LECTURE NOTE

ON

DEFORMATION BEHAVIOUR OF MATERIALS

5TH SEMESTER

COMPILED BY: SIDDHARTH SANKAR JENA

DEPT. OF METALLURGICAL & MATERIALS ENGINEERING.

2020-21

DISCLAIMER

This document does not claim any originality and cannot be used as a substitute for prescribed textbooks. The information presented here is merely a collection from different reference books and internet contents. Various sources as mentioned at the reference of the document as well as freely available material from internet were consulted for preparing this document. The ownership of the information lies with the respective author or institutions. Further, this document is not intended to be used for commercial purpose and the faculty is not accountable for any issues, legal or otherwise, arising out of use of this document. The faculty members make no representations or warranties with respect to the accuracy or completeness of the contents of this document and specifically disclaim any implied warranties of merchantability or fitness for a particular purpose.

SYLLABUS

Module I:

Introduction: Elastic, plastic and visco-elastic deformation.

Continuum mechanics: Concepts of stress and strain in 3D stress and strain tensor, principal stresses and strains and principal axes, mean stress, stress deviator, maximum shear, equilibrium of stresses, equations of compatibility.

Plastic response of materials: a continuum approach: classification of stress-strain curves, yield criteria

Module II:

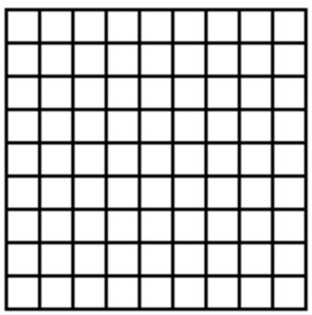
Plastic deformation of single crystals: Concepts of crystal geometry, lattice defects, deformation by slip, slip in a perfect lattice, slip by dislocation movement, critical resolved shear stress, deformation by twinning, stacking faults, deformation band and kink band, strain hardening of single crystal; stress-strain curves of fcc, bcc and hcp materials

MODULE-I

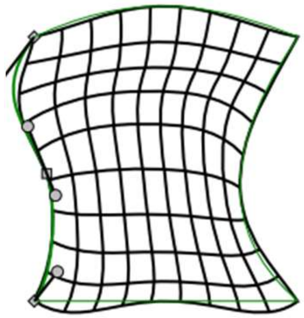
INTRODUCTION

- **Deformation behavior** is a branch of ***Mechanical Metallurgy***.

Branch of Metallurgy which deals with response & behavior of metals to load or force applied.



Before



After

- **Deformation** refers to the change in size or shape of an object.
- The mechanical metallurgists focus on the metals or materials used. To look for different mechanical testing and observe the materials under a microscope and find out why and how they behave under loading. Then use this information to design better applications.

TERMS

- ***continuous body:*** one which does not contain voids or empty spaces of any kind.
- ***Homogeneous:*** if the body has identical properties at all points.
- ***Isotropic:*** A body is considered to be isotropic with respect to some property when that property does not vary with direction or orientation.
- ***Anisotropic:*** A property which varies with orientation with respect to some system of axes.

STRESS & STRAIN

Stress is defined as a force applied per unit area. It is given by the formula

$$\sigma = \frac{F}{A}$$

- Where, **σ** is the stress applied
  F is the force applied
  A is the area of force application
- The unit of stress is $\mathbf{N/m^2}$

Strain is defined as change in length divided by original length

$$e = \frac{\Delta l}{l_o}$$

- Where, **e** is the strain
  Δl is change in length
  l_o is the change in length

Dimensionless.

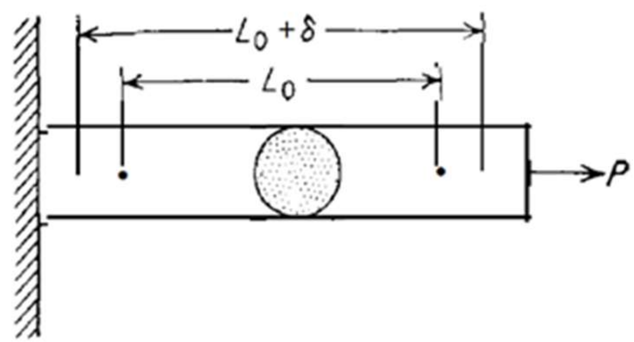


Figure **I-I** Cylindrical bar subjected to axial load.

ELASTICITY

- Elasticity deals with elastic stresses and strains, their relationship, and the external forces that cause them.
- An *elastic strain* is defined as a strain that disappears instantaneously once the forces that cause it are removed.
- The limiting load beyond which the material no longer behaves elastically is the ***elastic limit.***

HOOKE'S LAW

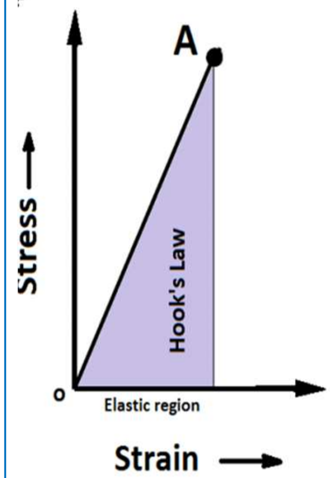
- In 1678, Robert Hooke performed experiments that demonstrated the proportionality between stress and strain

- For most materials, as long as the load does not exceed the elastic limit, the deformation is proportional to the load.
- Load vs. Deformation curve should be **linear.**

STRESS is proportional to STRAIN

But not all elastic materials follow Hooke's law of linearity.

Example-*Rubber*



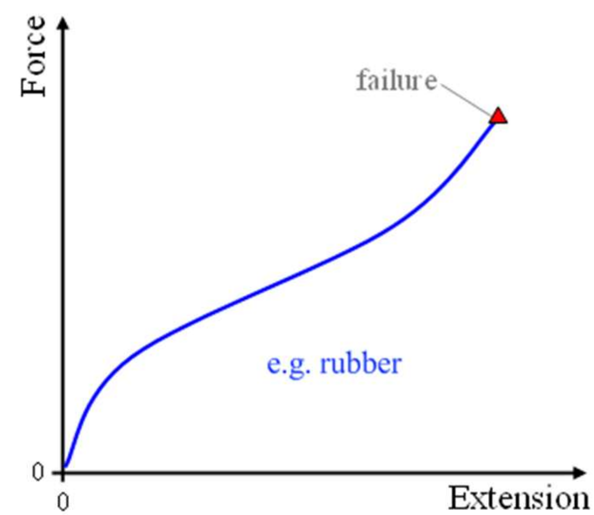
Stress ∝ Strain

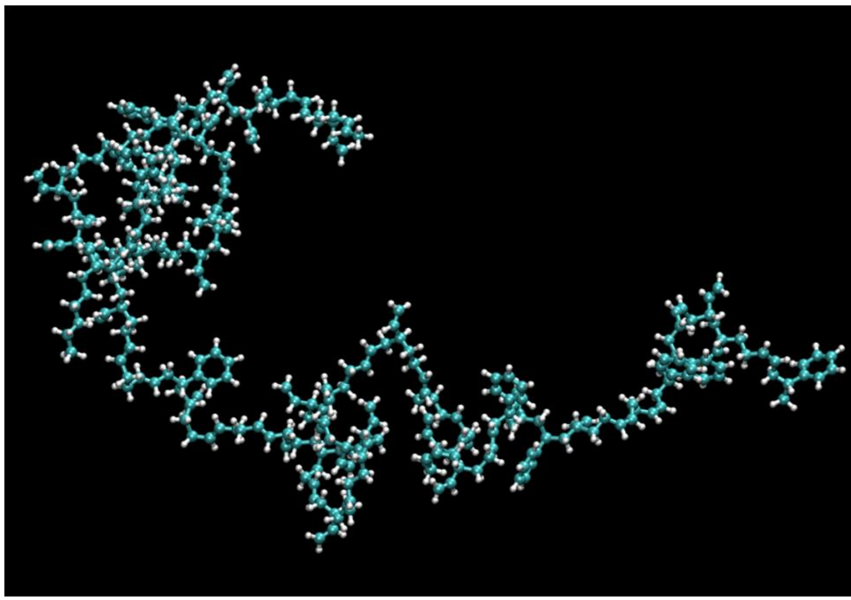
$$\sigma \propto \varepsilon$$

$$\sigma = E\varepsilon$$

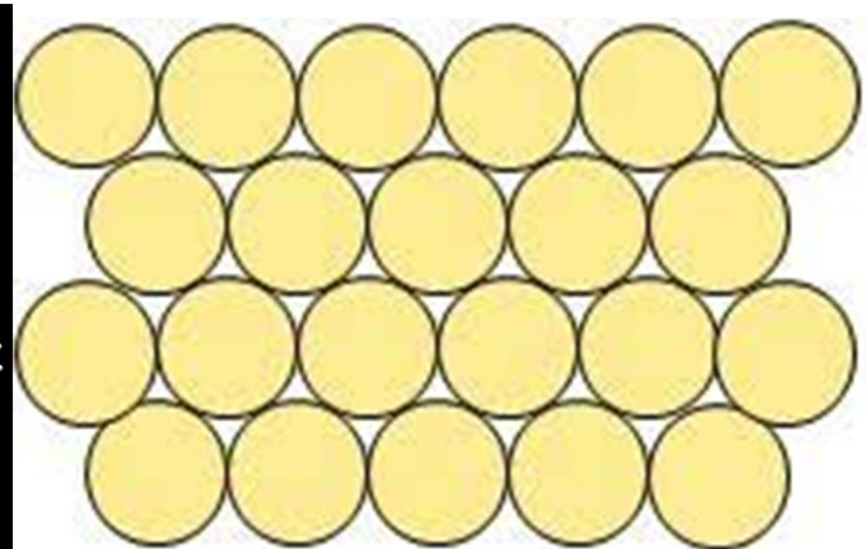
$$E = \frac{\sigma}{\varepsilon}$$

*E = Young's modulus of elasticity





Styrene Butadiene Rubber(molecular simulation)



Atoms in a Metal

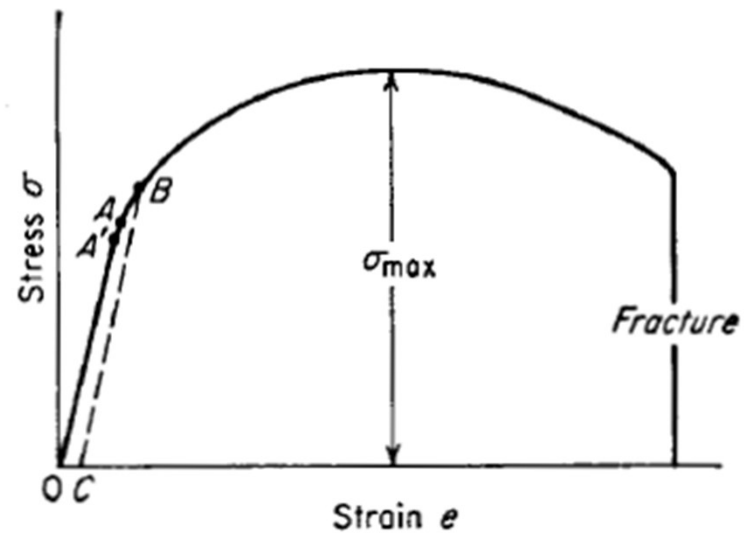
PLASTIC DEFORMATION

If the elastic limit is exceeded, the body will experience a permanent set or deformation when the load is removed. A body which is permanently deformed is said to have undergone ***plastic deformation.***

- OA is the elastic region with in which Hooke's law is obeyed.
- A is the elastic limit.
- A is replaced by A'(proportional limit) considering the sensitivity of strain measuring instrument. It is the stress at which stress strain curve deviates from linearity.
- For engineering purposes the limit of usable elastic behavior is described by **the *yield strength*,** point ***B***.

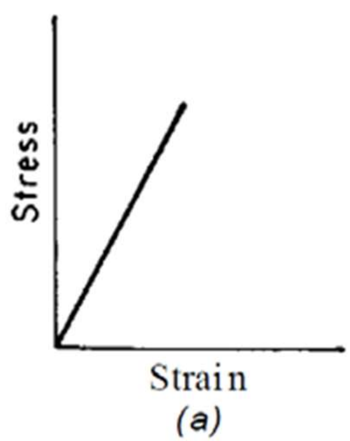
- The yield strength is defined as the stress which will produce a small amount of permanent deformation, generally equal to a strain of 0.002. This permanent strain, or offset, is **OC**.

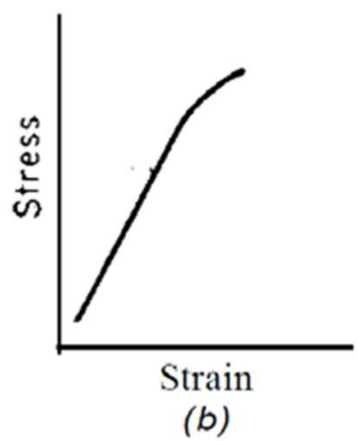
As plastic deformation increases metal becomes stronger and it reaches the maximum value(Ultimate Tensile Strength).

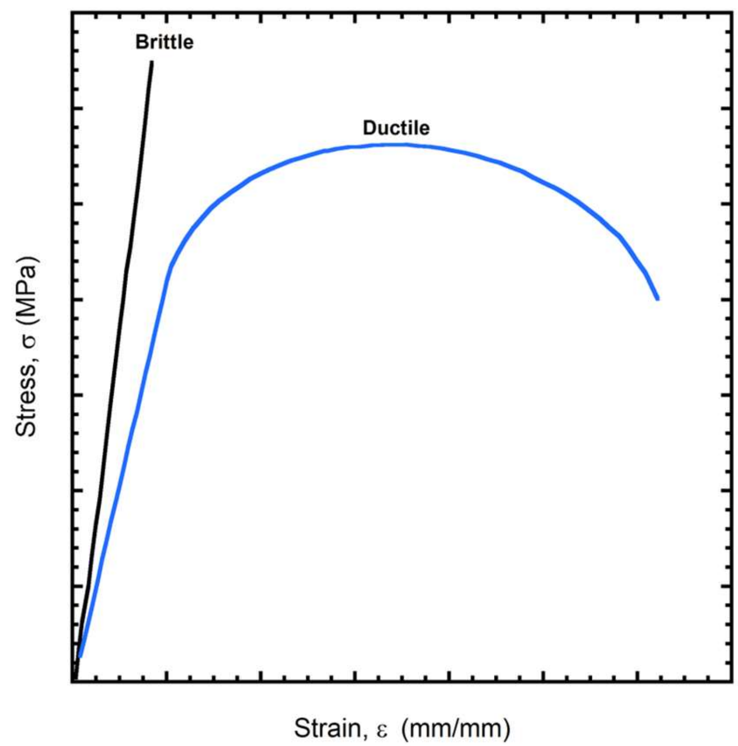


BRITTLE vs. DUCTILE BEHAVIOR

- **Brittle Materials:** A completely brittle material would fracture almost at the elastic limit. A brittle metal, such as white cast iron, shows some slight measure of plasticity before fracture
- **Ductile Materials:** Shows adequate plastic deformation. E.g. Cu, Al etc.

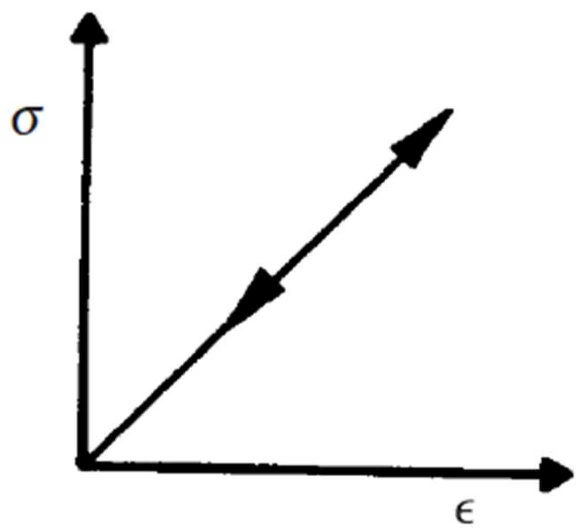




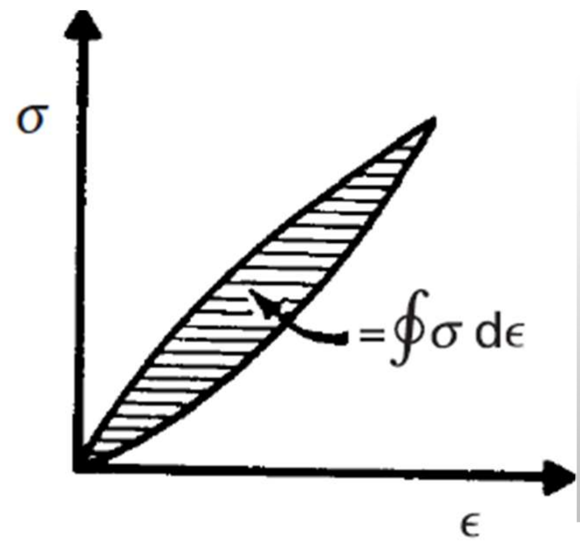


VISCOELASTIC DEFORMATION

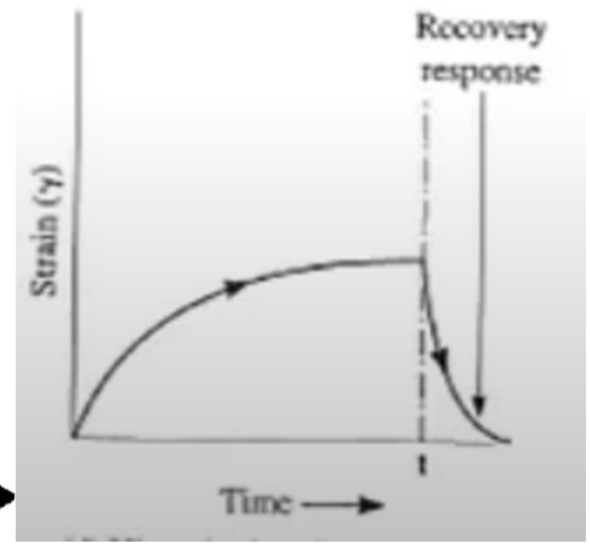
- **Viscoelasticity** is the property of materials that exhibit both viscosity & elastic characteristics when undergoing deformation.
- **Elastic behavior** :no energy is lost during a load–unload cycle)
- **Viscoelastic behavior**: energy equal to the shaded area is lost in a load–unload cycle.
- E.g. Polymers



Elasticity



Viscoelasticity



Strain returns to 0 with time

Factor of Safety

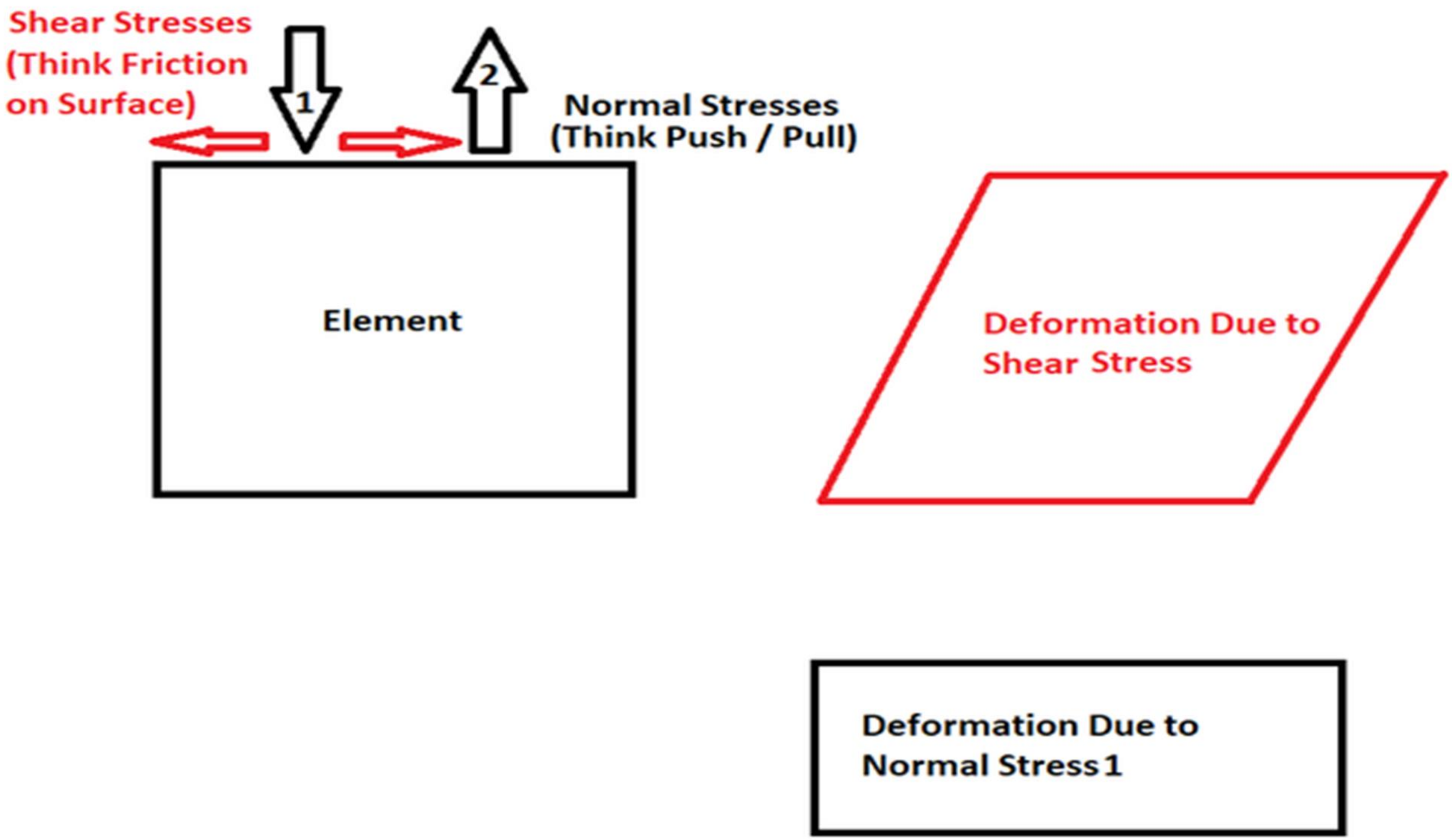
metals on the ultimate tensile strength σ_u. Values of working stress are established by local and federal agencies and by technical organizations such as the American Society of Mechanical Engineers (ASME). The working stress may be considered as either the yield strength or the tensile strength divided by a number called the *factor of safety*.

$$\sigma_w = \frac{\sigma_0}{N_0} \quad \text{or} \quad \sigma_w = \frac{\sigma_u}{N_u} \tag{1-5}$$

where σ_w = working stress
σ_0 = yield strength
σ_u = tensile strength
N_0 = factor of safety based on yield strength
N_u = factor of safety based on tensile strength

The value assigned to the factor of safety depends on an estimate of all the factors discussed above. In addition, careful consideration should be given to the consequences, which would result from failure. If failure would result in loss of life, the factor of safety should be increased. The type of equipment will also influence the factor of safety. In military equipment, where light weight may be a prime consideration, the factor of safety may be lower than in commercial equipment. The factor of safety will also depend on the expected type of loading. For static loading, as in a building, the factor of safety would be lower than in a machine, which is subjected to vibration and fluctuating stresses.

NORMAL STRESS & SHEAR STRESS



SHEAR STRAIN

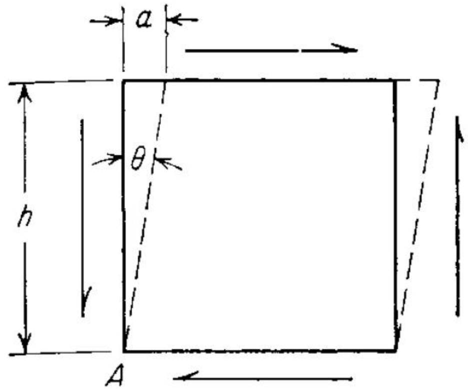


Figure 1-7 Shear strain.

between any two lines. The angular change in a right angle is known as *shear strain*. Figure 1-7 illustrates the strain produced by the pure shear of one face of a cube. The angle at A, which was originally 90°, is decreased by the application of a shear stress by a small amount θ. The shear strain γ is equal to the displacement a divided by the distance between the planes, h. The ratio a/h is also the tangent of the angle through which the element has been rotated. For the small angles usually involved, the tangent of the angle and the angle (in radians) are equal. Therefore, shear strains are often expressed as angles of rotation.

$$\gamma = \frac{a}{h} = \tan\theta = \theta \qquad (1\text{-}12)$$

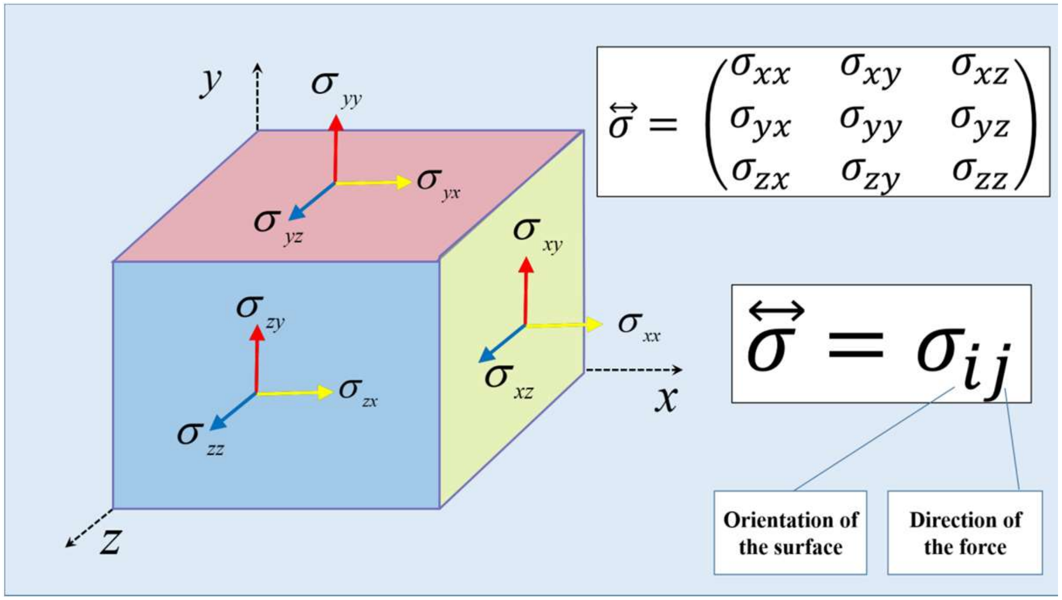
STRESS TENSOR

A **tensor** is a multi-dimensional array of numerical values that can be used to describe the physical state or properties of a material.

$$N = K^n$$

- **N=no. of components required to specify a tensor**
- **K= Dimension**
- **n=Rank**
- **Example:** For 3D, no. of components reqd. to specify the tensor=$\mathbf{3^2=9}$

- Rank 0-only magnitude-Scalar
- Rank 1-Magnitude & Direction-Vector
- Rank 2-Magnitude, Direction & Plane-Stress



STATE OF STRESS IN 2D

- For any state of stress it is always possible to define a new coordinate system which has axes perpendicular to the planes on which the maximum normal stresses act and on which no shearing stresses act. These planes are called the ***principal planes***, and the stresses normal to these planes are the ***principal stresses***. For two-dimensional plane stress there will be two principal stresses and which occur at angles that are 90° apart.
- When shear stress is zero, ***principal strain*** occurs.
- For the general case of stress in three dimensions there will be three principal stresses, and
- Maximum & Minimum Principal stresses for 2D state of stress is given by:

$$\left.\begin{matrix}\sigma_{\max} = \sigma_1 \\ \sigma_{\min} = \sigma_2\end{matrix}\right\} = \frac{\sigma_x + \sigma_y}{2} \pm \left[\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right]^{1/2}$$

- Maximum shear stress

$$\tau_{\max} = \pm\left[\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right]^{1/2}$$

STATE OF STRESS IN 3D

$$\begin{vmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{vmatrix}$$

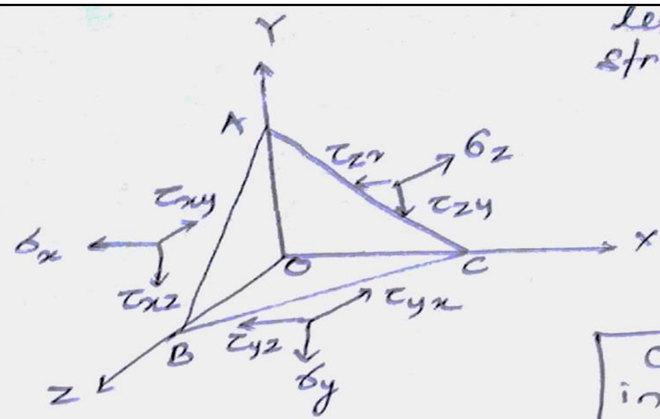
Solution of the determinant results in a cubic equation in σ.

$$\sigma^3 - (\sigma_x + \sigma_y + \sigma_z)\sigma^2 + \left(\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_x\sigma_z - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{xz}^2\right)\sigma$$
$$-\left(\sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{xz} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{xz}^2 - \sigma_z\tau_{xy}^2\right) = 0$$

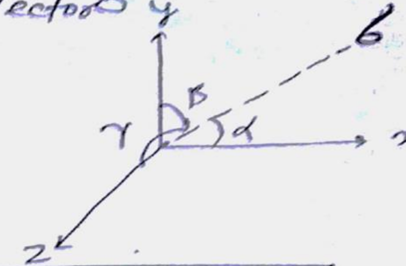
$$\sigma_1 \geq \sigma_2 \geq \sigma_3$$

Plane perpendicular to principal direction has no shear stress…

Where, $\tau_{xy} = \tau_{yx}$, $\tau_{yz} = \tau_{zy}$, $\tau_{zx} = \tau_{xz}$



Let's assume the resulting stress Vector 'σ'



Cosine components
in X-dir^n, $l = \cos\alpha$
Y-dir^n, $m = \cos\beta$
Z-dir^n, $n = \cos\gamma$

The components of σ along each axes are S_x, S_y & S_z
$S_x = \sigma l$, $S_y = \sigma m$, $S_z = \sigma n$

Let Area of $ABC = A$,
If we take Projection of this area on X-plane,
then Area of $AOB = Al$
Similarly Area of $BOC = AM$, $AOC = An$
Summation of the forces in the x-dir^n

$$\sigma Al - \sigma_x Al - \tau_{yx} Am - \tau_{zx} An = 0$$

Similarly

$$\cancel{-\tau_{xy}} + \sigma Am - \sigma_y Am - \tau_{xy} Al - \tau_{zy} An = 0$$

$$\& \quad \sigma An - \sigma_z An - \tau_{xz} Al - \tau_{yz} Am = 0$$

The nontrivial sol^n can be obtained by determinant of the coefficients of l, m & n equal to zero

$$\begin{vmatrix} \sigma-\sigma_x & -\tau_{yx} & -\tau_{zx} \\ -\tau_{xy} & \sigma-\sigma_y & -\tau_{zy} \\ -\tau_{xz} & -\tau_{yz} & \sigma-\sigma_z \end{vmatrix} = 0$$

or

$$\begin{vmatrix} \sigma_x & +\tau_{yx} & +\tau_{zx} \\ +\tau_{xy} & \sigma_y & +\tau_{zy} \\ +\tau_{xz} & +\tau_{yz} & \sigma_z \end{vmatrix} = 0$$

$$\boxed{\text{sol}^n = \sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0}$$

$I_1 = \sigma_x + \sigma_y + \sigma_z$

$I_2 = \begin{vmatrix} \sigma_x & \tau_{yx} \\ \tau_{xy} & \sigma_y \end{vmatrix} + \begin{vmatrix} \sigma_y & \tau_{zy} \\ \tau_{yz} & \sigma_z \end{vmatrix} + \begin{vmatrix} \sigma_x & \tau_{zx} \\ \tau_{xz} & \sigma_z \end{vmatrix}$

$I_3 = \begin{vmatrix} \text{Total Determinant} \end{vmatrix}$

PRINCIPAL STRESSES

$$\sigma_{i,j} = \begin{bmatrix} \sigma_{11} & 0 & 0 \\ 0 & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \end{bmatrix} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

$$\sigma_1 \geq \sigma_2 \geq \sigma_3$$

- **MAXIMUM SHEAR STRESS**

Plane perpendicular to principal direction has no shear stress…

Since according to convention σ_1 is the algebraically greatest principal normal stress and σ_3 is the algebraically smallest principal stress, τ_2 has the largest value of shear stress and it is called the *maximum shear stress* $\tau_{\max}$.

$$\boxed{\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2}} \qquad (2\text{-}21)$$

QUESTION:

GIVEN:

- $\sigma 1$=200 MPa in tensile direction
- $\sigma 2$=150 MPa in tensile direction
- $\sigma 3$=300 MPa in Compressive direction

Find Maximum Shear Stress.

MOHR'S CIRCLE IN 3D

- It gives a geometrical representation of the equations that shows the transformation of stress components to different sets of axes.
- Shear stress is maximum in case of uniaxial tension or compression.
- The bi-axial and tri-axial stresses reduce the shear stresses thus ductility also reduced.
- Thus brittle fracture is associated with triaxial stresses that generated at a notch or stress raiser.

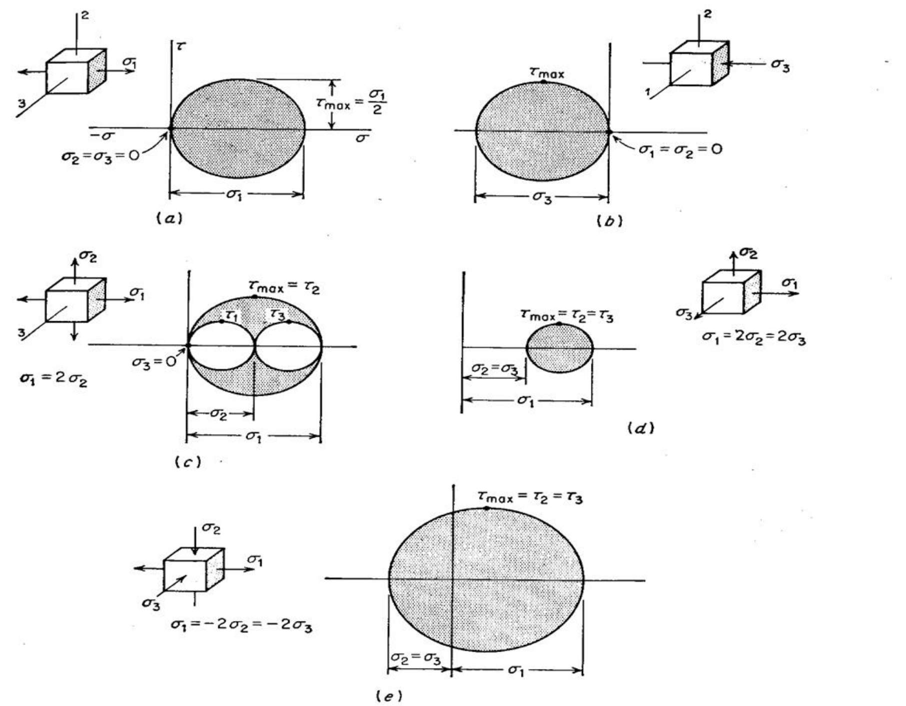
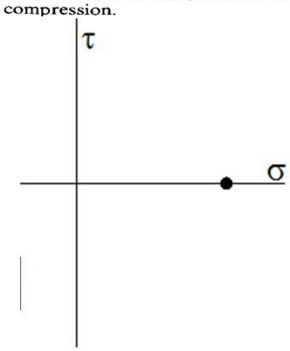


Figure 2-11 Mohr's circles (three-dimensional) for various states of stress. (*a*) Uniaxial tension; (*b*) uniaxial compression; (*c*) biaxial tension; (*d*) triaxial tension (unequal); (*e*) uniaxial tension plus biaxial compression.



Equi-triaxial tension

HYDROSTATIC STRESS

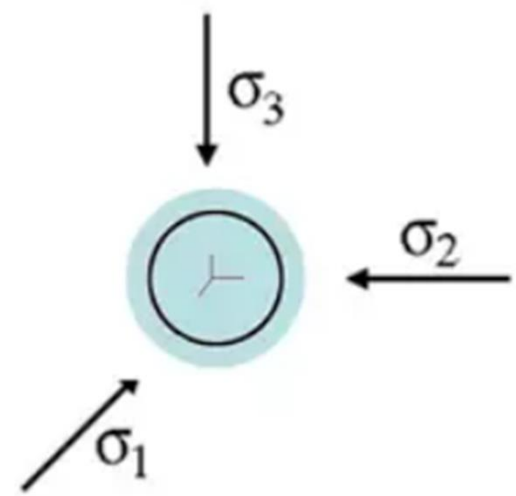
- Hydrostatic state of stress is when a material is subjected to equal normal stresses along all three coordinate axes with shear stresses all zero. An example is a body immersed in a fluid
- $\sigma 1= \sigma 2 =\sigma 3$
- These Principal Stresses

may be all tensile or compressive

The hydrostatic or mean stress is given by

$$\sigma_m = \frac{\sigma_{kk}}{3} = \frac{\sigma_x + \sigma_y + \sigma_z}{3} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

Hydrostatic compressive stress



$\sigma_1 = \sigma_2 = \sigma_3$

HYDROSTATIC STRESS

Involves only in **elastic volume changes** and does not cause plastic deformation.

$$\begin{bmatrix} \sigma_m & 0 & 0 \\ 0 & \sigma_m & 0 \\ 0 & 0 & \sigma_m \end{bmatrix}$$

The hydrostatic or mean stress is given by

$$\sigma_m = \frac{\sigma_{kk}}{3} = \frac{\sigma_x + \sigma_y + \sigma_z}{3} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

STRESS DEVIATOR

- It involves shearing stresses and **causes plastic deformation**
- **TOTAL STRESS = HYDROSTATIC STRESS + STRESS DEVIATOR**
- **STRESS DEVIATOR = TOTAL STRESS - HYDROSTATIC STRESS**

$$\sigma'_{ij} = \sigma_{ij} - \sigma_m \delta_{ij}$$

Hydrostatic Stress & Stress Deviator

Hydrostatic & Deviotric Stresses

Total Stress = Hydrostatic Stress + Stress Deviator

$$\sigma = \sigma_m + \sigma'_{ij}$$

$$\Rightarrow \begin{vmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{vmatrix} = \begin{vmatrix} \sigma_m & 0 & 0 \\ 0 & \sigma_m & 0 \\ 0 & 0 & \sigma_m \end{vmatrix} + \sigma'_{ij}$$

$$\Rightarrow \sigma'_{ij} = \begin{vmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{vmatrix} - \begin{vmatrix} \sigma_m & 0 & 0 \\ 0 & \sigma_m & 0 \\ 0 & 0 & \sigma_m \end{vmatrix}$$

$$\Rightarrow \sigma'_{ij} = \begin{vmatrix} \sigma_x - \sigma_m & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y - \sigma_m & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z - \sigma_m \end{vmatrix}$$

where, $\sigma_m = \dfrac{\sigma_x + \sigma_y + \sigma_z}{3}$

$$\Rightarrow \text{Stress Deviator } (\sigma'_{ij}) = \begin{vmatrix} \dfrac{2\sigma_x - \sigma_y - \sigma_z}{3} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \dfrac{2\sigma_y - \sigma_z - \sigma_x}{3} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \dfrac{2\sigma_z - \sigma_x - \sigma_y}{3} \end{vmatrix}$$

→ As stress is a 2nd rank tensor, it has Principal axes, so ~~it~~ has the deviotric stress.

→ The Principal Values can be obtained by finding the roots of the equation.

$$\boxed{(\sigma')^3 - J_1 (\sigma')^2 - J_2 \sigma' - J_3 = 0}$$

Where; J_1 = Sum of diagonals of the matrix

$$\Rightarrow J_1 = (\sigma_x - \sigma_m) + (\sigma_y - \sigma_m) + (\sigma_z - \sigma_m)$$

J_2 = Sum of Principal minors

$$J_2 = \tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2 - \sigma'_x\sigma'_y - \sigma'_y\sigma'_z - \sigma'_z\sigma'_x$$

$$= \frac{1}{6}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2)\right]$$

J_3 = Determinant of the matrix.

VOLUMETRIC STRAIN

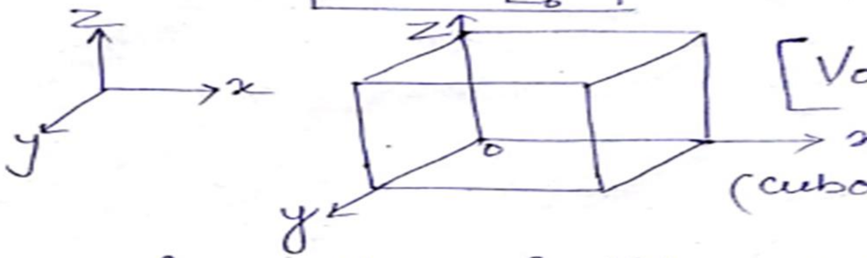
[Volumetric Strain]

We know the linear strain $= \dfrac{\Delta L}{L_0}$

But in a 3D cube, Volume also changes on application of stresses.

$\Rightarrow \boxed{\varepsilon = \dfrac{\Delta L}{L_0}}$ or in x-dirⁿ, $\boxed{\varepsilon_x = \dfrac{dx}{x}}$

[Volume 'V' $= xyz$]



Small change in Volume $= dV = d(xyz)$

There, Volumetric strain $= \dfrac{dV}{V}$

$$dV = d(xyz)$$

$$\Rightarrow \quad dV = yz\,dx + xz\,dy + xy\,dz$$

$$\Rightarrow \quad \frac{dV}{V} = \frac{yz\,dx}{xyz} + \frac{xz\,dy}{xyz} + \frac{xy\,dz}{xyz}$$

$$\Rightarrow \quad \frac{dV}{V} = \frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z}$$

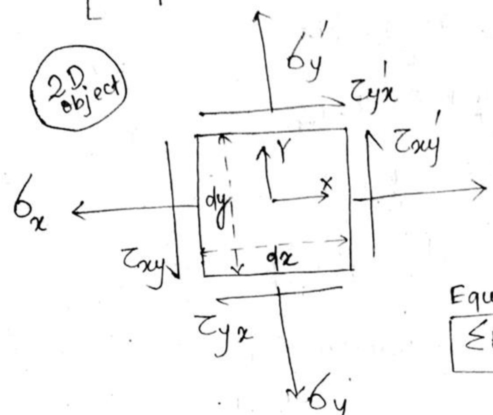
$$\Rightarrow \text{(Dilation)} \quad \boxed{\Delta = \varepsilon_x + \varepsilon_y + \varepsilon_z}$$

[Hydrostatic or Mean Strain $\Rightarrow \varepsilon_m = \dfrac{\Delta}{3} = \dfrac{\varepsilon_x + \varepsilon_y + \varepsilon_z}{3}$]

EQUATION OF STRESS EQUILIBRIUM

[Equations of stress Equilibrium]

(2D object)

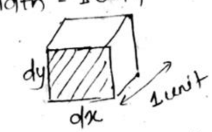


Equilibrium Conditions

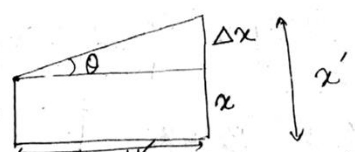
$$\boxed{\Sigma F_x = \Sigma F_y = \Sigma F_z = 0}$$

To find out the linear variation of the stresses.

Consider a cube with width = 1 unit



When one stress is applied on one side, but it does not remain same on the other side of the object



$$x' = x + \Delta x$$
$$= x + \tan\theta \cdot y$$
$$\Rightarrow x' = x + \frac{dx}{dy} \cdot y$$

$$\left[\begin{array}{l} \tan\theta = \frac{\Delta x}{y} \\ \Rightarrow \Delta x = \tan\theta \cdot y \end{array}\right]$$
$\tan\theta = \text{slope}$

This how we can represent the linear changes of the applied stresses.

$$\sigma_x' = \sigma_x + \frac{\partial \sigma_x}{\partial x} \cdot dx$$
$$\tau_{xy}' = \tau_{xy} + \frac{\partial \tau_{xy}}{\partial x} \cdot dx$$
$$\sigma_y' = \sigma_y + \frac{\partial \sigma_y}{\partial y} \cdot dy$$
$$\tau_{yx}' = \tau_{yx} + \frac{\partial \tau_{yx}}{\partial y} \cdot dy$$

Equations of equilibrium $\Sigma F_x = 0$
$\Sigma F_y = 0$

$\Sigma F_x = 0$

$$\Rightarrow \left[\sigma_x + \frac{\partial \sigma_x}{\partial x} \cdot dx\right](dy \cdot 1) + \left[\tau_{yx} + \frac{\partial(\tau_{yx})}{\partial y} \cdot dy\right](dx \cdot 1)$$
$$- \sigma_x \cdot (dy \cdot 1) - \tau_{yx} (dx \cdot 1) + X = 0$$

($\because X$ = other forces that act on the body)

$$\Rightarrow \frac{\partial \sigma_x}{\partial x} \cdot dx\,dy + \frac{\partial(\tau_{yx})}{\partial y} \cdot dy \cdot dx + X = 0$$

$$\Rightarrow \boxed{\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + X = 0}$$

Similarly $\Sigma F_y = 0$

$$\Rightarrow \boxed{\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + Y = 0}$$

(Y = Body forces)

These are the equations of stress equilibrium

These can be represented in matrix form

$$\begin{bmatrix} \sigma_x & \tau_{yx} \\ \tau_{xy} & \sigma_y \end{bmatrix} \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{bmatrix} = \begin{bmatrix} -X \\ -Y \end{bmatrix}$$

For 3D object

$$\begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix} = \begin{bmatrix} -X \\ -Y \\ -Z \end{bmatrix}$$

Equations of Stress Equilibrium

CONSTITUTIVE EQUATIONS FOR ELASTIC MATERIALS

- **Normal Stress**
- Acts normal to the plane.

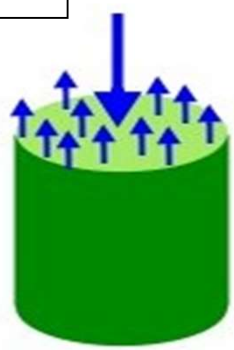
Stress ∝ Strain

$$\sigma \propto \varepsilon$$

$$\sigma = E\,\varepsilon$$

$$E = \frac{\sigma}{\varepsilon}$$

$^{*}E$ = Young's modulus of elasticity



Normal Stress

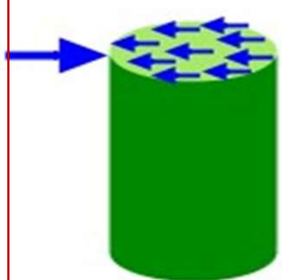
- **Shear Stress**
- Lies on plane and acts parallel to the plane

The shearing stresses acting on the unit cube produce shearing strains.

$$\tau_{xy} = G\gamma_{xy} \qquad \tau_{yz} = G\gamma_{yz} \qquad \tau_{xz} = G\gamma_{xz}$$

$$\boxed{G = \frac{\tau}{\gamma}}$$

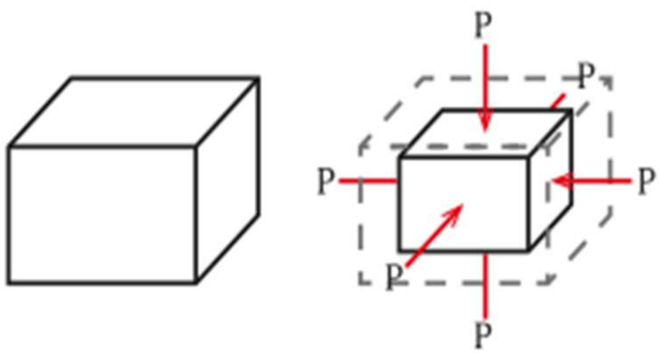
- **G=Shear Modulus or Modulus of rigidity.**



Shear Stress

Hydrostatic Stress & Bulk Modulus

- Bulk Modulus=$\frac{Hydrostatic\ Stress}{Change\ in\ Volume}$
- K=$\frac{\sigma_m}{\Delta}$



$$K=\frac{\sigma_m}{\Delta}=\frac{-p}{\Delta}=\frac{1}{\beta}$$

β is the compressibility.

- K=Bulk Modulus or Modulus of elasticity

Poisson’s Ratio

While a tensile force in the x direction produces an extension along that axis, it also produces a contraction in the transverse y and z directions. The transverse strain has been found by experience to be a constant fraction of the strain in the longitudinal direction. This is known as *Poisson's ratio*, denoted by the symbol ν.

$$\boxed{\varepsilon_y = \varepsilon_z = -\nu\varepsilon_x = -\frac{\nu\sigma_x}{E}} \qquad (2\text{-}63)$$

Only the absolute value of ν is used in calculations. For most metals the values[2] of ν are close to 0.33.

Stress	Strain in the x direction	Strain in the y direction	Strain in the z direction
σ_x	$\varepsilon_x = \dfrac{\sigma_x}{E}$	$\varepsilon_y = -\dfrac{\nu\sigma_x}{E}$	$\varepsilon_z = -\dfrac{\nu\sigma_x}{E}$
σ_y	$\varepsilon_x = -\dfrac{\nu\sigma_y}{E}$	$\varepsilon_y = \dfrac{\sigma_y}{E}$	$\varepsilon_z = -\dfrac{\nu\sigma_y}{E}$
σ_z	$\varepsilon_x = -\dfrac{\nu\sigma_z}{E}$	$\varepsilon_y = -\dfrac{\nu\sigma_z}{E}$	$\varepsilon_z = \dfrac{\sigma_z}{E}$

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right]$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_z + \sigma_x)\right]$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$

The shearing stresses acting on the unit cube produce shearing strains.

$$\tau_{xy} = G\gamma_{xy} \qquad \tau_{yz} = G\gamma_{yz} \qquad \tau_{xz} = G\gamma_{xz}$$

$$\boxed{\varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z)}$$

Relationship between the Elastic Constants

- The elastic constants we discussed about are **E,G,K** and **ν**(poisson's ratio)
- **There are two independent elastic constants.**
- **Different relationships may be derived from these Elastic constants.**

$$\varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z)$$

$$\Delta = \frac{1-2\nu}{E} 3\sigma_m$$

$$K = \frac{\sigma_m}{\Delta} = \frac{E}{3(1-2\nu)}$$

$$E = \frac{9K}{1 + 3K/G} \qquad \nu = \frac{1 - 2G/3K}{2 + 2G/3K}$$

$$G = \frac{3(1-2\nu)K}{2(1+\nu)} \qquad K = \frac{E}{9 - 3E/G}$$

$$G = \frac{E}{2(1+\nu)}$$

Question (GATE 2014)

What is the hydrostatic stress for the state of stress represented by σ_{ij} given below? ______

$$\sigma_{ij} = \begin{vmatrix} 100 & 50 & 50 \\ 50 & 125 & 75 \\ 50 & 75 & 75 \end{vmatrix}$$

ANS: 100

Question(GATE 2018)

Consider the following stress state imposed on a material:

$$\sigma = \begin{bmatrix} 90 & 50 & 0 \\ 50 & -20 & 0 \\ 0 & 0 & 140 \end{bmatrix} \text{MPa.}$$

If the material responds elastically with a volumetric strain $\Delta = 3.5 \times 10^{-4}$, what is its bulk modulus?

(A) 150 GPa (B) 350 GPa (C) 200 GPa (D) 400 GPa

ANS: C

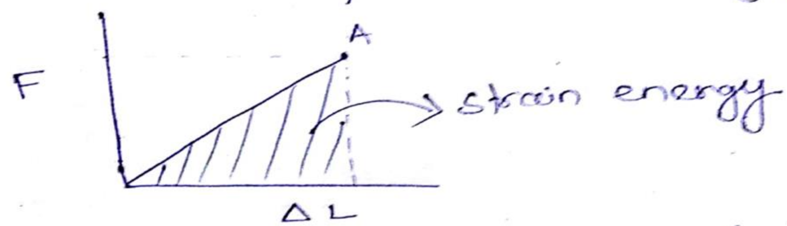
STRAIN ENERGY

[Strain Energy]

→ It is the energy that is stored when elastic deforming a body.

→ This energy is recovered upon unloading or release of applied load.

→ Area under Load - deformation curve gives the value of strain energy.



Area under curve = strain energy (U)

$\Rightarrow \quad U = \frac{1}{2} F \times \Delta L$

$\Rightarrow \quad U = \frac{1}{2} \frac{F}{A} \cdot A \times \frac{\Delta L}{L} \times L$

$\Rightarrow \quad U = \frac{1}{2} \sigma \varepsilon (A \cdot L) = \frac{1}{2} \sigma \varepsilon \times V$

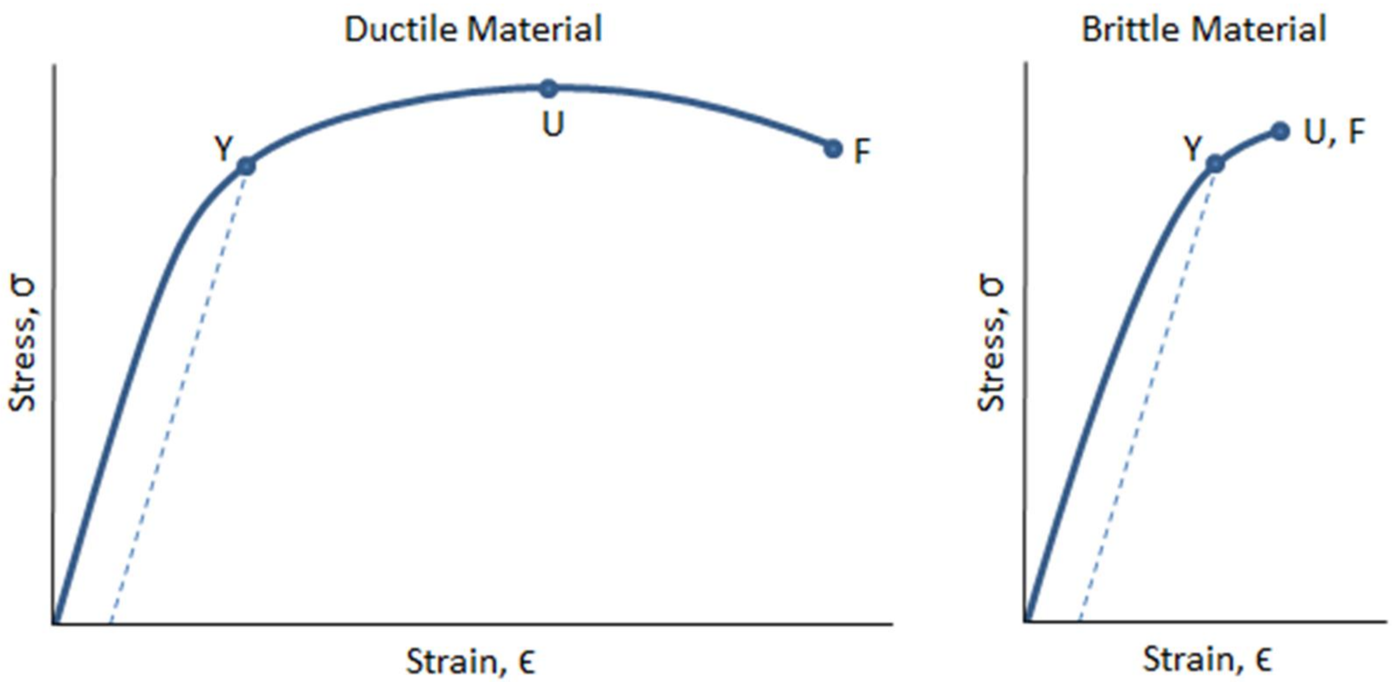
$\Rightarrow \quad \frac{U}{V} = \frac{1}{2} \sigma \varepsilon$

$\Rightarrow$ strain energy per unit volume (U_0)

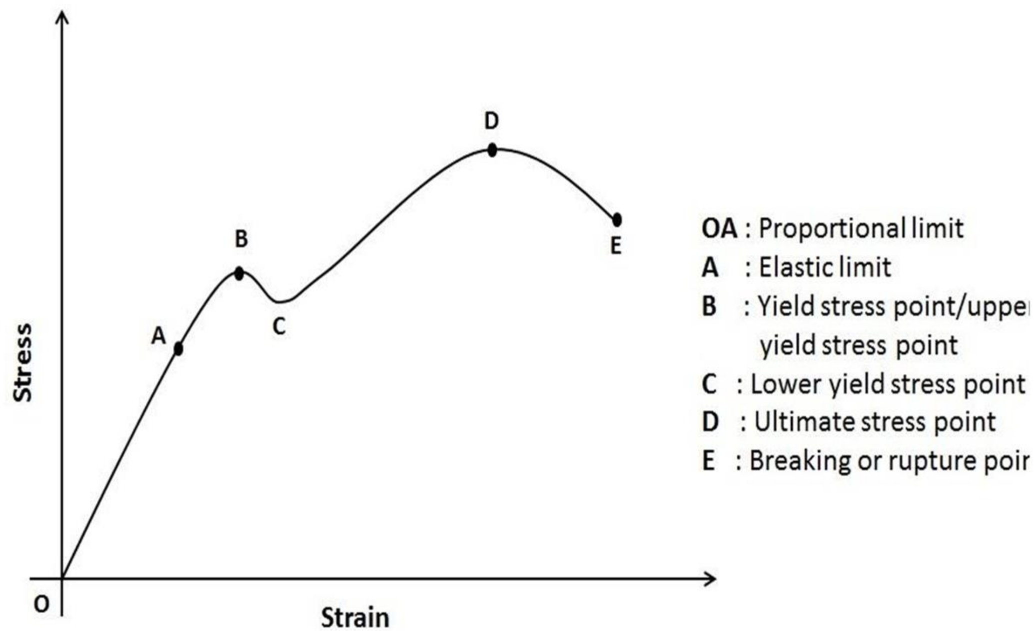
$\Rightarrow \quad \boxed{U_0 = \frac{1}{2} \sigma \varepsilon} = \boxed{\frac{1}{2} \frac{\sigma^2}{E} = \frac{1}{2} \varepsilon^2 E}$

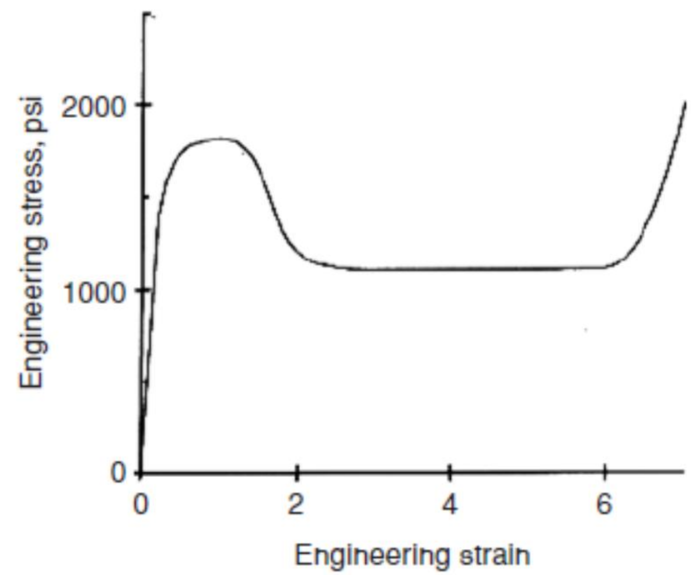
For shear stress, $\boxed{U_0 = \frac{1}{2} \tau \gamma = \frac{1}{2} \frac{\tau^2}{G} = \frac{1}{2} \gamma^2 G}$

Classification of Stress-Strain Curves



- Brittle Materials do not show plastic deformation where as Ductile Materials do.

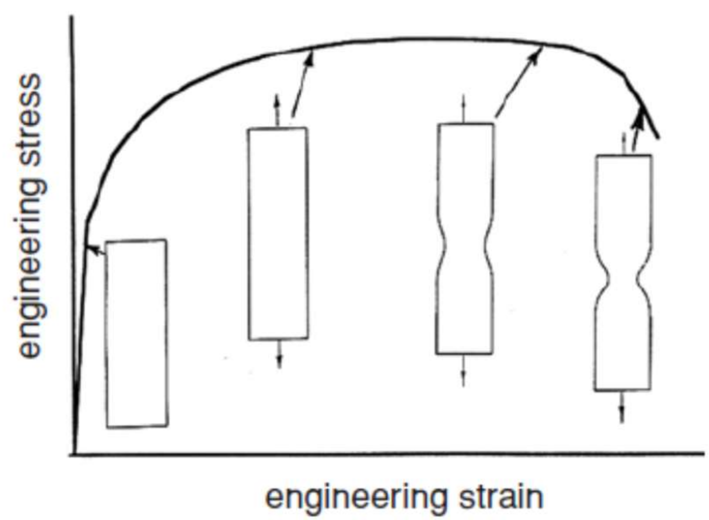




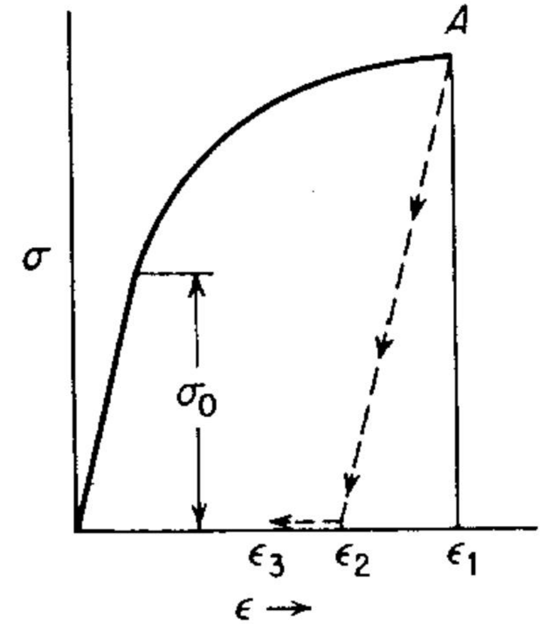
Low Carbon Steel

Polymer

NECKING



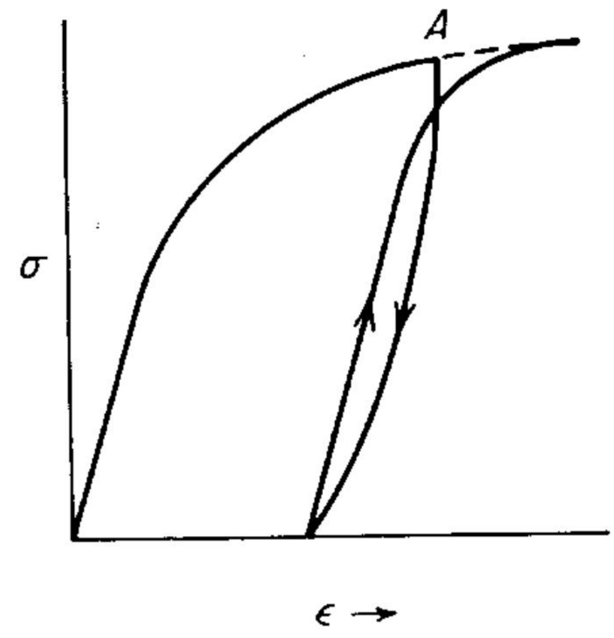
FLOW CURVES



(a)

A, when the load is released the total strain will immediately decrease from ε_1 to ε_2 by an amount σ/E. The strain decrease $\varepsilon_1 - \varepsilon_2$ is the *recoverable elastic strain*. However, the strain remaining is not all permanent plastic strain. Depending upon the metal and the temperature, a small amount of the plastic strain $\varepsilon_2 - \varepsilon_3$ will disappear with time. This is known as *anelastic behavior*. Generally the anelastic strain is neglected in mathematical theories of plasticity.

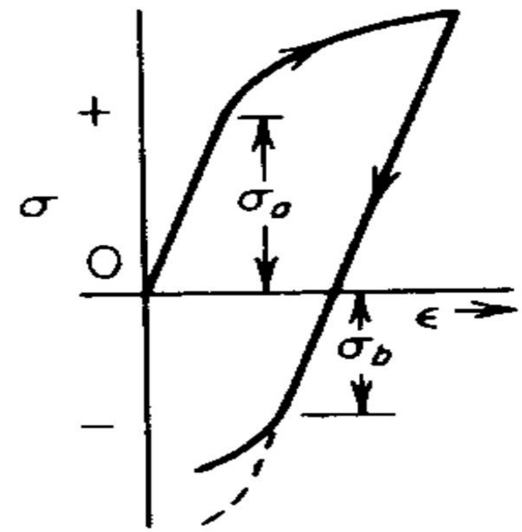
FLOW CURVES



(b)

Usually the stress-strain curve on unloading from a plastic strain will not be exactly linear and parallel to the elastic portion of the curve (Fig. 3-1*b*). Moreover, on reloading the curve will generally bend over as the stress approaches the original value of stress from which it was unloaded. With a little additional plastic strain the stress-strain curve becomes a continuation of what it would have been had no unloading taken place. The hysteresis behavior resulting from unloading and loading from a plastic strain is generally neglected in plasticity theories.

FLOW CURVES



(c)

If a specimen is deformed plastically beyond the yield stress in one direction, e.g., in tension, and then after unloading to zero stress it is reloaded in the opposite direction, e.g., in compression, it is found that the yield stress on reloading is less than the original yield stress. Referring to Fig. 3-1*c*, $\sigma_b < \sigma_a$. This dependence of the yield stress on loading path and direction is called the *Bauschinger effect*. The Bauschinger effect is commonly ignored in plasticity theory, and it is usual to assume that the yield stress in tension and compression are the same.

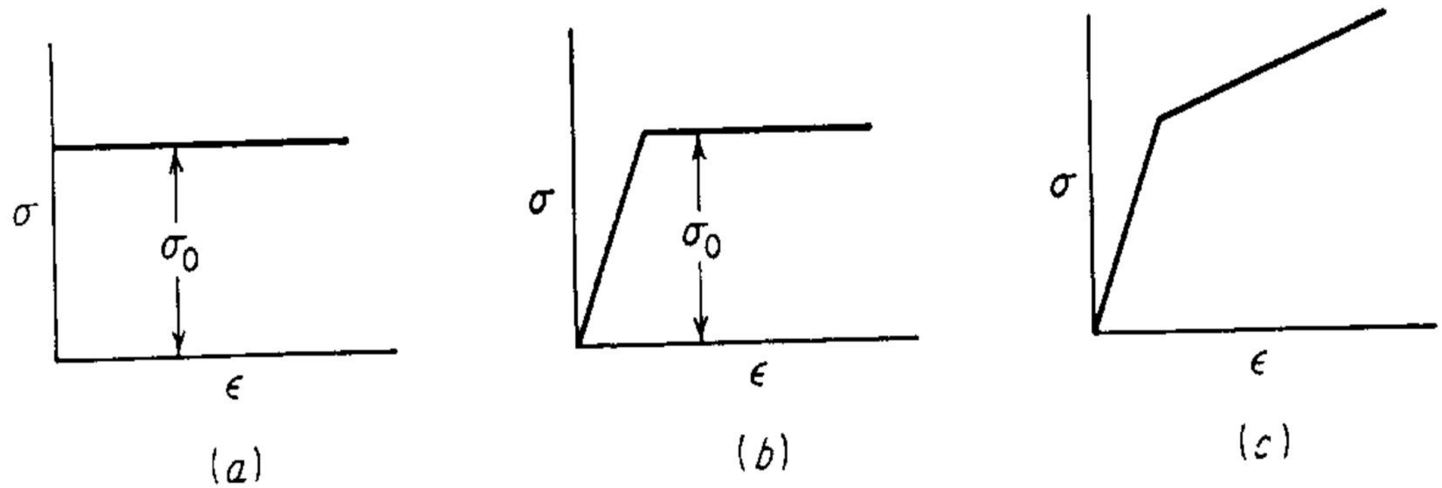


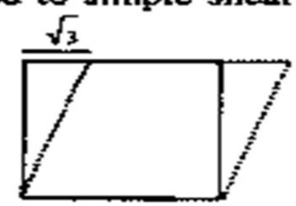
Figure 3-2 Idealized flow curves. (*a*) Rigid ideal plastic material; (*b*) ideal plastic material with elastic region; (*c*) piecewise linear (strain-hardening) material.

Questions for Practice

The engineering stress-strain curve for a ceramic material is

(A) parabolic (B) exponential (C) logarithmic (D) linear

A square of 9 mm^2 area is subjected to simple shear displacement $\sqrt{3}$ mm along x-direction, as shown below



The shear strain imparted will be

(A) 1/3 (B) $1/\sqrt{3}$ (C) $\sqrt{3}$ (D) 3

An aluminium alloy rod of diameter 15 mm and length 120 mm is subjected to a tensile load of 35,000 N along its axis. The Young's modulus and Poisson's ratio for aluminium are 70 GPa and 0.33 respectively.

Q.50 The reduction in diameter on the application of tensile load is
(A) 0.011 mm (B) 0.014 mm (C) 0.018 mm (D) 0.021 mm

Q.51 The elastic strain energy is approximately

(A) 200 kJ m^{-3} (B) 240 kJ m^{-3} (C) 280 kJ m^{-3} (D) 320 kJ m^{-3}

YIELD CRITERIA

- **Von Mises' Criterion**

$$\sigma_0 = \frac{1}{\sqrt{2}}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}$$

$$\boxed{\sigma_0 = \frac{1}{\sqrt{2}}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6\left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2\right)\right]^{1/2}}$$

Yielding occurs when value right side of the equation exceeds the yield stress in uniaxial tension.

- **Maximum Shear Stress or Tresca Criterion**

$$\boxed{\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \tau_0 = \frac{\sigma_0}{2}}$$

Yielding occurs when maximum shear stress reaches the value of the shear stress in the uniaxial tension test.

Yield Criteria

Why this Criteria??

It Predicts, at what combination of stresses a material starts yielding (start of plastic-Deformation).

Total stress = Hydrostatic Stress + Stress Deviator

(Doesnot cause plastic deformation) (Causes plastic deformation)

Von Mises' or Distortion-Energy Criterion

* Von-Mises Proposed that yielding would occur when the second variant of the stress deviator J_2 exceeded some critical value.

$\boxed{J_2 = K^2}$ $\quad J_2 = \frac{1}{6}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]$ --- (i)

To find K, we need to assume the uniaxial-loading, ($\sigma_1=\sigma_0$, $\sigma_2=\sigma_3=0$)

Putting these in (i)

$$\Rightarrow J_2 = \frac{1}{6}\left[\sigma_0^2+\sigma_0^2\right] = \frac{1}{3}\sigma_0^2 = K^2$$

$$\Rightarrow \boxed{\sigma_0 = \sqrt{3}K} \Rightarrow K = \frac{\sigma_0}{\sqrt{3}} \Rightarrow K^2 = \frac{\sigma_0^2}{3} = J_2 \quad \text{---(ii)}$$

Using (ii) in (i)

$$\frac{\sigma_0^2}{3} = \frac{1}{6}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]$$

$$\Rightarrow \sigma_0^2 = \frac{1}{2}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]$$

$$\Rightarrow \boxed{\sigma_0 = \frac{1}{\sqrt{2}}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]^{1/2}}$$

Where, no shear stress acts.

Also, $\boxed{\sigma_0 = \frac{1}{\sqrt{2}}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2+6\left(\tau_{xy}^2+\tau_{yz}^2+\tau_{xz}^2\right)\right]^{1/2}}$

(So, yielding will occur, when right side value of the equation exceeds the yield stress (σ_0) in uniaxial tension)

To identify 'K', consider the pure shear condition

$$\sigma_1 = -\sigma_3 = \tau, \quad \sigma_2 = 0$$

Put these in eqn for J_2

$$\Rightarrow \sigma_1^2+\sigma_1^2+4\sigma_1^2 = 6K^2$$

$$\Rightarrow \boxed{\sigma_1 = K} \rightarrow \text{Pure shear}$$

$$\boxed{\sigma_0 = \sqrt{3}K} \rightarrow \text{For Uniaxial tension}$$

So, Von Mises' criterion predicts that the yield stress in pure stress shear (torsion) is less than in uniaxial tension.

Maximum Shear Stress or Tresca Criterion

This Criterion assumes, yielding occurs when the maximum shear stress reaches the value of shear stress in uniaxial-tension test.

$$\boxed{\tau_{max} = \frac{\sigma_1-\sigma_3}{2}} \qquad \sigma_1 = \text{largest}, \; \sigma_3 = \text{Smallest}$$

For uniaxial tension, $\sigma_1=\sigma_0$, $\sigma_2=\sigma_3=0$

So, $\tau_{max} = \frac{\sigma_1-\sigma_3}{2} = \tau_0 = \frac{\sigma_0}{2}$

Thus, $\boxed{\sigma_1 - \sigma_3 = \sigma_0}$ --- (i)

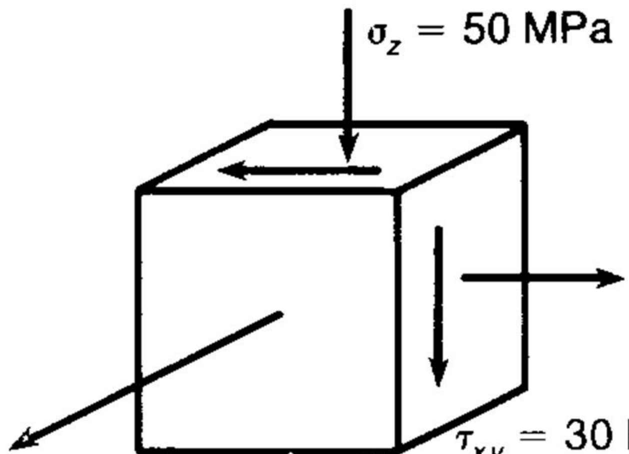
For pure shear, $\sigma_1 = -\sigma_3 = K$, $\sigma_2 = 0$

Putting in (i) $\sigma_1 - \sigma_3 = 2K = \sigma_0$

$$\Rightarrow \boxed{K = \frac{\sigma_0}{2}}$$

QUESTION:

Example Stress analysis of a spacecraft structural member gives the state of stress shown below. If the part is made from 7075-T6 aluminum alloy with $\sigma_0 = 500$ MPa, will it exhibit yielding? If not, what is the safety factor?



$$\sigma_0 = \frac{1}{\sqrt{2}}\left[\left(\sigma_x - \sigma_y\right)^2 + \left(\sigma_y - \sigma_z\right)^2 + \left(\sigma_z - \sigma_x\right)^2 + 6\left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2\right)\right]^{1/2}$$

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \tau_0 = \frac{\sigma_0}{2}$$

- **Calculate the yield stresses(σ_0) using Von Mises and Tresca criterion.**
- **Will it yield by this state of stresses?**

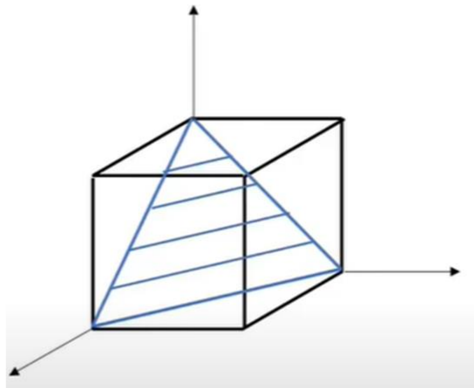
ANS: Put $\boldsymbol{\sigma_x=200, \sigma_y=100}$ **&** $\boldsymbol{\sigma_z= -50}$ in Von Mises equation and put $\boldsymbol{\sigma_1=\sigma_x=200, \sigma_3=\sigma_z= -50}$ in Tresca equation

- **224MPa**(by Von Mises criteria) & **250MPa** by Tresca criteria.
- It **will not yield** as the calculated yield stresses are much less than that of the given yield stress.(**224<500 and 250<500**)

OCTAHEDRAL SHEAR STRESS

For such a geometric body, the angle between the normal to one of the faces and the nearest principal axis is 54°44′, and the cosine of this angle is $1/\sqrt{3}$. This is equivalent to {111} plane in an fcc crystal lattice.

The stress acting on each face of the octahedron can be resolved[1] into a normal octahedral stress σ_{oct} and an octahedral shear stress lying in the octahedral plane, τ_{oct}. The normal octahedral stress is equal to the hydrostatic component of the total stress



$$\boxed{\sigma_{oct} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} = \sigma_m} \tag{3-32}$$

The octahedral shear stress τ_{oct} is given by

$$\tau_{oct} = \tfrac{1}{3}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2} \tag{3-33}$$

so we need to relate it to the axial yield strength σ_0. For a given material under axial load where $\sigma_1 = \sigma_0$ and $\sigma_2 = \sigma_3 = 0$, we assume that yielding occurs when the octahedral shear stress is equivalent to the octahedral stress criterion. This means we can combine Eq. 2 and 4 to get the octahedral stress criterion in terms of the yield strength:

$$\boxed{\tau_{h0} = \tau_h = \tfrac{1}{3}\sqrt{(\sigma_0 - 0)^2 + (0-0)^2 + (0-\sigma_1)^2} = \tfrac{\sqrt{2}}{3}\sigma_0} \tag{5}$$

With $\sigma_0 = \frac{3}{\sqrt{2}}\tau_{h0}$, we expect to observe yielding in a material under 3-D loading when, as before, we combine Eq. 2 and 4 to get

$$\sigma_0 = \tfrac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \tag{6}$$

OCTAHEDRAL SHEAR STRESS

Since the normal octahedral stress is a hydrostatic stress, it cannot produce yielding in solid materials. Therefore, the octahedral shear stress is the component of stress responsible for plastic deformation. In this respect, it is analogous to the stress deviator.

Since Eq. (3-34) is identical with the equation already derived for the distortion-energy theory, the two yielding theories give the same results. In a sense, the octahedral theory can be considered the *stress equivalent* of the distortion-energy theory. According to this theory, the octahedral shear stress corresponding to yielding in uniaxial stress is given by

$$\boxed{\tau_{oct} = \frac{\sqrt{2}}{3}\sigma_0 = 0.471\sigma_0} \qquad (3\text{-}35)$$

OCTAHEDRAL SHEAR STRAIN

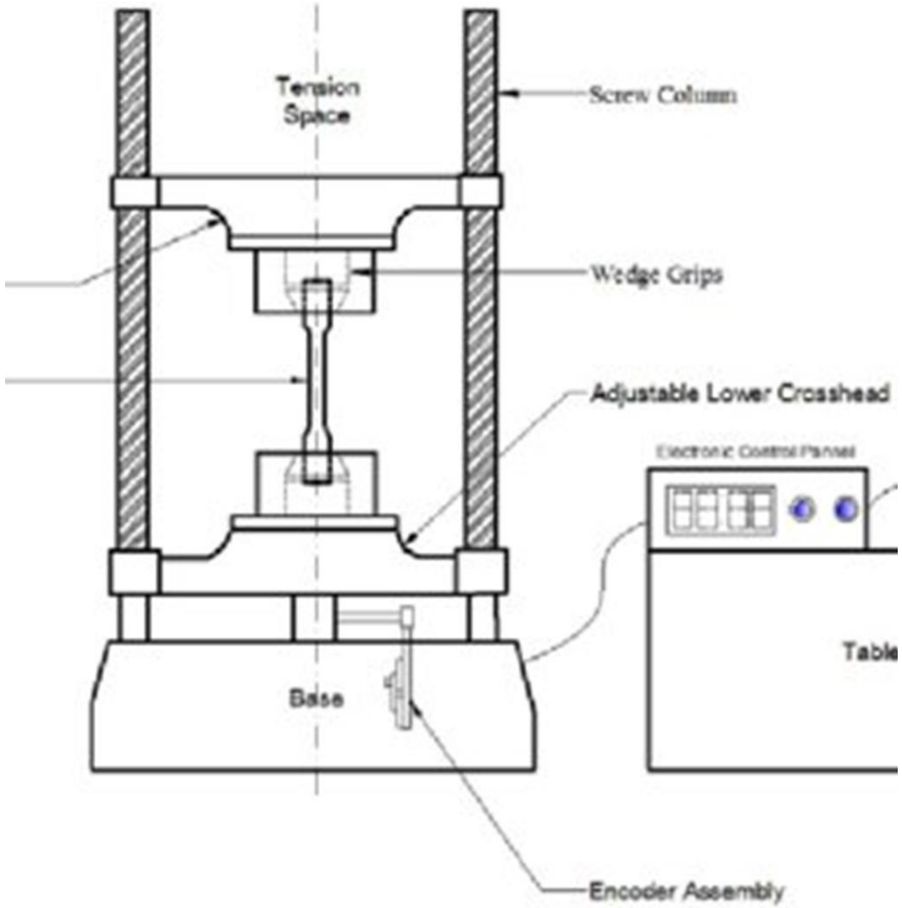
Octahedral strains are referred to the same three-dimensional octahedron as the octahedral stresses. The octahedral linear strain is given by

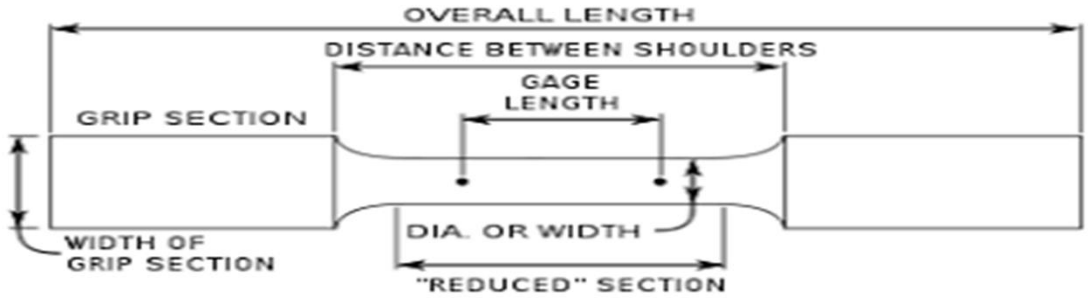
$$\varepsilon_{oct} = \frac{\varepsilon_1 + \varepsilon_2 + \varepsilon_3}{3} \tag{3-36}$$

Octahedral shear strain is given by

$$\gamma_{oct} = \frac{2}{3}\left[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2\right]^{1/2}$$

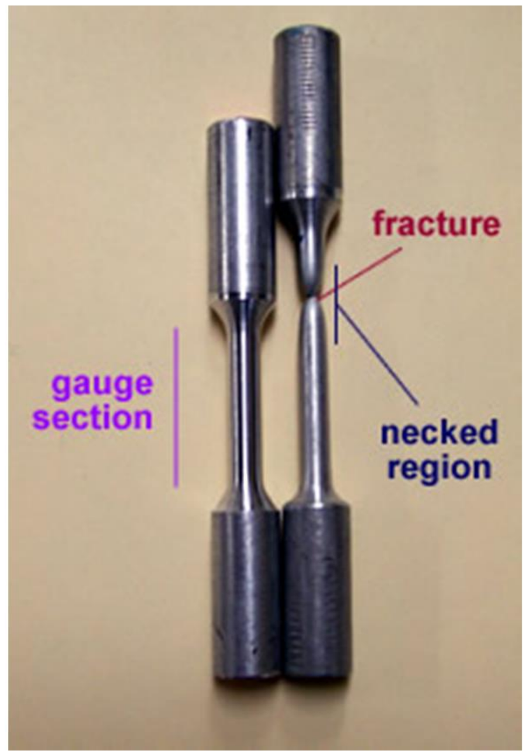
TENSILE TEST

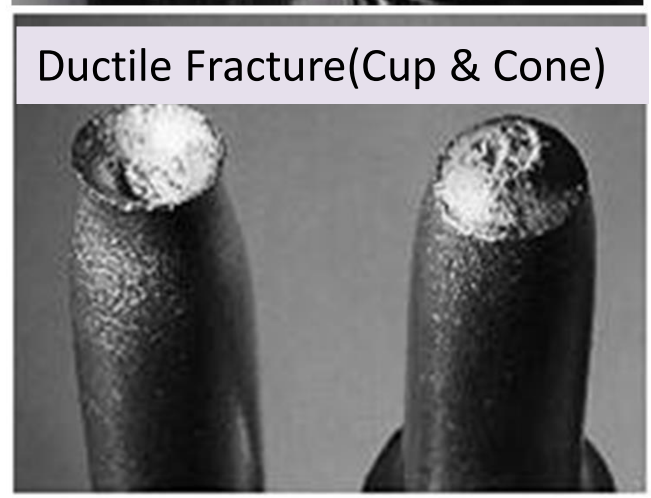



OVERALL LENGTH
DISTANCE BETWEEN SHOULDERS
GAGE LENGTH
GRIP SECTION
DIA. OR WIDTH
WIDTH OF GRIP SECTION
"REDUCED" SECTION


Brittle Fracture


brittle fracture
deformation fracture


fracture
gauge section
necked region


Ductile Fracture(Cup & Cone)

ASSIGNMENT

1. Identify the basic types of strain:
(a) Tensile strain (b) Volumetric strain (c) Shear Strain (d) Compressive strain
2. Hooke's Law is applicable up to:
(a)Yield Point (b) Ultimate Tensile Stress (c) Elastic Limit (d) Fracture point
3. Poisson's ratio is given by:

(a)Longitudinal strain/Transverse strain (b) Shear strain/Lateral Strain (c) Transverse strain / Longitudinal strain (d) Volumetric strain/Shear strain
4. A rod, 100 cm long and of diameter 4 cm is subjected to an axial load of 22 kN. The stress in N/m2 is.:

(a)1.75×107 (b) 4.37×106 (c) 1.75×103 (d) 4.37×103
5. The property of a material by which it can be rolled into thin sheets, is called:

(a)Elasticity (b)Malleability (c) Ductility (d) Viscosity
6. Briefly write about Tensile stress , Tensile strain, Shear Stress, Shear strain.
7. Draw stress-strain curves for brittle & ductile materials.
8. Which component of stress tensor causes plastic deformation?
9. A stress of 200MPa is applied in longitudinal direction of a steel rod, find the transverse strains (Assume isotropic condition and **ν**(poisson's ratio)=0.33, E=207 GPa).
10. A uniaxial stress is applied on a rod which resulted in elastic deformation with a strain of 0.025. Find the volumetric strain if **ν**(poisson's ratio)=0.33.
11. Find the Bulk modulus by using the data given in question no. 9.
12. A steel wire of cross sectional area 2mm2 and length 1m is stretched to 1.02m. Find the strain energy if E=200GPa.
13. Find stress deviator using the following stress tensor.

$$\sigma_{ij} = \begin{bmatrix} 100 & 50 & 50 \\ 50 & 125 & 75 \\ 50 & 75 & 75 \end{bmatrix}$$

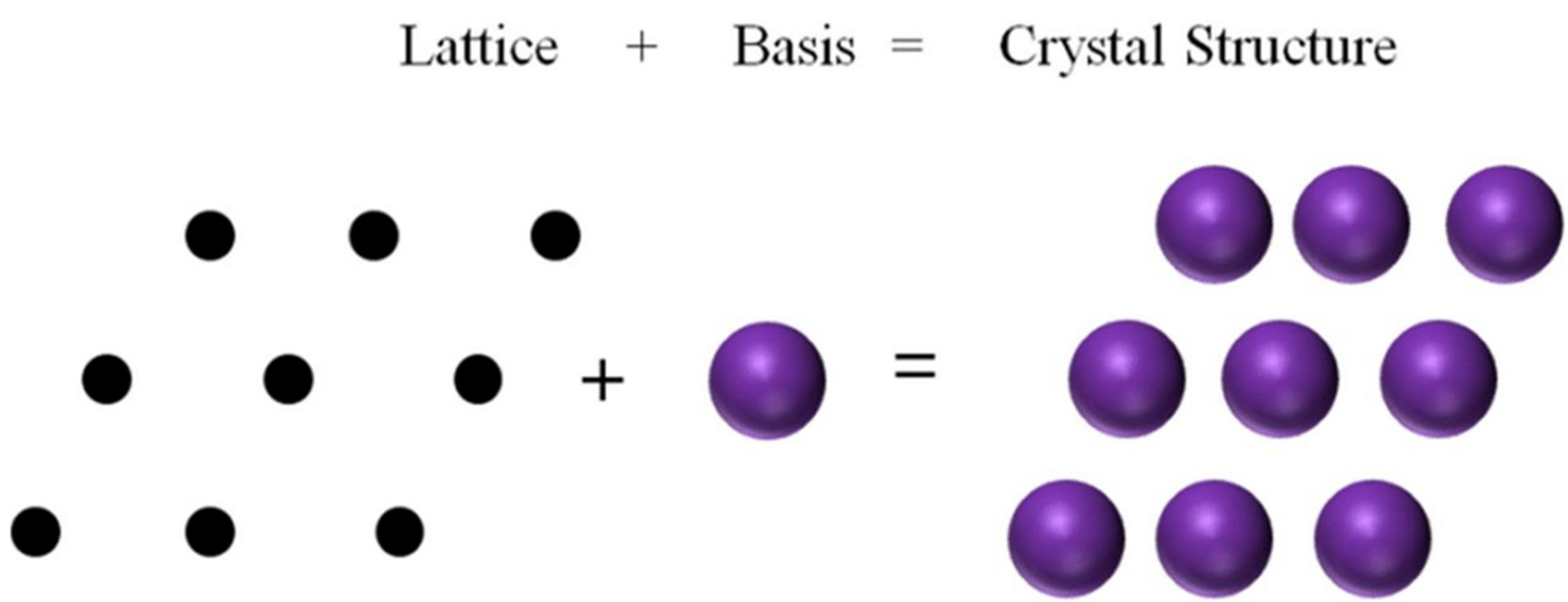
14. Find the principal stresses (in MPa) for the given state of stress.

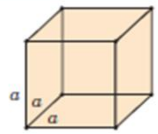
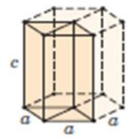
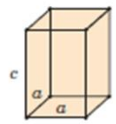
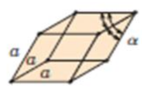
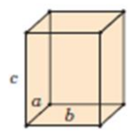
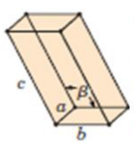
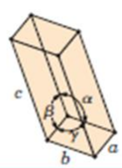
$$\begin{bmatrix} 0 & -48 & 0 \\ -48 & 40 & 0 \\ 0 & 0 & -56 \end{bmatrix} \text{MPa}$$

MODULE-II

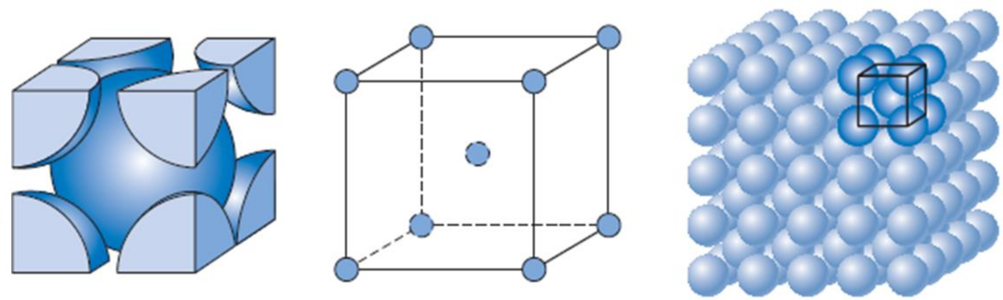
CRYSTAL GEOMETRY

- **Crystal:** Regular 3D patterns of atoms in space.
- **Lattice:** Distribution of points in 3D in such a way that every point has identical surroundings.
- Basis or Motif: Putting one or more atoms at a lattice point is called the basis or motif.



Crystal System	*Axial Relationships*	*Interaxial Angles*	*Unit Cell Geometry*
Cubic	$a = b = c$	$\alpha = \beta = \gamma = 90°$	
Hexagonal	$a = b \neq c$	$\alpha = \beta = 90°, \gamma = 120°$	
Tetragonal	$a = b \neq c$	$\alpha = \beta = \gamma = 90°$	
Rhombohedral (Trigonal)	$a = b = c$	$\alpha = \beta = \gamma \neq 90°$	
Orthorhombic	$a \neq b \neq c$	$\alpha = \beta = \gamma = 90°$	
Monoclinic	$a \neq b \neq c$	$\alpha = \gamma = 90° \neq \beta$	
Triclinic	$a \neq b \neq c$	$\alpha \neq \beta \neq \gamma \neq 90°$	

- ***Unit Cell:***
  - The atomic order in crystalline solids indicates that small groups of atoms form a repetitive pattern, thus, in describing crystal structures, it is often convenient to subdivide the structure into small repeat entities called **unit cells.**



BCC unit cell & aggregate of atoms

Metal	*Crystal Structure*[a]	*Atomic Radius*[b] *(nm)*	*Metal*	*Crystal Structure*	*Atomic Radius (nm)*
Aluminum	FCC	0.1431	Molybdenum	BCC	0.1363
Cadmium	HCP	0.1490	Nickel	FCC	0.1246
Chromium	BCC	0.1249	Platinum	FCC	0.1387
Cobalt	HCP	0.1253	Silver	FCC	0.1445
Copper	FCC	0.1278	Tantalum	BCC	0.1430
Gold	FCC	0.1442	Titanium (α)	HCP	0.1445
Iron (α)	BCC	0.1241	Tungsten	BCC	0.1371
Lead	FCC	0.1750	Zinc	HCP	0.1332

William D. Callister, Material science materials science and engineering.

Crystallographic Planes

- A crystallographic **plane** is specified in terms of length of its intercepts on the three axes, measured from the origin of the coordinate axes.
- **Miller Indices:** Planes and Directions will be specified w.r.t. the axes in terms of Miller Indices.

Qus: But, how to find Miller indices??

Ans: Just follow the simple steps!!

1. Find the intercepts along the axes.
2. Take reciprocals of these intercepts.
3. Change the reciprocals into smallest integers.
4. Enclose the integers in parentheses ().

- **Example:**

1. **Intercepts along the axes:**

   2, 3, 2

2. **Reciprocals of these:**

   1/2, 1/3, 1/2

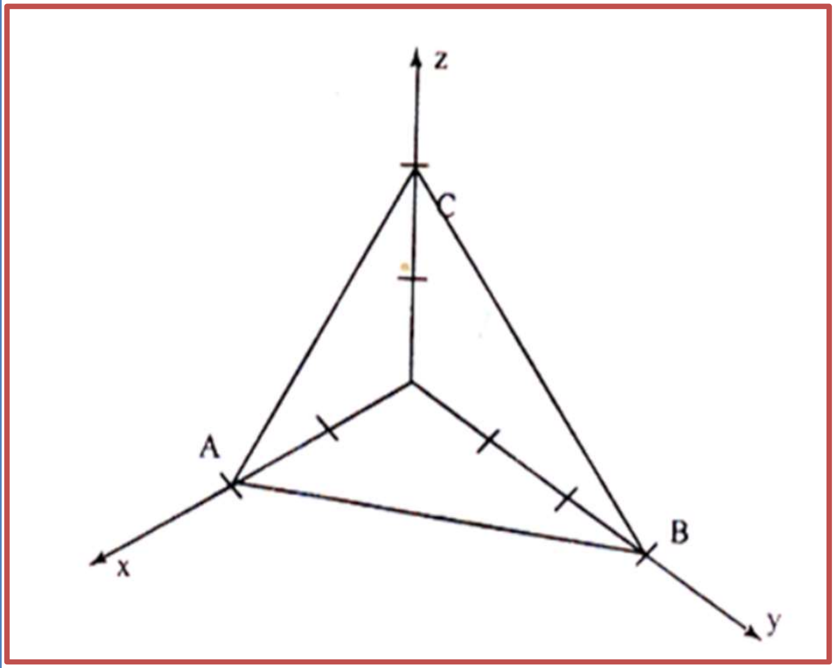
3. **Changing to integers:**

   (Find the least common denominator & multiply this with the reciprocals)

   3 2 3

4. **Enclosed in parentheses:**

   (323) ⟶ Miller Indices



How to draw a plane if Miller indices are given:

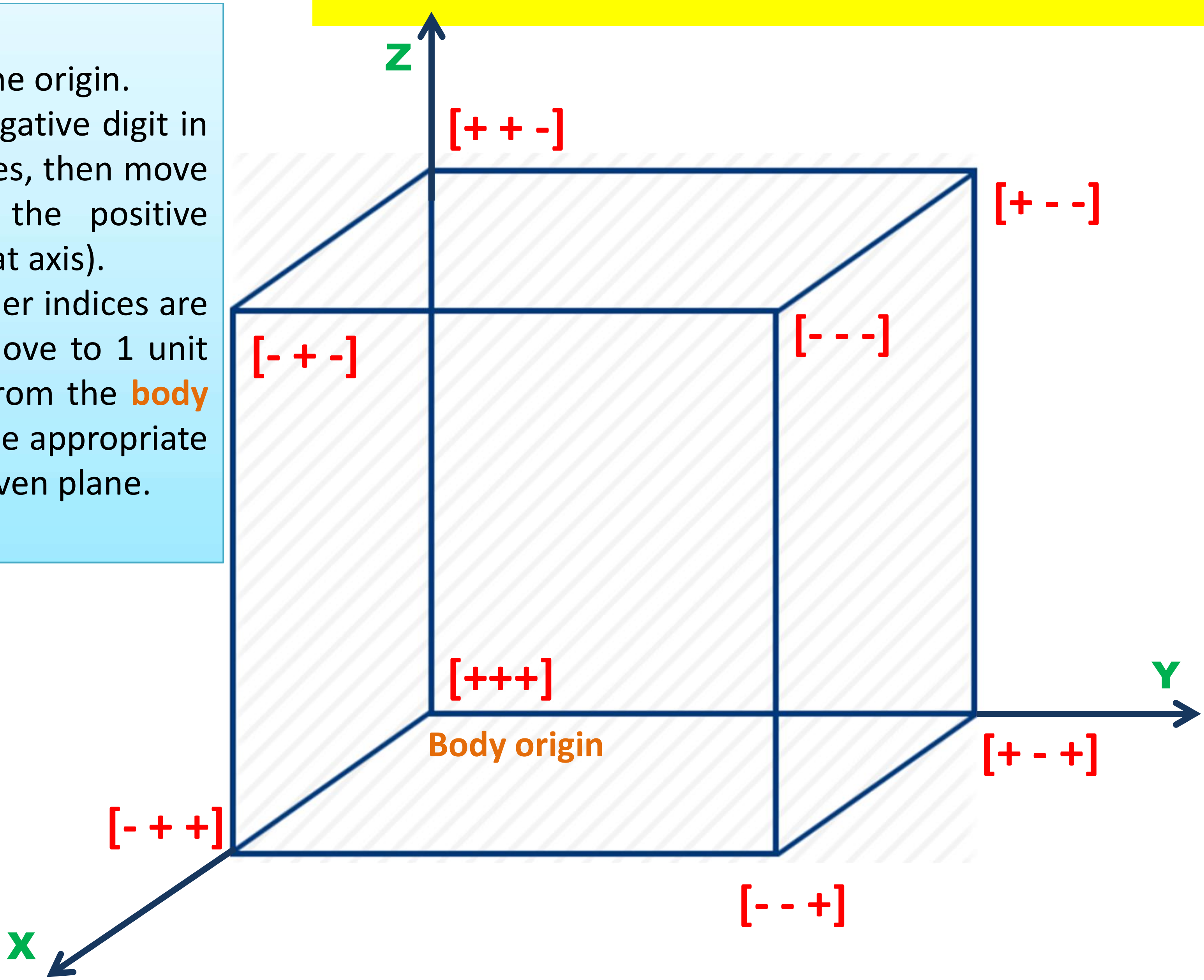
- **Find the reciprocals.**
- **Choose an appropriate origin.**
- **Mark the intercepts on each axis.**
- **Join them by straight lines.**

- **Draw planes using the given miller indices:**
- **$(\bar{2}10)$, (112), $(1\bar{2}\bar{3})$**

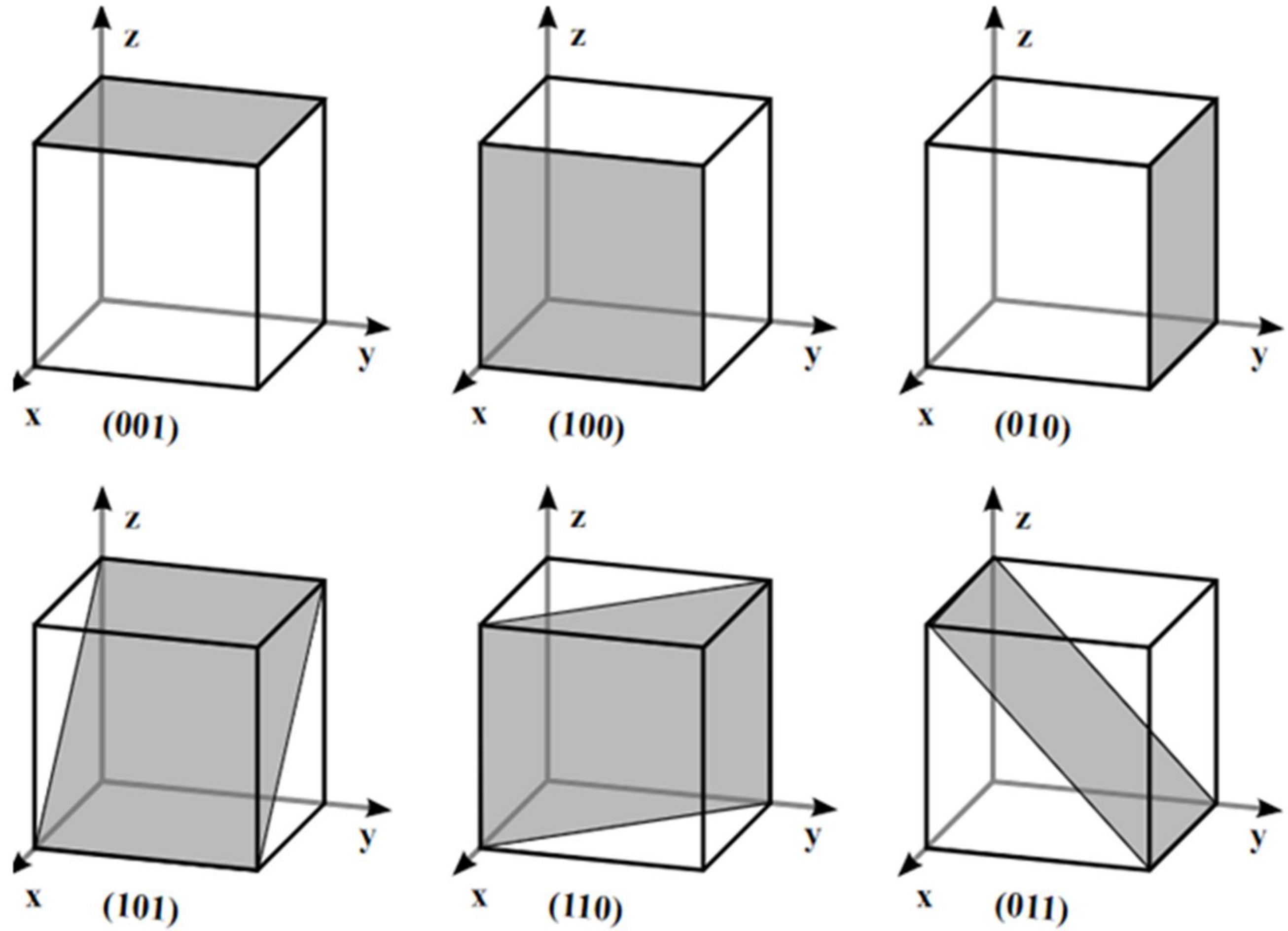
Origins for Possible Coordinates

Trick:

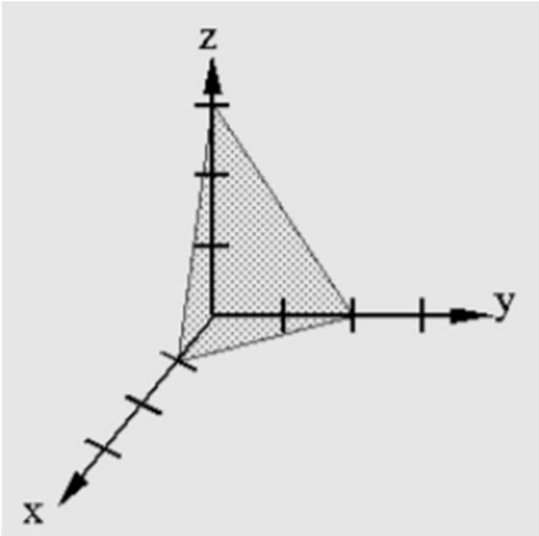
To Determine the origin.
(If there is a negative digit in the miller indices, then move the origin to the positive direction for that axis).
e.g. if given miller indices are ($\bar{1}$ 1 1), then move to 1 unit in X-direction from the **body origin** to find the appropriate origin for the given plane.

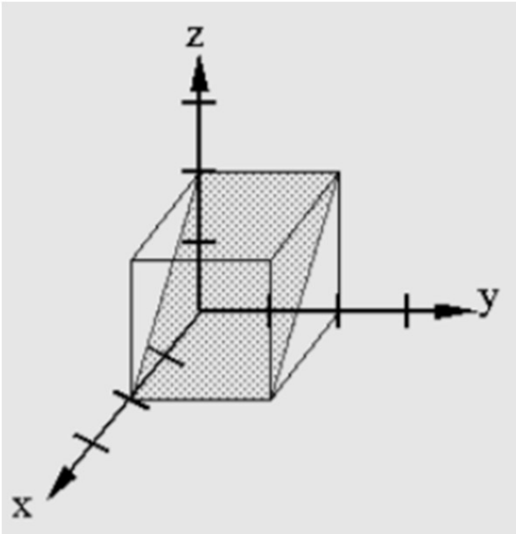


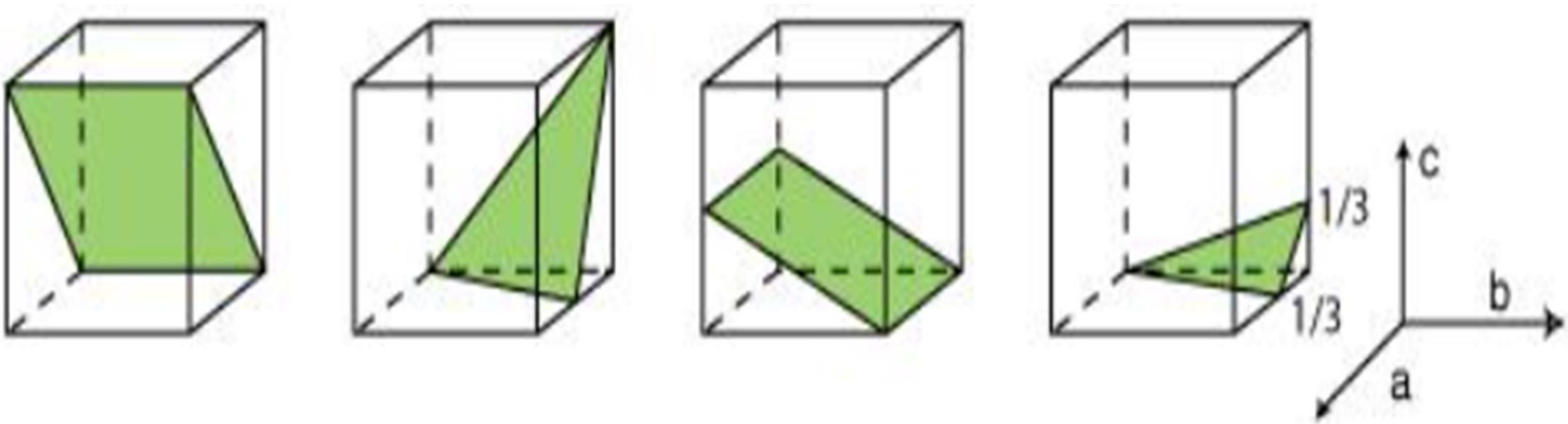
EXAMPLES



Find the Miller Indices

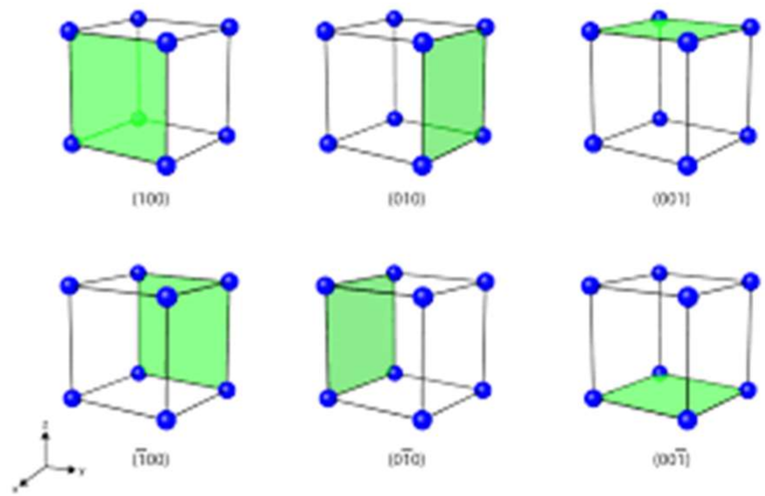






Family of Planes

- The bar over one of the integers indicates that the plane intersects one of the axes in a negative direction.
- E.g. There are six crystallographically equivalent planes of the type (100), any one of which can have the indices: (100), (010), (001), $(\bar{1}00)$,$(0\bar{1}0)$ $(00\bar{1})$
- These all can be considered as a group or ***family of planes*** and represented as **{100}.**



Crystallographic Directions

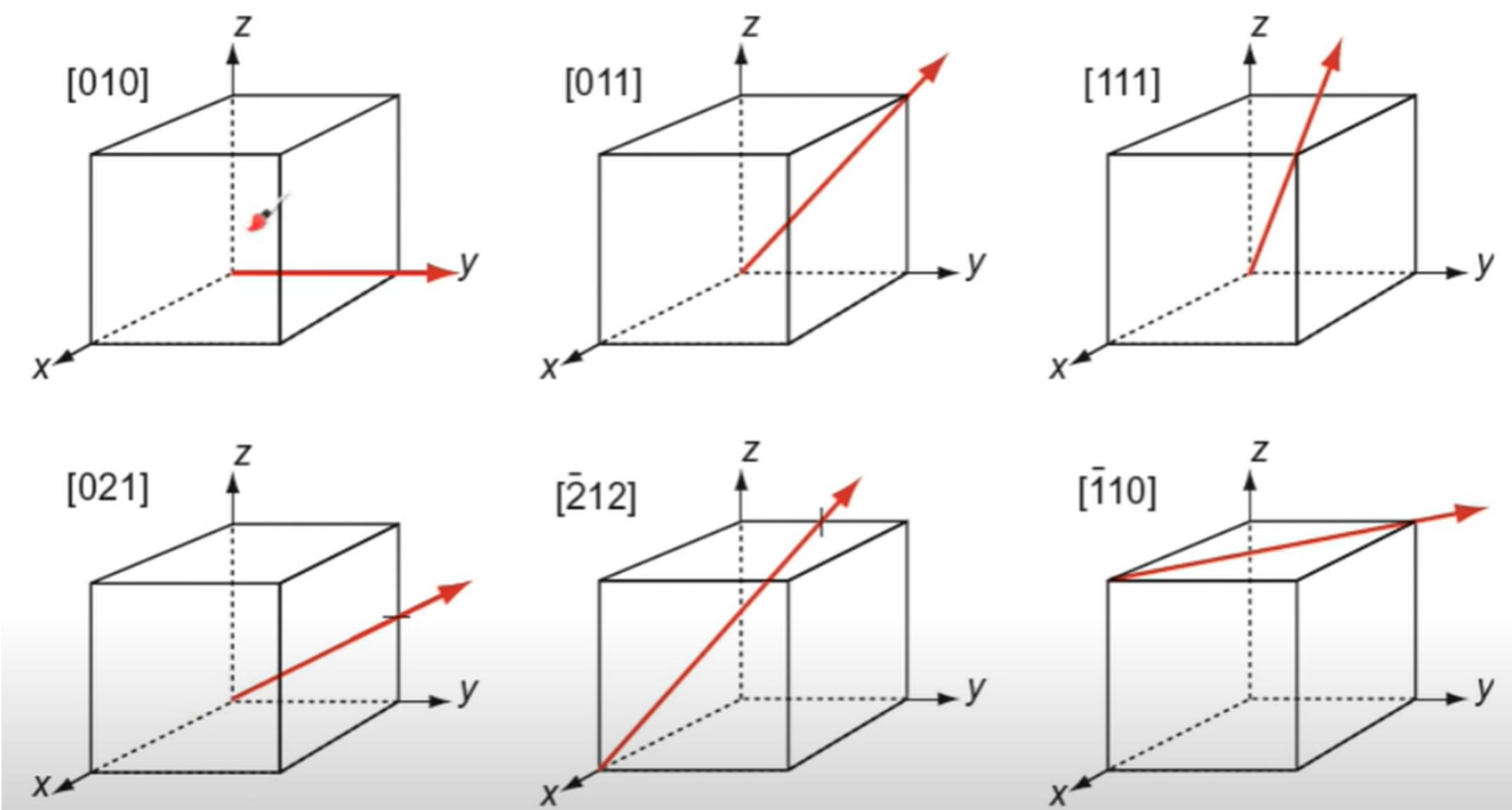
- Crystallographic Directions are indicated by integers in brackets: [uvw]
- Reciprocals are not used in determining directions.
- Family of directions is designated as: <uvw>

How to find a direction:

1. Draw X, Y & Z axes.
2. Determine the origin. (If there is a negative no in the direction vector, then move the origin to the positive direction for that axis).
3. Determine the end point.(Take common factor if any of the indices greater than 1.)
4. Connect the origin with the end point to get the direction.

E.g. [212] $\Longrightarrow$ 2[1 ½ 1]

Examples



Simple Relationships

For cubic systems there is a set of simple relationships between a direction $[uvw]$ and a plane (hkl) which are very useful.

1. $[uvw]$ is normal to (hkl) when $u = h$; $v = k$; $w = l$. [111] is normal to (111).
2. $[uvw]$ is parallel to (hkl), i.e., $[uvw]$ lies in (hkl), when $hu + kv + lw = 0$. $[11\bar{2}]$ is a direction in (111).
3. Two planes $(h_1k_1l_1)$ and $(h_2k_2l_2)$ are normal if $h_1h_2 + k_1k_2 + l_1l_2 = 0$. (001) is perpendicular to (100) and (010). (110) is perpendicular to $(1\bar{1}0)$.
4. Two directions $u_1v_1w_1$ and $u_2v_2w_2$ are normal if $u_1u_2 + v_1v_2 + w_1w_2 = 0$. [100] is perpendicular to [001]. [111] is perpendicular to $[11\bar{2}]$.
5. Angles between planes $(h_1k_1l_1)$ and $(h_2k_2l_2)$ are given by

$$\cos\theta = \frac{h_1h_2 + k_1k_2 + l_1l_2}{\left(h_1^2 + k_1^2 + l_1^2\right)^{1/2}\left(h_2^2 + k_2^2 + l_2^2\right)^{1/2}}$$

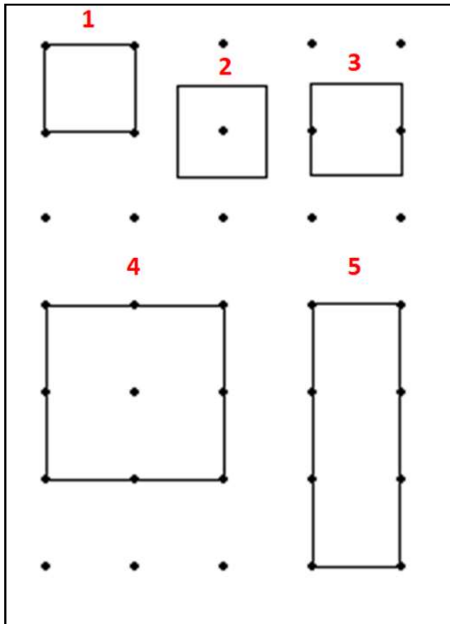
QUESTIONS FOR PRACTICE

The miller indices of the direction common to the planes (111) and (110) in a cubic system is

(A) [111] (B) [110] (C) $[\bar{1}10]$ (D) $[\bar{1}11]$

For a simple cubic unit cell with unit vectors **i**, **j** and **k**, the angle between lattice vectors [100] and [111] in degrees is

(A) 35.2 (B) 54.7 (C) 60 (D) 90

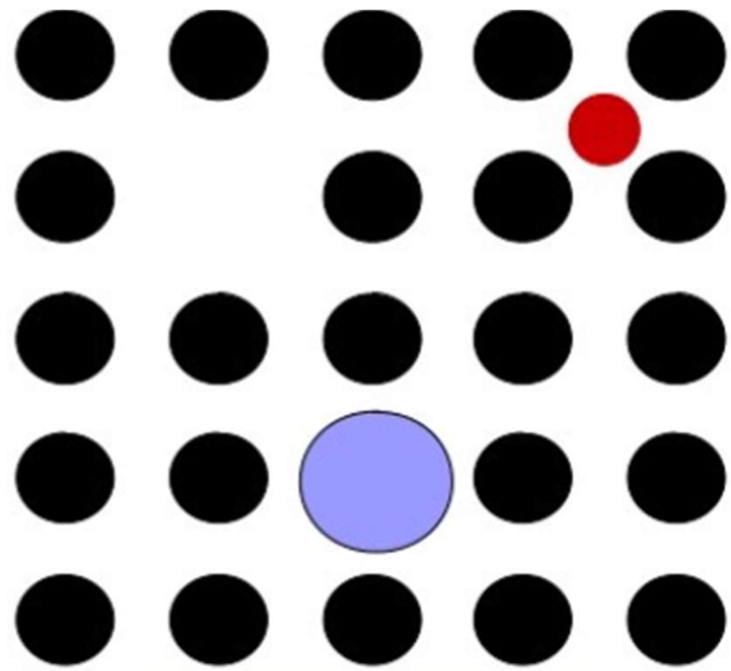


IDENTIFY THE PRIMITIVE CELLS

- **FOR A RHOMBOHEDRAL CRYSTAL** Given, **Axis ‘a’=5 Å**, Angle between the axes **‘α’=50º**. Find Axes **‘b’** and **‘c’** and angles **β** and **γ.**

Lattice Defects

- The defects or imperfections are the occasional disruptions in the periodicity within the crystals.
- As crystal structure is a geometric concept, it is natural to classify the defects based on the basic geometry:



- **1. Point defects (Zero dimensional)**
- **2. Line defects (One dimensional)**
- **3. Surface defects (Two dimensional)**
- **4. Volume defects (Three dimensional or 3D)**

Properties in terms of Sensitivity to Defect

- **Structure Insensitive Properties:**
  - The properties of the crystalline solid which are hardly or not affected by the presence of defects in crystal.

- **Structure Sensitive Properties:**
  - The properties of the crystalline solid which are profoundly affected by the presence of defects in crystal.

Structure-insensitive	Structure-sensitive
Elastic constants	Electrical Conductivity
Melting point	Semiconductor properties
Density	Yield stress
Specific heat	Fracture strength
Coefficient of thermal expansion	Creep strength

Point Defects

- Point defects is a defect of dimensions like a point.
- **Types:**
- Vacancy exists when a atom is missing from its normal lattice position.
- In pure crystals, small no of vacancies are created by thermal excitation.
- Vacancies are **thermodynamically stable** at temperatures greater than absolute zero.
- At equilibrium, the fraction of lattices that are vacanct at a given temperature is given by,

$$\boxed{\frac{n}{N} = e^{-E_s/kT}}$$

- n= no. of vacant sites
- N= no. of lattice sites
- Es=energy required to move an atom from interior of crystal.
- k= Boltzmann Constant

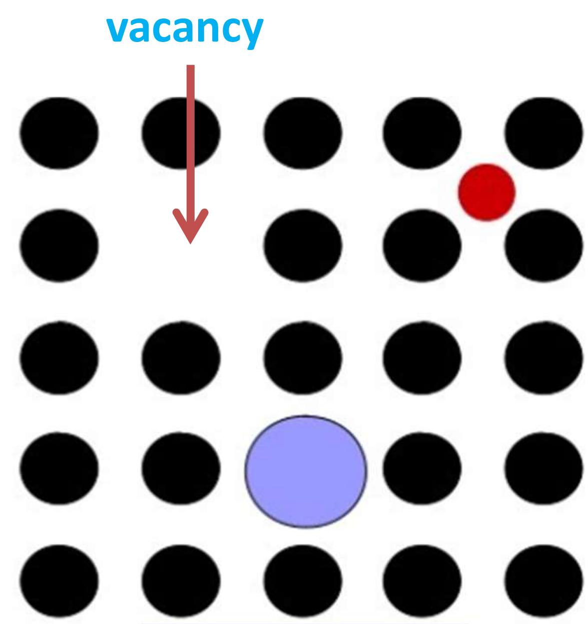


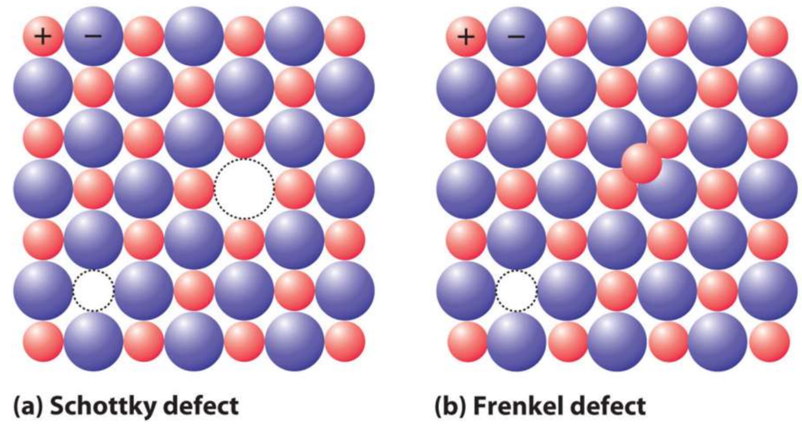
Table 4-3 Equilibrium vacancies in a metal

Temperature, °C	Approximate fraction of vacant lattice sites
500	1×10^{-10}
1000	1×10^{-5}
1500	5×10^{-4}
2000	3×10^{-3}

$E_s \approx 1$ ev $(= 0.16 \times 10^{-18}$ J)

Point Defects

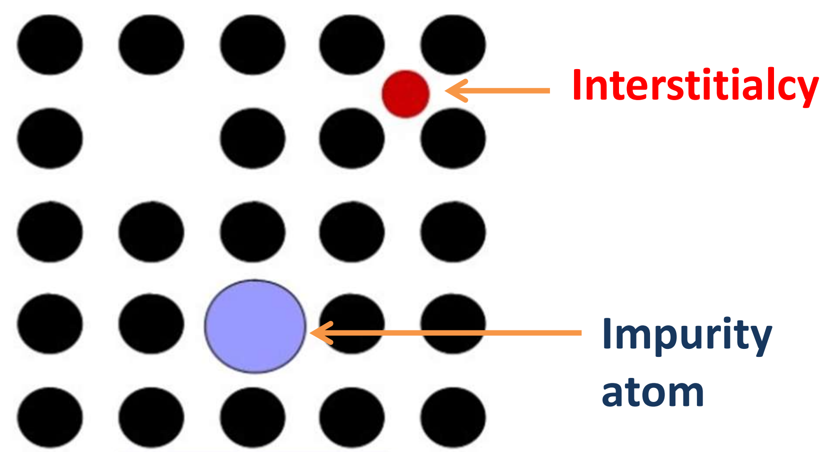
- **Schottky defect:**
- When equal no. of cations & anions are missing from their regular lattice positions.
- **Frenkel defect:**
- When an atom is shifted from a normal lattice site and is forced in to interstitial.



- **Interstitialcy**

Occurs when an atom is trapped inside the interstitial position.

- **Impurity Atoms:**
- The presence of impurity atoms at normal lattice sites or at interstitial positions create local disturbances in the lattice.



Reference Books

- Mechanical Metallurgy by G.E. Dieter.
- Physical Metallurgy by Vijendra Singh.
- Materials Science & Engineering: An Introduction by William D. Callister