FLUID MECHANICS AND HYDRAULIC MACHINE (RME3C002)

MODULE-1

INTRODUCTION

Scope of fluid mechanics and its development as a science Physical property of fluid: Density, Specific gravity, Specific Weight, Specific volume, Surface tension and capillarity, viscosity, compressibility and bulk modulus, Fluid classification

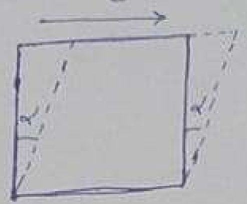
FLUID STATICS

Pressure, Pascal's Law, Pressure variation for incompressible fluid, atmosphere pressure, absolute pressure, gauge pressure and vacuum pressure, manometer. Hydrostatic process on submerged surface on a horizontal submerged plane surface, force on a vertical submerged plane surface. Buoyancy and floatation, Archimedes principle, stability of immersed and floating bodies, determination of metacentric height.

Fluid →

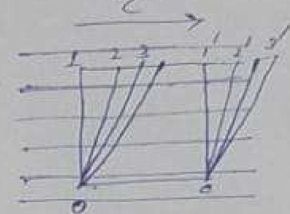
→ Things which can flow (liquid & gas)

→ If tangential force applied a fluid, then fluid will continously deforming



(Solid)

stress $\propto$ strain



(Fluid)

stress $\propto$ rate of strain

Concept of Continuum / Continous Medium →

λ → mean free path

L → Characteristic dimension of the problem.

$\frac{\lambda}{L}$ → Knudsen Number

$\frac{\lambda}{L} < 0.01$ (Continuum is valid)

$0.01 < \frac{\lambda}{L} < 0.1$ (Slip flow)

$0.1 < \frac{\lambda}{L} < 10$ (Transition Flow)

$\frac{\lambda}{L} > 10$ (Free molecular Flow)

Physical Property of fluid :

① Mass Density (ρ) →

It is defined as mass per unit volume at a specified pressure & temperature in the fluid.

$4^\circ C \rightarrow 1\ atm \rightarrow \rho_w \rightarrow 1000\ kg/m^3$

$20^\circ C \rightarrow 1\ atm \rightarrow \rho_w \rightarrow 998\ kg/m^3$.

$20^\circ C \rightarrow 100\ atm \rightarrow \rho_w \rightarrow 1.007\ kg/m^3$.

$20^\circ C \rightarrow 1\ atm \rightarrow \rho_{air} \rightarrow 1.23\ kg/m^3$

$0^\circ C \rightarrow 100\ KPa \rightarrow \rho_{air} \rightarrow 1.27\ kg/m^3$.

② Specific Weight / weight density (w) →

$$w = \frac{wt}{vol}$$

$$\Rightarrow \boxed{w = \frac{mg}{V} = \rho g}$$

$$w_{H_2O} = \rho_{H_2O} \times g = 1000 \times 9.81 = 9810 \ N/m^3.$$

③ Relative Density (R.D) →

$$RD = \frac{\rho_1}{\rho_2}$$

④ Specific Gravity (SG)

$$(SG)_f = \frac{\rho_f}{\rho_{standard}}$$

$\rho_w = 1000 \ kg/m^3$ $\qquad$ $\rho_{air} = 1.27 \ kg/m^3$.

Liquid → $S_f = \dfrac{\rho_f}{\rho_w}$

$$S_f = \frac{\rho_f \times g}{\rho_w \times g}$$

$$\boxed{S_f = \frac{w_f}{w_w}}$$

① 1 lt of oil wt is 7 N, calculate the following.

i) value in m^3, in cm^3 & mm^3.

ii) mass of the oil.

iii) Desity of the oil.

iv) sp. wt of the oil

v) Sp. Gravity of the oil.

vi) sp. volume of oil.

Given: Volume = 1lit (lt)

$W = 7N$.

$1000\ lt = 1m^3$

$1 lt = \frac{1}{1000} m^3$

i) $v = 1lt = 0.001\ m^3 = 0.001\ 100cm^3$

$= 1000\ cm^3 = 10^6\ mm^3$

ii) $7 = m \times 9.81$

$m = 0.713 kg.$

iii) $\rho = \frac{m}{V} = \frac{0.713}{0.001} = 713\ kg/m^3$

iv) $w = \rho \times g$

$= 713 \times 9.81\ N/m^3$

$= 7000\ N/m^3$

v) $S = \frac{\rho_{oil}}{\rho_{H_2O}} = \frac{713}{1000} = 0.713$

vi) $v_s = \frac{1}{713}\ m^3/kg.$

② 1 lt of Petrol: sp. gravity is = 0.7, Determine volume in m^3 & cm^3, mass density of petrol in kg/m^3 & gm/ml, wt. density of the petrol, mass of the petrol, wt. of the petrol, sp. volume of the petrol

$V = 1$ lit

Sp. Gravity $S_p = 0.7$

i) $V = 0.001\, m^3 = 1000\, cm^3$

ii) $S_p = \frac{\rho_p}{\rho_{H_2O}}$

$\Rightarrow \rho_p = 0.7 \times 1000 = 700\, kg/m^3$

iii) $w_p = \rho_p \times g$

$= 700 \times 9.81$

$= 6867\, N/m^3$

iv) $\rho_p = \frac{m}{0.001}$

$\Rightarrow m = 0.7\, kg$

v) $W = mg$

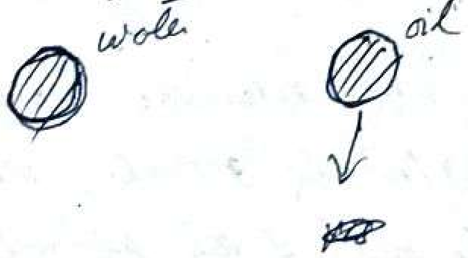
$= 0.7 \times 9.81 = 68.67\, N$.

vi) $v_s = \frac{1}{\rho_p} = \frac{1}{700}\, m^3/kg$

Viscocity →

It is the property of the fluid due to which the fluid flow is resisted. This property is due to intermolecular attraction of the fluids due to their own molecules & adjaint contact other body molecules. It is similar to coefficient of friction betn solid bodies.

Ex; vis of oil > vis of H_2O



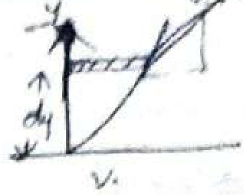
→ Viscocity is a ~~drag~~ drag force which opposes fluid motion. All the real fluid exibit viscocity property.

Newton's Law of Viscosity

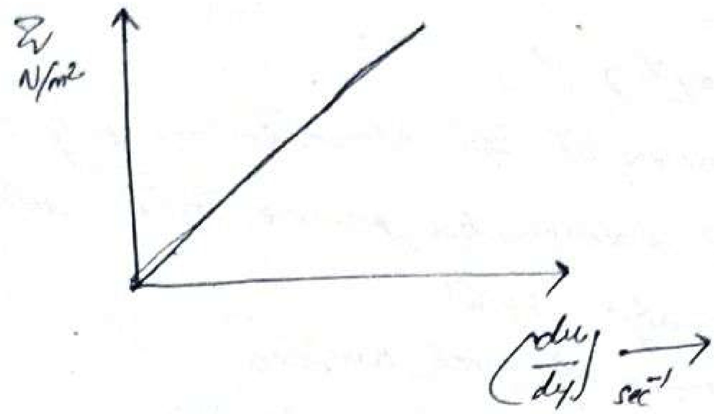
It states that shear stress of the fluid layer is proportional to the rate of change of velocity of fluid layer w.r.to the distance betn 2 relative layers.

$\tau = F/A$

Layer flow
or
Laminar flow.
or
Viscous flow.



$\frac{du}{dy}$ = vel. gradient
or
shear strain rate
or
deformation rate



$\tau \propto \frac{du}{dy}$

$\Rightarrow \tau = \mu \frac{du}{dy}$

$\mu \rightarrow$ coefficient of viscosity or Dynamic viscosity. Absolute viscosity.

Units of μ

$N/m^2 = \mu \left(\frac{m/sec}{m}\right)$

$N/m^2 \cdot sec = \mu$

$\Rightarrow Pa \cdot sec = \mu = F^1 L^{-2} T^{-1}$

$\mu = N/m^2 \cdot sec$

$= \frac{kg \cdot m/sec^2}{m^2} \cdot sec$

$= kg/m \cdot sec = M^1 L^{-1} T^{-1}$

(3) An incompressible fluid (a) $\nu = 7.4\times10^{-7}$ m²/sec & Sp. Gravity of oil is 0.88 then what is the value of mass density, wt. density & dynamic viscosity of the oil?

1) $S_{oil} = \frac{\rho_{oil}}{\rho_{water}}$

$\Rightarrow 0.88 = \frac{\rho_{oil}}{1000}$

$\Rightarrow \rho_{oil} = 880\, kg/m^3$

2) $w_{oil} = \rho_{oil} \times g$

$= 880 \times 9.81\ N/m^3$

$= 8632\ N/m^2$

3) $\nu = \mu/\rho$

$\Rightarrow 7.4\times10^{-7} = \frac{\mu}{880}$

$\Rightarrow \mu = 6.51\times10^{-4}\ kg/m\text{-}sec.$

(4) The shear stress developed in a lubricating oil of viscosity ~~of~~ 0.0985 Pa-sec. filled betn 2 parallel plates 1cm apart & moves with a relative velocity of 2 m/sec.

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$\tau = \mu \frac{du}{dy}$

$= 0.0985 \times \frac{2\ m/sec}{0.01\ m}$

$= 19.62\ N/m^2$.

(5) An incompressible fluid $\nu = 0.74$ cm²/sec, Sp. Gravity 0.88 is held betn 2 parallel plates. If the top plate is moved with a velocity 0.5 m/sec. while the bottom plate is held stationary, the fluid reaches a linear velocity in the gap of 0.5 mm betn the plates, then shear stress in parallel at the surface of the plate

Compressibility of fluid →

To compress the gases a small amount of force is sufficient because of loose force of attraction between gas molecules where as in case of liquid large amount of force required to compress liquid molecule.

→ Compressibility is the reciprocal of Bulk modulus of Elasticity (K)

→ Bulk modulus of Elasticity (K) = $\frac{\text{Increase in pressure}}{\text{Decrease in vol. strain}}$

$$K = \frac{+\Delta P}{+\left(\frac{\Delta V}{V}\right)} = \frac{+\Delta P}{+\left(\frac{\Delta \rho}{\rho}\right)}$$

(P) A mercury of Bulk modulus 2 MPa subjected to pressure change of 200 N/cm², Then the change in volume in % is ________

Ans: $K = \frac{\Delta P}{\frac{\Delta V}{V}}$

$$\Rightarrow 2 \times 10^6 \, N/m^2 = \frac{200 \times 10^4 \, N/m^2}{\frac{\Delta V}{V}}$$

$$\Rightarrow \frac{\Delta V}{V} = 1 = 100\%$$

Surface Tension →

Property of the fluid (mostly liquid) exibits a tensile force at the free surface of the liquid.

→ This property is due to intermolecular forces of the own fluid molecules and two different liquid molecules.

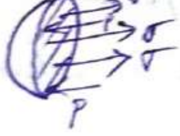
→ Surface Tension $= \frac{\text{Force}}{\text{Unit Length}}$ (N/m)

→ " " $= \frac{\text{Energy}}{\text{Unit Area}} \left(\frac{N \cdot m}{m^2}\right) = J/m^2$

Relation between Surface Tension (σ) & Pressure (P) ⇒

Case - I → Liquid droplet

$$P = \frac{F}{\pi/4 \, d^2} \quad \ldots (1)$$



$$\sigma = \frac{F}{L} = \frac{F}{\pi d} \quad \ldots (2)$$

From (1) & (2) $\quad P \times \pi/4 \, d^2 = \sigma \times \pi d$

$$\Rightarrow \boxed{P = \frac{4\sigma}{d}}$$

Case - (2) Soap Bubble

$$P = \frac{F}{\pi/4 \, d^2} \quad — (1)$$

$$\sigma = \frac{F}{2(\pi d)} \quad — (2)$$

(As inside & outside surface of bubble exibit surface Tension)

From (1) & (2), $\boxed{P = \frac{8\sigma}{d}}$

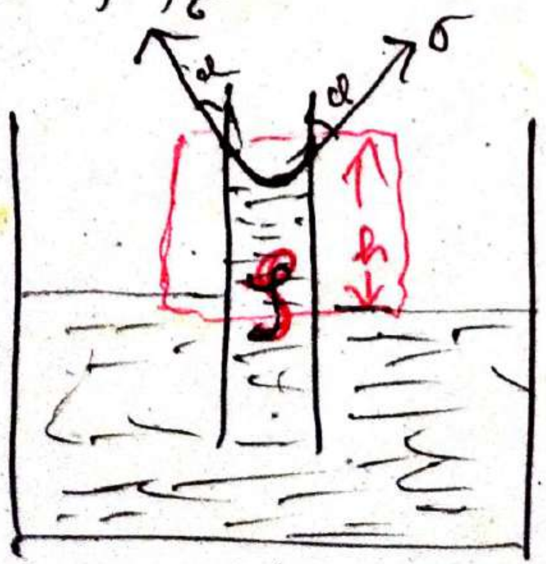
$= 0.0175\ N$

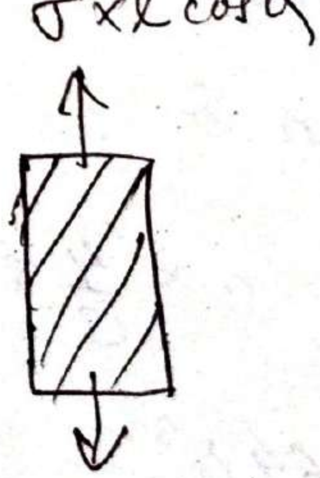
Cappillarcity →

Cappilarity is the phenomenon of liquid due to which liquids rises in narrow tubes.

Ex; Trees of Plant getting H_2O through the different branches of a tree.

– Inner Diameter of pipe < 12 mm exibits capillarity Rise due & size of the pipe.





$W = mg$

$= \rho V g$

$= \rho A h g$

Expression for capillarity in a narrow pipe →

Let d = Inside diameter of narrow pipe (m)

ρ = Density of fluid (kg/m³)

σ = Surface Tension of fluid (N/m)

θ = Angle of contact with vertical surface

Self wt of fluid = Surface Tension of fluid

$$\Rightarrow \rho A h g = \sigma l \cancel{\sin\theta} \cos\theta$$

$$\Rightarrow \rho \times \frac{\pi}{4} d^2 \times h \times g = \sigma \times \pi d \times \cancel{\sin\theta} \cos\theta$$

$$\Rightarrow h = \frac{4\sigma \cos\theta}{\rho g d}$$

$$h = \frac{4\sigma \cos\theta}{\gamma d}$$ where γ = Sp. weight or weight density

$$\Rightarrow \boxed{h \simeq \frac{4\sigma}{\gamma d}}$$ if $\theta < 20^\circ$

$$h \propto \frac{1}{d}$$

$$\Rightarrow \boxed{hd = c}$$

Q. ~~3×45~~

... viscosity

Types of Fluids -

1) Ideal Fluid
2) Real Fluid
3) Newtonian Fluid.
4) Non Newtonian Fluid.
5) Plastic Flow Fluid.
6) Thixotropic fluid.

1) Ideal Fluid

- incompressible
- zero viscosity (No shear stresses or forces.)
- It is imaginary Fluid for comparison process.
- No Fluid in universe possess Ideal Fluid properties.

2) Real Fluid →

- one whose has viscosity property & subjected to small amount of shear stresses
- Viscosity property counted.

Types of Real Fluid

i) Newtonian Fluid → Fluid which obeys Newton's law of viscosity (shear stress proportional shear strain Rate).

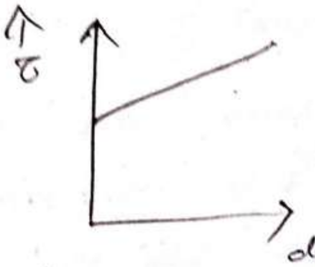
Ex: H_2O, Hg, Light viscous oil.

ii) Nonnewtonian Fluid
- Doesn't obey Newton's law of viscosity.

- $\tau \neq \mu \frac{du}{dy}$.

Types of Nonnewtonian Fluid < Dilatent, pseudoplastic

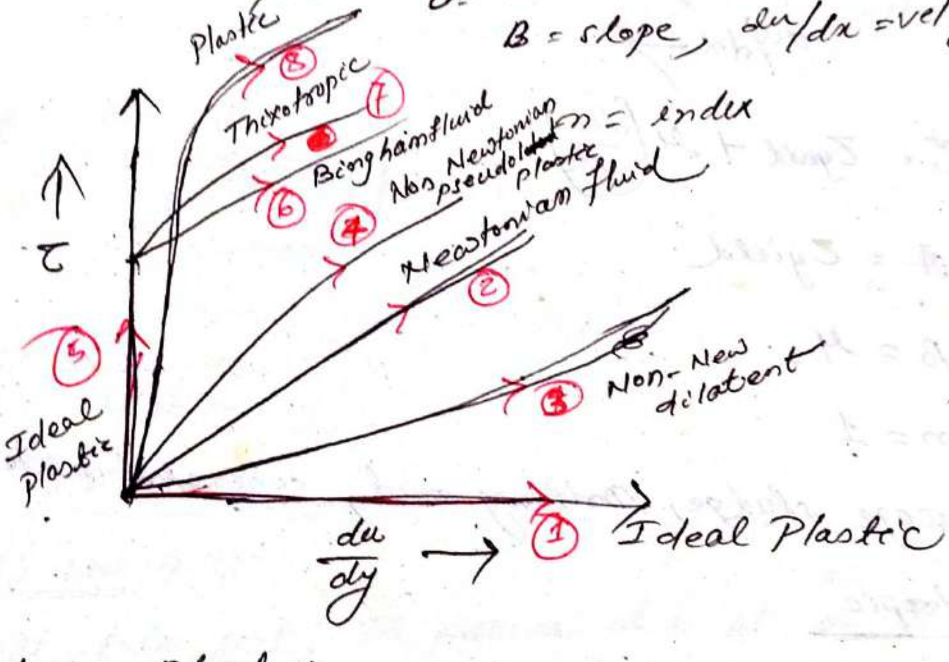
a) Dilatent Fluid.
All the fluids follow fluid power law.



$y = c + mx$

$$\boxed{\tau = A + B\left(\frac{du}{dy}\right)^n}$$ ← Fluid Power law.

A = const
B = slope, du/dx = vel. gradient
n = index



① Newtonian Fluid ⇒

$A = 0$
$B = \mu$
$n = 1$

$\tau = 0 + \mu\left(\frac{du}{dy}\right)^1$

$\Rightarrow \boxed{\tau = \mu \frac{du}{dy}}$

② Dilatent Fluid

Ex: Sugar in water, salt in water, Rice starch, Butter.
Any aceous solⁿ

$A = 0$

$B = \mu$

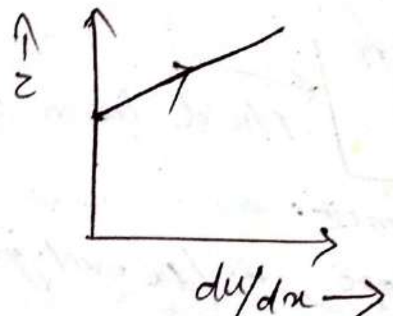
$n > 1$

Pseudo ~~Fluid~~ Plastic →

$A = 0,\ B = \mu,\ n < 1$

Ex; Milk, Blood, Paper pulp, Fruit juice.

Bingham Fluid



$$\tau = \tau_{yeild} + \mu\left(\frac{du}{dx}\right)$$

$A = \tau_{yield}$

$B = \mu$

$n = 1$

Ex; Sewage sludge, Drilling mud, concrete mixture.

Thixotropic

$A = \tau_{yeild}$

$B = \mu$

$n < 1$

Ex; Lipstics solⁿ, Printer link.

Viscoocity changes with temp →

i) viscocity of liquids decrese with increase temp

ii) viscocity of ~~liquid~~ gas increases with increase temp

10/11/2010 FLUID STATICS

1) Pressure Measurement (Manometry)
2) Forces acting on submerged body
3) Buyanancy (Forces acting on Floating & submerged bodies)

Fluid statics →

It is a branch of FM which deals the fluid under Rest (No shear is acting). When fluid at Rest the following forces involves 1) Body Force (selfwt of the fluid), $W = mg$ (N)

2) Force due to static pressure ($F_P = P \times A = \rho g h$ a)

→ Fluid statics based on

i) Hydrostatic Law & ii) Pascal's Law

It states that, " The rate of increase of pressure in the vertical dirn along the gravitational const 'g' is equal to the wt of the fluid at that pt".

i.e $\boxed{\frac{dP}{dh} = \gamma = w = \rho g}$

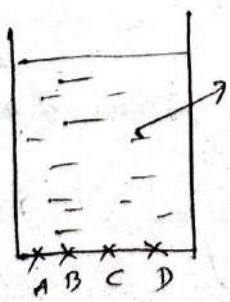
$\Rightarrow dP = \rho g dh$

$\Rightarrow \int dP = \int \rho g dh$

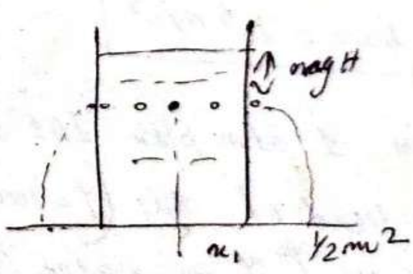
$\Rightarrow \boxed{P = \rho g h = wh}$

ii) Pascal's Law →

It states that, "The pressure at a pt for the given fluid is equal in all dirn."



$P_A = P_B = P_C = P_D = \rho_{wat}\, g\, h_{water}$



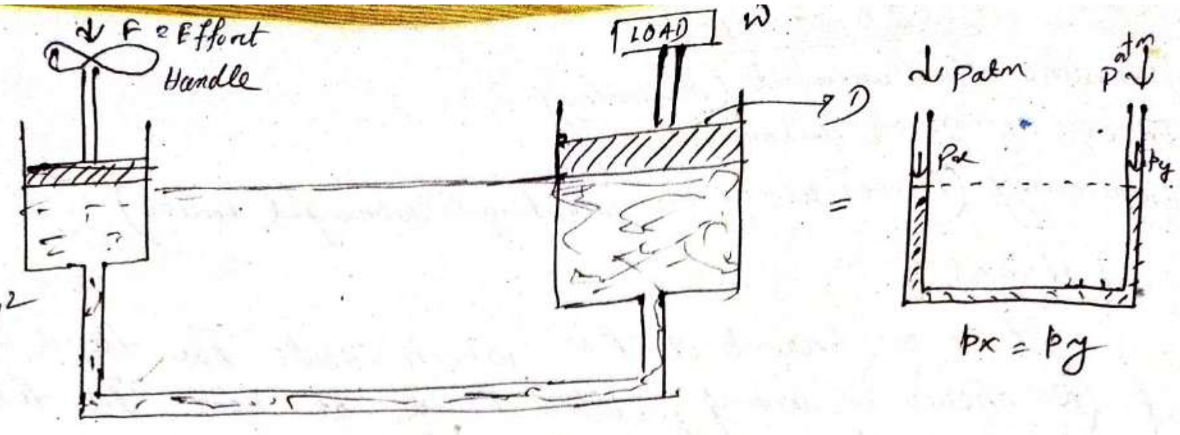
$gH = \frac{v^2}{2}$

$v = \sqrt{2gH}$

N

Application of Pascal's Law & Hydrostatic Law

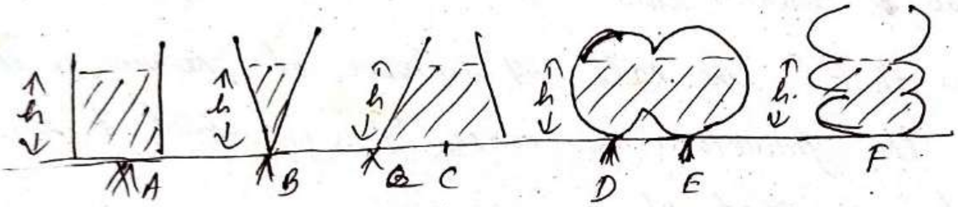
① Hydraulic Press
② Hydraulic Jack
③ Hydraulic accumulator



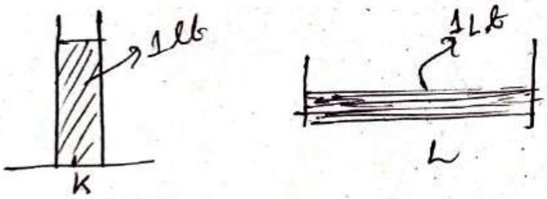
$p = \frac{F}{a} = \frac{F}{\pi/4\, d^2}$

(Hyd. Press)

Case - 1 (Pistons are same level)



$P_A = P_B = P_C = P_D = P_E = \cdots = \rho g h.$



$P_K > P_L$.

Units:

$P = 101.325\ KPa =$

$P_{atm} = 1.013\ bar$

$$\boxed{\begin{array}{l} N/m^2 = \text{Pascal} \\ bar = 10^5\ N/m^2 \end{array}}$$

1) Express 1 atm press 101.325 KPa into i) column of the air ($\rho_{air} = 1.2\ kg/m^3$)

ii) Head of H_2O ($\rho = 1000\ kg/m^3$)

iii) Head of sea water (ρ 1025 kg/m³)

iv) Head of Hg ($S = 13.6$)

v) " " oil ($S = 0.8$)

vi) " " steel cylinder (dia = 1m, $S = 7.85$)

p = 101 325 ...

i) $\rho g h_{air} = P$

$\Rightarrow h = \frac{P}{\rho g}$

$= \frac{101.325 \times 10^3}{1.2 \times 9.81}$

$= 8605$ mts of air. $= 8.6$ kms of air

ii) $\rho g h_w = P$

$\Rightarrow h_w = \frac{P}{\rho g}$

$= \frac{101.325 \times 10^3}{1000 \times 9.81}$

$= 10.3$ mts of H_2O.

iii) $\rho g h_{s.w} = P$

$\Rightarrow h_{sw} = \frac{P}{\rho g}$

$= \frac{101.325 \times 10^3}{1025 \times 9.81}$

$= 10.08$ mts of seawater.

iv) $S_{Hg}\, g\, h_{Hg} = P$

$\Rightarrow S_{Hg} \times \rho_{H_2O} \times g \times h_{Hg} = P$

$\Rightarrow h_{Hg} = \frac{101.325 \times 10^3}{13.6 \times 1000 \times 9.81}$

$= 0.76$ m

$= 76$ cm of Hg

$= 760$ mm of Hg.

v) $S_{oil} \times \rho_{H_2O} \times g \times h_{oil} = P$

$\Rightarrow h_{oil} = \frac{101.325 \times 10^3}{0.8 \times 1000 \times 9.81}$

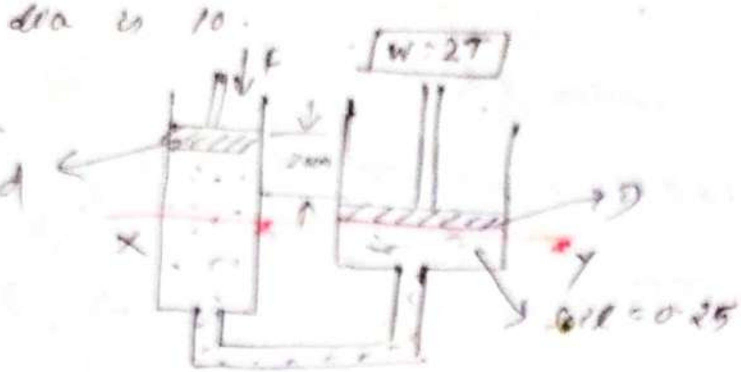
$= 12.9$ mts of oil

vi) $P = \rho_{steel} \times g \times h_{steel}$

$101.325 \times 10^3 = 7.85 \times 1000 \times 9.81 \times h_{steel}$

$\therefore h_{steel} = 1.31$ mts of steel

① Determine the Effort Required to lift the 2 tonnes loads by using Hydraulic press. The oil of specific gravity = 0.825. The position of Plunger 20 cm above the piston. The Ratio of Piston to plunger dia is 10.



$W = 2$ tonnes

$= 2 \times 1000 \times 9.8$

$= 19620$ (N)

$S_{oil} = \frac{\rho_{oil}}{\rho_{water}}$

$\Rightarrow 0.825 = \frac{\rho_{oil}}{1000}$

$\Rightarrow \rho_{oil} = 825\ kg/m^3$

$h_{oil} = h_{left\ side} = 0.2\,m$.

According to principle of Manometry.

$P_X = P_Y$

$\Rightarrow \frac{F}{a} + \rho_{oil}\, g\, h_{oil} = \frac{W}{A}$

$\Rightarrow \frac{F}{a} + 825 \times 9.81 \times 0.2 = \frac{19620}{\pi/4\,(D^2)}$

$\Rightarrow \frac{F}{\pi/4\, d^2} + 825 \times 9.81 \times 0.2 = \frac{19620}{\pi/4\,(D^2)}$

Let $\frac{D}{d} = 10$

$\Rightarrow D = 10d$

$\Rightarrow \frac{F}{d^2} - \frac{19620}{100d^2} = -825 \times 9.81 \times 0.2$

$\Rightarrow \frac{100F - 19620}{100d^2} = -825 \times 9.81 \times 0.2$

Assume d is some value otherwise it is impossible

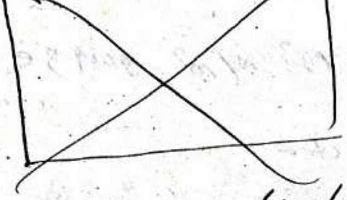
Pressure Measuring Device →

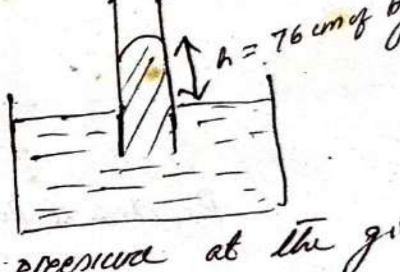
1) simple Manometer.
2) U-Tube Manometer.
3) Differential U-Tube Manometer
4) Bourdon pressure gauge.
5) Bellows etc.

→ Pressure is a measured quantity which can be sensed by fluids changing one form of energy into another form of energy. Hence it is a Transducer.

→ Following are devices.

1) Barometer → it measures atmospheric pressure.





Sudden fall in Barometer indicates, that pressure at the given location is falling down which causes cyclone

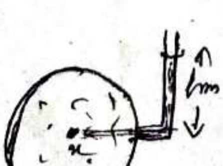
$$h = \frac{4\sigma_w \cos\theta}{\rho g d_w} \quad (\theta = 0 \text{ for } H_2O)$$

$$\Rightarrow \frac{1mm}{1000} = \frac{4 \times (73 \times 5 \times 10^{-3})}{1000 \times 9.81 \times d_w}$$

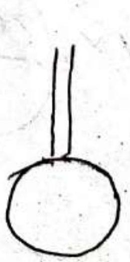
$$\Rightarrow d_w = 0.03\ m = 30\ mm.$$

Simple Manometer →

It is a vertical tube which is connected to a point where pressure is to be measured.



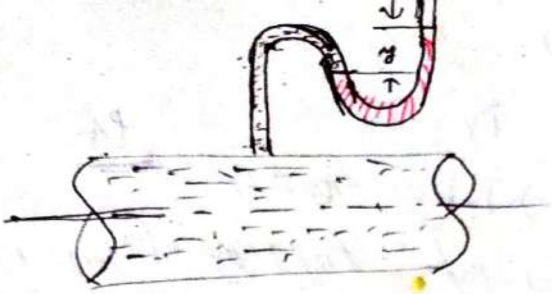
$P_x = \rho_M g h_m$

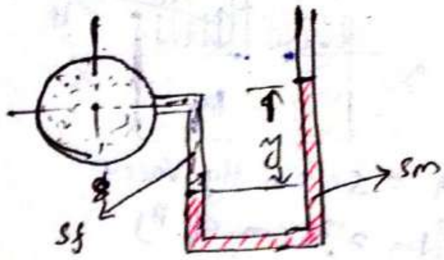


U Tube Manometer →

U. tube manometer used for measuring large pressures at a point provided one end of the tube is open other end is connected to a point where pressure is to be measured

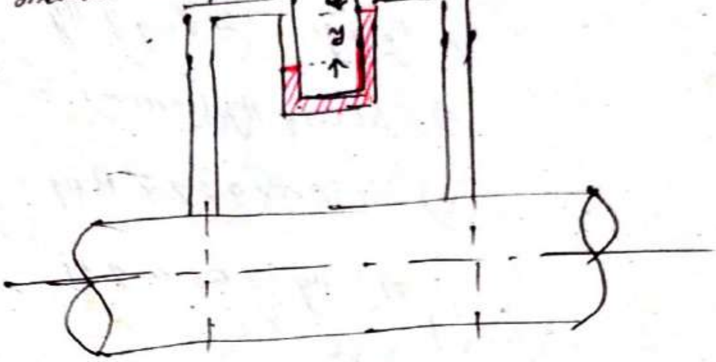
Ex!





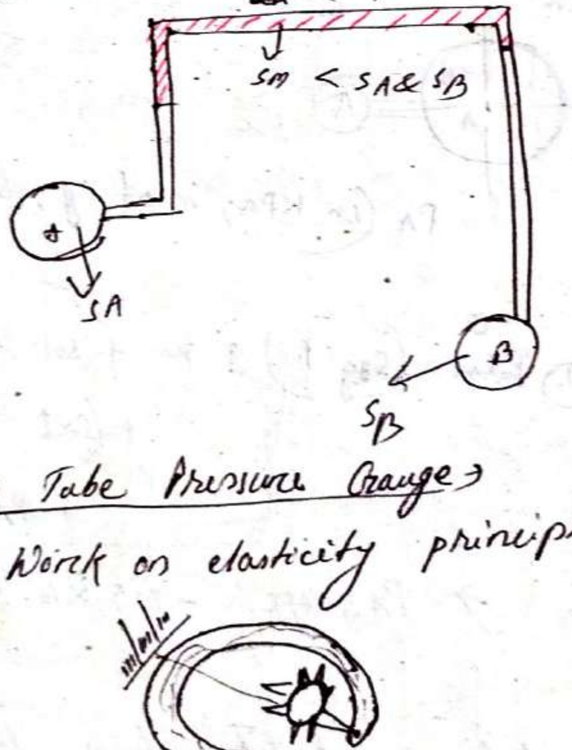
3) Differential Manometer →

It measures pressure betn 2 points (differential pressure)



4) Inverted Differential Manometer →

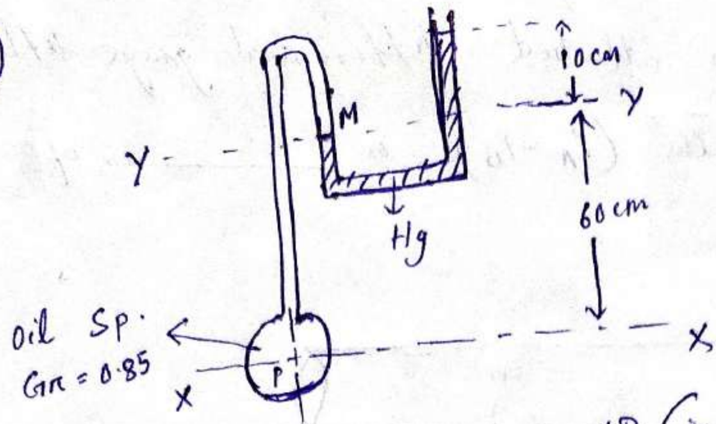
It measures pressure betn 2 points provided pressures are low below atmospheric.



5) Burdon Tube Pressure Gauge →

Work on elasticity principle.

(P)



What is the gauge pressure at P (in terms of oil column) & also find the pressure at M.

$$P_P = \rho_{oil}\, g\, h_{oil} + \rho_{Hg}\, g\, h_{Hg}$$

$$= 850 \times 9.81 \times 0.6 + 13600 \times 9.81 \times 0.1$$

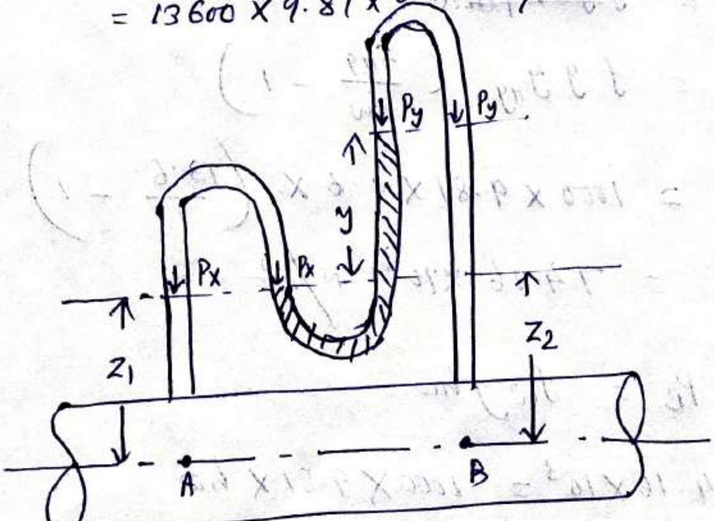
$$= 18.34 \times 10^3 \ N/m^2$$

$$18.34 \times 10^3 = 850 \times 9.81 \times h_{oil}$$

$$\Rightarrow h_{oil} = 2.2 \text{ met. of oil}$$

$$P_M = \rho_{Hg} \times g \times h_{Hg}$$

$$= 13600 \times 9.81 \times 0.1 \ N/m^2$$



$$P_A = P_x + g \rho_w Z_1 \quad \text{--- (1)}$$

$$P_B = P_y + \rho_w g (y + Z_2)$$

$$P_x = P_y + \rho_{Hg}\, g\, y_{Hg}$$

$$P_A - P_B = P_x + \rho_w g Z_1 - P_y - \rho_w g (y_{Hg} + Z_2)$$

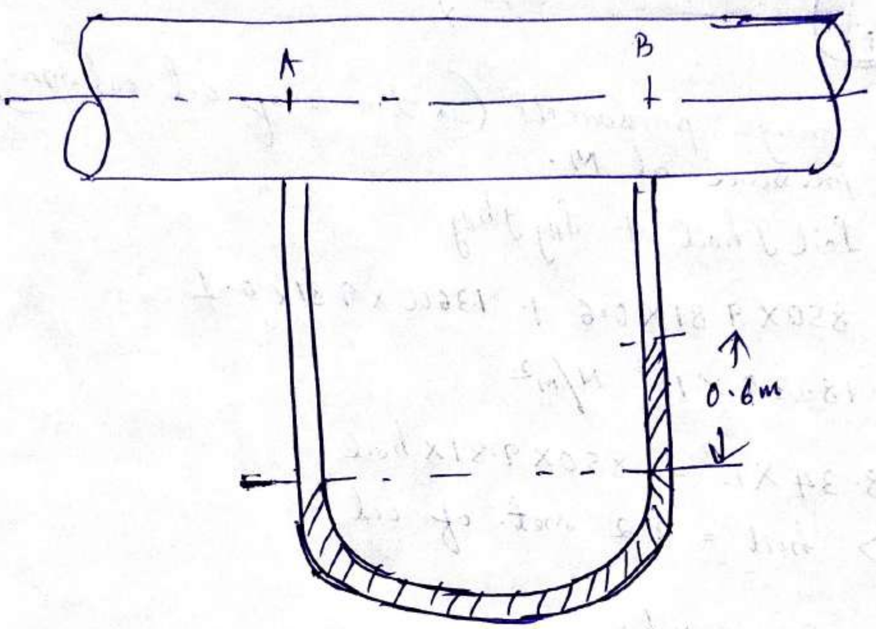
$$= \cancel{P_y} + \rho_{Hg} g y + \rho_w \cancel{g Z_1} - \cancel{P_y} - \rho_w g y_{Hg} - \rho_w g \cancel{Z_2}$$

$$= g y_{Hg} (\rho_{Hg} - \rho_w)$$

$$\boxed{P_A - P_B = \rho_w g y_{Hg} \left(\frac{\rho_{Hg}}{\rho_w} - 1\right)}$$

$$\boxed{P_A - P_B = \rho_w g h_{Hg} \left(\frac{S_{Hg}}{S_w} - 1\right)}$$

(P) ~~A~~ Pipe carrying water attached differential gauge deflection mercury gauge is 0.6m, the $(P_A - P_B)$ is ______ m of water.



$P_A - P_B = \cancel{\rho g h_{fluid}}$

$= \rho g y_{Hg} \left(\frac{S_{Hg}}{S_w} - 1 \right)$

$= 1000 \times 9.81 \times 0.6 \times \left(\frac{13.6}{1} - 1 \right)$

$= 74.16 \times 10^3 \ N/m^2$

$P_A - P_B = \rho_w g h_w$

$\Rightarrow 74.16 \times 10^3 = 1000 \times 9.81 \times h_w$

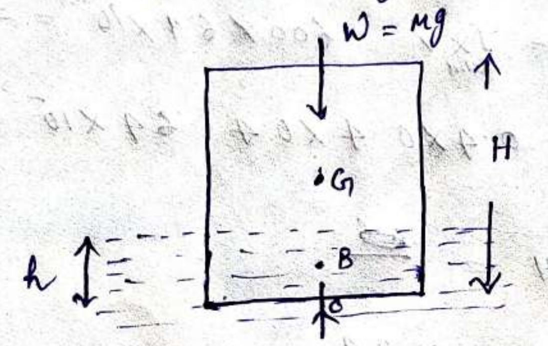
$\Rightarrow h_w = 7.56$ m of H_2O.

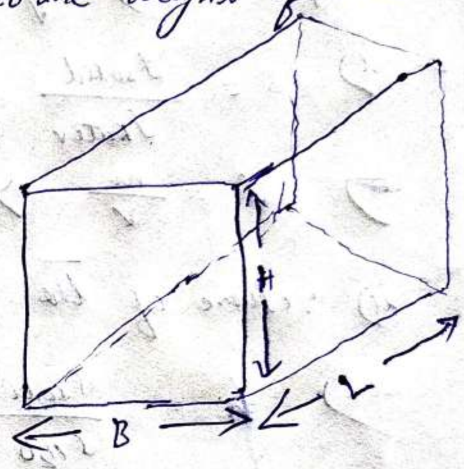
Buyancy →

→ Buyancy is the physical phenomenon due to which bodies floating or submerged in the fluids

→ Buyancy is treated as a force always acts vertically against the gravity.

→ The Magnitude of Buyancy force equal to the weight of the fluid displaced by the body.





$OG = H/2$, $OB = h/2$

B = Centre of Buyancy, G = Centre of Gravity

F_B = Buyancy force (↑) due to fluid (upthrust force)

$= \text{weight of fluid displaced}$

$= m_{fluid} \times g = \rho_{fluid} \times V_{fluid} \times g$

$= \rho_{fluid} \times B \times L \times h \times g$

$$\boxed{F_B = \rho_{fluid} \times (A_{c/s}) \times h \times g}$$

$\sum F_y = 0$

$\Rightarrow W_{body} - F_B = 0$

$\Rightarrow W_{body} = F_B$

$\Rightarrow m_{body} \times g = F_B$

$\Rightarrow \rho_{body} \times V_{body} \times g = F_B$

$\Rightarrow \rho_{body} \times (A_{c/s} \times H) \times g = \rho_{fluid} \times A_{c/s} \times g$

$\Rightarrow \rho_{body} \times H = \rho_{fluid} \times h$

$\Rightarrow \boxed{\frac{\rho_{body}}{\rho_{fluid}} \times H = h}$ $\Rightarrow \boxed{\frac{S_{body}}{S_{fluid}} \times H = h}$

(P) A Stone size 0.4m in cube form floats in water. The gravity of the porous stone is 0.6. Determine the following
i) Mass of the stone ii) volume of the stone iii) Density of the stone iv) Depth of immersion of the cube in H_2O v) Volume of H_2O displaced by the floating cube vi) Weight of H_2O displaced vii) Buyancy Force viii) Gauge Pressure at the bottom

i) $S = \frac{\rho_{solid}}{\rho_{water}} \Rightarrow 0.6 = \frac{\rho_{solid}}{1000} \Rightarrow \rho_{solid} = 600\ kg/m^3$

iii) $\rho = \frac{m}{V} \Rightarrow m = \rho_{solid} \times V = 600 \times 64 \times 10^{-3} = 38.4\ kg$.

ii) volume of the cube $= 0.4 \times 0.4 \times 0.4 = 64 \times 10^{-3}\ m^3$.

iv) $h = \frac{\rho_{solid}}{\rho_{H_2O}} \times H =$

$= \frac{0.6}{1} \times 0.4 = 0.24$.

v) volume of H_2O displaced $= (\forall) = B \times L \times h = 0.4 \times 0.4 \times 0.24 = 0.0384\ m^3$.

vi) ~~Buyancy force (FB) =~~
Weight of water displaced $= m_{water} \times g$

$= \rho_{water} \times \forall \times g$

$= 1000 \times 0.0384 \times 9.81$

$= 376.704\ N$.

vii) Buyancy force (F) = Weight of Fluid displaced.

$= 376.704\ N$.

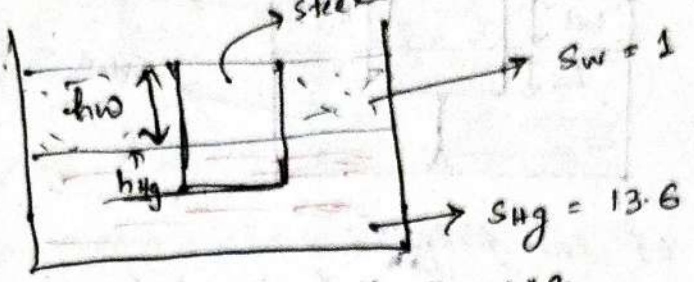
viii) Gauge Pressure at the bottom ($P_{gauge\ bottom}$)

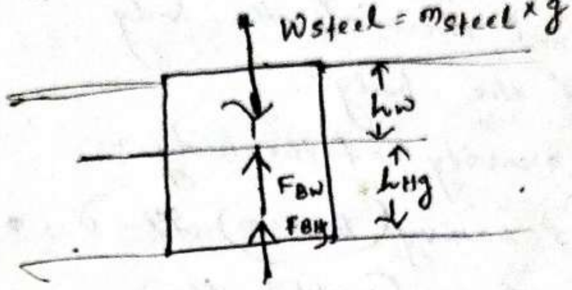
$= \rho_{water} \times g \times h$

$= 1000 \times 9.81 \times 0.24$

$= 2.35 \times 10^3\ N/m^2$.

Q/ A solid body (steel), $s = 7.85$, $0.5m \times 2m \times 1m$ $(B \times L \times H)$ is kept in 2 fluid vessels shown in the fig. First fluid is Hg, 2nd Fluid is H_2O. What is depth of immersion of steel in Hg & H_2O





$W_{steel} = F_{B\,Hg} + F_{B\,W}$

$\Rightarrow m_{steel} \times g = W_{Hg}$ displaced $+ W_{H_2O}$ displaced

$= m_{Hg} \cdot g + m_{H_2O} \cdot g$

$\Rightarrow m_{steel} \times g$

$\Rightarrow m_{steel} = m_{Hg} + m_{H_2O}$

$\Rightarrow \rho_{steel} \times V_{steel} = \rho_{Hg} \forall_{Hg} + \rho_w \forall_{H_2O}$

$$\Rightarrow \frac{\rho_{steel}}{\rho_w} \times V_{steel} = \frac{\rho_{Hg}}{\rho_w} \forall_{Hg} + \frac{\rho_w}{\rho_w} \forall_{H_2O}$$

$\Rightarrow S_{steel} \cdot V_{steel} = S_{Hg} \forall_{Hg} + S_w \forall_{H_2O}$

$\Rightarrow S_{steel} \times \cancel{(B \times L)} H = S_{Hg} \cancel{(B \times L)} h_{Hg} + S_w \cancel{(B \times L)} h_w$

$$\Rightarrow \boxed{S_{steel} \times H = S_{Hg} \times h_{Hg} + S_w \times h_w}$$

$\Rightarrow 7.85 \times 1 = 13.6 \times h_{Hg} + 1 \times h_w$ —— (1)

$H = h_w + h_{Hg}$

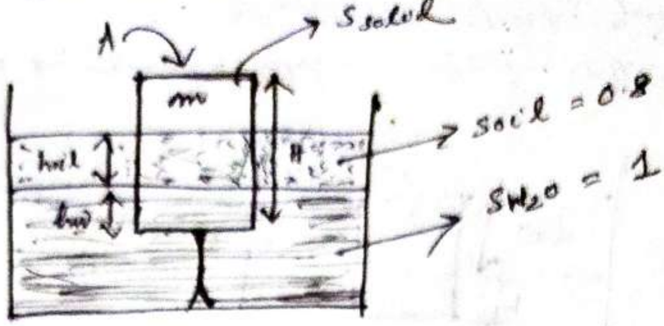
$1 = h_w + h_{Hg} \Rightarrow h_w = (1 - h_{Hg})$

So (1) $7.85 = 13.6\, h_{Hg} + (1 - h_{Hg})$

$\Rightarrow h_{Hg} = 0.543\ m$

$h_w = 1 - 0.543\ m = 0.457\ m$

Connected or Floating Submerged Bodies



m = mass of the body

$A_{c/s}$ = Cross sectional area of the body.

H = Height of the body.

S = Specific gravity of the body.

S_W = Density of heavy (bottom) fluid.

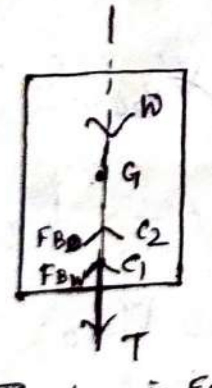
S_{oil} = Density of top (light) fluid.

h_W = Depth of Bottom fluid in m

h_{oil} = " " Top fluid

T = Tension in the string.

FBD :



(System in Equilibrium)

$W + T = F_{B_W} + F_{B_{oil}}$

$\Rightarrow mg + T = m_W g + m_{oil} g$

$= \rho_W \forall_W g + \rho_{oil} \forall_{oil} \times g$

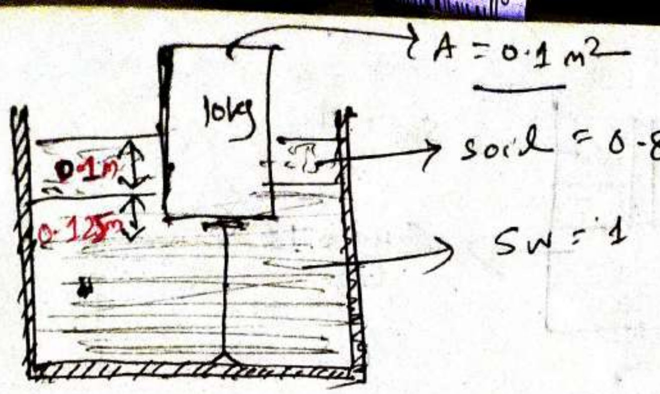
$\Rightarrow mg + T = \rho_W \cdot A_{c/s} h_W g + \rho_{oil} \times A \times h_{oil}$

Q1 A cylinder of mass 10 kg & c/s area 0.1 m² is tied down with a string in a vessel containing 2 different liquids shown in the fig. Density of bottom fluid (H_2O) = 1000 kg/m³, Gravity of the top fluid (oil) = 0.8, & S_{H_2O} = 1 - Determine

98 GATE

i) Gauge pressure at the bottom of the cylinder. (2.011 kN/m²)

ii) Tension in the string (N) 103 N.



a) $P_{BOTTOM} = \rho_w g h_w + \rho_{oil} \times g \times h_{oil}$

$= 1000 \times 9.81 \times 0.125 + 800 \times 9.81 \times 0.1$

$= 2.011 \times 10^3 \, N/m^2$.

b) $mg + T = \rho_w A_{c/s} h_w g + \rho_{oil} \times A_{c/s} \times h_{oil} \times g$

$\Rightarrow 10 \times 9.8 + T = 1000 \times 0.1 \times 0.125 \times 9.81 + 800 \times 0.1 \times 0.1 \times 9.81$

$\Rightarrow T = 103 \, N$.

Q11 A Mettalic cube of each side 10 cm, $\rho = 6.8 \, gm/cc$ is floating in liquid Hg ($\rho_{Hg} = 13.6 \, gm/cc$) with 5 cm height of the cube exposed above the Hg level. H_2O (Density = 1 gm/cc) is filled over this, to submerge cube fully. The New height of the cube exposed above the Hg level is

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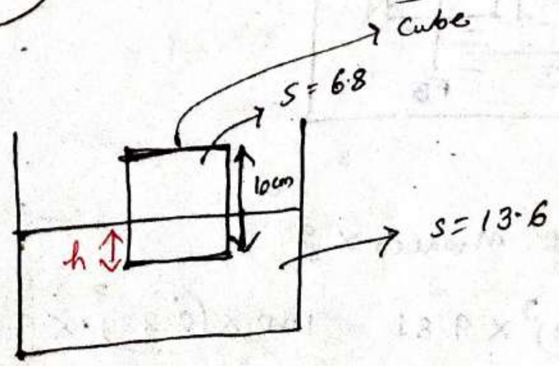
a) 4.6 cm b) 5.4 cm c) 5 cm d) 5.8 cm.

Case-I

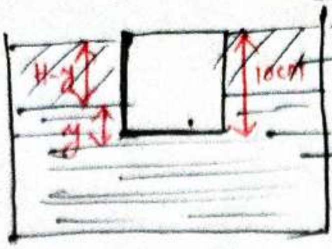


$h = \frac{S_{solid}}{S_{Liq}} \times H$

$= \frac{6.8}{13.6} \times 10$

$= \frac{1}{2} \times 10 = 5 \, cm$

Case - II :



~~H = H-y + ...~~

$$S_{solid} \times H = S_{Hg} \times h_{Hg} + S_W (H-y)$$

$$6.8 \times 10 = 13.6 \times y + 1(10-y)$$

$$\Rightarrow 68 = 13.6y + 10 - y$$

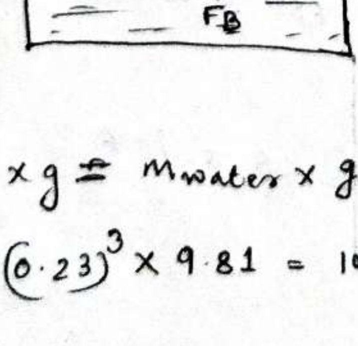
$$\Rightarrow 58 = 12.6y$$

$$\Rightarrow y = 4.6 \text{ cm}$$

$$H - y = 10 - 4.6 = 5.4 \text{ cm.}$$

Q// A cube of wood ~~has~~ $(S = 0.6)$ has 230 mm each side when is floating in H_2O. Estimate the Magnitude & dirn of force F Required to hold the wood completely submerged in the H_2O. a) 47.74 N b) 64.64 N c) 87.13 N d) 96.72 N

IES



$$F + W = F_B$$

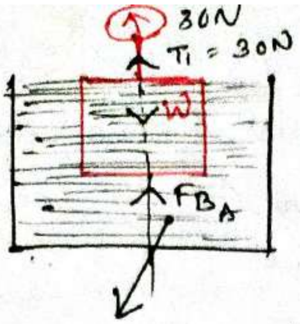
$$\Rightarrow F + M_{body} \times g = M_{water} \times g$$

$$\Rightarrow F + 600 \times (0.23)^3 \times 9.81 = 1000 \times (0.23)^3 \times 9.81$$

Fully submerged condn is given

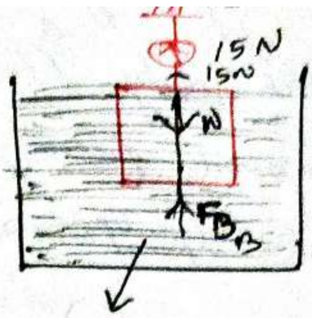
$$\Rightarrow F = 47.74$$

Q// A body weighting ~~12N~~ 30N & 15 N when weighted under submerged in 2 different liquids of RDs 0.8 & 1.2 Respectively. What is the volume of the body



Fluid A

$S_A = 0.8$



Fluid B

$S_B = 1.2$

$W = T_1 + F_{BA}$

$W = 30 + F_{B_A}$ —— (1)

$W = T_2 + F_{B_B}$

$W = 15 + F_{B_B}$ —— (2)

From (1) & (2)

$F_{B_B} - F_{B_A} = 30 - 15 = 15\,N$

$W_B - W_A = 15\,N$

$m_B \cdot g - m_A \cdot g = 15\,N$

$\Rightarrow \rho_B\, v_B \cdot g - \rho_A \times v_A \times g = 15\,N$

$\Rightarrow 1200 \times v_B \times g - 800 \times v_A \times g = 15\,N$

$v_A = v_B = v$ = volume of body submerged.

$(1200 - 800) \times 9.81 \times v = 15$

$v = 3.82 \times 10^{-3}\, m^3$

$= 3.82$ lit

Note: For floating body depth of immersion depends upon gravity of the fluid where the body float.

Example: Boat moved from river water to sea water boat rises due to moves up uptrust devlopped by sea H_2O

($S_{sea\,H_2O} > S_{River\,H_2O}$)

5) $S_{solid} = \dfrac{W_{AIR}}{W_{AIR} - W_{H_2O}}$

=

An object weighing 100N in air was found to be 80N when fully submerged in H_2O. The relative density of the object is ____

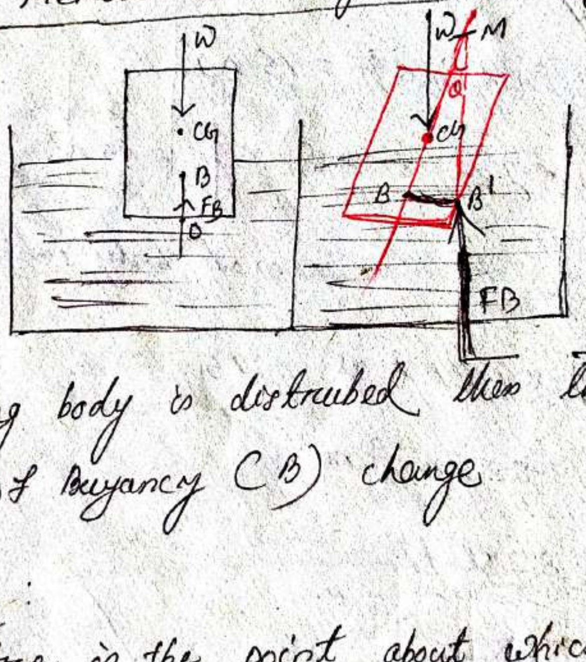
a) 5 b) 4 c) 2.5 d) 1.25

4) $h = \frac{3.4}{13.6} \times H$

$h = \frac{1}{4} \times H$

$h = 0.25\,H$

Metacentre & Metacentric Height of Floating Body



→ When a Floating body is distrubed then the body tilts due to which centre of Buyancy (B) change.

METACENTRE:

Metacentre is the point about which a body ossicalates when it is tilted by small angle. The metacentre point is obtained by intersecting Normal axis of the body and the action of Buyancy Force.

Metacentric height (GM)

It is the distance measured from centre of gravity of the solid body to Metacentre point

Significance of Metacentre

i) If the point M above the G then the floating body ~~will~~ is said to be stable equilibrium.

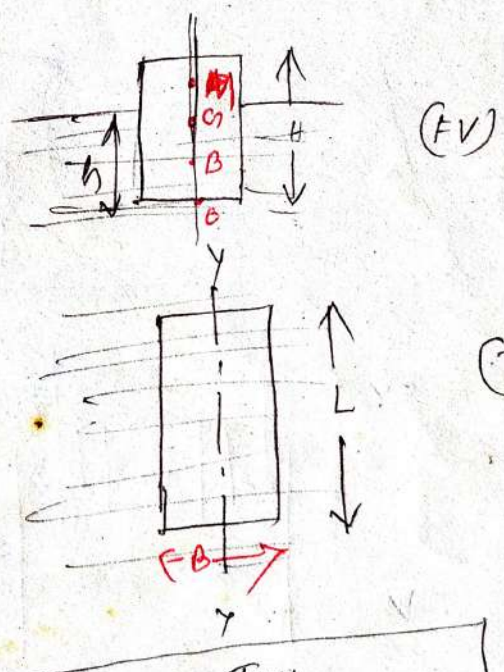
ii) If the point M is below G then floating body will be unstable equilibrium (Body overturns)

iii) If point M ~~below the G then~~ coincides with then floating body is said to Neutral Equilibrium.

iv) In order to have more stability the point M should be away from (above) the Gravity G.

Ex: There are 2 bodies A & B, it was estimated that the metacentric height for body A & ~~body~~ B are 2m & 3m respectively. It is concluded that body B is more stable in floating cond? since metacentric height is more than B' metacentric ~~b~~ height of body A.

Analytic Method to find Metacentric Height



$$\boxed{GM = \frac{I_{YY}}{\forall} - BG}$$

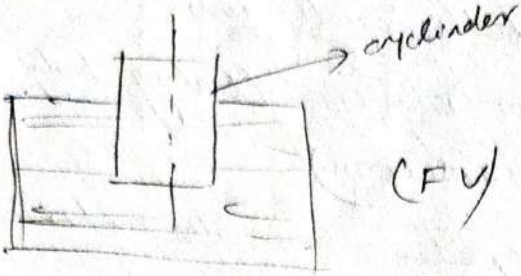
$$\forall = B \times L \times h$$

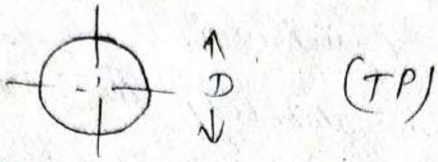
$$I_{YY} = \frac{L \times B^3}{12}$$

$BG = OG - OB$

$= H/2 - h/2$

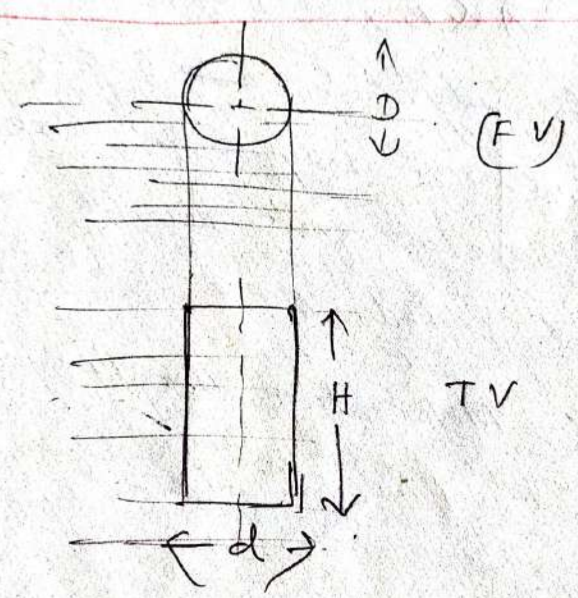
Case ①

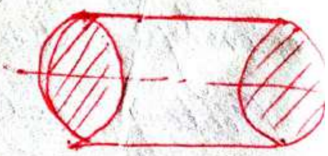


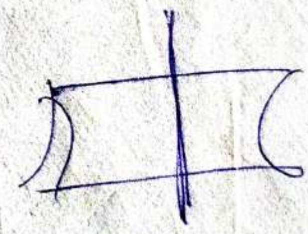


$I_{yy} = \pi/64 \, d^4$

Case – II

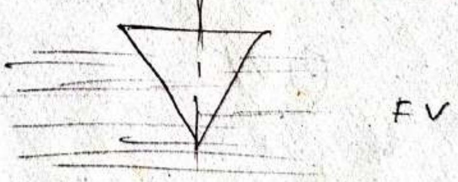


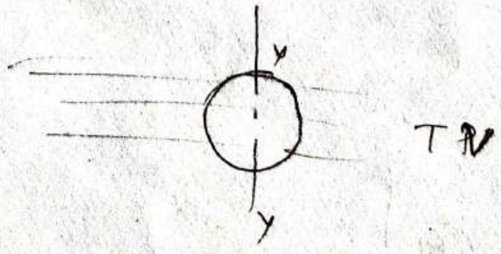




$I_{yy} = \frac{Hd^3}{12}$

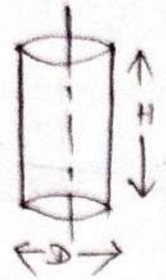
Case – III

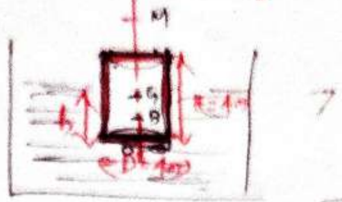


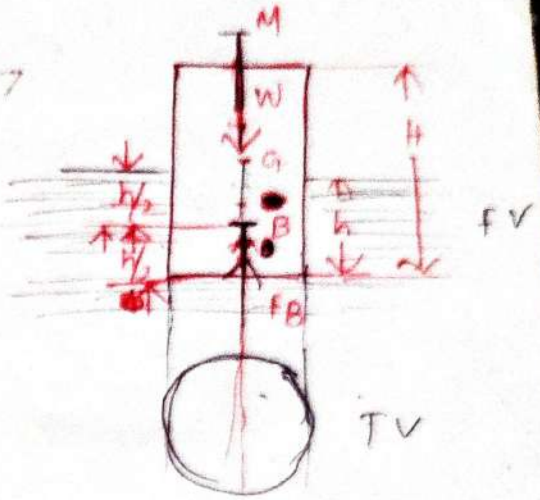


$I_{yy} = \frac{\pi}{64} d^4$

A solid cylinder of diameter 4m & height 4m kept in water. The specific gravity of the solid is 0.6 & it is floating in H_2O with its axis i) vertical
ii) Horizontal.
State the cond^n of the solid equilibrium (stable or unstable)?







$H = 4m, D = 4m$

$$h = \frac{S_{solid}}{S_{H_2O}} \times H$$
$$= \frac{0.6}{1} \times 4$$
$$= 2.4\ m.$$

$$GM = \frac{I}{\forall} - BG$$

where $I = \frac{\pi}{64} D^4$
$$= \frac{\pi}{64} (4)^4$$
$$= 4\pi\ m^4$$

$$\forall = A_{c/s} \times h$$
$$= \frac{\pi}{4} D^2 \times 2.4$$
$$= \frac{\pi}{4} \times (4)^2 \times 24$$
$$= 9.6\pi\ m^3$$

$$BG = H/2 - h/2$$
$$= 4/2 - \frac{2.4}{2}$$
$$= 2 - 1.2$$
$$= 0.8$$

$$GM = \frac{4\pi}{9.6\pi} - 0.8 = -3.83\ m$$

CONCLUSION : -ve sign indicated Metacentre below C.G.
Hence the cylinder on vertical axis is unstable

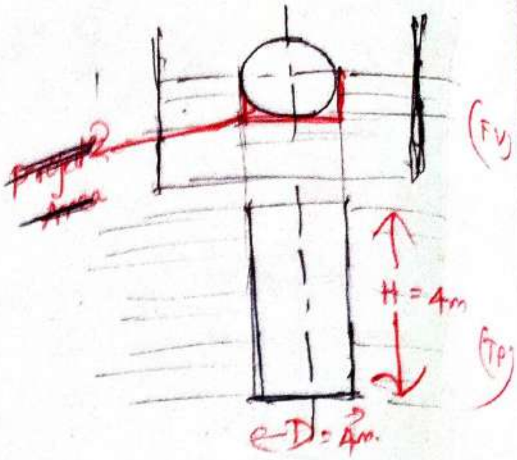
Case - II :

$$GM = \frac{I}{\forall} - BG$$

$$I = \frac{HD^3}{12}$$

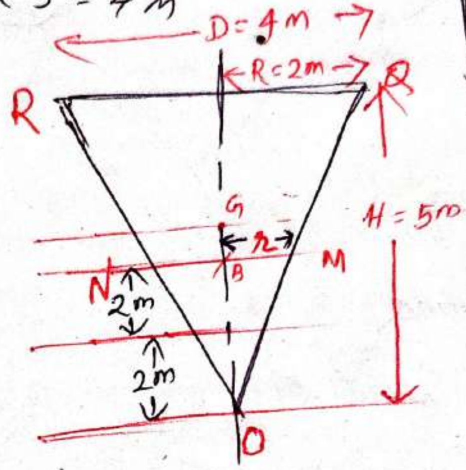
$$= \frac{4 \times 4^3}{12}$$

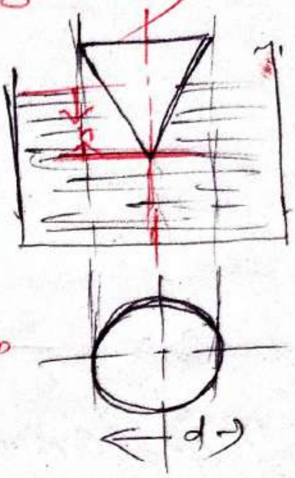
$$= 21.33 \ mm^2$$



A solid cone floats in H_2O with its apex downwards. The gravity of the solid cone is 0.8. Base diameter is 4m. & height is 5m. Determine (i) Depth of immersion, condition of the stability of the cone (volume of cone $= \frac{1}{3}\pi R^2 H$)

i) $h = \frac{S_{cone}}{S_{water}} \times H = \frac{0.8}{1} \times 5 = 4\,m$





From similar Δle

$$\frac{R}{H} = \frac{r}{h}$$

$$\Rightarrow \frac{2}{5} = \frac{r}{4}$$

$$\Rightarrow r = 1.6\,m$$

$$\forall = \frac{1}{3} \times \pi r^2 \times h = \frac{1}{3} \times \pi \times (1.6)^2 \times 4 = 3.413\pi\,m^3$$

$$BG = OG - OB$$

$$= \frac{3}{4}H - h/2$$

$$= 3/4 \times 5 - 4/2$$

$$= 1.75\,m$$

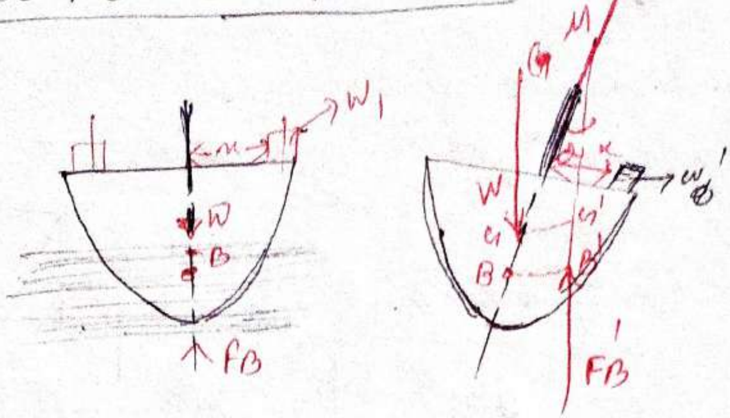
$$I = \frac{\pi}{64} d^4 = \frac{\pi}{64}(4^4) = 4\pi\,m^4$$

$$GM = \frac{I}{\forall} - BG$$

$$= \frac{4\pi}{3.413\pi} - 1.75$$

$$= -0.057$$

Experimental Method to find GM ⇒



$$\tan\theta = \frac{GG'}{GM}$$

$$GG' = GM \tan\theta$$

Momement about M

$$W \times GG' = W' \times x$$

$$W \times (GM \tan\theta) = W' \times x$$

$$\Rightarrow \boxed{GM = \frac{W'x}{W \tan\theta}}$$

(13) Material $GM = \dfrac{50 \times 6}{10{,}000 \times \tan 3^\circ} = 0.57\,m.$

Caclulation of Natural Frequency of the floating Body

The floating body is in SHM, for which

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{g\,(GM)}{K^2}}\ Hz}$$

Where GM = Meta centric height (m)

K = Radious gyration (m)

$$I = AK^2$$

$$K = \sqrt{\frac{I}{A}}$$

all Estimate the natural frequency of the floating ship with metacentric height 70 cm & least Radious of gyration is 8 m & also state time period for the

~~are~~ ~~at~~ oscillation.

Given $f_n = \frac{1}{2\pi}\sqrt{\frac{9.81 \times 0.7}{(8)^2}}$

$= 0.05\ Hz$

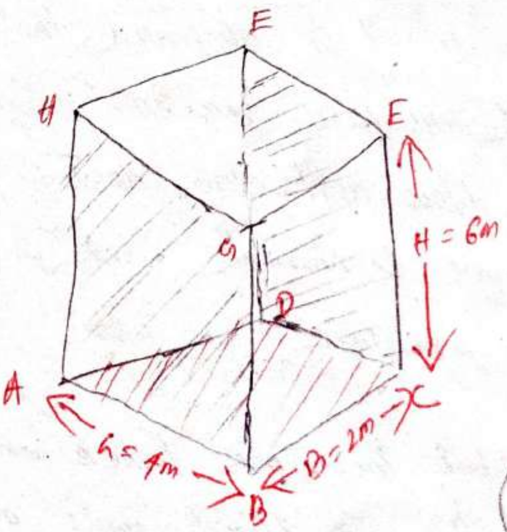
$\therefore$ time period $= \frac{1}{f_n} = 19.2$ sec.

Hydrostatic Force on submerged bodies

For design of dams, design of hydraulic gates, submarines, jet pumps etc the hydrostatic force is to be ~~ea~~ estimated.

- Hydrostatic force is the normal force due to ~~ø~~ Normal pressure (compressive stress) acting on the plane surface.

Ex; A Tank is filled with H_2O shown in the fig.



i) ~~ABCD~~ ABCD = Floor Area. — 1 No

ii) CDEF = ABGH ——— 2 NO

iii) BCFG = ADGE

Q① What is the fluid pressure on ABCD plane

$P_{ABCD} = \rho g H \quad (h = H)$

$= 1000 \times \cancel{9.81}^{10} \times 6$

$= 60\ KN/m^2$

Q② $F_{ABCD} = \cancel{60 \times A_{ABCD}}\ P_{ABCD} \times A_{ABCD}$

(Force on the floor due to pressure) $= 60 \times (2 \times 4)$

$= 480\ KN$

(3) Force due to self wt. of water on Floor ABCD

$= \omega_{water} = m_w \times g$

$= \rho_w \times V_w \times g = \rho_w \times (A_{c/s} \times H) g$

$= 1000 \times (2 \times 4) \times 6 \times 10 \, N$

$= 480 \, KN$

(4) Pressure force on Large vertical surface

$F_{P_{\text{Large vertical}}} = (\rho g H) \times A_{\text{vertical surface}}$

$= 60 \times 24 \, m^2 = 1440 \, KN$

(5) Pressure for on smallers vertical surfaces

$= 60 \frac{KN}{m^2} \times 2 \times 6 = 720 \, KN$

Note: From the above example it is observed that the forces acting different normal surfaces varies even pressure at a point is same. The flow restriction creates pressure. The product of the pressure & Normal area is called pressure force or Total pressure.

Hydrostatic forces nothing but Pressure forces under static fluid action. No shear acting on fluid acts on the particles. When the fluid at Rest following forces involving

1) Self wt of the fluid particle.
2) Normal force due to Normal pressure on the particle surface (Horizontal, vertical & inclined)

Terms Used

1. Total Pressure.
2. Centre of Pressure.
3. Horizontal Force component.
4. Vertical force component
5. Resultant force on hydrostatic fluid.

① Total Pressure →

It is the force developped due to hydrostatic pressure on the plane surfaces which are in contact with fluid.

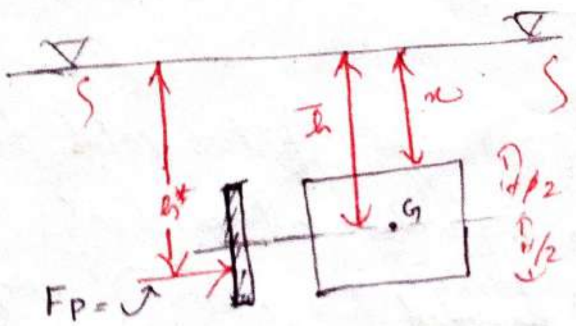
The magnitude of Total pressure is equal to p pressure × Normal Area.

i.e $F = P \times A_N$

where P = Hydrostatic fluid pressure $= \rho g \bar{h}$

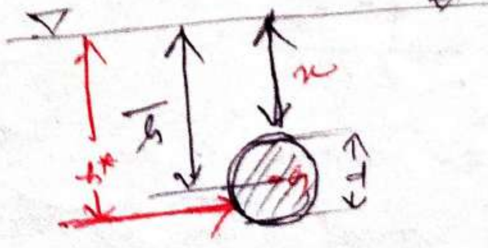
where $\bar{h}$ = distance from free surface of the liquid to the centroid of the submerged plane surface.

Ex :- ①



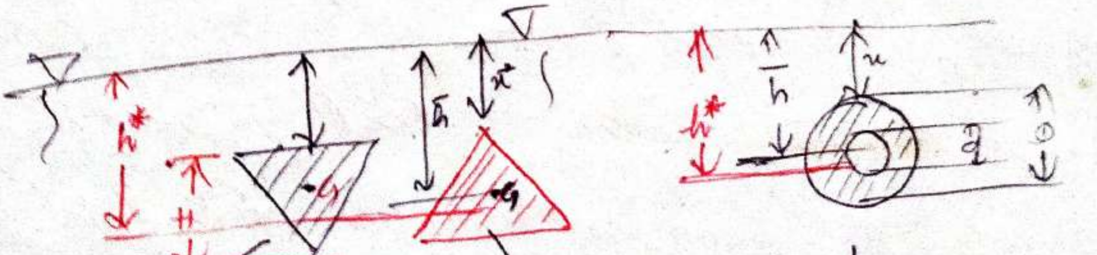
$\bar{h} = x + H/2$.

②



$\bar{h} = x + d/2$

③



$h = x + H/3$ $\qquad h = x + 2/3\, H$ $\qquad \bar{h} = x + D/2$

(2) Centre of Pressure for the submerged Plane surfaces in any position is defined as the pt where total pressure (Pressure force) concentrated or acts.

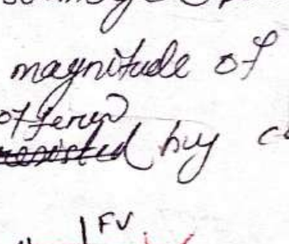
$$\boxed{h^* = \text{Centre of Pressure} = \bar{h} + \frac{I}{A\cdot\bar{h}}}$$

(3) F_H is a vector quantity force due to scalar quantity pressure acting on Normal Surface

- For the horizontal surface it is equal to the pressure due to depth of the fluid × Normal Area.

$$\boxed{F_H = (\rho g \bar{h}) \times A}$$

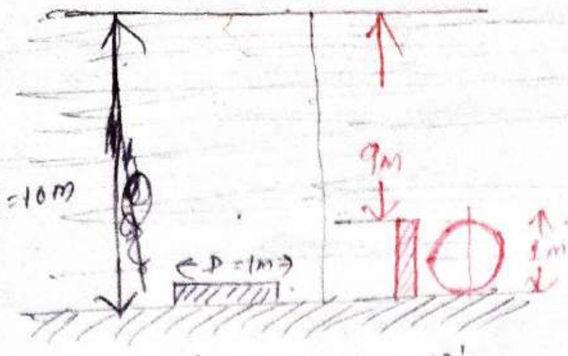
(4) Vertical force for submerged bodies valid for curved surfaces only. The magnitude of vertical force = wt of the fluid offered by curved surface



Q(1) A circular lamina of diameter 1m is submerged in a fluid of Gravity 0.85. What is the a) pressure of the circular lamina is horizontal position at depth 10m.

b) What is the total pressure acting on the circular lamina

c) If circular lamina is placed in vertical position then what is the pressure acting on it & total pressure acting on it.



(i) Circular lamina in horizontal plane

(ii) Circular lamina in vertical plane

a) $p = \rho g \bar{h} = 850 \times 10 \times 10$
$= 85000 \ N/m^2$.

b) F_p = Total Pressure = Pressure due to pressure
$= P \times AN$
$= 85000 \times \frac{\pi}{4}(1)^2$

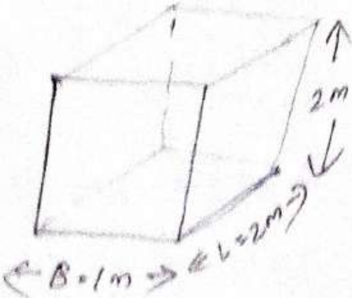
$= 66.75 \ KN$

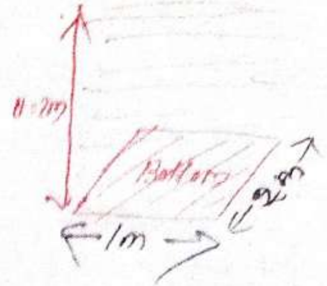
c) $P = \rho g \bar{h}$
$= 850 \times 10 \times \left[x + \frac{d}{2}\right]$
$= 850 \times 10 \times \left[9 + \frac{1}{2}\right]$
$= 850 \times 10 \times 9.5$
$= 80.75 \ KN/m^2$

d) $F_p = P_{(E)} \times A$
$= 80.75 \times \frac{\pi}{4}(1)^2$
$= 63.42 \ KN$.

② A Rectangular H_2O Tank filled with H_2O completely with $L \times B \times H$ ($2m \times 1m \times 2m$). The Ratio of hydrostatic forces at the bottom to that any larger vertical surface is a

a) 1/2 b) 1 c) 2 d) 4





$F_P = P \times A$

$= \rho g \bar{h} \times A$

$= 1000 \times \cancel{9.81}^{10} \times 2 \times (1 \times 2)$

$= 40\ KN.$

Large vertical surface

$F = \rho g \bar{h} A$

$= 1000 \times g \times \left(\frac{2}{2}\right) \times 2 \times 2$

$= 1000 \times 10 \times 1 \times 4$

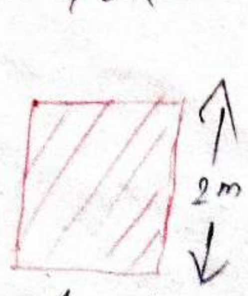
$= 40\ KN$

$$\frac{F_{BOTTOM}}{F_{VERTICAL\ SURFACE}} = \frac{40\ KN}{40\ KN} = 1$$

(5) In the above problem find, ~~Total pressure at the bottom~~

$$\frac{F_{bottom}}{F_{smaller\ vertical\ surface}} = ?$$

$F_{bottom} = 40\ KN.$



$F_{S.V.S} = \rho g \bar{h} A$

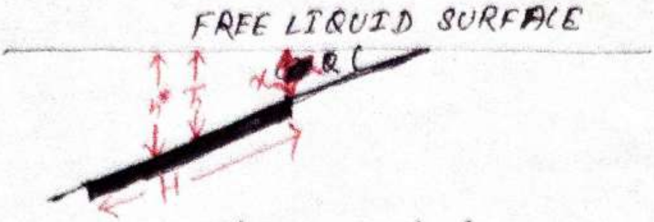
$= 1000 \times 10 \times \left(\frac{H}{2}\right) \cdot (B \times H)$

$= 1000 \times 10 \times 1 \times 1 \times 2$

$= 20\ KN$

$$\frac{F_{bottom}}{F_{SVS}} = \frac{40\ KN}{20\ KN} = 2$$

Inclined Plane Surface Submerged in liquids →



A = Total Area of Inclined Surface.

$\bar{h}$ = Depth of CG of inclined area from free surface.

h^* = Distance of centre of pressure from free surface of liquid

Q = Angle Made by the plane of the surface with free liquid surface.

$$\boxed{P = \rho g \bar{h}}$$

$$\boxed{F = \rho g A \bar{h}}$$

$$\boxed{\bar{h} = \alpha + \frac{H}{2} \sin Q}$$

$$\boxed{h^* = \frac{I \sin^2 Q}{A\bar{h}} + \bar{h}}$$

Curved Surface →

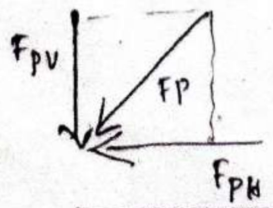
For the submerged curved surface, the hydrostatic force analysis involves two components.

(I) Horizontal Component. (F_{PH})

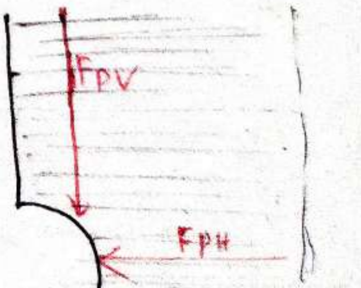
(II) Vertical Component (F_{PV})

→ Horizontal component is similar to vertical plane surface analysis.

→ For calculating vertical force component the total weight of the fluid supported by curved surface



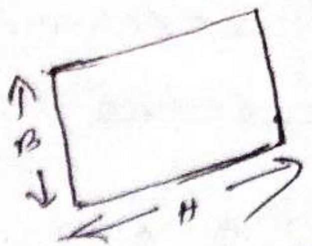
$$F_P = \sqrt{F_{P_H}^2 + F_{P_V}^2}$$



24) $x = 0$ $\quad \theta = 60°$

$B = 0.75\ m$ $\quad S = 0.85$

$H = 2.4\ m.$



A Rectangular plate $0.75m \times 2.4m$ is immersed in a liquid of relative density 0.85 with its 0.75 m side horizontal and just at the liquid surface. If the plane of the plate makes an angle of 60° with the horizontal, pressure force in kN & centre of pressure in mm

$$\bar{h} = \frac{H}{2} \sin\theta$$

$$= \frac{2.4}{2} \sin 60$$

$$= 1.039\ m.$$

$$F = \rho g A \bar{h}$$

$$= 0.85 \times 1000 \times 9.81 \times 0.75 \times 2.4 \times 1.039$$

$$= \cancel{18.350\ KN}\ \ 15.594\ KN$$

$$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$$

$$I_G = \frac{B \times H^3}{12}$$

$$= \frac{0.75 \times (2.4)^3}{12}$$

$$= 0.864\ m^4$$

$$h^* = \frac{0.864 \times \sin^2 60}{0.75 \times 2.4 \times 1.039} + 1.039$$

~~0.4096~~

$$= 1.385\ m.$$