MODULE-4

- Stress-strain analysis in 2D and 3D
- Failure criteria of Intact Rock and Rock Mass
- Effect of anisotropic behavior of rock

(Intact Rock) CHILE

(i) C = Continous
(ii) H = Homogenous Material
(iii) I = Isotropic
(same physical property in ~~all direction~~ 3 axis)
(iv) Ex: Abrasion, electrical conductivity, poisson's Ratio.
(v) LE = linear Elastic

(Rock Mass) DIANE

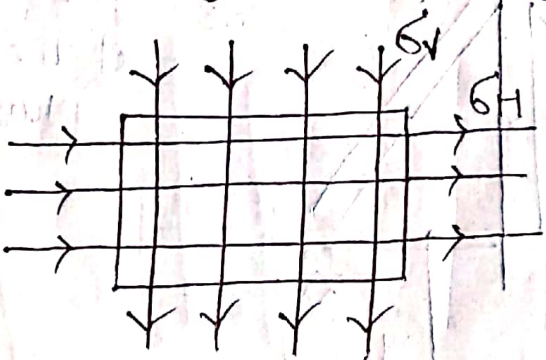
(i) D = Discontinous
(ii) I = In-homogenous Rock
(iii) A = Anisotropic
(physical properties changes in all direction)
(iv) NE = Non elastic

2D stress

Insitu stress
- pre excavation or pre mining stress

Induced stress

(Insitu stress are the stress which developed due to weight of overlying strata.)



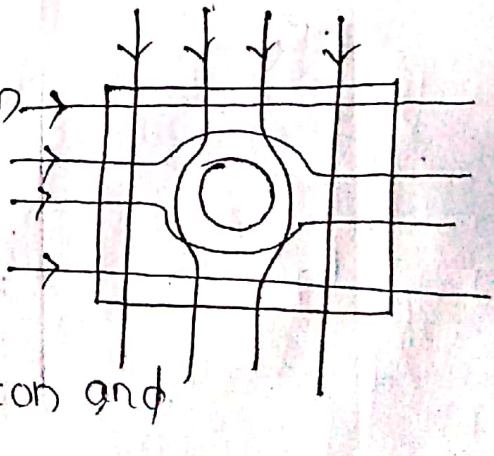
$\frac{\sigma_V}{\sigma_H} = K = \frac{\sigma H}{\sigma V}$

At greater depth $\frac{\sigma H}{\sigma HV} > 1$

$\Rightarrow \frac{\sigma H}{\sigma V} > 1$

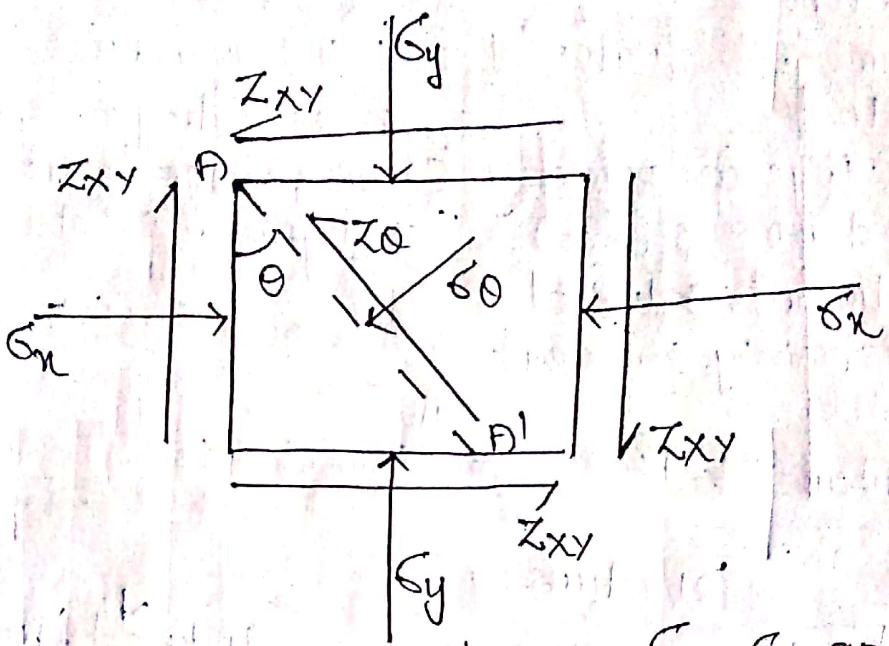
$\boxed{\sigma H > \sigma V}$

Induced stress

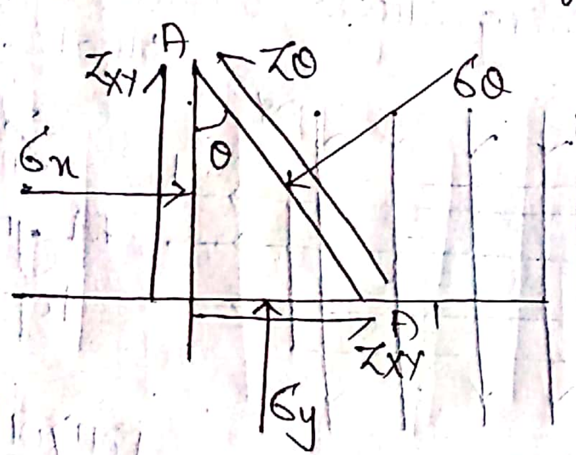


(i) An excavation is made in rock mass, deformation occurs around the excavated boundaries and the stresses are redistributed based on geometry of the excavation and properties of the rock.
(ii) In Actually in practical condition, stresses act or failure are in 3D form like slope stability analysis, tunneling etc.

(iii) But due to simplicity of analysis we reduced the 3D form into 2D form,

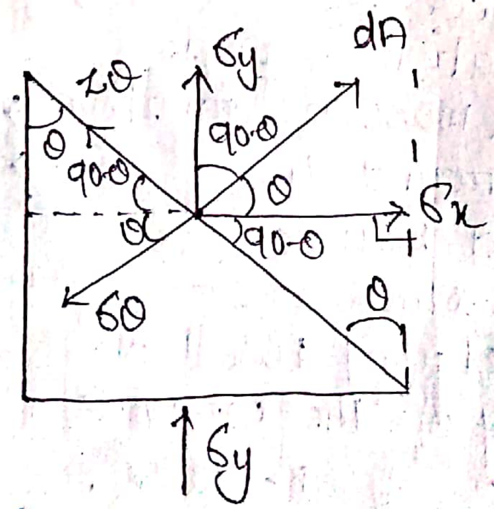


Let assume the stresses σ_x, σ_y and τ_{xy} are the non stress and acting on a small element as shown in figure.



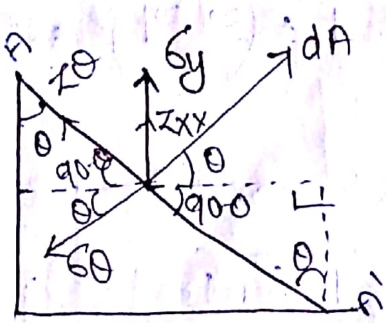
Let dA = Area of the failure plane

$\Sigma F_x =$



$$\Sigma F_x = \sigma_x dA\cos\theta + \tau_{xy} dA\cos\theta - \sigma_\theta dA\cos\theta - \tau_\theta dA\cos(90-\theta)$$

$$= \sigma_x dA\cos\theta + \tau_{xy} dA\cos\theta - \sigma_\theta dA\cos\theta - \tau_\theta dA\sin\theta \longrightarrow (A)$$



$$\Sigma F_y = \sigma_y dA\sin\theta + \tau_{xy} dA\sin\theta - \sigma_\theta dA\cos(90-\theta) + \tau_\theta dA\sin(90-\theta)$$

$$= \sigma_y dA\sin\theta + \tau_{xy} dA\sin\theta - \sigma_\theta dA\overset{\sin}{\cancel{\cos}}\theta + \tau_\theta dA\cos\theta \longrightarrow (B)$$

By rearranging equation (A) and (B) by trignometric operation we will get

$$\sigma_\theta = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta \longrightarrow (1)$$

$$\tau_\theta = \left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta - \tau_{xy}\cos 2\theta \longrightarrow (2)$$

σ_θ and τ_θ act on an arbitrary plane AA'.

It is also clear that σ_θ and τ_θ are periodic after θ exceeds $180°$.

This can be verified by substituting θ with $(\theta+180°)$ in eqn (1) and (2).

The normal and shear stress on the perpendicular plane AA' can be evaluated by substituting θ with ~~θ+9~~ $(\theta+90°)$ in eqn (1) and (2).

$$\sigma_\theta = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\therefore \sigma_{\theta+90} = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos(180+2\theta) + \tau_{xy}\sin(180+2\theta)$$

$$= \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)(-\cos 2\theta) - \tau_{xy}\sin 2\theta$$

$$\Rightarrow \sigma_{\theta+90} = \left(\frac{\sigma_x+\sigma_y}{2}\right) \cancel{+} - \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$$

Similarly,

$$\tau_\theta = \left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta - \tau_{xy}\cos 2\theta$$

$$\Rightarrow \tau_{\theta+90} = \left(\frac{\sigma_x-\sigma_y}{2}\right)\sin(180+2\theta) - \tau_{xy}\cos(180+2\theta)$$

$$\Rightarrow \tau_{\theta+90} = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

Now,

$$\sigma_\theta + \sigma_{\theta+90} = \sigma_x+\sigma_y = \text{Constant}$$

$$\Rightarrow \boxed{\sigma_1 + \sigma_2 = \sigma_x + \sigma_y}$$

Here σ_1 and σ_2 are known as principal stress.

Principal plane

(i) The plane which have no shear stress are called principal plane.

(ii) These plane carry only normal stress.

Principal stress

(i) The stress acting on the principal plane is called principal stress.

(ii) The plane carrying the max^m normal stress is called major principal plane and the corresponding principal stress is called major principal stress.

(iii) The plane carrying minimum normal stress is called minor principal plane and the corresponding stress is called minimum minor principal stress.

We know,

$$\boxed{\sigma_\theta = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta} \rightarrow (1)$$

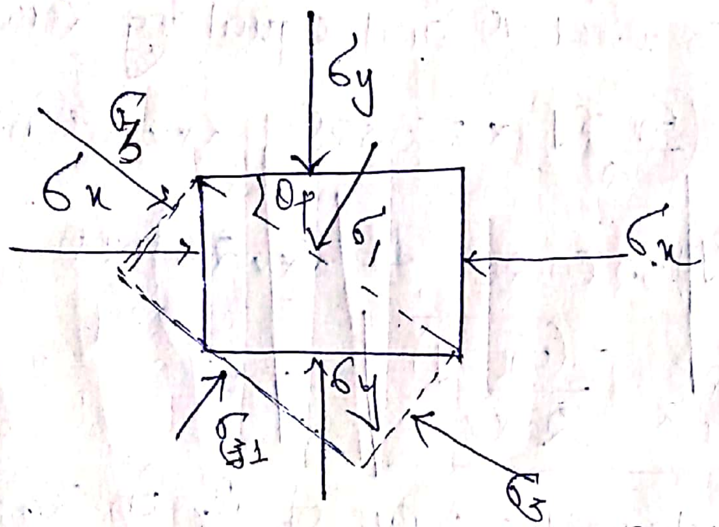
$$\boxed{\tau_\theta = \left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta - \tau_{xy}\cos 2\theta} \rightarrow (2)$$

We know for principal plane $\tau_\theta = 0$,

$$\Rightarrow \left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta = \tau_{xy}\cos 2\theta$$

$$\Rightarrow \boxed{\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x-\sigma_y}}$$

Where θ_p is the angle between horizontal axis and principal stress direction.



Here θ_p takes two values which are 90° apart. These two values represents the orientation of major and minor principal stresses.

Substituting the value of θ_p in eqn (1) we get

$$\boxed{\sigma_1 = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + (\tau_{xy})^2}}$$

$$\boxed{\sigma_3 = \left(\frac{\sigma_x+\sigma_y}{2}\right) - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + (\tau_{xy})^2}}$$

Overall principal stress

$$\boxed{\sigma_{1,3} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}}$$

σ_1 = Major principal stress
σ_3 = Minor principal stress

Maximum principal shear stress

We know,

$$\boxed{\tau_\theta = \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta - \tau_{xy}\cos 2\theta}$$

Isotropic rock specimen, failure due to compression loading would occur along max^m principal shear stress.

Diff. τ_θ w.r.t θ and equating zero, we get

$$\frac{d\tau_\theta}{d\theta} = \left(\frac{\sigma_x - \sigma_y}{2}\right) \times 2\cos 2\theta + \tau_{xy}\, 2\sin 2\theta$$

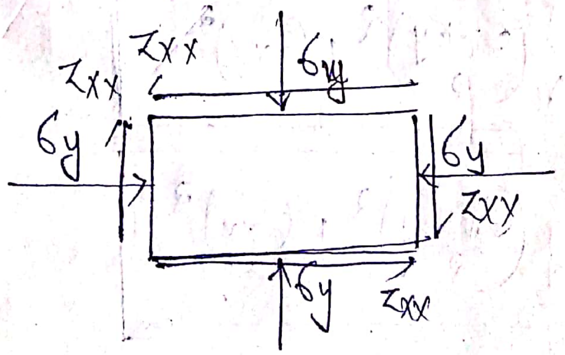
$$\Rightarrow \left(\frac{\sigma_x - \sigma_y}{2}\right) \times 2\cos 2\theta = -\tau_{xy}\, 2\sin 2\theta$$

$$\Rightarrow \boxed{\tan 2\theta_\tau = \frac{-(\sigma_x - \sigma_y)}{2\tau_{xy}}} \rightarrow (i)$$

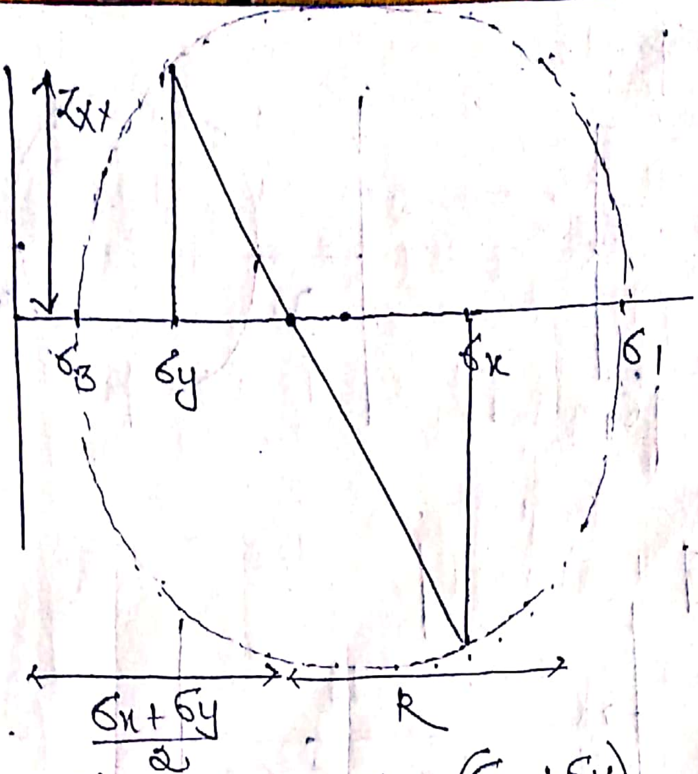
Substituting the value of θ_τ in τ_θ we get,

$$\boxed{\tau_{\theta_{max}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}}$$

*



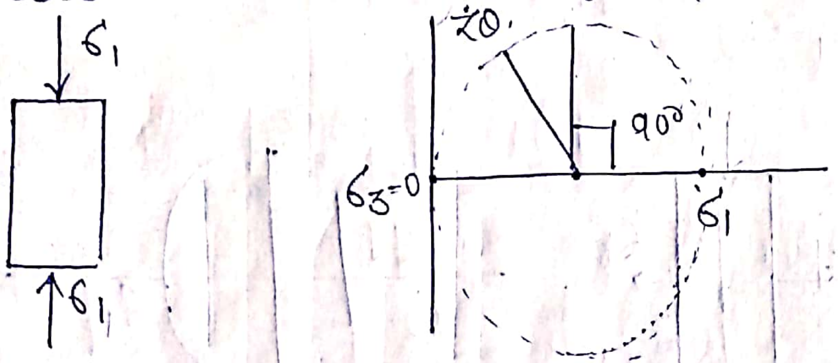
Let ($\sigma_x > \sigma_y$)

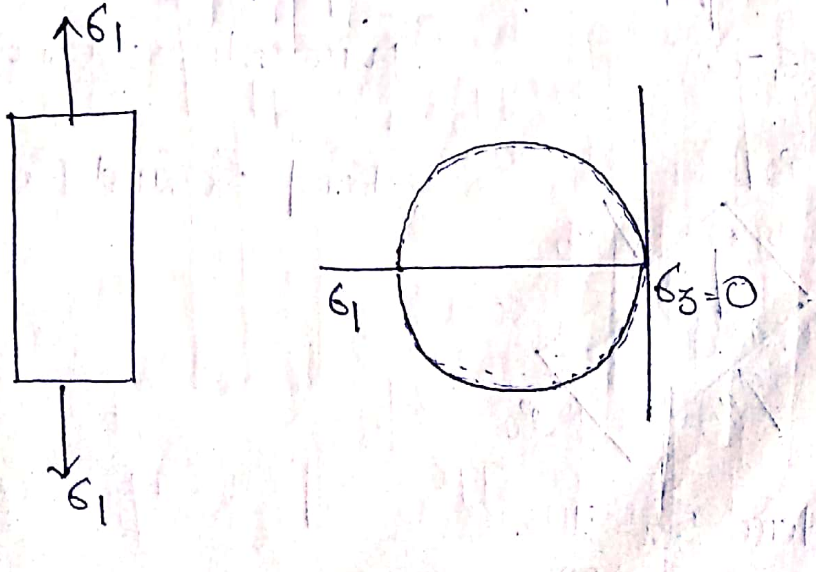


$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + R \qquad \sigma_3 = \left(\frac{\sigma_x + \sigma_y}{2}\right) - R$$

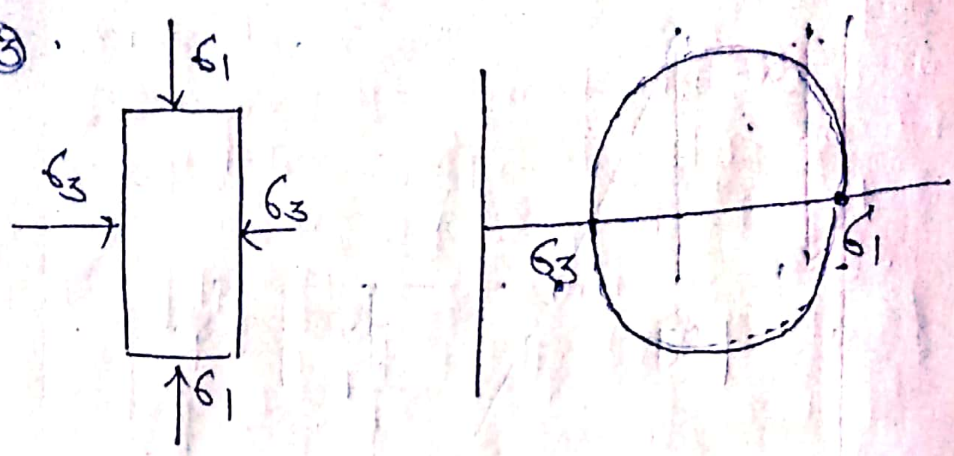
Special stress condn.

$\tau_0 = max$

1 

2 

(3)



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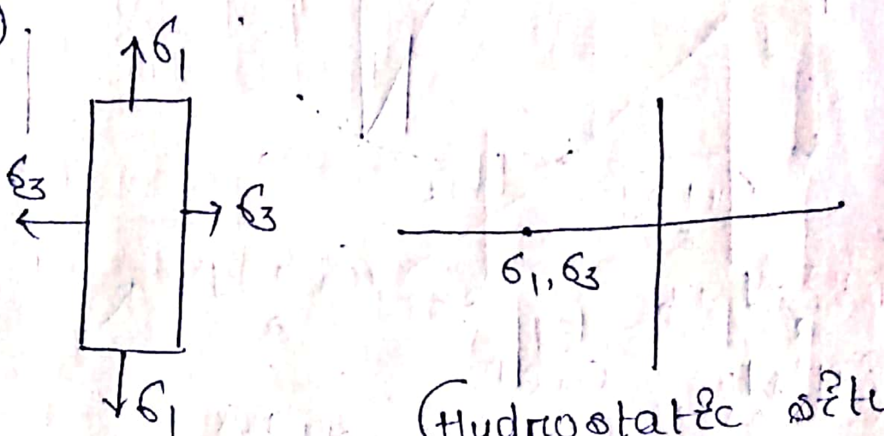
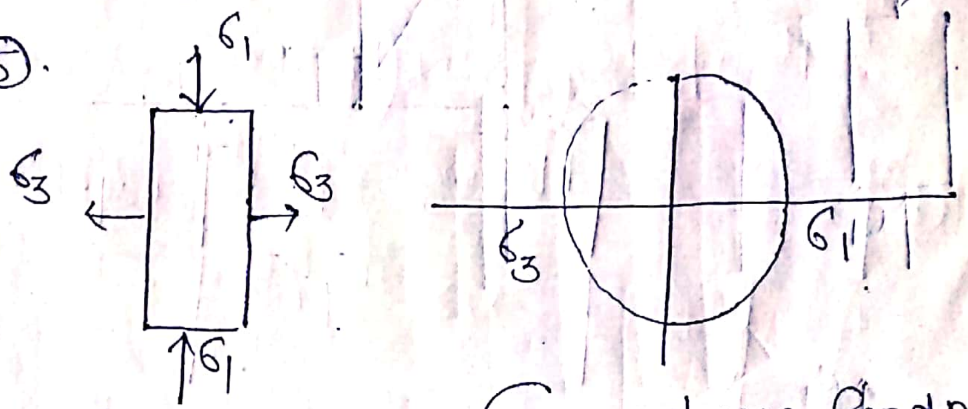


$(\sigma_1 = \sigma_3)$

(Hydrostatic situation)

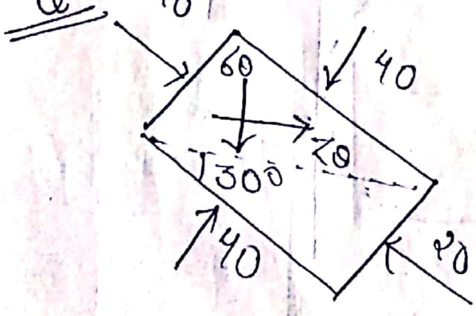
(5)



$(\sigma_1 = \sigma_3)$

(pure stress condn)

Q



Find σ_θ and τ_θ?

Ans Here $\sigma_x = 40$

$\sigma_y = 20.$

$\theta = 30^0$

$$\sigma_\theta = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta + \tau_{yx}\sin 2\theta$$

$$= \left(\frac{40+20}{2}\right) + \left(\frac{40-20}{2}\right)\cos 60 + 0$$

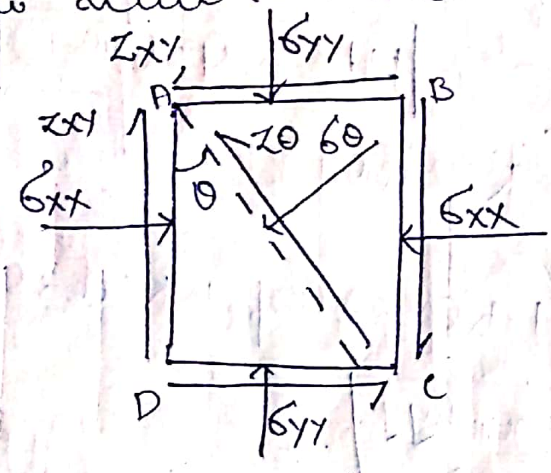
$$= 35$$

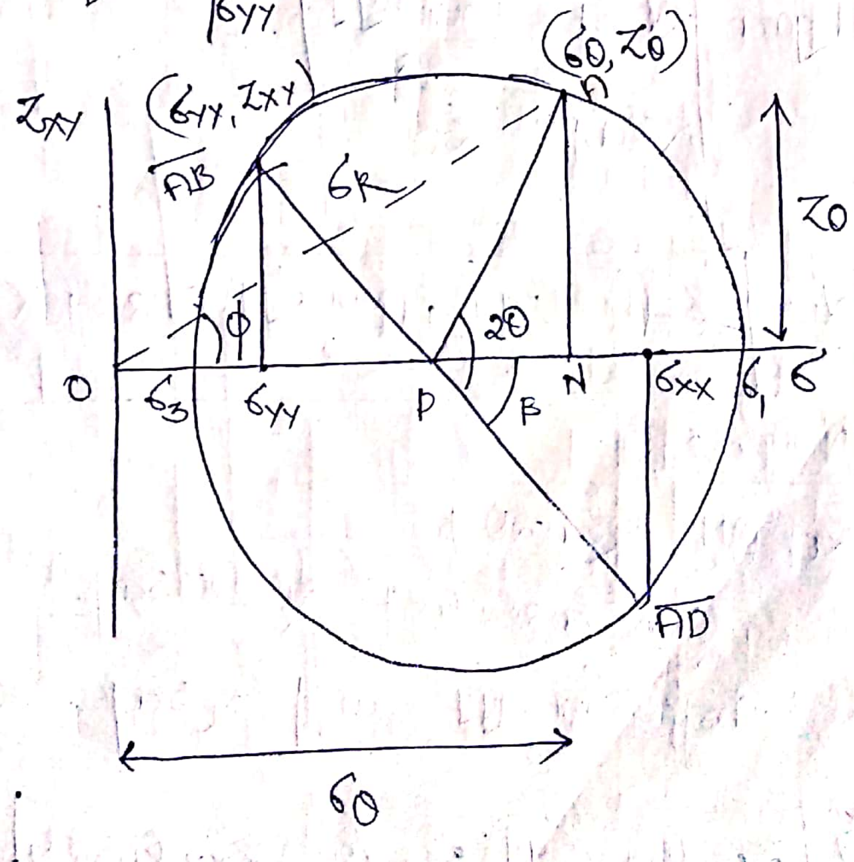
$$\tau_\theta = \left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta - \tau_{xy}\cos 2\theta$$

$$= \left(\frac{40-20}{2}\right)\sin 60 - 0$$

$$= 8.66$$

Mohr Columb (MC) - Graphical solution





If we examine eqn (i) and (ii) we see that this is the same eqn which we have already mathematically derived.

Now Max^m shear stress $= R = \frac{\sigma_1 - \sigma_3}{2}$

$$= \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + (\tau_{xy})^2}$$

$\sigma_{1,3}$ $R = OP \pm R$

$$= \frac{\sigma_{xx} + \sigma_{yy}}{2} \pm \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + (\tau_{xy})^2}$$

Q The vertical stress and horizontal stress on the middle of the coal pillar is found to be 15 Mpa and 3 Mpa respectively. The shear stress is 2.5 Mpa as shown in fig. Determine the principal stresses and its dirn. Estimate the max^m shear stress and its direction?

Ans Given $\sigma_{xx} = 15$ Mpa $\tau_{xy} = 2.5$ Mpa

$\sigma_{yy} = 3$ Mpa

We know, $\sigma_1 = \left(\frac{\sigma_{xx} + \sigma_{yy}}{2}\right) + \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + (\tau_{xy})^2}$

$$= \left(\frac{15+3}{2}\right) + \sqrt{\left(\frac{15-3}{2}\right)^2 + (2.5)^2}$$

$= 15.5$ Mpa

$$\sigma_3 = \left(\frac{\sigma_{xx} + \sigma_{yy}}{2}\right) \mp \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + (\tau_{xy})^2}$$

$$= \left(\frac{15+3}{2}\right) - \sqrt{\left(\frac{15-3}{2}\right)^2 + (2.5)^2}$$

$= 2.5$ Mpa

We know,

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_{xx}-\sigma_{yy}}$$

$$\Rightarrow \theta_p = \frac{1}{2}\tan^{-1}\left(\frac{2\times 2.5}{15-3}\right)$$

$\Rightarrow \boxed{\theta_p = 11^0 18'}$ Direction of principal stress.

$$\tau_{max} = \sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + (\tau_{xy})^2} \text{ or } \frac{\sigma_1-\sigma_2}{2}$$

$$= \sqrt{\left(\frac{15-3}{2}\right)^2 + (2.5)^2} \qquad = \frac{15.5-2.5}{2}$$

$$= 6.5 \text{ Mpa} \qquad = 6.5 \text{ Mpa}$$

Max^m shear stress will act at an angle

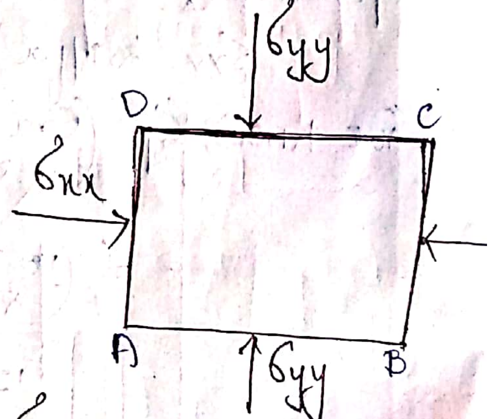
Relationship between 2D stress and strain

Consider a 2D figure ABCD, subjected to two mutually perpendicular stress σ_{xx} and σ_{yy}.

Where,

σ_{xx} = Normal stress in x-dir^n

σ_{yy} = Normal stress in Y-dir^n



Along σ_{xx} dir^n

Longitudinal strain $= \frac{\sigma_{xx}}{E}$ in X dir^n

Lateral strain in Y dir^n $= -\mu\frac{\sigma_{xx}}{E}$

Along σ_{yy} dir^n

Longitudinal strain in Y dir^n $= \frac{\sigma_{yy}}{E}$

Lateral strain in X dir^n $= -\mu\frac{\sigma_{yy}}{E}$

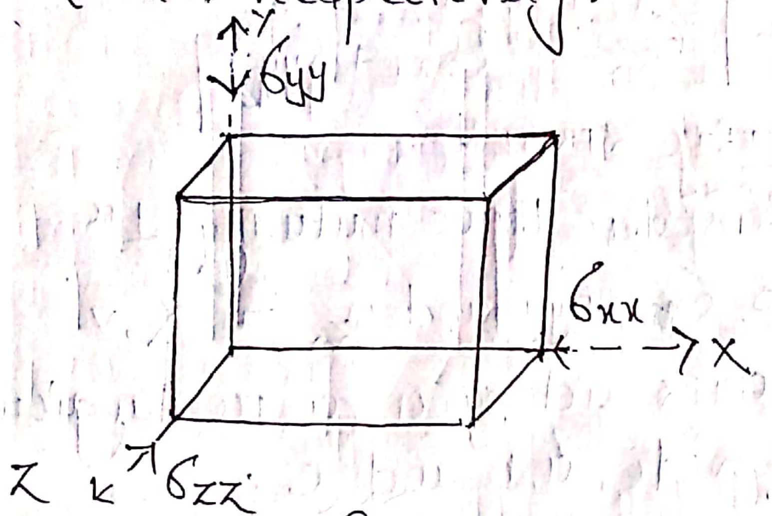
Let σ_{xx} and ε_y be the nol strain along x axis and y axis.

$$\therefore \varepsilon_x = \frac{\sigma_{xx}}{E} - \mu \frac{\sigma_{yy}}{E} = \frac{1}{E}[\sigma_{xx} - \mu \sigma_{yy}]$$

$$\varepsilon_y = \frac{\sigma_{yy}}{E} - \mu \frac{\sigma_{xx}}{E} = \frac{1}{E}[\sigma_{yy} - \mu(\sigma_{xx})]$$

Relationship between 3D stress and strain

Consider a three dimensional body subjected to three mutually perpendicular stresses σ_{xx}, σ_{yy} and σ_{zz} in X, Y and Z axis respectively.



Along σ_{xx} dirn

Longitudinal strain in X axis $= \frac{\sigma_{xx}}{E}$

Lateral strain in Y and Z axis

$$= -\mu \frac{\sigma_{yy}}{E}, -\mu \frac{\sigma_{zz}}{E}$$

Along σ_{yy} dirn

Longitudinal strain in Y axis $= \frac{\sigma_{yy}}{E}$

Lateral strain in Z and X axis $= -\mu \frac{\sigma_{zz}}{E}, -\mu \frac{\sigma_{xx}}{E}$

Along σ_{zz} direction in Z-direction

Longitudinal strain $= \frac{\sigma_{zz}}{E}$

Lateral strain $= -\mu \frac{\sigma_{xx}}{E}$, $-\mu \frac{\sigma_{yy}}{E}$ in x and y direction

Let ϵ_x, ϵ_y and ϵ_z be the net strain in x, y and z direction,

$$\epsilon_x = \frac{1}{E}\left[\sigma_{xx} - \mu(\sigma_{yy}+\sigma_{zz})\right]$$

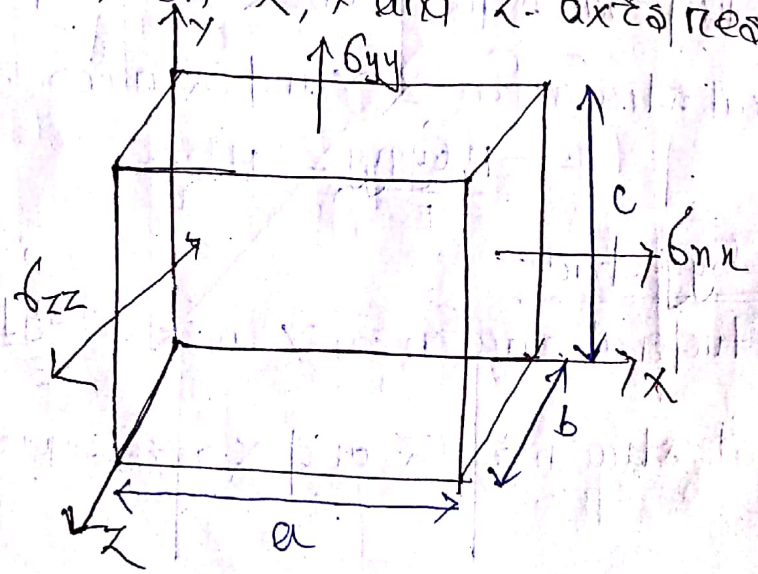
$$\epsilon_y = \frac{1}{E}\left[\sigma_{yy} - \mu(\sigma_{zz}+\sigma_{xx})\right]$$

$$\epsilon_z = \frac{1}{E}\left[\sigma_{zz} - \mu(\sigma_{xx}+\sigma_{yy})\right]$$

VOLUMETRIC STRAIN

Let us consider three mutually $\perp^r$ stresses σ_{xx}, σ_{yy} and σ_{zz} in x, y and z axis acting on a rectangular body of side a, b and c.

Let ϵ_x, ϵ_y and ϵ_z be the net strain in x, y and z axis respectively.



We know, $V = abc$

partially derivative on both sides.

$$dV = d(a)\,bc + ac\,d(b) + ab\,d(c)$$

Dividing V on both sides,

$$\frac{dV}{V} = \frac{bc\,d(a)}{V} + \frac{ac\,d(b)}{V} + \frac{ab\,d(c)}{V}$$

$$\Rightarrow \epsilon_V = \frac{bc\,d(a)}{abc} + \frac{ac\,d(b)}{abc} + \frac{ab\,d(c)}{abc}$$

$$\Rightarrow \epsilon_V = \frac{d(a)}{a} + \frac{d(b)}{b} + \frac{d(c)}{c}$$

$$\Rightarrow \boxed{\epsilon_V = \epsilon_x + \epsilon_y + \epsilon_z} \rightarrow (i)$$

Now substituting the value of ϵ_x, ϵ_y and ϵ_z on eqn (i) we get

$$\Rightarrow \epsilon_V = \frac{1}{E}\left[\sigma_{xx} - \mu(\sigma_{yy} + \sigma_{zz})\right] + \frac{1}{E}\left[\sigma_{yy} - \mu(\sigma_{zz} + \sigma_{xx})\right] + \frac{1}{E}\left[\sigma_{zz} - \mu(\sigma_{xx} + \sigma_{yy})\right]$$

$$= \frac{1}{E}\left[(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}) - 2\mu(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})\right]$$

$$\Rightarrow \epsilon_V = \frac{(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})(1 - 2\mu)}{E}$$

<u>For One dimensional</u> $\sigma_{yy} = \sigma_{zz} = 0$

$$\epsilon_V = \frac{\sigma_{xx}(1 - 2\mu)}{E}$$

<u>For two dimensional</u> $\sigma_{zz} = 0$

$$\epsilon_V = \frac{(\sigma_{xx} + \sigma_{yy})(1 - 2\mu)}{E}$$

Bulk Modulous (K)

When a body is subjected to three mutually perpendicular stresses of equal intensity, then the ratio of direct stresses to Volumetric strain is known as bulk Modulous.

$$\boxed{K = \frac{\sigma}{\epsilon_V}}$$

Relationship between E, K and μ

We know,

$$\epsilon_V = \frac{1}{E}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})(1-2\mu)$$

In hydrostatic stress condition,

$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \sigma \text{ (say)}$$

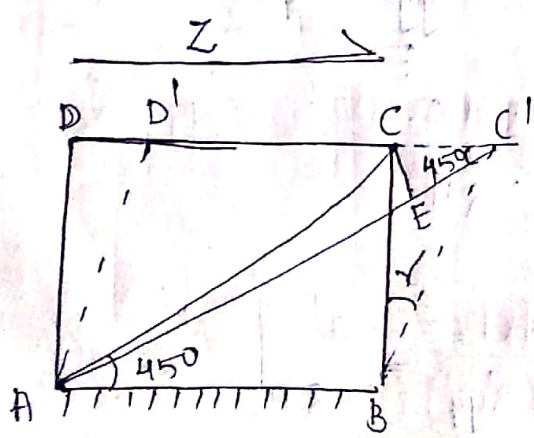
$$\Rightarrow \boxed{\epsilon_V = \frac{3\sigma(1-2\mu)}{E}}$$

Again, $K = \dfrac{\sigma}{\epsilon_V} = \dfrac{\sigma}{\frac{3\sigma(1-2\mu)}{E}}$

$$\Rightarrow K = \frac{E}{3(1-2\mu)}$$

$$\Rightarrow \boxed{E = 3K(1-2\mu)}$$

Relationship between E, K and G



Consider a square element ABCD under the action of shear stress τ as shown in figure.

Here shear strain $\boxed{(\gamma) = \frac{CC'}{BC}} \rightarrow (i)$

We know $\boxed{\gamma = \frac{\tau}{G}} \rightarrow (ii)$

$\gamma = \frac{CC'}{BC} \Rightarrow \boxed{CC' = BC \times \gamma} \rightarrow (iii)$

Comparing eqn (ii) and (iii)

$\boxed{CC' = \frac{\tau}{G} \times BC} \rightarrow (iv)$

Now diagonal strain on AC $= \frac{\text{Change in length}}{\text{Original length}}$

$\Rightarrow \epsilon = \frac{EC'}{AE}$

$\Rightarrow \boxed{\epsilon = \frac{EC'}{AC}}$

In $\triangle CEC'$,

$\cos 45^\circ = \frac{EC'}{CC'}$

$\Rightarrow \boxed{EC' = CC' \cos 45^\circ}$

Again,

In $\triangle ABC$,

$\cos 45^\circ = \frac{AB}{AC}$

$\Rightarrow \boxed{AC = \frac{AB}{\cos 45^\circ}}$

$\therefore \epsilon = \frac{EC'}{AC} = \frac{CC' \cos 45^\circ}{\frac{AB}{\cos 45^\circ}} = \frac{\frac{\tau}{G} \times BC \times (\cos 45^\circ)^2}{AB}$

$\Rightarrow \boxed{\epsilon = \frac{\tau}{2G}} \rightarrow (v)$

The effect of shear stress τ will cause a tensile strain on diagonal AC and compressive strain on diagonal BD.

Tensile strain on AC due to $\tau = \frac{\tau}{E}$

Compressive strain on BD $= \mu \frac{\tau}{E}$

Total strain $(e) = \frac{\tau}{E}(1+\mu)$

$\Rightarrow \boxed{e = \frac{\tau}{E}(1+\mu)} \rightarrow (\text{vi})$

Comparing eqn (v) and (vi).

$\Rightarrow \frac{\cancel{\tau}}{2G} = \frac{\cancel{\tau}}{E}(1+\mu)$

$\Rightarrow \boxed{E = 2G(1+\mu)}$

Or

Applying pure stress condition i.e.

$\boxed{\sigma_1 = -\sigma_3 = \tau = \sigma}$

We know,

$e = \frac{1}{E}[\sigma_1 - \mu(\sigma_2 + \sigma_3)]$

$\Rightarrow e = \frac{1}{E}[\sigma_1 + \sigma_3 \mu]$

$\Rightarrow \boxed{e = \frac{1}{E} \times \sigma(\mu+1)} \rightarrow (\text{vi})$

$\therefore$ Comparing eqn (v) and (vi)

$\frac{\tau}{2G} = \frac{\sigma}{E}(\mu+1)$

$\Rightarrow \boxed{E = 2G(1+\mu)}$

Relation between G and K:

$$3K(1-2\mu) = 2G(1+\mu)$$

$$\Rightarrow \boxed{K = \frac{2G}{3}\frac{(1+\mu)}{(1-2\mu)}}$$

* The general hook's law relating stress-strain tensors can be written in the matrix Form.

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix} = \frac{1}{E}\begin{bmatrix} 1 & -\mu & -\mu & 0 & 0 & 0 \\ -\mu & 1 & -\mu & 0 & 0 & 0 \\ -\mu & -\mu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(1+\mu) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(1+\mu) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(1+\mu) \end{bmatrix} \cdot \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix}$$

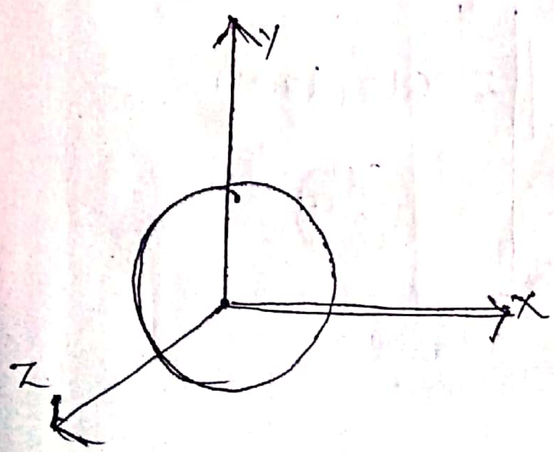
$$\varepsilon = [C]\sigma$$

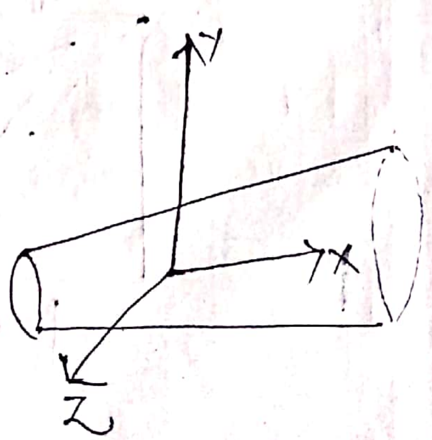
Compliance Matrix

*. In case of principal stress

$$\begin{bmatrix} \varepsilon_{1} \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix} = \begin{bmatrix} 1 & -\mu & -\mu \\ -\mu & 1 & -\mu \\ -\mu & -\mu & 1 \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \end{bmatrix}$$

Relationship between horizontal and Vertical stress





We know, $\varepsilon_{zz} = \frac{1}{E}\left[\sigma_{zz} - \mu(\sigma_{yy} + \sigma_{xx})\right]$

Here $\sigma_{xx} = \sigma_{zz} = \sigma_h$ (since there is no load in z direction therefore ε_{zz})

$\sigma_{yy} = \sigma_v$

$\Rightarrow \varepsilon_{zz} = \frac{1}{E}\left[\sigma_h - \mu(\sigma_v + \sigma_h)\right]$

$\Rightarrow 0 = \frac{1}{E}\left[\sigma_h(1-\mu) - \mu\sigma_v\right]$

$\Rightarrow \sigma_h(1-\mu) = \mu\sigma_v$

$\Rightarrow \boxed{\frac{\sigma_h}{\sigma_v} = \frac{\mu}{1-\mu}}$ This is the required relationship.

Failure in Rock

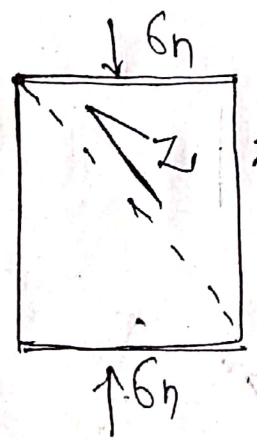
Intact Rock | Rock Mass

* It obeys Mohr's column Failure.
* It is also known as shear failure.
* $\boxed{|\tau| = c + \sigma_n \tan\phi}$

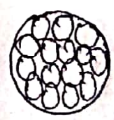
Where c = ~~Adhesion~~ Cohesion (particle to particle attraction)

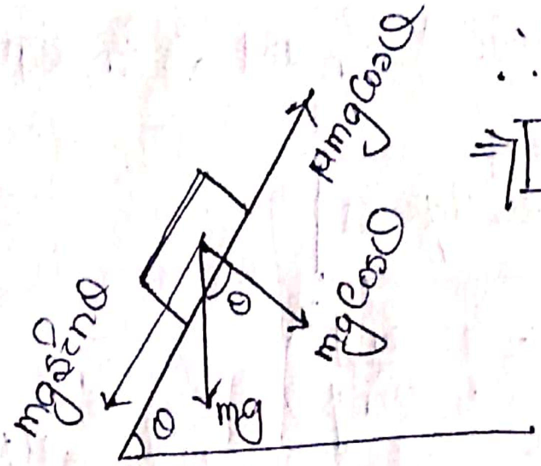
ϕ = Angel of internal friction.

* It obeys Hoek Brown's Failure (HB)



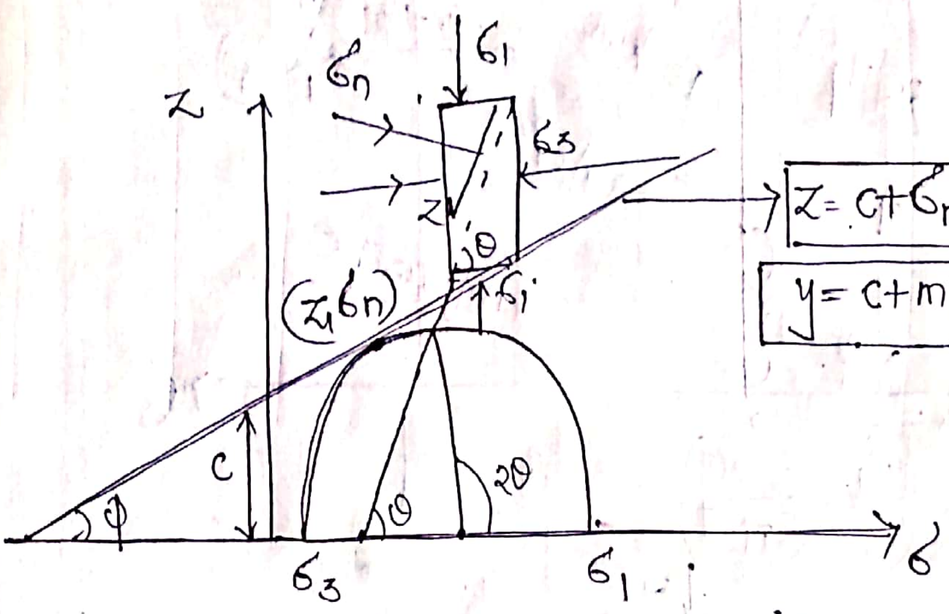
$\tau = c + \sigma_n \tan\phi$





$\therefore mg\sin\theta = \mu mg\cos\theta$

$\Rightarrow \boxed{\mu = \tan\theta}$



$\boxed{\tau = c + \sigma_n \tan\phi}$

$\boxed{y = c + mx}$

* c and φ are the physical property of the rock.

* Mohr Column suggested that the shear stress develope in the rock is related to Cohesion, angle of internal friction (φ) of the material and also depend upon the normal stress acting on the body.

We know, $\boxed{|\tau| = c + \sigma_n \tan\phi} \rightarrow A.$

Again $\sigma_n = \left(\frac{\sigma_1+\sigma_3}{2}\right) + \left(\frac{\sigma_1-\sigma_3}{2}\right)\cos 2\theta$

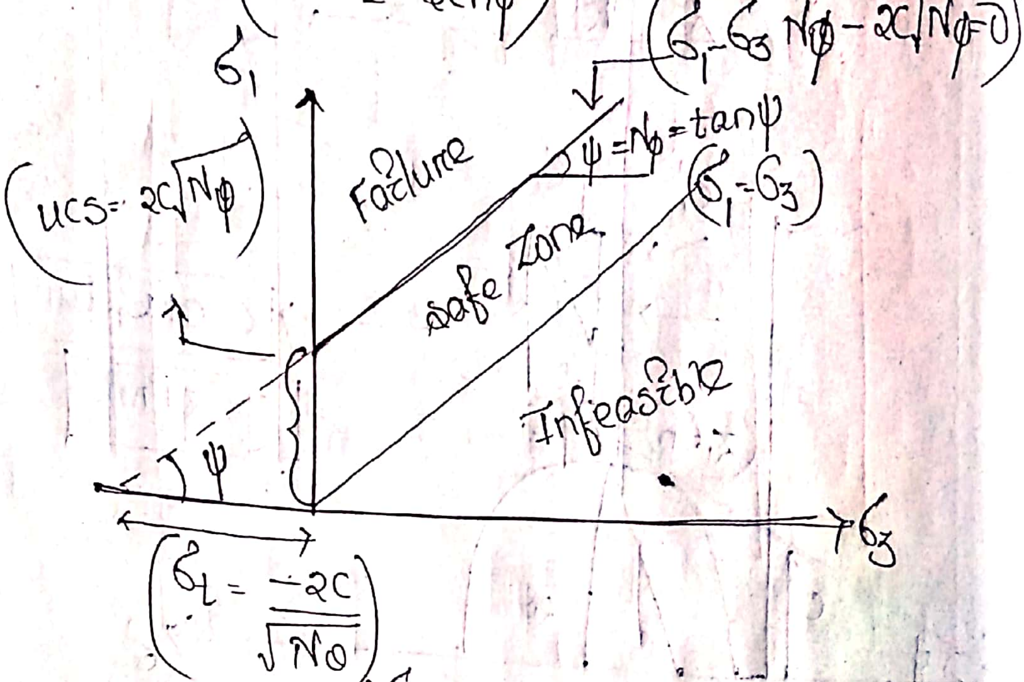
$\tau_0 = \left(\frac{\sigma_1-\sigma_3}{2}\right)\sin 2\theta$

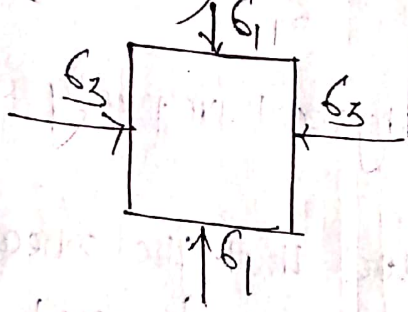
Where $\boxed{2\theta = 90+\phi}$

put the value of σ_n and τ_θ in eqn A,
We get

$$\boxed{\sigma_1 - \sigma_3 N_\phi - 2c\sqrt{N_\phi} = 0}$$

Where $\left(N_\phi = \dfrac{1+\sin\phi}{1-\sin\phi}\right)$





Case-I

(For evaluation of UCS ie $\sigma_3 = 0$)

$$\sigma_1 - \sigma_3 N_\phi - 2c\sqrt{N_\phi} = 0$$

$$\rightarrow \boxed{\sigma_1 = 2c\sqrt{N_\theta} = UCS}$$

Case-II (For evaluation of tensile stress)
$(\sigma_1 = 0)$

$$\sigma_1 - \sigma_3 N_\theta - 2c\sqrt{N_\theta} = 0$$

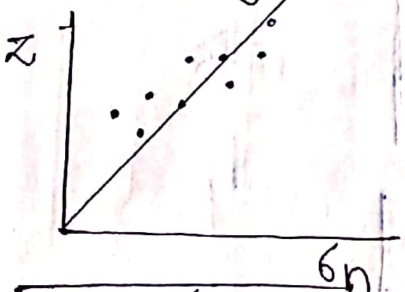
$$\rightarrow \boxed{\sigma_3 = \frac{-2c}{\sqrt{N_\theta}}}$$

$$\frac{\sigma_c}{\sigma_t} = \frac{2c\sqrt{N_\phi}}{\frac{-2c}{\sqrt{N_\phi}}} = -N_\phi$$

$$\Rightarrow \boxed{\frac{\sigma_c}{\sigma_t} = -N_\phi}$$

Evaluation of C and ϕ

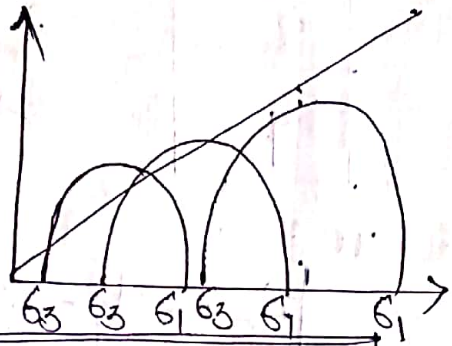
Direct sheare Test (sand, clay etc)



$$\boxed{\tau = c + \sigma_n \tan\phi}$$

τ			
σ_n			

Tri-axial tens. Lost (solid rock)



$$\boxed{\sigma_1 - \sigma_3 N_\phi - 2c\sqrt{N_\phi} = 0}$$

σ_1			
σ_3			

Q.1

τ	344	516
σ_n	336	648

in Kilopascal

Find C and ϕ.

Ans $\tau = c + \sigma_n \tan\phi$

$\Rightarrow \tau_1 = c + \sigma_{n_1} \tan\phi$

$\Rightarrow \boxed{344 = c + 336 \tan\phi} \rightarrow (i)$

$\tau_2 = c + \sigma_{n_2} \tan\phi$

$\Rightarrow \boxed{516 = c + 648 \tan\phi} \rightarrow (ii)$

ean (ii) − (i)

$$516 = c + 648 \tan\phi$$
$$344 = c + 336 \tan\phi$$
$$\underline{-\quad -\quad -}$$
$$172 = 312 \tan\phi$$
$$\Rightarrow \phi = 28^\circ 51'$$

Putting the value of ϕ in ean (i),

$$344 = c + 336 \tan(28^\circ 51')$$
$$\rightarrow \boxed{C = 158.9}\ \text{kpa}$$

Q. A series of triaxial test of rock sample revels the following result, Determine Compressive strength and tensile strength.

Test NO	Confining stress (σ_3 in Mpa)	Failure stress (σ_1 in Mpa)
1	2	45
2	4	56
3	6	63
4	8	75

Ans We Know,

$$\boxed{\sigma_1 - \sigma_3 N_\phi - 2C\sqrt{N_\phi} = 0}$$

Now

$$\boxed{45 - 2N_\phi - 2C\sqrt{N_\phi} = 0} \longrightarrow (i)$$

$$\boxed{56 - 4N_\phi - 2C\sqrt{N_\phi} = 0} \longrightarrow (ii)$$

$$\boxed{63 - 6N_\phi - 2C\sqrt{N_\phi} = 0} \longrightarrow (iii)$$

$$\boxed{75 - 8N_\phi - 2C\sqrt{N_\phi} = 0} \longrightarrow (iv)$$

Eqn (ii) − Eqn. (i)

$$56 - 4N_\phi - 2C\sqrt{N_\phi} = 0$$

$$45 - 2N_\phi - 2C\sqrt{N_\phi} = 0$$

$$- \quad + \quad +$$

$$11 - 2N_\phi = 0$$

$$\Rightarrow \boxed{N_\phi = 5.5}$$

$$N_\phi = \frac{1+\sin\phi}{1-\sin\phi}$$

$\Rightarrow 5.5 - 5.5\sin\phi = 1 + \sin\phi$

$\Rightarrow 4.5 = 6.5\sin\phi$

$\Rightarrow \boxed{\phi = 43^\circ.481}$

Now, $45 - 2\times 5.5 - 2c\sqrt{5.5} = 0$

$\Rightarrow c = \frac{45 - 2\times 5.5}{2\sqrt{5.5}}$

$\Rightarrow \boxed{c = 7.24}$

$\sigma_c = 2c\sqrt{N_\phi} = 2\times 7.24 \times \sqrt{5.5} = 33.95$ Mpa

$\sigma_t = \frac{2c}{\sqrt{N_\phi}} = \frac{2\times 7.24}{\sqrt{5.5}} = 6.17$ Mpa.

Hoek-Brown (HB) Failure Criteria (Rock Mass)

(Latest update-2002)

$$\boxed{\phi = \sigma_1 - \sigma_3 \cdot \sigma_{ci}\left[m_b \frac{\sigma_3}{\sigma_{ci}} + s\right]^a = 0}$$

Where, σ_1 and σ_3 are principal stresses

m_b, s and a are rock properties.

σ_{ci} = Uniaxial Compressive stress.

$$\boxed{m_b = m_i \exp\left[\frac{GSI-100}{28-14D}\right]} \quad m_b\ (5-40)$$

$$\boxed{s = \exp\left[\frac{GSI-100}{9-3D}\right]} \quad D\ (0-1)$$

$$\boxed{a = \frac{1}{2} + \frac{1}{6}\left[\exp\left(\frac{-GSI}{15}\right) - \exp\left(\frac{-20}{3}\right)\right]}$$

D = Degree of disturbance due to blasting

* Without disturbance, D=0
*. Disturbance of poor rock mass after blasting = 0.9.

CASE-01

Compressive strength of Rock Mass (σ_{cm})
(Here $\sigma_3 = 0$)

Now, $\sigma_1 - \sigma_3 - \sigma_{ci}\left[m_b \frac{\sigma_3}{\sigma_{ci}} + s\right]^a = 0$

$\Rightarrow \sigma_1 - \sigma_{ci}[s]^a = 0$

$\Rightarrow \boxed{\sigma_{cm} = \sigma_{ci}[s]^a}$

CASE-02

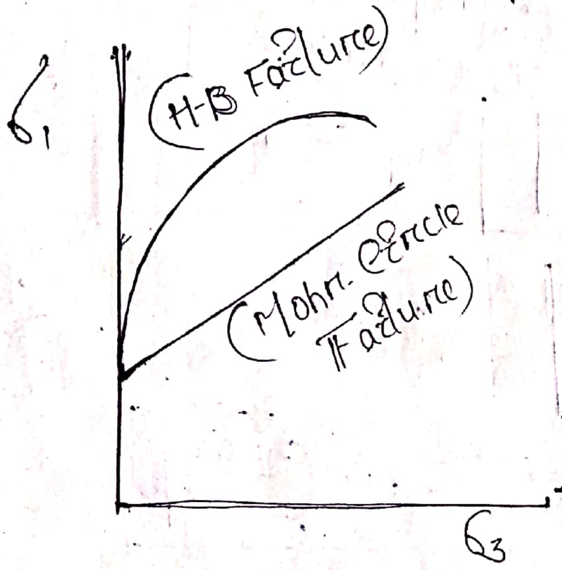
(Tensile stress (σ_{tm}) of Rock Mass)
Here ($\sigma_1 = \sigma_3 = \sigma_{tm}$)

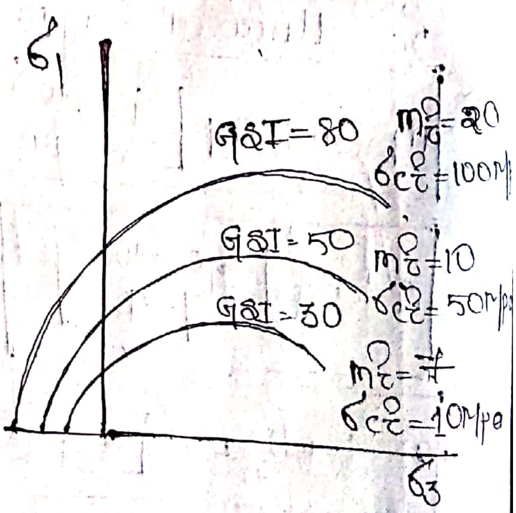
Now, $\sigma_1 - \sigma_3 - \sigma_{ci}\left[m_b \frac{\sigma_3}{\sigma_{ci}} + s\right]^a = 0$

$\Rightarrow \sigma_{ci}\left[m_b \frac{\sigma_{tm}}{\sigma_{ci}} + s\right]^a = 0$

$\Rightarrow m_b \frac{\sigma_{tm}}{\sigma_{ci}} = -s$

$\Rightarrow \boxed{\sigma_{tm} = \frac{s\sigma_{ci}}{m_b}}$





Q. Determine the compressive strength and tensile strength of rock mass of GSI = 56, UCS of the intact rock = 90 Mpa, $m_i = 12$, $D = 0$.

Ans * $\sigma_{cm} = \sigma_{ci} [s]^a$

Here $\sigma_{ci} = 90$ Mpa

$s = \exp\left[\frac{GSI - 100}{9 - 3D}\right]$

$= e^{\left[\frac{56-100}{9}\right]}$

$= 7.52 \times 10^{-3}$

$a = \frac{1}{2} + \frac{1}{6}\left[\exp\left(\frac{-GSI}{15}\right) - \exp\left(\frac{-20}{3}\right)\right]$

$= 0.503$

$\therefore \sigma_{cm} = 90 \times [7.52 \times 10^{-3}]^{0.503}$

$= 7.69$ Mpa.

* $\sigma_{tm} = \frac{s\,\sigma_{ci}}{m_b}$

$m_b = m_i \exp\left[\frac{GSI - 100}{28 - 14D}\right]$

$= 12 \exp\left[\frac{56-100}{28}\right]$

$= \cancel{1.91}\ 2.49$

$\Rightarrow \sigma_{tm} = \frac{7.52 \times 10^{-3} \times 90}{2.491}$

$\boxed{\sigma_{tm} = 0.271}$ Mpa