z+\Delta z,t)}{\partial t}$$

$$-\frac{I(z+\Delta z) - I(z,t)}{\Delta z} = G V(z+\Delta z,t) + C\frac{\partial V(z+\Delta z,t)}{\partial t}$$

...ng $\Delta z \to 0$

$$-\frac{\partial I(z,t)}{\partial z} = G V(z,t) + C\frac{\partial V(z,t)}{\partial t}$$

$$\boxed{-\frac{\partial I}{\partial z} = GV + C\frac{\partial V}{\partial t}} \quad \text{--- (2)}$$

Now the sol^n of eq^n (1) and (2) can be ~~done~~

$$V(z,t) = \text{Real}[V(z)\, e^{i\omega t}] \quad \text{---} (3)$$

$$I(z,t) = \text{Real}[I(z)\, e^{i\omega t}] \quad \text{---} (4)$$

Now we can apply phasor technique in eq^n (1), (2), we get

$$-\frac{dV}{dz} = (R + i\omega L)\, I \quad \text{---} (5)$$

$$-\frac{dI}{dz} = (G + i\omega C)\, V \quad \text{---} (6)$$

Diff^n w.r.t (5) z, we get

$$\frac{d^2V}{dz^2} = -(R + i\omega L)\frac{dI}{dz}$$

$$\frac{d^2V}{dz^2} = (R + i\omega L) \times (G + i\omega C)\, V$$

$$\boxed{\frac{d^2V}{dz^2} = \gamma^2 V} \quad \text{---} (1) \quad \text{where } \gamma^2 = (R + i\omega L)(G + i\omega C)$$

$$\gamma = \sqrt{(R + i\omega L)(G + i\omega C)} = \text{propagation const.}$$

The sol^n of the above eq^n is given by

$$\boxed{V(z) = A e^{-\gamma z} + B e^{\gamma z}} \quad \text{---} (7)$$

A, and B are two arbitrary const which can be determined from boundary condition

From eq^n (5)

$$I = \frac{-\frac{dV}{dz}}{(R + i\omega L)}$$

$R + i\omega L = Z$ = is the series impedance of the line per unit length

$$I(z) = -\frac{1}{(R + i\omega L)} \times \left(-\gamma A e^{-\gamma z} + B\gamma e^{\gamma z}\right)$$

$(G + i\omega C) = Y$ is the shunt admittance of the line per unit length

$$I(z) = \frac{\gamma}{(R + i\omega L)}\left[A e^{-\gamma z} - B e^{\gamma z}\right]$$

$$I(z) = \frac{\sqrt{(R + i\omega L)(G + i\omega C)}}{R + i\omega L}\left[A e^{-\gamma z} - B e^{\gamma z}\right] = \sqrt{\frac{G + i\omega C}{R + i\omega L}}$$

$$\boxed{I(z) = \frac{1}{Z_0}\left[A e^{-\gamma z} - B e^{\gamma z}\right]} \quad \text{where } Z_0 = \sqrt{\frac{R + i\omega L}{G + i\omega C}}$$

$Z_0 = \sqrt{\frac{j\ldots}{G\ldots}}$ ... impedance in di...

The sol^n for the line voltage and line current represent as the superposition of (+) and (−) waves, i.e. wave propagating in the positive z and negative z dir^n respectively. They are completely analogous to the sol^ns for the electric and magnetic fields in the medium between the conductors of the line.

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

The ch. impedance analogous to intrinsic

The propagation const $\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$

~~But for~~

But for a lossless line i.e. for a line consisting of perfect dielectric medium bet^n the conductors. $G=0$.

$$\gamma = \sqrt{j\omega L \times j\omega C} = j\omega\sqrt{LC}$$

$$\alpha + j\beta = j\omega\sqrt{LC}, \quad \alpha = 0, \quad \beta = \omega\sqrt{LC}.$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{L/C}$$

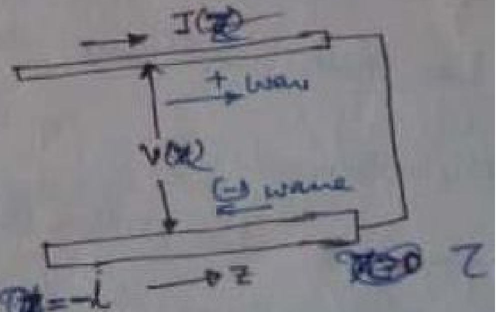
$$\boxed{Z_0 = \sqrt{L/C}}$$

$$V_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

$$\boxed{V_p = \frac{1}{\sqrt{LC}}}$$

Short-Circuited Line and Frequency Behaviour:

Let us now consider a lossless line short-circuited at the far end $z=0$. We shall assume that the line is driven by a voltage generator at frequency ω at the left end $z=-l$ so that waves are set up on the lines. The short ckt at



The voltage across the short ckt has to be zero.

Thus $V(0) = 0$.

Applying boundary cond^n to the general sol^n for V

$$V = Ae^{-j\beta z} + Be^{+j\beta z}$$

$V(0) = Ae^{-j\beta 0} + Be^{j\beta 0}$, $A+B=0$

$B=-A$, so we found that short cut gives rise to a (-) r wave whose voltage is exactly the neg of the (+)ve incident wave voltage at short ckt

$V(z) = Ae^{-j\beta z} - Ae^{+j\beta z} = -2jA\sin\beta z$

$I(z) = \frac{1}{Z_0}(Ae^{-j\beta z} + Ae^{j\beta z}) = \frac{2A}{Z_0}\cos\beta z$

The real voltage and current are then given by

$v(z,t) = \text{Real}[V(z)e^{j\omega t}] = \text{Real}[-2jA\sin\beta z \cdot e^{j\omega t}]$

$= \text{Real}[2e^{-j\pi/2}Ae^{j\theta}\sin\beta z \cdot e^{j\omega t}]$ $\cos(\omega t+\theta$ $\sin($

$$\boxed{v(z,t) = 2A\sin\beta z\sin(\omega t+\theta)} \quad (1)$$

$I(z,t) = \text{Real}[I(z)e^{j\omega t}] = \text{Real}\left[\frac{2Ae^{j\theta}}{Z_0}\cos\beta z\, e^{j\omega t}\right]$

$$\boxed{I(z,t) = \frac{2A}{Z_0}\cos\beta z\cos(\omega t+\theta)} \quad (2)$$

when we cont $A = Ae^{j\theta}$ $-j = e^{-j\pi/2}$

The instantaneous power flow is given by

$P(z,t) = v(z,t)\,I(z,t)$

$= \frac{4A^2}{Z_0}\sin\beta z\cos\beta z\sin(\omega t+\theta)\cdot\cos(\omega t+\theta)$

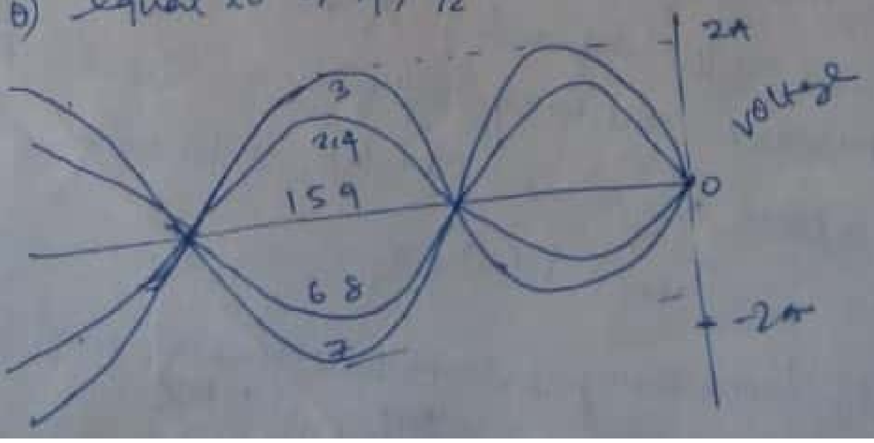
$$\boxed{P(z,t) = \frac{A^2}{Z_0}\sin 2\beta z\cdot\sin 2(\omega t+\theta)} \quad (3)$$

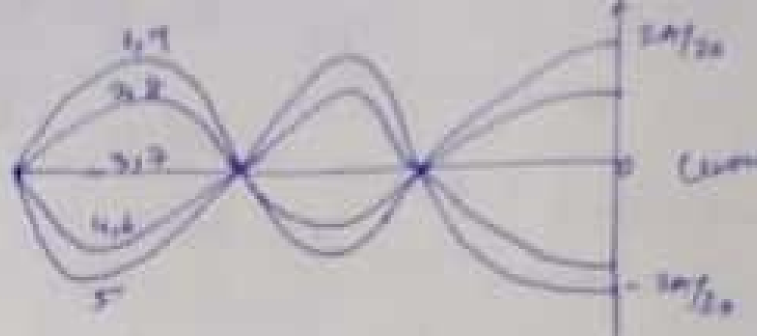
Eqn (1), (2), (3) are the voltage, current and power flow on the short-circuited line, which shows the variation of each of the quantities with distance from the short ckt for several values as shown ~~of different values of~~ besides the curve

The numbers 1, 2, 3 .... 9 besides the curves corresponding to values represent the order of the curves

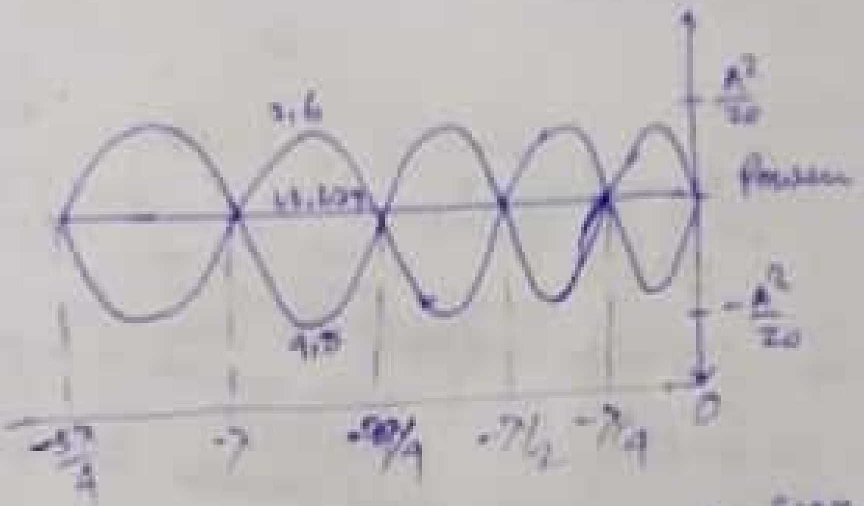
of $(\omega t+\theta)$ equal to $0, \pi/4, \pi/2 \ldots\ldots 2\pi$.

1, 2, 3, 4, 5
0 $\pi/4$ $\pi/2$ $3\pi/4$
$\omega t+\theta =$
$\omega t+\theta =$
$\omega t+\theta =$
1, 5, 9 →





The variation of voltage, current and power flow associated with a standing wave on a short-circuited transmission line



(1) The line voltage amplitude is zero for $\sin\beta z = 0$, $\beta z = 0, \pi, 2\pi \ldots$
$\Rightarrow \beta z = -m\pi$, $m = 1,2,3 \ldots$ $z = -\frac{m\lambda}{2}$, $m = 1,2,3$
ie multiples of $\lambda/2$ from the short ckt.

(2) The line current amplitude is zero for values of z given by
$\cos\beta z = 0$, $\beta z = \pi/2, 3\pi/2, 5\pi/2 \ldots$ $\beta z = -(2m+1)\frac{\pi}{2}$, $m = 0,1,2,3 \ldots$
$z = -(2m+1)\lambda/4$, $m = 0,1,2 \ldots$ ie the odd multiples of $\lambda/4$
from short ckt.

(3) The power flow amplitude is zero for values z given by
$\sin 2\beta z = 0$, $\beta z = -\frac{m\pi}{2}$, $m = 1,2,3 \ldots$, $z = -\frac{m\lambda}{4}$, $m = 1,2,3$
ie multiples of $\lambda/4$ from the short ckt.

Thus, the phenomenon on the short-circuited line is one in which the voltage, current and power flow oscillate sinusoidally with time with different amplitudes at different locations on the line unlike in the case of traveling waves in which a given point on the waveform progresses in distance with time

Since the ... these waves are known as standing ...

Now we can find out input impedance of the short circuited line of length l by taking the ratio of the complex line voltage to the complex line current at the input $z = -l$. Then

$$Z_{in} = \frac{V(-l)}{I(-l)} = \frac{-2jA\sin\beta(-l)}{\frac{2A}{Z_0}\cos\beta(-l)} = jZ_0\tan\beta l = jX_{is}$$

$$Z_{in} = jZ_0\tan\beta l$$

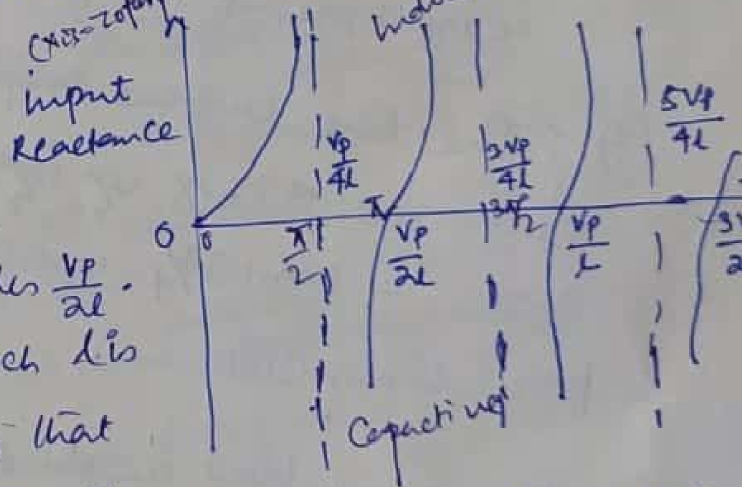
The value of $\tan\beta l$ can vary from $-\infty$ to ∞, so the input impedance of a lossless line can also be either purely inductive or purely capacitive depending on the value of βl.

When the length of a short cuted line is very short, $\beta l \ll 1$, we have input impedance $Z_{in} = jX_{is} = jZ_0\beta l = j\sqrt{\frac{L}{C}}\,\omega\sqrt{LC}\,l = j\omega Ll$ which is the impedance of Ll henries.

$$\boxed{Z_{in} = jZ_0\tan\frac{2\pi f}{v_p}l}$$

Hence the input impedance of the short circuit line is purely reactive.

As the frequency is varied from a low value upward, the input reactance changes from inductive to capacitive and back to inductive and so on.



"Variation of the input reactance of a short circuited transmission line with frequency"

The input reactance is zero for values of frequency equal to multiples $\frac{v_p}{2l}$. These are the frequencies for which l is equal to multiples of $\lambda/2$ so that the line voltage is zero at the input and hence the input sees a short cut.

The input reactance is infinity for values of frequency equal to odd multiples of $\frac{v_p}{4l}$. These are the frequencies for which l is equal to odd multiples of $\frac{\lambda}{4}$ so that the lines current is zero at the input and hence the input sees as open cut.

$$\alpha = \sqrt{RG}$$
$$\beta = \omega\sqrt{LC}$$

(*b*) Phase velocity $v = \dfrac{\omega}{\beta} = \dfrac{1}{\sqrt{LC}} = f\lambda$

(*c*) Characteristic impedance

$$Z_0 = R_0 + jX_0 = \sqrt{\frac{R\,(1 + j\omega L/R)}{G\,(1 + j\omega C/G)}}$$
$$= \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}$$
$$R_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}, \quad X_0 = 0$$

The phase velocity is independent of frequenc
nstant β linearly depends on frequency signals wil
e have λ and v independent of frequency.

The distortionless line has non-vanishing atte
s constant velocity and constant real characteristic
ss line). A lossless line is also distortionless line
ne is not necessarily lossless. I
ave lossl

... transmission line is said to be lossless if both its conductor and dielectric are lossless, $R = 0$, $G = 0$. For such a line

(a) Propagation constant : $\gamma = \alpha + j\beta = j\omega\sqrt{LC}$

$$\alpha = 0 \qquad (9.63)$$

$$\beta = \omega\sqrt{LC} \qquad (9.64)$$

(b) Phase velocity $\quad v = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} \qquad (9.65$

(c) Characteristic impedance :

$$Z_0 = R_0 + jX_0 = \sqrt{\frac{L}{C}}$$

$$Z_0 = R_0 = \sqrt{\frac{L}{C}} \qquad (9.6$$

$$X_0 = 0 \qquad (9.6$$

2. Distortionless line ($R/L = G/C$)

A distortionless line is one in which the attenuation constant α is frequency independent while the phase constant β is linearly depende on frequency. For a distortionless line, the condition applicable is

$$\boxed{\frac{R}{L} = \frac{G}{C}} \qquad (9.$$

for both γ and Z_0.

(a) Propagation constant :

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)\left(\frac{RC}{L} + j\omega C\right)}$$

$$\gamma = \sqrt{\frac{C}{L}}\,(R + j\omega L)$$

$$\alpha = R\sqrt{\frac{C}{L}} \qquad ($$

$$\beta = \omega\sqrt{LC} \qquad ($$

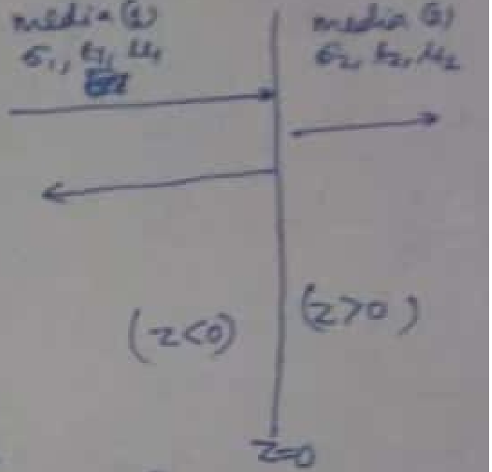
α does not depend on frequency and β is linear function of frequenc

or $$\gamma = \sqrt{RG\left(1 + \frac{j\omega L}{R}\right)\left(1 + \frac{j\omega C}{G}\right)}$$

$$= \sqrt{RG}\left(1 + \frac{j\omega C}{G}\right) = \alpha + j\beta$$

Reflection and Transmission of Uniform plane wave (Normal incidence)

Let us consider that a wave is incident on a boundary bet^n two different media, a reflected wave is produced, and if the second medium is not perfect conductor, a transmitted wave is set up.



Let for a uniform plane wave, the media '1' and media '2' are characterized by $\sigma_1, \mu_1, \epsilon_1$ and $\sigma_2, \mu_2, \epsilon_2$ respectively.

For media '1'. $\eta_1 = \sqrt{\dfrac{j\omega\mu_1}{\sigma_1 + j\omega\epsilon_1}}$, $\gamma_1 = \sqrt{j\omega\mu_1(\sigma_1 + j\omega\epsilon_1)}$

and media '2' $\eta_2 = \sqrt{\dfrac{j\omega\mu_2}{\sigma_2 + j\omega\epsilon_2}}$, $\gamma_2 = \sqrt{j\omega\mu_2(\sigma_2 + j\omega\epsilon_2)}$

Here the electric field to be x dir^n and the magnetic to be in the y dir^n. then

E_i = electric field intensity of the incident
E_r = " " of the reflected
E_t = " " of the transmitted

$$E_i = \eta_1 H_i \quad (1)$$
$$E_r = -\eta_1 H_r \quad (2)$$
$$E_t = \eta_2 H_t \quad (3)$$

Since the total field in media '1' consists of both incident and reflected wave, whereas media '2' has only transmitted wave.

$$E_i + E_r = E_t$$

$$H_t = \frac{E_t}{\eta_2} = \frac{1}{\eta_2}[E_i + E_r]$$

$$H_i + H_r = \frac{1}{\eta_2}[E_i + E_r]$$

$$\frac{E_i}{\eta_1} - \frac{E_r}{\eta_1} = \frac{1}{\eta_2}[E_i + E_r] \qquad \left[\frac{1}{\eta_1} - \frac{1}{\eta_2}\right]E_i = \left[\frac{1}{\eta_1} + \frac{1}{\eta_2}\right]E_r$$

$$\frac{\eta_2 - \eta_1}{\eta_1\eta_2} E_i = \frac{\eta_1+\eta_2}{\eta_1\eta_2} E_r \qquad \boxed{\frac{E_r}{E_i} = \frac{\eta_2-\eta_1}{\eta_2+\eta_1}} \text{ --- } (4)$$

$$\boxed{T = \frac{E_r}{E_i} = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = \text{Reflection Co-efficient}}$$

But we know that
$$E_i + E_r = E_t$$
$$1 + \frac{E_r}{E_i} = \frac{E_t}{E_i}$$

$$\frac{E_t}{E_i} = 1 + \frac{E_r}{E_i} = 1 + \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = \frac{2\eta_2}{\eta_1+\eta_2}$$

$$\boxed{\frac{E_t}{E_i} = \frac{2\eta_2}{\eta_1+\eta_2}} = \tau = \text{Transmission Co-efficient}$$

$$\tau = \frac{E_t}{E_i} = \frac{2\eta_2}{\eta_1+\eta_2} \qquad \boxed{\tau = 1+}$$

From (1), (2),
$$\frac{E_r}{E_i} = -\frac{H_r}{H_i} \qquad \frac{H_r}{H_i} = -\frac{E_r}{E_i} = -\left[\frac{\eta_2-\eta_1}{\eta_2+\eta_1}\right] = \frac{\eta_1-\eta_2}{\eta_1+\eta_2}$$

(1), (3)
$$\frac{E_t}{E_i} = \frac{\eta_2 H_t}{\eta_1 H_i}$$

$$\frac{H_t}{H_i} = \frac{\eta_1}{\eta_2}\frac{E_t}{E_i} = \frac{\eta_1}{\eta_2} \times \frac{2\eta_2}{\eta_1+\eta_2}$$

$$\boxed{\frac{H_t}{H_i} = \frac{2\eta_1}{\eta_1+\eta_2}}$$

1. If $\eta_1 = \eta_2$, $T = 0$, $\tau = 1$. The incident wave is entirely transmitted. This type of situation occurs when the two media have the same value of the material parameter.
2. If $\sigma_1 = \sigma_2 = 0$, perfect dielectric (both media), both media have same permeability (μ), then

$$T = \frac{\sqrt{\mu/\epsilon_2} - \sqrt{\mu/\epsilon_1}}{\sqrt{\mu/\epsilon_2} + \sqrt{\mu/\epsilon_1}} = \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}} \qquad \boxed{\tau = \frac{2\sqrt{\epsilon_1}}{\sqrt{\epsilon_1}+\sqrt{\epsilon_2}}}$$