① Structural Analysis 1 Dt-17/4/2020
Civil Engg.

Module-1

Concept of Determinate and Indeterminate Structures

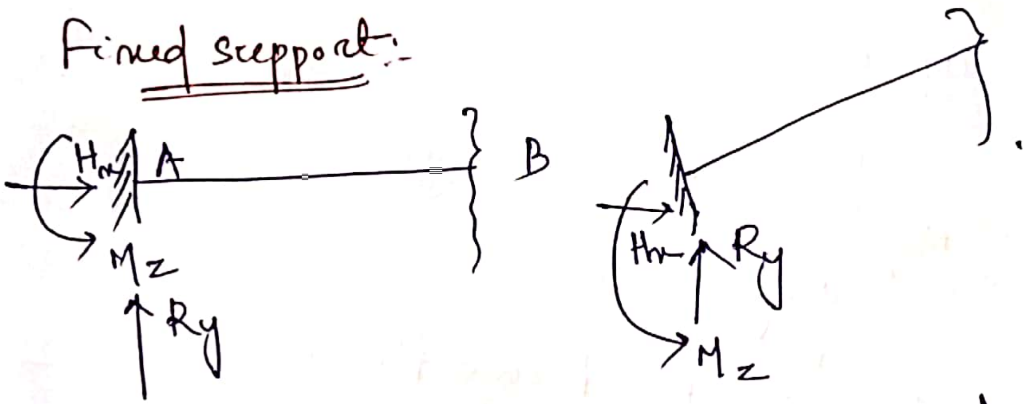
(1) Support system:-

2D supports:-

For 2D supports there are 3 types of supports

(i) Fixed support
(ii) Hinge support
(iii) Roller support.

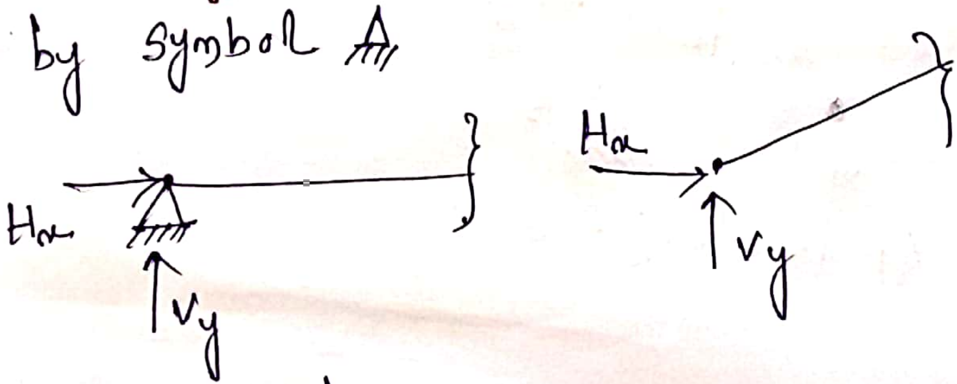
Fixed support:-



A 2D fixed support, there can be 3 reactions

(i) one vertical reaction (V_y)
(ii) one horizontal reaction (H_x)
(iii) one moment reaction (M_z)

(II) **Hinge support:-** Hinge support is represented by by symbol

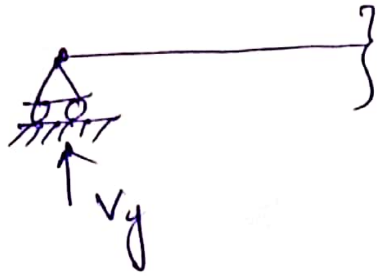


No. of reactions = 2

At hinge support, there can be two supports
(i) one horizontal reaction (H_x)
(ii) one vertical reaction (V_y).

Roller support:-

Roller support is represented by  and 



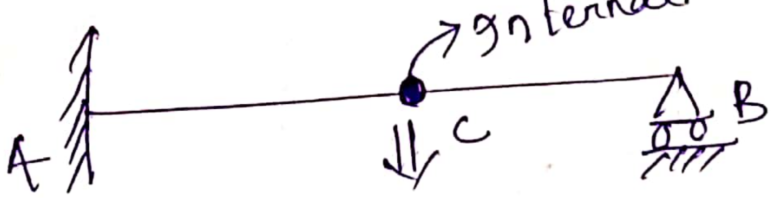
No. of reactions for roller support = 1

Guided Roller support

At guided support there can be two reactions
(i) one vertical reaction (V_y).
(ii) one moment reaction (M_z)

2D Internal hinge:-

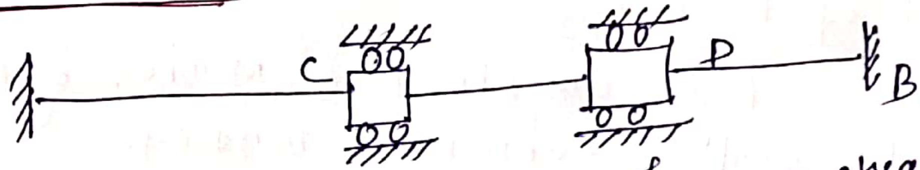
At internal hinge bending moment will be zero.



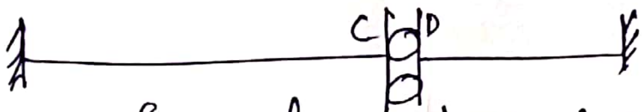
At this $BM_C = 0$.

(2)

Internal Roller :-



At internal roller either axially force or shear force will be zero. (axial force at C & D is zero)



Shear force at C and D $S_C = S_D = 0$.

3-D Supports

(a) Fixed support

At 3D fixed support there can be 6 reactions such as (i) 3 reactions R_x, R_y and R_z

(ii) 3 Moment reactions M_x, M_y and M_z.

- The fixed support are also called Built in support.

(b) 3-D hinged support

At 3D hinged support 3 reactions are there.

(i) R_x

(ii) R_y

(iii) R_z

3-D hinged support are k/w as ball and socket joint.

(c) Roller support :-

At 3D Roller support there can be only 1 externally independent reaction which is perpendicular to contact surface.

Structure:-

Elements of structure

(1) **Beams** :- Beams are structural members by which is predominantly subjected to bending.

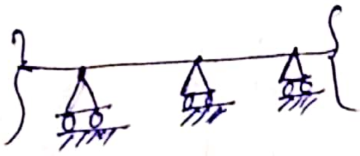
Beams are classified as

(a) Simply supported beam 

(b) Cantilever beam 

(c) Propped cantilever beam 

d) Fixed beam 

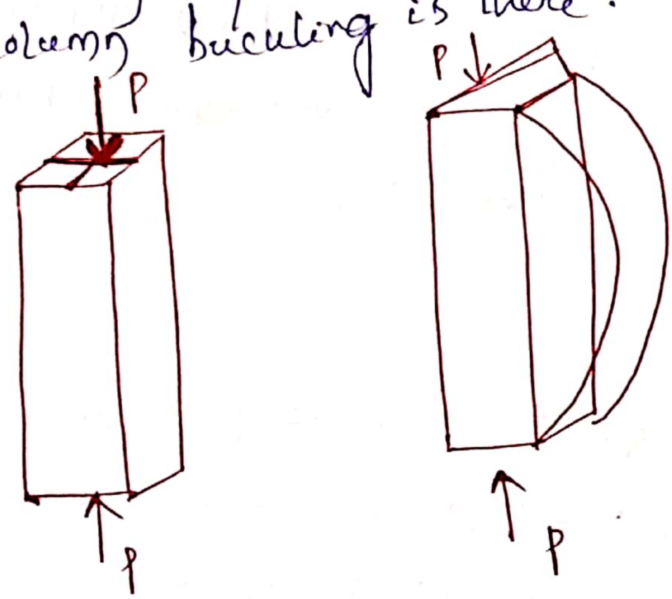
(e) Continuous beam 

(2) **Column** :-

A vertical compression member which is slender and straight.

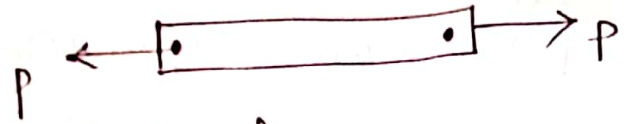
→ Generally columns are subjected to axial compression and bending moment as shown in figure.

→ In column buckling is there.



Tie Members

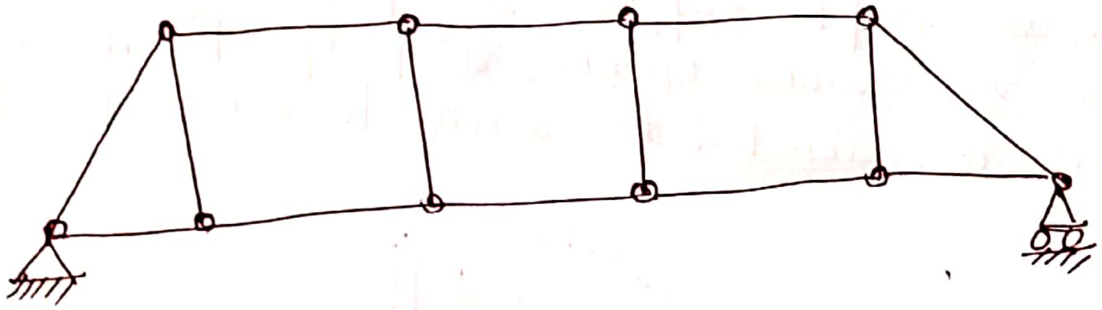
The members are tension members of trusses which are subjected to axial tensile force.



Types of structures

1) **Trusses:-** A truss is constructed from pin jointed slender members usually arranged in triangular manner.

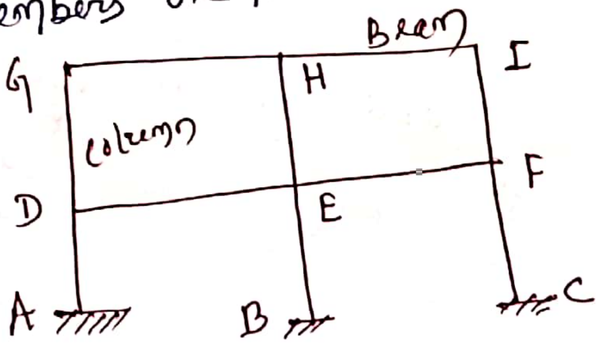
→ In trusses loads are applied on joints due to which each member of truss subjected to only axial force, either axial compression or axial tension.



(2) **Frames:-**

A frame is constructed from either pin jointed or fixed jointed beam and columns.

→ Generally loads are applied on beams and this loading causes axial force, shear force and bending to the members or frame.

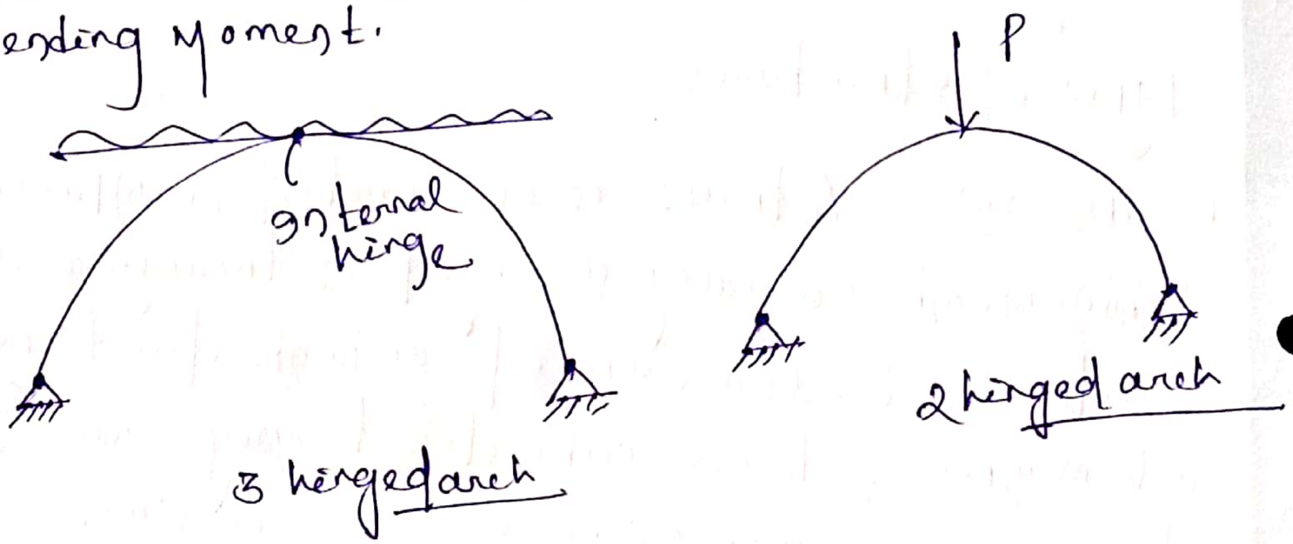


Frames

Arches

Arches are used in bridges, dome roof, auditorium where span of structures are relatively more due to external Loading.

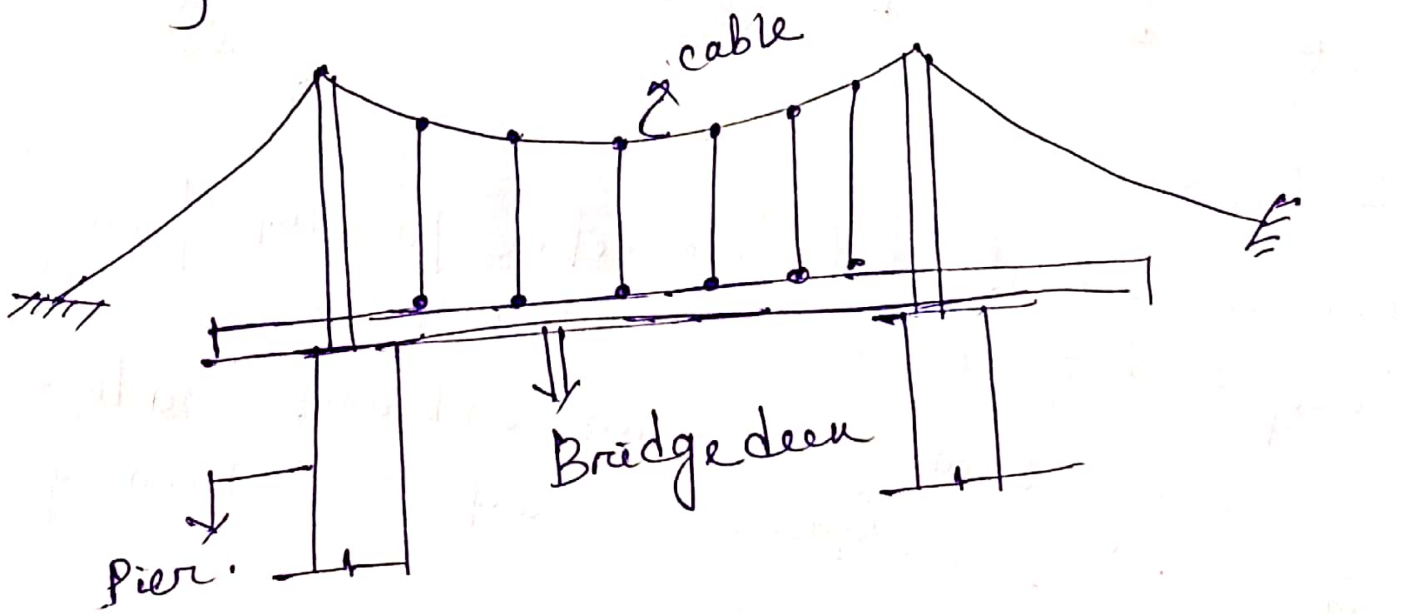
→ Arch can be subjected to axial compression, shear force or bending moment.



Cables:

Cables are used to support Long span bridges.

→ cables are flexible members and due to external Loading it is subjected to axial tension only.



[cable and bridge]

Stability of structure:-

Structural stability is the Major concern of the Structural designer.

→ To ensure the stability, a structure Must have enough support reaction along with proper arrangement of Members.

The overall stability of structures can be divided into

(i) External stability

(ii) Internal stability.

External stability:-

for stability of 2D structure following 3 conditions should be satisfied.

(i) $\Sigma F_x = 0$

(ii) $\Sigma F_y = 0$

(iii) $\Sigma M_z = 0$.

3 independent External reaction for 2D structure.

→ for 3D structures there should be minimum 6 independent external reaction.

and 6 equation of static equilibrium

(i) $\Sigma F_x = 0$ (iv) $\Sigma M_x = 0$

(ii) $\Sigma F_y = 0$ (v) $\Sigma M_y = 0$

(iii) $\Sigma F_z = 0$ (vi) $\Sigma M_z = 0$.

Internal stability:-

→ For the internal stability, no part of the structure can move relative to the other part so that geometry of the structure is preserved.

→ Mechanism is formed when there are 3 colinear hinges.

for 2D truss the minimum no. of members needed for geometric stability are

$$\boxed{M = 2j - 3}$$

for 3D structure/ truss $\boxed{M = 3j - 6}$.

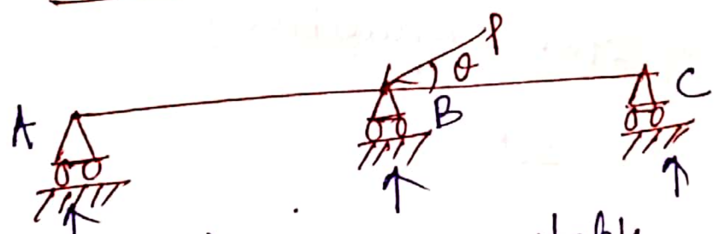
Note

→ for stability in 3D structures all the reactions should be nonparallel, non concurrent and nonparallel.

→ for 2D structure all ~~Mer~~ reactions should n't be parallel.

Examples of Stability of structures

(1)

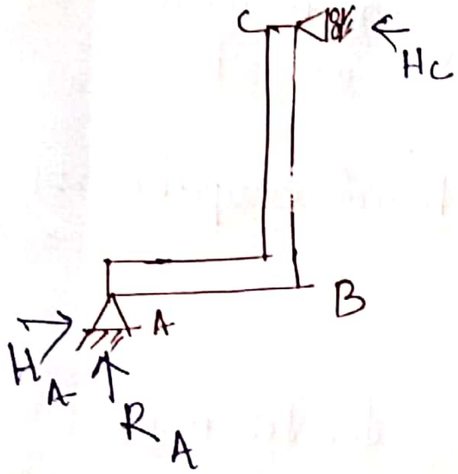


This structure is unstable because all reactions are parallel.

→ R_A and R_B and R_C are parallel as (Roller support).

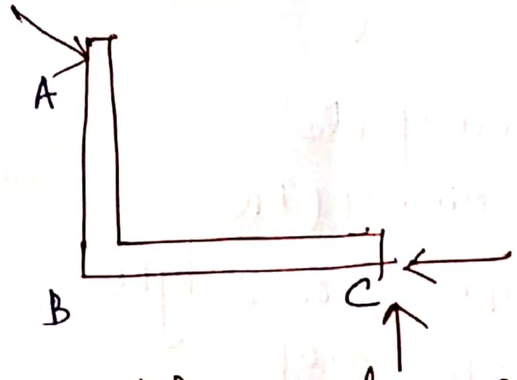
(5)

Example-2



This member is stable since reactions are non parallel and non concurrent.

(3)



Since all three reactions are concurrent at 'C' so this member is unstable.

Statically determinate and indeterminate structure

Statically determinate structure

A structure is said to be determinate if conditions of static equilibrium $\begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma M_z = 0 \end{cases}$ are sufficient to analyse the structure.

Note

Indeterminate structure bending moment and shear force are independent of properties of material and c/s area.

→ No stresses are induced due to lack of fit and temperature changes.

Statically Indeterminate structure

A structure is said to be statically indeterminate if conditions of static equilibrium aren't sufficient to analyse the structure.

→ To analyze the structure additional compatibility conditions are required.

Degree of Indeterminacy:-

The degree of indeterminacy can be devided into 2 types.

(1) Static Indeterminacy (D_s)

(2) Kinematic Indeterminacy (D_k)

Again static determinacy is 2 types

(1) External Indeterminacy

(2) Internal "

Static Indeterminacy:-

Those structures which can't be analyse using equation of static equilibrium alone are called as Indeterminate structure or hyper static structure.

→ To analyse these structures extra equation are required which is called compatibility equation.

External static indeterminacy (D_{se})

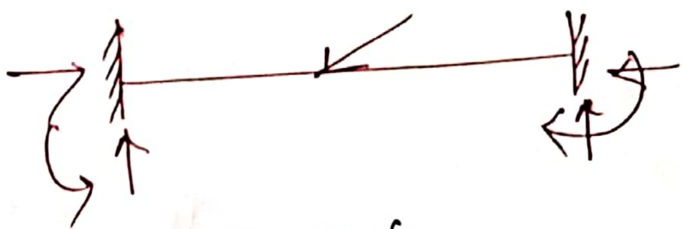
External static indeterminacy is equal to no. of independent external reactions in excess to available equilibrium condition for static equilibrium

$$\boxed{D_{se} = r_e - r}$$

where r_e = Total no. of independent support reaction
r = Total no. of available equation of static equilibrium.

for 2D ⇒ equation of equilibrium = 3
for 3D ⇒ " " = 6.

Case-1 :- (2D beam subjected to general Loading)

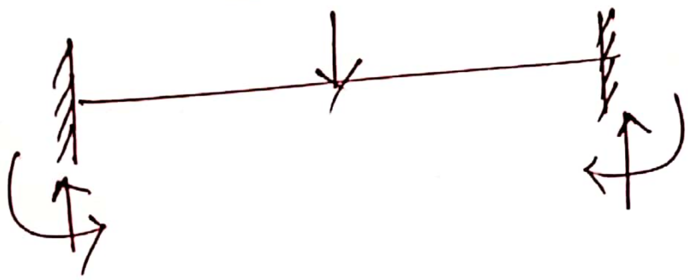


Here $r_e = 6$
$r = 3$.

$D_{se} = r_e - r = 6 - 3 = 3$.

For general Loading system a fixed beam is statically indeterminate to 3rd degree.

Case-2 :- (2D beam vertical Loading)



$r_e = 4$
$r = 2$
$D_{se} = r_e - r$
$= 4 - 2 = 2$

Here beam is indeterminate to 2nd degree.

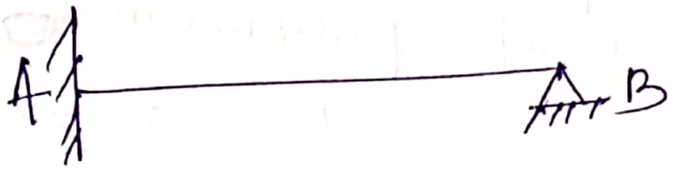
Note

for general Loading, the external determinacy D_{se} is given by

$$D_{se} = R_e - 3 \rightarrow \text{for 2D}$$

$$D_{se} = R_e - 6 \rightarrow \text{for 3D.}$$

Example-1 For the structure shown in figure, determine degree of static indeterminacy (D_{se})

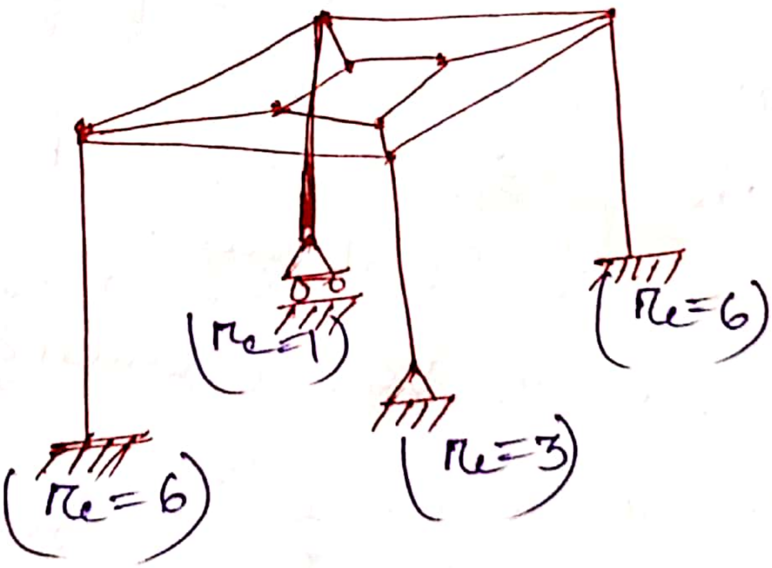


Solution

$R_e = 5$ $\qquad D_{se} = R_e - r$

$r = 3$ $\qquad\qquad = 5 - 3 = 2.$

Q For space frame as shown in figure Determine D_{se}.



for fixed beam (3D space) reaction = 6
for pinned / hinged = 3
for Roller = 1
Total Reaction $r_e = 6+1+3+6 = 16$
for general Loading $DS_e = r_e - 6$
$= 16 - 6 = 10$

Note

$$\begin{bmatrix} 2D \rightarrow r_e - 3 \\ 3D \rightarrow r_e - 6 \end{bmatrix}$$

- Since all reaction are non parallel and non current hence given frame is stable and indeterminate to 10th degree.

Internal static indeterminacy (DS_i)

Case-1:- pin jointed plane frame (2D truss):

$$\boxed{DS_i = m - (2j - 3)}$$

- If $DS_i = 0$, Truss is internally determinate and such trusses are called as perfect truss.
If $DS_i > 0$ it is internally indeterminate and over stiff.
If $DS_i < 0$ internally deficient and geometrically unstable.

Case-2:- pin jointed space frame:-

$$\boxed{DS_i = m - 3j - 6}$$

m = total no. of members
j = Total no. of joints.

Case-3:- 2D and 3D Rigid frames:-

for 2D rigid frame, $D_{Si} = 3C$.
for 3D rigid frame, $D_{Si} = 6C$.
where C = No. of closed loop.

Note:- In above analysis all the joints are considered rigid. If some of the joints are hybrid (hinged) then $\boxed{D_{Si} = 3C - r_R}$ ⇒ for (2D)

$\boxed{D_{Si} = 6C - r_R}$ ⇒ for 3D.

C = closed loop
r_R = Total no. of internal reaction released.

Note:- formulae for static indeterminacy

External static indeterminacy :- (D_{Se})

(1) In general degree of external static indeterminacy

$\boxed{D_{Se} = r - e}$ r = No. of external reaction
e = No. of equilibrium equation

(2) plane frame or 2D frame $\boxed{D_{Se} = r - 3}$
(3) space frame or 3D frame $\boxed{D_{Se} = r - 6}$

Internal static indeterminacy :- (D_{Si})

(1) Pin jointed plane frame or truss (2D) $\boxed{D_S = M - (2j - 3)}$
(2) Pin jointed space frame = $M - (3j - 6)$
(3) Rigid jointed plane frame $D_{Si} = 3C - r'$
or $D_{Si} = 3C - r_R$

(8)

(3) Rigid jointed plane frame $\boxed{DS_i = 3C - r_r}$

(4) Rigid jointed space frame $\boxed{DS_i = 6C - r_r}$

where m = total no. of members

j = total no. of joints

C = Total no. of cuts required for open configuration / closed loop

r_r = No. of reaction relesed for hybrid joints or internal hinge.

Total degree of static indeterminacy

$$\boxed{D_S = DS_i + DS_e}$$

DS_i = internal static indeterminacy

DS_e = External static indeterminacy.

final formulae

(1) Pin jointed plane frame $(D_S = m + r - 2j)$ (2D truss)
(2D truss)

(2) Pin jointed space frame $(D_S = m + r - 3j)$ (3D truss)

(3) Rigid jointed plane frame = $(3m + r - 3j)$

or $(r - 3) + (3C - r_r)$

(4) Rigid jointed space frame = $(6m + r - 6j)$

<u>or</u> $(r - 6) + (6C - r_r) = D_S$

Alternative approach

(1) 2D truss. $\boxed{D_s = m + r - 2j}$

m = No. of members.
r = No. of external reactions.
j = No. of joints.

If $D_s > 0$ = statically indeterminate but stable
$D_s = 0$ = statically determinate
$D_s < 0$ = statically unstable.

(2) space or 3D truss = $\boxed{D_s = m + r - 3j}$

(3) 2D rigid frames :- $\boxed{D_s = 3m + r - 3j}$

When all joints are rigid.

$D_s = (3m + r) - (3j - r_r)$ (when some joints are hybrid).

(4) 3D Rigid frames

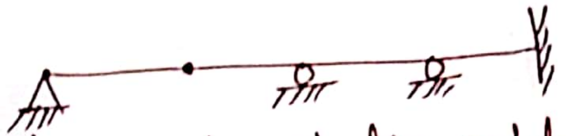
$D_s = (6m + r) - 6j \Rightarrow$ when all joints are rigid.

$D_s = (6m + r) - (6j - r_r) \Rightarrow$ when some joints are hybrid.

(9)

Examples

Beam



Determine the static indeterminacy of beam,

Ans:- Here the structure is 2D frame.

$m = 4$ total reaction $(R) = 2+1+1+3 = 7$

$j = 5$

Degree of static indeterminacy for hybrid joint

R_R = no. of reaction released $= 2-1 = 1$

$3m + R - (3j - R_R)$

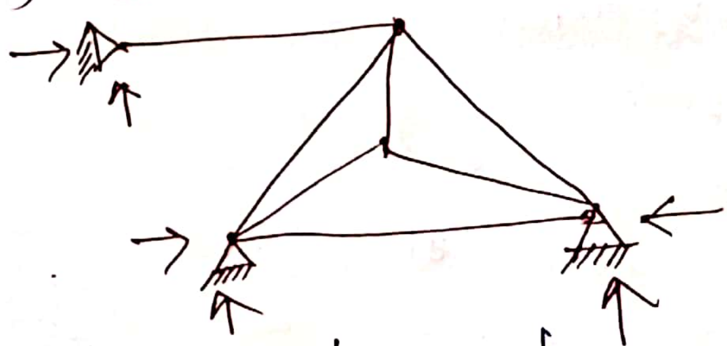
$\Rightarrow 3 \times 4 + 7 - (3 \times 5 - 1)$

$\Rightarrow 12 + 7 - (15 - 1) = 5$.

Note:- $D_S = 3m + R - 3j$ when joints are rigid

$D_S = 3m + R - (3j - R_R)$ when joints are hybrid.

(2) Truss



Solution 1st approach

D_{Se} = external static indeterminacy $\boxed{R - e}$

$= 4 + 2 - 3$

$= 6 - 3 = 3$.

D_{Si} = internal static indeterminacy

Scanned with CamScanner

$D_{S_i} = M - 2j - 3$

$\begin{bmatrix} m = 7 \\ j = 5 \end{bmatrix}$ r = No. of reaction = 6

$D_{S_i} = 7 - 2\times5 - 3 = 0$

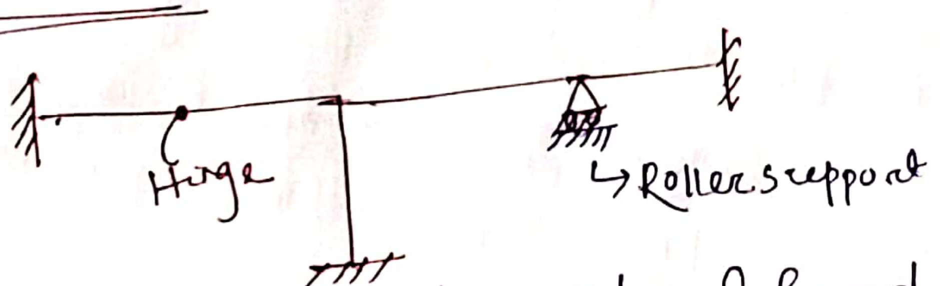
$D_S = D_{S_i} + D_{S_e} = 3 + 0 = 3$

2nd approach:- Direct formula of degree of static indeterminacy of truss

$D_S = M + r - 2j$

$= 7 + 6 - (2\times5) = 3.$

(3) Rigid frame:-



As we know for hybrid or internal hinged frame

$D_S = (3m + r) - 3j - r_R$

m = No. of member = 5

j = No. of joint = 6

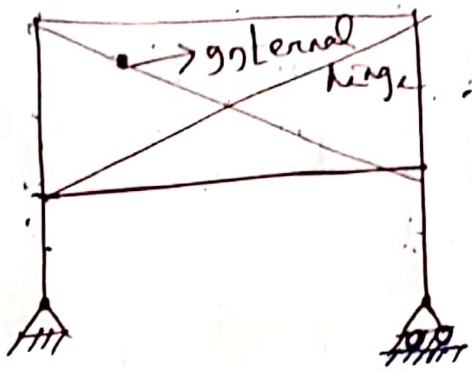
r = No. of reaction = $3+3+3+1 = 10$.

r_R = Reaction released = as in internal hinge two members are connected

so $r_R = 2 - 1 = 1$

$D_S = (3m + r) - 3j - r_R$

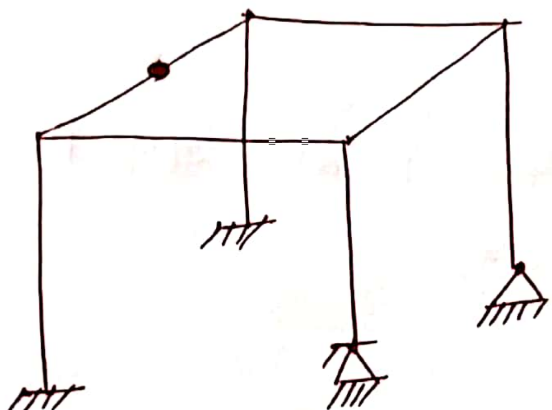
$= (3\times5 + 10 - 3\times6 - 1) = 6.$ Ans

* 

This is the 2-D frame. for 2D frame with hybrid joint or internal hinge the degree of static indeterminacy $(D_s = 3m + r - 3j - r_r)$

m = no. of member = 11
j = no. of joint = 8
r = no. of reaction = 3
r_r = reaction relesed = 1

$$D_s = 3m + r - 3j - r_r$$
$$= 3 \times 11 + 3 - 3 \times 8 - 1 = 11$$

* 

for this 3D frame or space frame determine D_s ?

In 3D
for fixed = 6
hinged = 3
} reaction

Solution:- m = no. of member = 9
r = no. of reaction = $6+6+3+3$
$= 18$

$r_r = 3(m'-1)$
$= 3(2-1) = 3$

m' = no. of member connected to internal hinge.

Note:- for 2D → r_r (reaction relesed) is $(m'-1) = \Sigma(m'-1)$

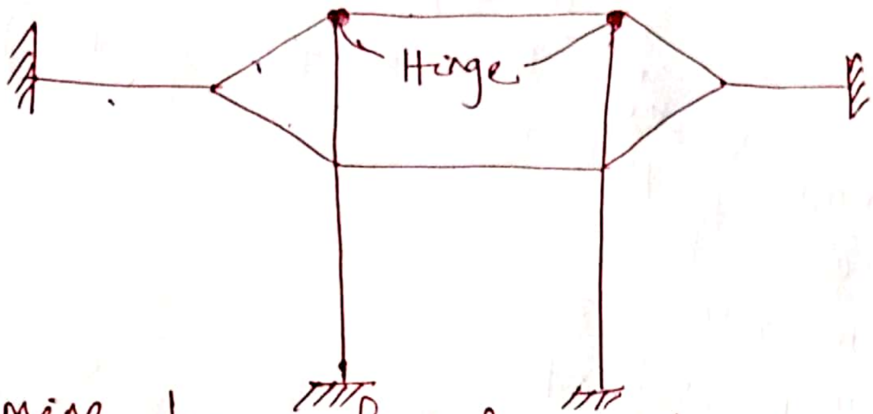
for 3D, $r_r = 3(m'-1) = \Sigma 3(m'-1)$.

Degree of static indeterminacy

$$D_s = 6m + r - 6j - r_r$$

$$= 6 \times 9 + 18 - 6 \times 9 - 3 = 15$$

Q



Determine degree of static indeterminacy for this Plane frame)

m = No. of member = 12

$r = 12$

$j = 10$

r_r = reaction relesed for 2D = $\Sigma(M'-1)$

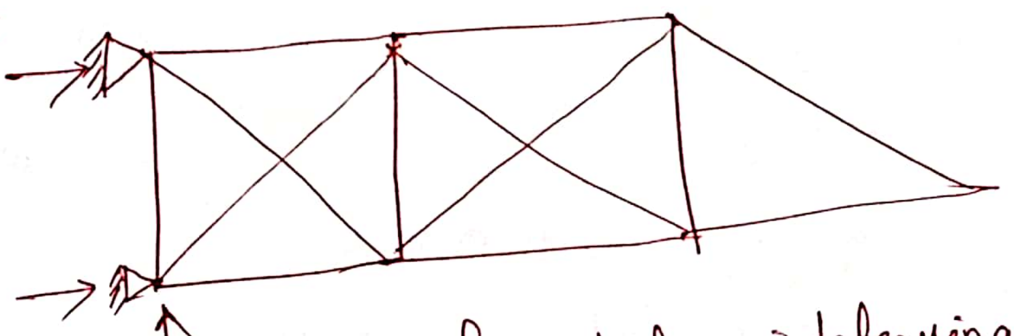
$= (3-1) + (3-1)$

$= 2 + 2 = 4.$

$$D_s = 3m + r - 3j - r_r$$

$$= 3 \times 12 + 12 - 3 \times 10 - 4 = 14.$$

Q



Calculate degree of static indeterminacy for cantilever truss.

(11)

Solution

$m = 13$

$j = 7$

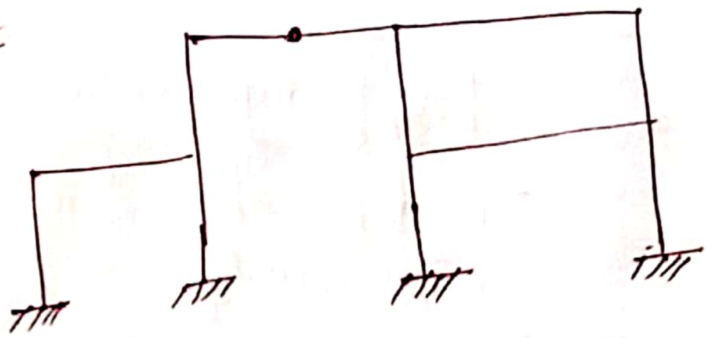
$r = 4$

$D_s = m + r - 2j$

$= 13 + 4 - 2\times 7 = 3.$

$\left[\begin{array}{l} \text{for 2D truss} = m + r - 2j \\ \text{3D "} = m + r - 3j \end{array}\right]$

Q.



$D_s = ?$

Solution:-

$m = 12$

$j = 12$

$r = 12$

$r_r = \Sigma(m' - 1) = 2 - 1 = 1$

$m' =$ no. of members connected to internal hinge

$D_s = 3m + r - 3j - r_r$

$= 3\times 12 + 12 - 3\times 12 - 1$

$= 11.$

Note

$m' =$ no. of members for internal hinge

for 2D $= \boxed{\Sigma(m'-1) = r_r}$ $\quad r_r =$ no. of reaction released

for 3D $= \boxed{\Sigma 3(m'-1) = r_r}$

Degree of Kinematic Indeterminacy

Degree of Kinematic indeterminacy refers to the total no. of independent available degree of freedom at all joints.

→ The D.O.F or Kinematic indeterminacy may be defined as total no. of displacement component at all joints.

Types of Joint		Possible degree of freedom
1. 2D truss joint	→	Two degree of freedom Δx and Δy
2. 3D truss joint	→	3 degree of freedom Δx, Δy and Δz.
3. 2D rigid joint	→	3 degree of freedom Δx, Δy, Δz
4. 3D rigid joint	→	6 degree of freedom $\begin{bmatrix} \Delta x & \Delta y & \Delta z \\ \theta x & \theta y & \theta z \end{bmatrix}$

Degree of Kinematic Indeterminacy

(a) for plane truss (2D truss)

$$\Rightarrow \boxed{D_k = 2j - r_e}$$

$r_e = R =$ No. of reaction (external)

(b) Space truss (3D truss)

$$\boxed{D_k = 3j - r_e}$$

(c) Rigid jointed plane frame $\boxed{D_k = 3j - r_e}$

(d) Rigid Jointed space frame $\boxed{D_k = 6j - r_e}$

Special case

In rigid frame, if some of the members are axially rigid then axial displacements in such members may n't be available. Hence DOF reduced

for 2D rigid jointed frame $\boxed{D_k = 3j - r_e - m''}$

for 3D rigid jointed frame $\boxed{D_k = 6j - r_e - m''}$

j = no. of joints
r_e = No. of external reactions
m'' = No. of axially rigid members.

* There are 2 types of beam
  (i) Axially extensible
  (ii) Axially inextensible.

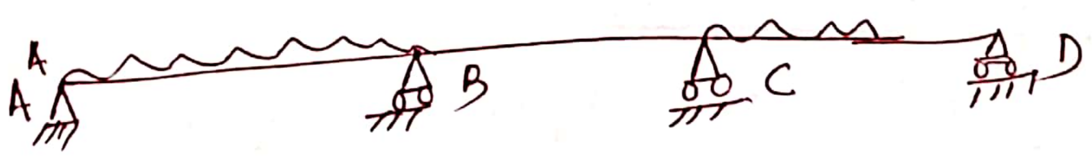
or members are with extensible and inextensible are discussed.

Equilibrium condition

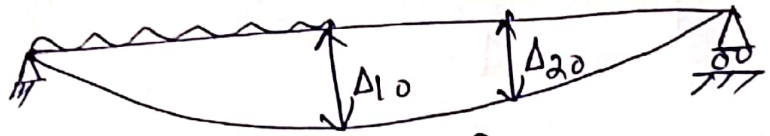
As we all discussed before there are 3 equation of equilibrium

such as
$$\left.\begin{aligned} \Sigma F_x &= 0 \\ \Sigma F_y &= 0 \\ \Sigma M_z &= 0 \end{aligned}\right\}$$

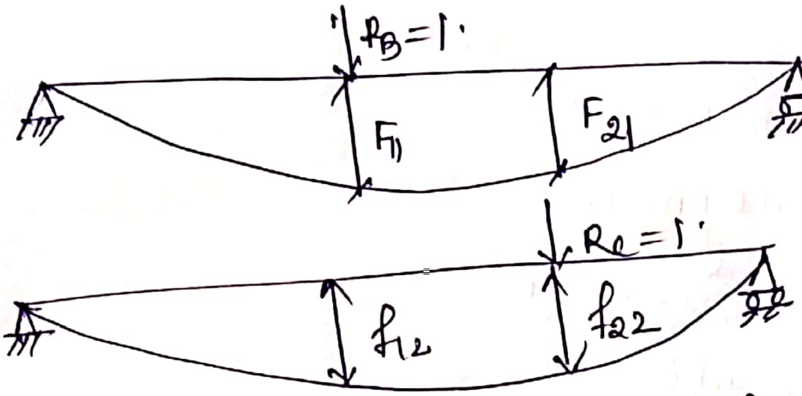
How compatibility equations are developed :-



The indeterminate structure is converted into determinate structure.



when we remove R_B and R_C we get deflection at R_B and R_C such as Δ_{10} and Δ_{20}.



f_{11} = deflection at 1 due to unit load at 1

f_{21} = deflection at 2 due to unit load at 1

f_{12} = deflection at 1 due to unit load at 2

f_{22} = deflection at 2 due to unit load at 2

Compatibility conditions are required for the analysis of indeterminate structure.

$$\Rightarrow \quad 0 = \Delta_{10} \times 1 + R_B \times f_{11} + R_C F_{12} \quad \text{——— (1)}$$

$$\Rightarrow \quad 0 = \Delta_{20} + R_B \times f_{21} + R_C \times F_{22} \quad \text{——— (2)}$$

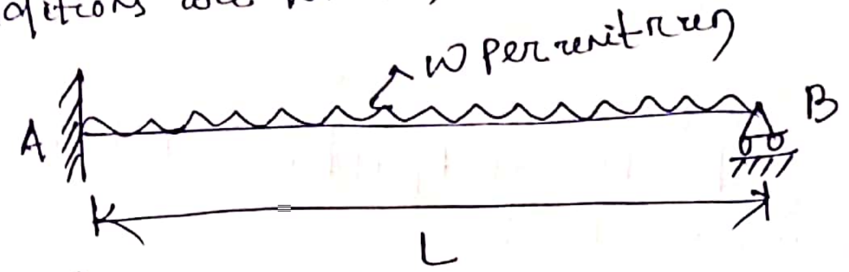
we know $\boxed{\text{deflection} \propto \text{Load}}$

these two equation are compatibility equation.

Consistent Deformation Method :-

This is the basic method to analyse redundant structures taking force or reaction as unknown.

→ To find unknown force or reactions suitable compatibility conditions are written.



- For above propped cantilever $D_S = R_e - 3$
  $= (3+1) - 3$
  $= 1.$

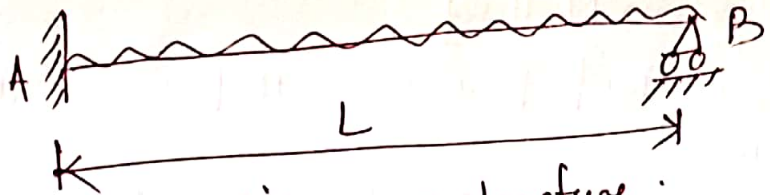
for fixed beam reaction $= 3$
" Roller " " $= 1$
" Hinged " " $= 2$.

Hence above beam is statically indeterminate to first degree. So shearforce and Bending moment at any section can't be obtained by using equations of static equilibrium alone.

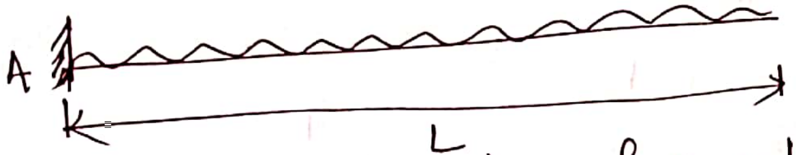
- → If we consider Reaction R_B as redundant and remove restraint offered by R_B. Then remaining cantilever is statically determinate which is known as primary structure.

for a cantilever

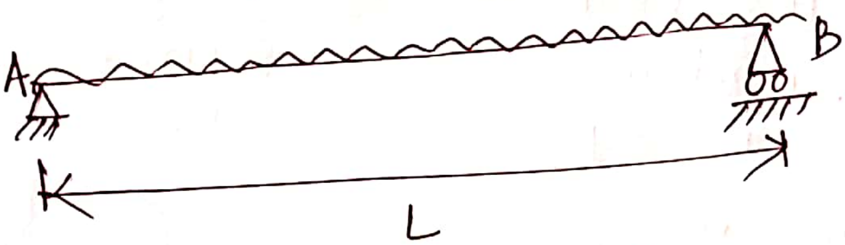
$$\boxed{\begin{aligned} D_S &= 3 - 0 \\ &= 3 \end{aligned}}$$



(i) given structure.



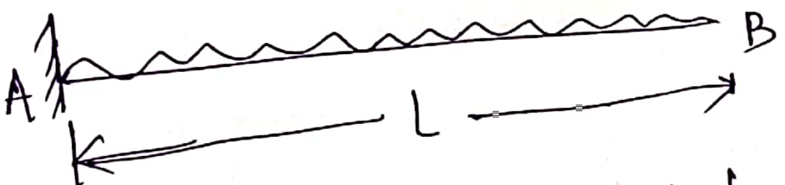
(II) Primary structure if R_B is taken as redundant
Determinate structure.



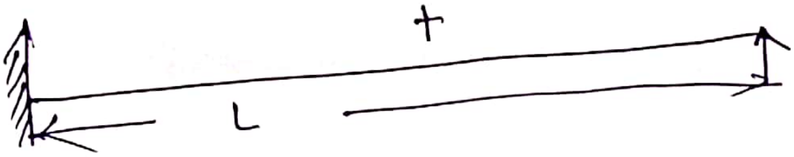
(III) Primary structure, if M_A taken as redundant.

Consider figure (II) as primary structure

(i) structure subjected to given loading.



(II) Primary subjected to given loading.



(Structure subjected to R_B)

(14)

Since at B, there is a unyielded support, Hence net vertical deflection at B will be zero.

$\Delta_B = 0 \longrightarrow$ (compatibility condition)

$$\frac{WL^4}{8EI} - \frac{R_B L^3}{3EI} = 0$$

$$\boxed{R_B = 3/8\, WL}$$

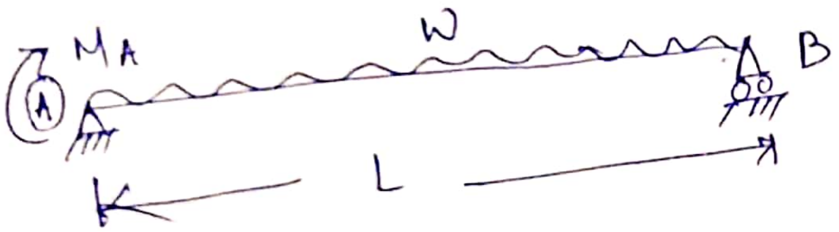
Since end A in actual structure is fixed, hence at end A net rotation (slope) will be zero.

$\theta_A = 0 \longrightarrow$ (compatibility condition)

$$\theta_A + \theta_A' = 0$$

$$\Rightarrow \frac{WL^3}{24EI} + \frac{M_A L}{3EI} = 0$$

$$\Rightarrow \boxed{M_A = -WL^2/8}$$



=

$\theta_A = \frac{WL^3}{24EI}$

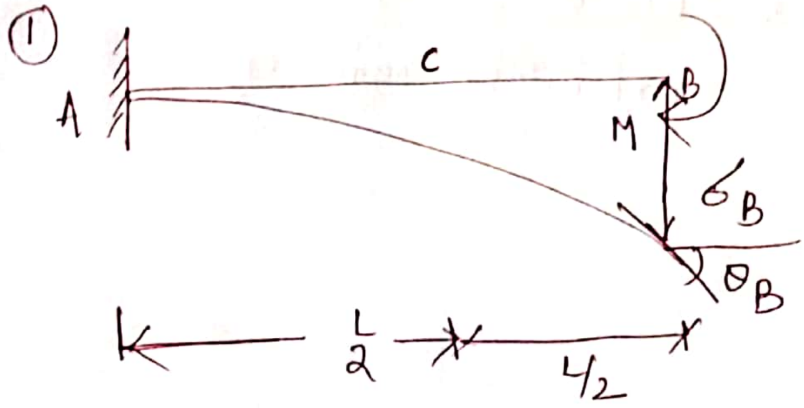
+

$\theta'_A = M_A L / 3EI$

Some Important Beam Displacement:-

Displacement

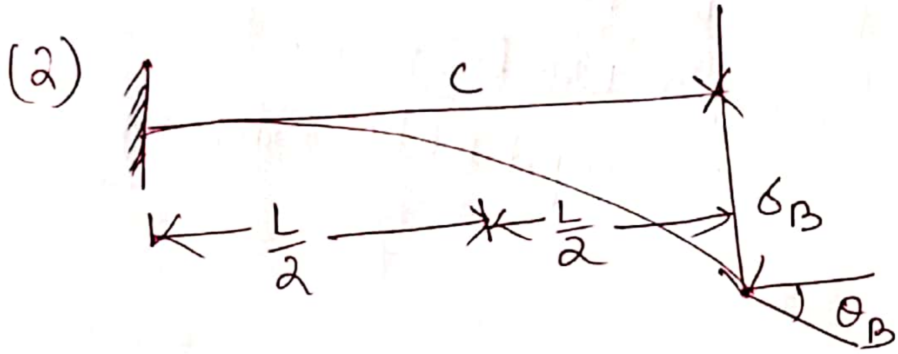
(1)



$$\theta_B = \frac{ML}{EI}$$

$$\delta_B = \frac{ML^2}{2EI}$$

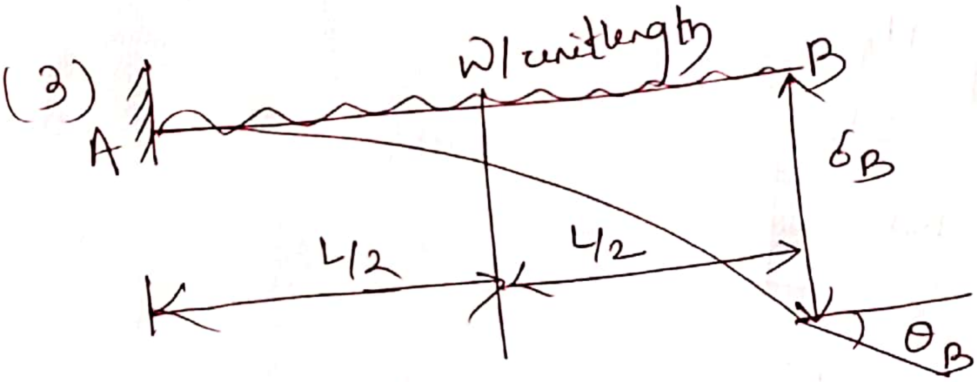
(2)



$$\theta_B = \frac{PL^2}{2EI}$$

$$\delta_B = \frac{PL^3}{3EI}$$

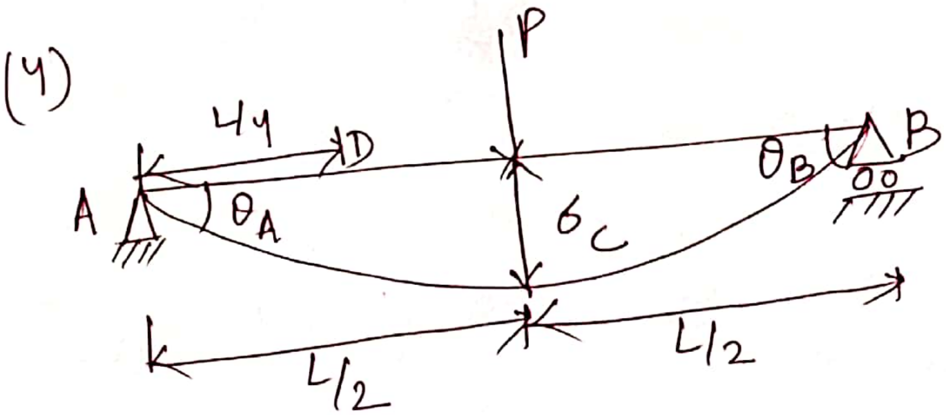
(3)



$$\theta_B = \frac{wL^3}{6EI}$$

$$\delta_B = \frac{wL^4}{8EI}$$

(4)

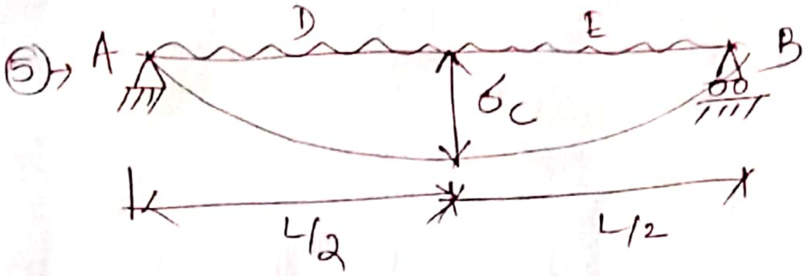


$$\theta_A = \frac{PL^2}{16EI}$$

$$\theta_B = \frac{-PL^2}{16EI}$$

$$\delta_C = \frac{PL^3}{48EI}.$$

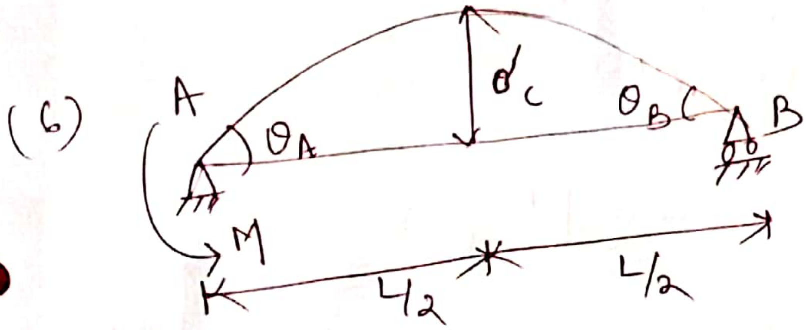
(14)

(5)


$\theta_A = \frac{WL^3}{24EI}$

$\theta_B = \frac{-WL^3}{24EI}$

$\delta_C = \frac{5}{384}\frac{WL^4}{EI}$

(6)


$\theta_A = \frac{-ML}{3EI}$

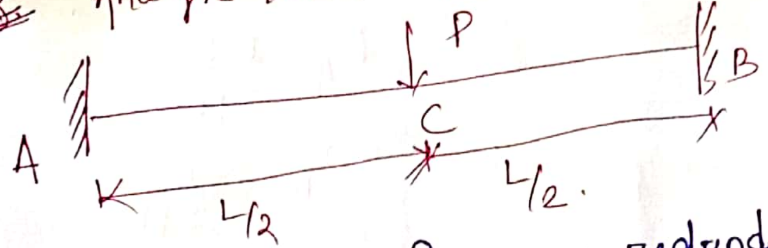
$\theta_B = \frac{ML}{6EI}$

$\delta_C = \frac{-ML^2}{16EI}$.

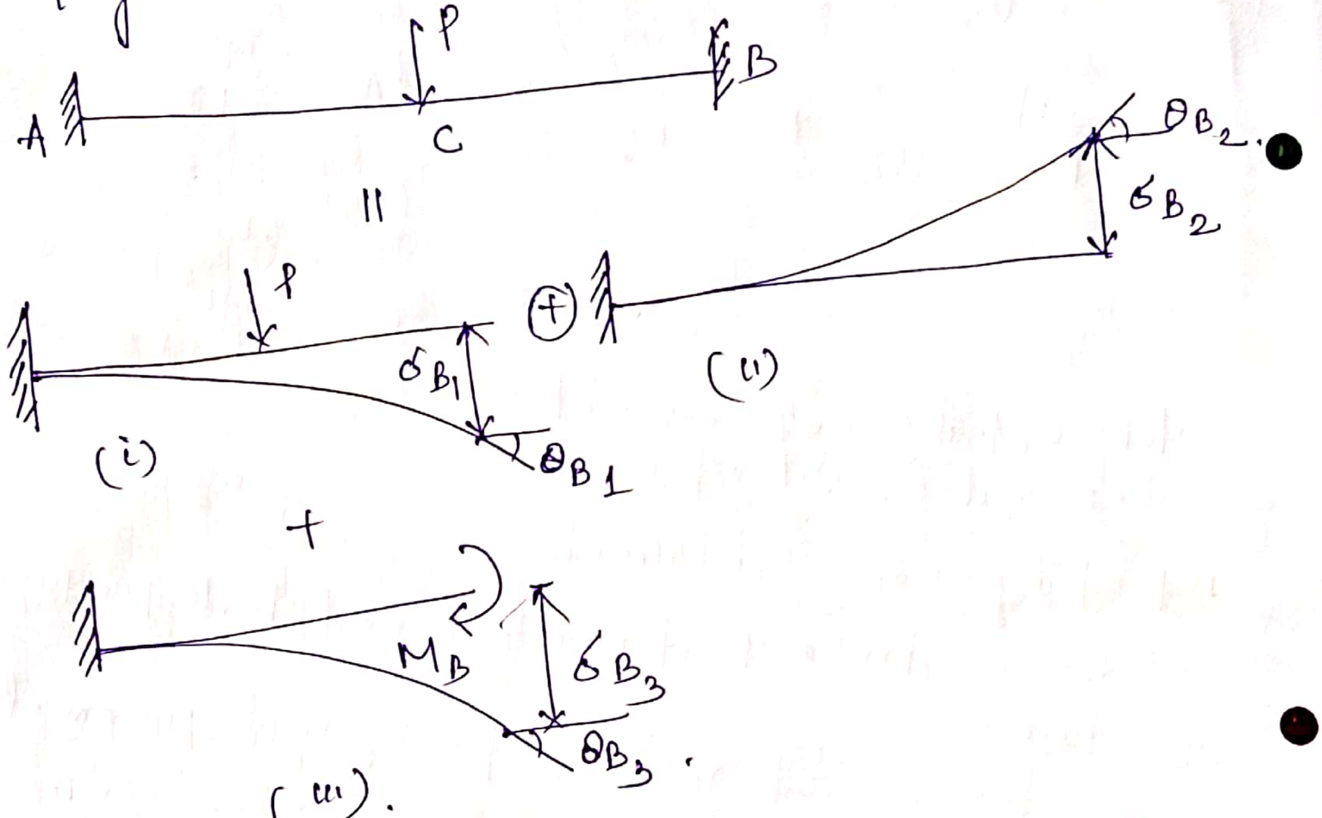
Steps of Analysis of consistent Deformation method

1) Find degree of indeterminacy
2) choose statically determinate and stable primary structure.
3) Identify the redundants and compute displacements of primary structure subjected to axial Loading.
4) By using compatibility conditions get the value of those assumed redundants.
5) Plot S.F and B.M diagram by superimposing effect of redundants and given loading on the primary structure.

Q. Analyse the fixed beam shown in figure



Let us assume M_B and R_B as redundant. The corresponding primary structure is shown below.



The primary structure is the combination of (i), (ii) and (iii).

Since end B is fixed in actual structure.

Hence $\theta_B = 0$ (first compatibility condition)

$\delta_B = 0$ (2nd compatibility condition)

Slope at end B

$$\theta_{B_1} = \frac{P(L/2)^3}{2EI} = +\frac{PL^2}{8EI} (\circlearrowleft)$$

$$\theta_{B_2} = \frac{-R_B L^2}{2EI} (\circlearrowright)$$

$$\theta_{B_3} = \frac{M_B L}{EI} (\circlearrowleft)$$

from compatibility condition (i) $\theta_B = 0$.

$$\Rightarrow \theta_{B_1} + \theta_{B_2} + \theta_{B_3} = 0.$$

$$\Rightarrow \frac{PL^2}{8EI} - \frac{R_B L^2}{2EI} + \frac{M_B L}{EI} = 0$$

$$\Rightarrow PL - 4R_B L + 8M_B = 0 \quad \text{—— (1)}.$$

Deflection at end B

$$\delta_{B_1} = \delta_C + \theta_C \times L/2.$$

$$= \frac{P(L/2)^3}{3EI} + \frac{PL^2}{8EI} \times L/2 = \frac{PL^3}{24EI} + \frac{PL^3}{16EI}.$$

$$\delta_{B_2} = \frac{-R_B L^3}{3EI} (\uparrow) \qquad \delta_{B_3} = \frac{+M_B L^2}{2EI} (\downarrow)$$

from Compatibility condition (ii)

$$\delta_B = 0$$

$$\Rightarrow \delta_{B_1} + \delta_{B_2} + \delta_{B_3} = 0.$$

$$\Rightarrow \frac{5}{48}\frac{PL^3}{EI} + \left(\frac{-R_B L^3}{3EI}\right) + \frac{M_B L^3}{2EI} = 0.$$

$$\Rightarrow 5PL - 16R_B L + 24M_B = 0 \quad \text{——— (ii)}$$

from eq.^{n} (i) we know $\boxed{4R_B L = PL + 8M_B}$

Substituting value of $4R_B$ in eq.^{n} (ii) we get

$$M_B = \frac{PL}{8}$$

$$\Rightarrow 4R_B L = PL + 8\left(\frac{PL}{8}\right)$$

$$\boxed{R_B = P/2}$$

* Conjugate Beam Method :-

An imaginary beam for which the load diagram is the $\frac{M}{EI}$ diagram.

* slope at a section of a given beam = shear force at a section of conjugate Beam.

Means

$\boxed{\text{Slope in real beam} = \text{S.F of conjugate beam}}$

* Deflection at a section of a given beam = Bending moment at a section of conjugate beam

Means

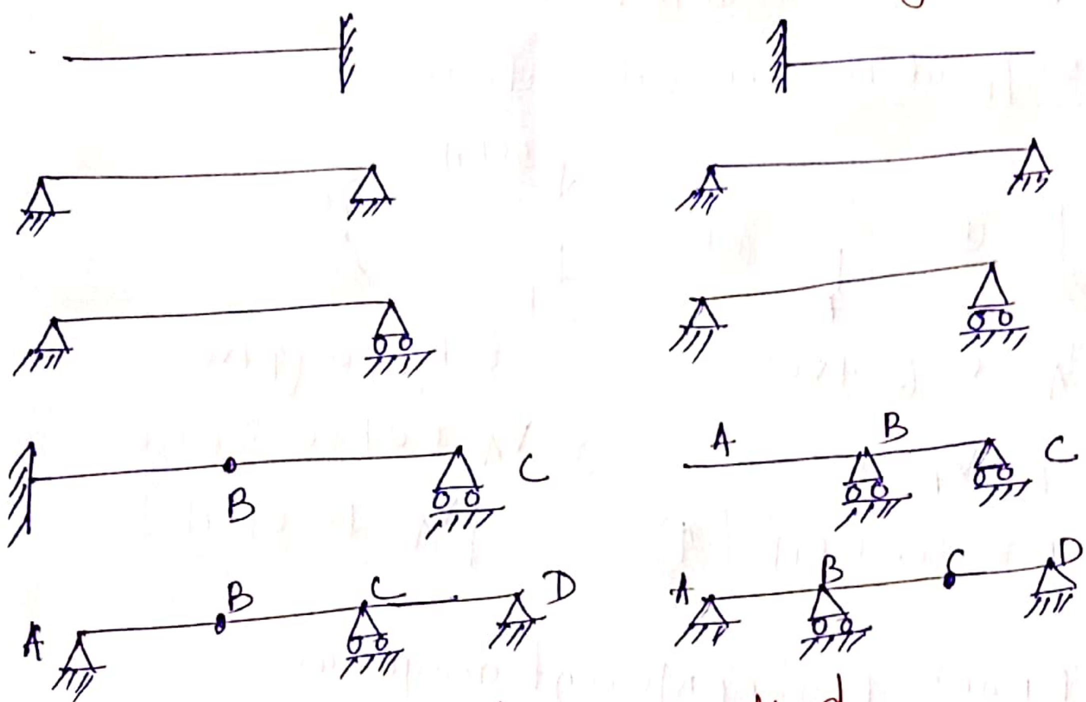
$\boxed{\text{Deflection in real beam} = \text{B.M of conjugate beam}}$

(16)

Real beam and their corresponding conjugate Beam

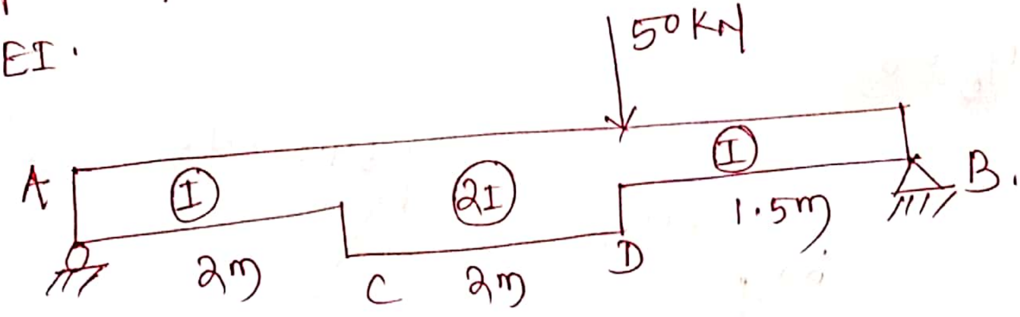
Real beam | Conjugate beam

(i)
(ii)
(iii)
(iv)
(v)



Problem using conjugate Beam method

Using conjugate beam method find the vertical deflection at D and slope at A for simply supported beam loaded as shown in figure in terms of EI.



Solution

Steps

(1) Support reaction calculation

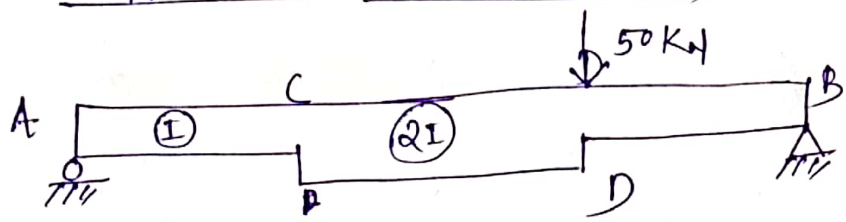
(2) Draw BMD

(3) Draw M/EI diagram.

(4) Draw conjugate beam and consider M/EI diagram as Loading on the beam.

(5) Find slope and deflection.

(1) Support reaction calculation



$\Sigma M_A = 0$ ($\circlearrowright$ +ve)

$\Rightarrow 50 \times 4 - V_B \times 5.5 = 0$

$\boxed{V_B = 36.36\ KN}$ ($\uparrow$)

$\Sigma F_y = 0$ ($\uparrow$ +ve)

$\Rightarrow V_A - 50 + 36.36 = 0$

$\boxed{V_A = 13.64\ KN}$

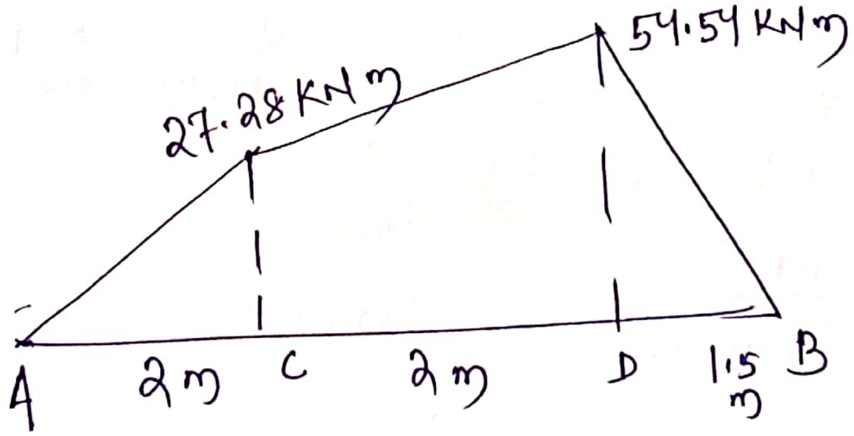
(2) Draw Bending Moment diagram

$BM_A = 0$

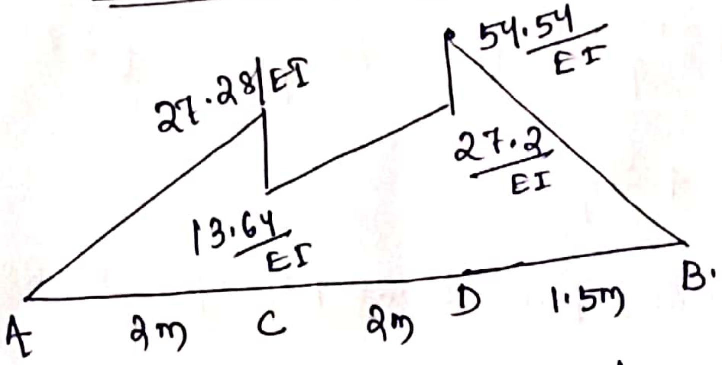
$BM_C = 13.64 \times 2 = 27.28\ KNm$

$BM_D = 36.36 \times 1.5 = 54.54\ KNm$

$BM_B = 0$

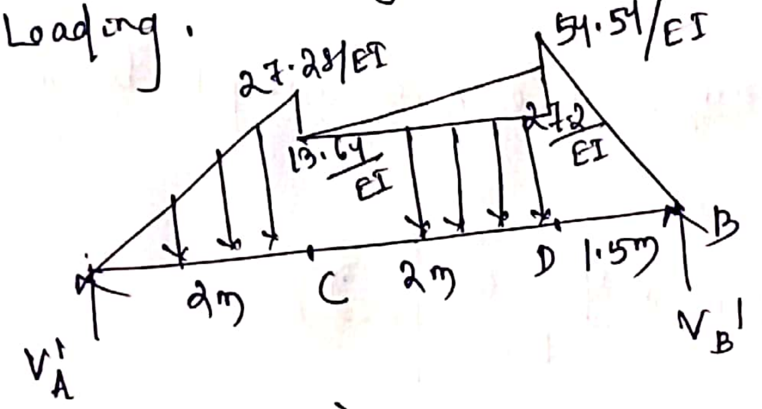


Step-3 Draw $\frac{M}{EI}$ diagram:-



[M/EI Diagram]

- Step-4 :- Draw conjugate beam and consider M/EI diagram as Loading.



$\Sigma M_A = 0$ (↺ +ve)

- $$\Rightarrow \left(\frac{1}{2}\times 2\times \frac{27.28}{EI}\right)\times\left(\frac{2}{3}\times 2\right) + \left(\frac{1}{2}\times 2\times \frac{13.6}{EI}\right)\times\left(2+\frac{2}{3}\times 2\right) + \left(2\times\frac{13.64}{EI}\right)\left(2+\frac{2}{2}\right) + \left(\frac{1}{2}\times 1.5\times\frac{54.54}{EI}\right)\left(4+\frac{1}{3}\times 1.5\right)$$

$$-V_B'\times 5.5 = 0.$$

$$\Rightarrow 5.5\,V_B' = \frac{1}{EI}\left[347.61\right]$$

$$\Rightarrow V_B' = \frac{347.61}{5.5\,EI}$$

$$\Rightarrow \boxed{V_B' = \frac{63.20}{EI}}$$

$\Sigma F_y = 0$ ($\uparrow$ +ve)

$\Rightarrow V_A' - \left(\frac{1}{2} \times 2 \times \frac{27.28}{EI}\right) - \left(\frac{1}{2} \times 2 \times \frac{13.6}{EI}\right) - \left(2 \times \frac{13.64}{EI}\right)$

$- \left(\frac{1}{2} \times 1.5 \times \frac{54.54}{EI}\right) + \frac{63.20}{EI}$

$\Rightarrow V_A' = \frac{1}{EI}\left[27.28 + 13.6 + 27.28 + 40.90 - 63.20\right]$

$\Rightarrow \boxed{V_A' = \frac{45.86}{EI}}$

Step-5 :-

θ_A in Real beam = S.F @ A in conjugate beam

S.F @ A left = 0

$\boxed{\text{S.F @ A Right} = \frac{45.86}{EI}}$ $(\circlearrowleft)$

Deflection (y_D) in real beam = B.M @ D in conjugate beam

$B.M_D = \frac{63.20}{EI} \times 1.5 - \left[\frac{1}{2} \times 1.5 \times \frac{54.54}{EI}\right]\left(\frac{1}{3} \times 1.5\right)$

$\boxed{B.M_D = \frac{74.35}{EI}}$ $(\downarrow)$.

Ans.

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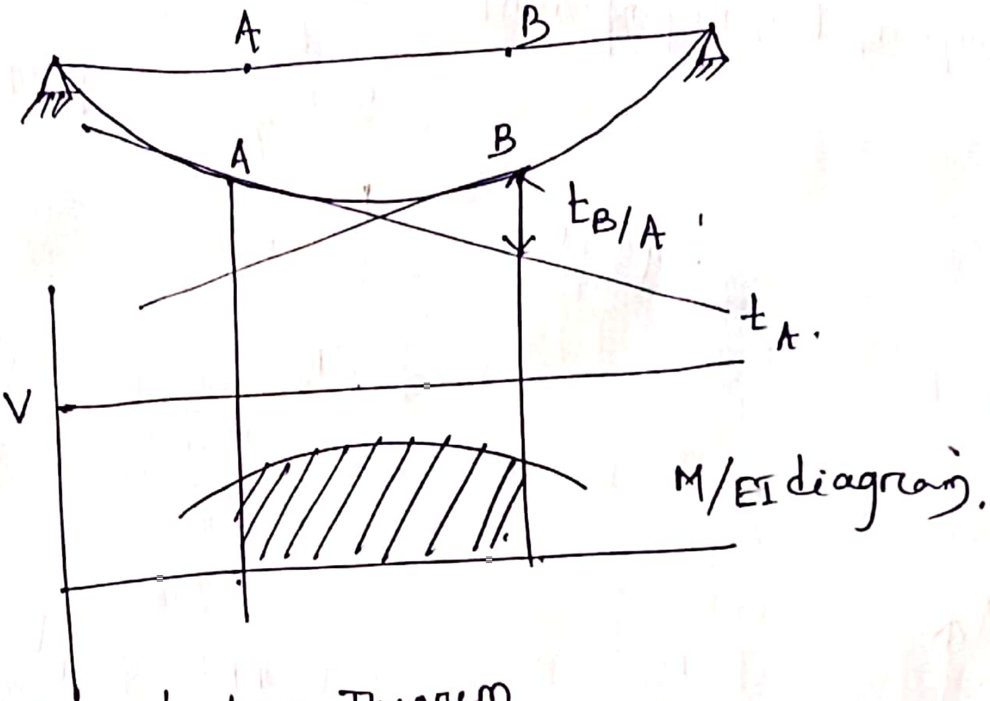
Moment - Area Method:

Moment Area Method is a semigraphical solution that relates slopes and deflections of the elastic curve to the area under the M/EI diagram and the moment of the area of the M/EI diagram respectively.

→ This method is particularly useful when deflection at a specific point on the beam is required.



First Moment Area Theorem

$$\theta_{B/A} = \int_A^B M/EI \, dx$$

This theorem states that change in slope between two point is equal to Area under M/EI diagram.

2nd Theorem of Moment area method

$$t_{B/A} = \bar{x}_B \int_A^B M/EI \, dx.$$

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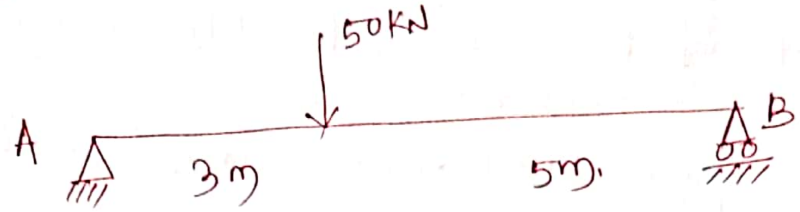
Deflection between two point = Area under M/EI diagram × C.G of that section.

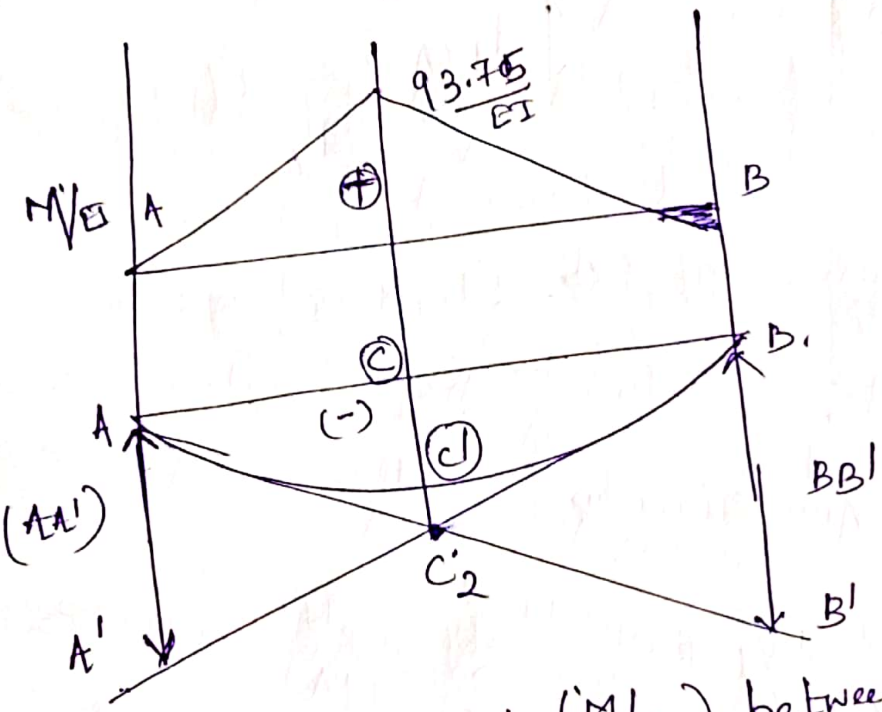
Example :



(1) Bending moment diagram

Maximum Bending Moment $= \frac{wab}{L}$

$= \frac{50 \times 3 \times 5}{8} = 93.75\,kNm$



AA' = Area Moment (M/EI) between A and B about A.

$= \left(\frac{1}{2} \times 8 \times \frac{93.75}{EI} \times \frac{3+8}{3}\right)$

Note

c.g of this Δ

$= \frac{a+L}{3}$ or $b + \frac{L}{3}$

$$= \frac{1375}{EI}$$

$BB' =$ Area Moment (M/EI) between B and A about B

$$= \left(\left(\frac{1}{2} \times 8 \times \frac{93.75}{EI} \times \frac{5+8}{3}\right)\right)$$

$$= \frac{1625}{EI}.$$

Slopes :- $\tan\theta_A = \theta_A = \frac{BB'}{AB} = \frac{1625/EI}{8}$

$$\theta_A = \frac{203.125}{EI} \text{ Rad}$$

$$\tan\theta_B = \theta_B = \frac{AA'}{AB}$$

$$= \frac{1375/EI}{8} (\curvearrowleft)$$

$$\Rightarrow \theta_B = \frac{171.875}{EI} \text{ Rad}. (\circlearrowright)$$

In triangle $AB'B$

$$\frac{BB'}{AB} = \frac{CC_2}{AC}$$

$$\Rightarrow 203.125 = \frac{CC_2}{3}$$

$$\Rightarrow CC_2 = \frac{609.375}{EI}$$

2nd theorem:

$C_1C_2 =$ Area Moment (M/EI) between C & A about C.

$$= \left[\frac{1}{2} \times 3 \times \frac{93.75}{EI} \times \frac{1}{3} \times 3\right]$$

$$= \frac{140.625}{EI}.$$

deflection @ c = δc. (but we know c to $c_1 = \delta c$)

$$\delta c = CC_1$$

$$= CC_2 - C_1C_2$$

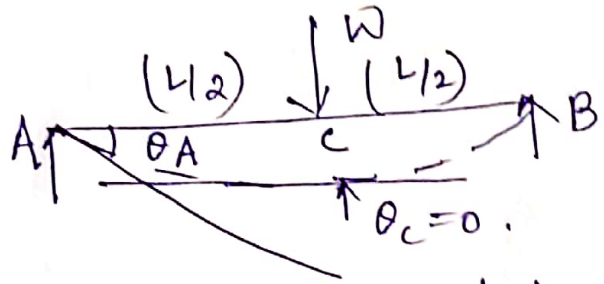
$$= \frac{609.375}{EI} - 140.625$$

$$= \frac{468.75}{EI} m \ (\downarrow).$$

Explanation of Moment Area Method:-

Mohr's Moment Area Method / Theorem 1 :-

This theorem states that the change of slope between the two points on an elastic curve / line is equal to the Area of M/EI diagram between those points.



The change in slope between A & C $(\theta_A - \theta_C)$

= Area of M/EI diagram between A & C.

$$\boxed{\theta_A - \theta_C = \frac{1}{2} \times \frac{WL}{4EI} \times \frac{L}{2} = \frac{WL^2}{16EI}}$$

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