Department of Civil Engineering

B.TECH – 5TH SEM

DCS

Module-I

Properties of concrete and reinforcing steel, Philosophy, concept and methods of reinforced concrete design, Introduction to limit state method: Limit state of collapse and limit state of serviceability. Application of Limit state method to rectangular beams for flexure, shear, bond and torsion.

Introduction

Reinforced concrete, as a composite material, has occupied a special place in the modern construction of different types of structures due to its several advantages. Due to its flexibility in form and superiority in performance, it has replaced, to a large extent, the earlier materials like stone, timber and steel. Further, architect's scope and imaginations have widened to a great extent due to its mouldability and monolithicity. Thus, it has helped the architects and engineers to build several attractive shell forms and other curved structures. However, its role in several straight line structural forms like multistoried frames, bridges, foundations etc. is enormous.

Concrete

Concrete is a product obtained artificially by hardening of the mixture of cement, sand, gravel and water in predetermined proportions.

Depending on the quality and proportions of the ingredients used in the mix the properties of concrete vary almost as widely as different kinds of stones.

Concrete has enough strength in compression, but has little strength in tension. Due to this, concrete is weak in bending, shear and torsion. Hence the use of plain concrete is limited applications where great compressive strength and weight are the principal requirements and where tensile stresses are either totally absent or are extremely low.

PROPERTIES OF FRESH CONCRETE

Workability

- Workability is the ease with which the concrete can be mixed, placed, consolidated and finished.
- Workable concrete is the one which exhibits very little internal friction between particle and particle or which overcomes the frictional resistance offered by the formwork surface or reinforcement contained in the concrete.

The factors affecting workability are given below:

- Water Content
- Mix Proportions
- Size of Aggregates
- Shape of Aggregates
- Surface Texture of Aggregate
- Grading of Aggregate

 Use of Admixtures

The following tests are commonly employed to measure workability.

- Slump Test
- Compacting Factor Test
- Flow Test
- Kelly Ball Test
- VeeBee Consistometer Test

Segregation

 Segregation can be defined as the separation of the constituent materials of concrete.

 A good concrete has all its constituents properly distributed to form a homogenous mixture. To ensure this, optimum grading, size, shape and surface texture of aggregates with optimum quantity of cement & water makes a mix cohesive. Such a concrete does not exhibit the tendency for segregation.

 Prime cause of segregation is the difference in specific gravity of constituents of concrete.

Segregation may be one of the following types:

1. Coarse aggregate separating out of the rest
2. Cement paste or cement-fine aggregate matrix separating out from coarse aggregate
3. Water separating out of the rest

The conditions that favour segregation are:

- Bad mix proportion
- Inadequate mixing
- Excessive compaction by vibration of wet mix
- Large height of dropping of concrete for placement
- Long distance conveyance of mix

Bleeding

Here, water from the concrete comes out to the top surface of the concrete after casting.

The conditions that favour bleeding are:

- Highly wet mix
- Bad mix proportion
- Inadequate mixing

Sometimes, the bleeding water is accompanied to the surface by certain quantity of cement, which forms a cement paste (known as *Laitance*) at the surface.

PROPERTIES OF HARDENED CONCRETE

Grade of concrete

Designated in terms of letter ‘M’ followed by a number. ‘M’ refers to mix; the number represents the 28-day characteristic compressive strength of concrete cubes (150mm) expressed in MPa.

Eg: M20 denotes the concrete mix with 28-day characteristic compressive strength of 20MPa.

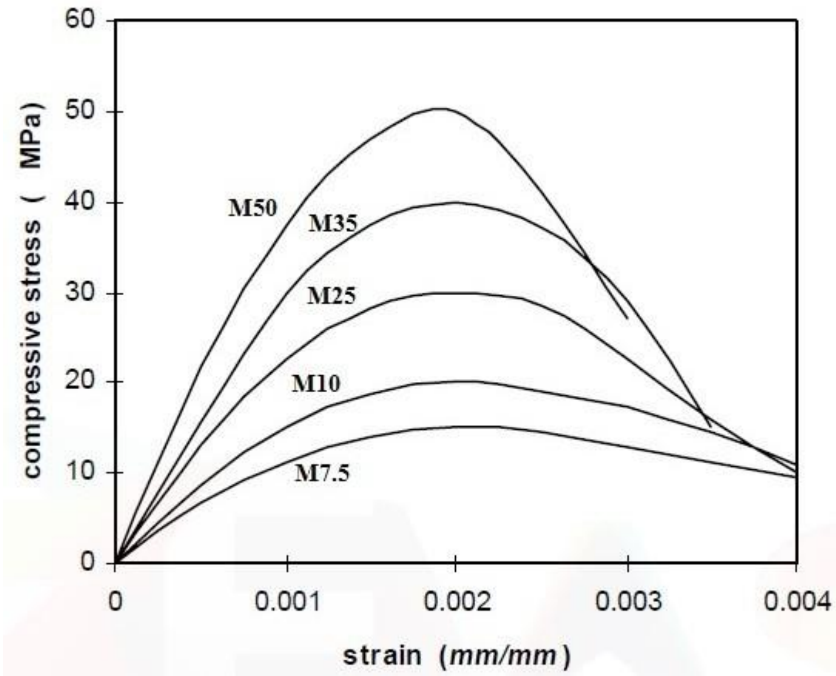
Minimum grade of concrete used is dictated by durability (the environment to which the structure is exposed to, expressed in terms of exposure conditions)

Exposure Condition	**Minimum grade of concrete for RCC works**
Mild	M20
Moderate	M25
Severe	M30
Very severe	M35
Extreme	M40

- Classification
    - *Ordinary concrete* – M10 to M20
    - *Standard concrete* – M25 to M55
    - *High Strength concrete* – M60 and above

Stress-strain curve of concrete

- Stress-Strain curves of concrete for various grades obtained from uniaxial compression tests are shown in above figure
- Maximum stress is attained by concrete at an approximate strain of 0.002
- The strain at failure is in the range 0.003 to 0.005
- The curves are linear within the initial portion of the curve. This is approximately true upto one-third of the maximum stress level, beyond which the non-linearity continues
- For higher grades of concrete, the initial portion of the stress-strain curve is steeper, but the failure strain is low. For low strength concrete, the initial slope of curve is gentle but has high failure strain. (observe the figure)



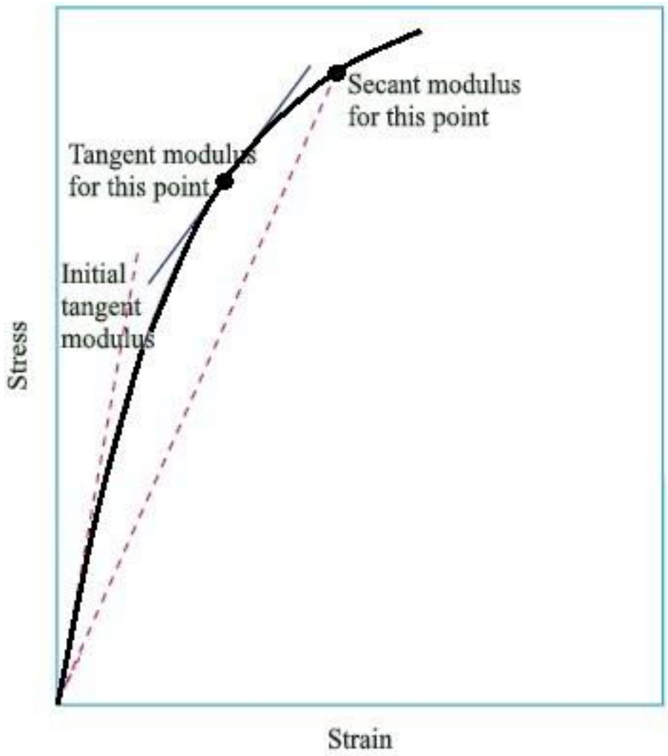
- ***Poisson Effect:*** Failure of concrete subject to uniaxial compression is primarily initiated by longitudinal cracks (cracks developed parallel to direction of loading) formed due to lateral expansion (because lateral fibres experience tensile stress) and finally lateral strain exceeds limiting tensile strain of concrete of 0.0001 to 0.0002. These longitudinal cracks generally occur at coarse aggregate-mortar interface.
- The descending part of Stress-Strain curve is attributed to the extensive microcracking in mortar. This is called *Strain-Softening* of Concrete.

Modulus of Elasticity of concrete

- Young's Modulus of Elasticity (equal to ratio of stress to strain, when the material is loaded within the linearly elastic limit) for concrete subjected to uniaxial compression, has validity only within the initial portion of the Stress-Strain curve. For concrete, there are 3 ways to determine the Modulus of Elasticity. This is shown in figure.
  - *Initial Tangent Modulus* – Slope of tangent at origin of curve; measure of Dynamic Modulus of Elasticity of concrete
  - *Tangent Modulus* – Slope of tangent at any point on the curve
  - *Secant Modulus* – slope of line joining origin & one-third of maximum

stress level; measure of Static Modulus of Elasticity of concrete

- Static Modulus of Elasticity of concrete – is applicable to static system of loads on structures
- Dynamic Modulus of Elasticity of concrete – is applicable when structure is subject to dynamic loads (wind & earthquake loads)



- Secant Modulus at one-third of maximum stress level represents the "Short-term Static Modulus of Elasticity of Concrete (E_c)". "Short-term" means the long term effects of creep & shrinkage are not considered.

- According to IS456,

$$E_c = 5000\sqrt{f_{ck}} \text{ N/mm}^2$$

Where f_{ck} is the 28-day characteristic compressive strength of 150mm concrete cubes. Thus, it should be noted that Modulus of Elasticity of concrete is a function of its strength.

Creep of concrete

Creep is another time dependent deformation of concrete by which it continues to deform, usually under compressive stress. The creep strains recover partly when the stresses are released.

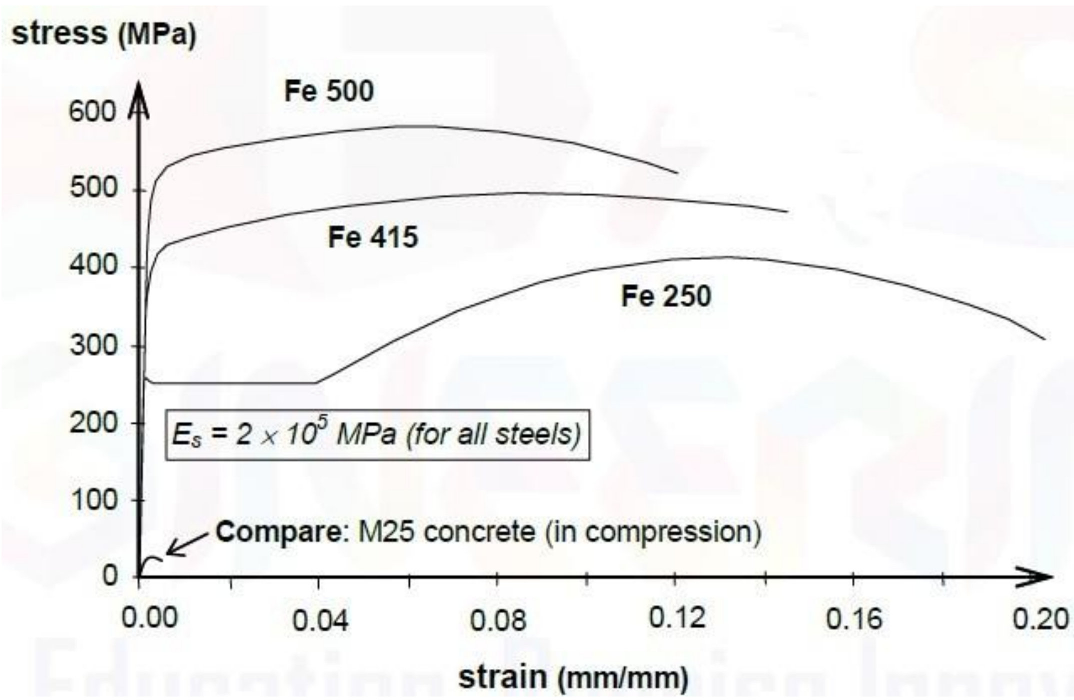
θ=creep co-efficient

$E_{cr} = Ec /(1+ \theta)$

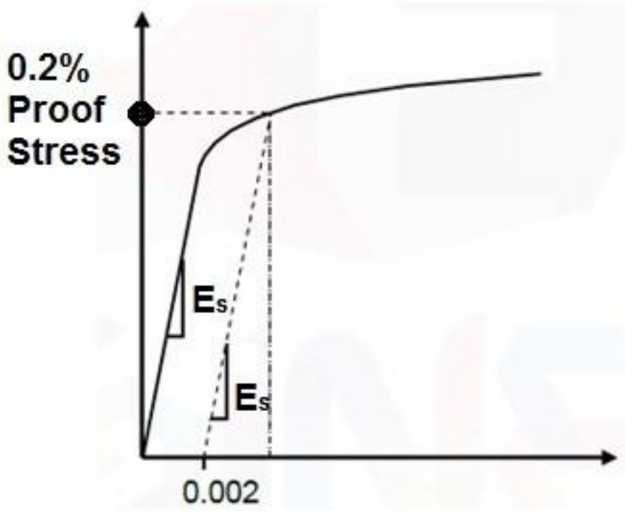
PROPERTIES OF STEEL

Stress-strain curve of reinforcing steel

- Reinforcing steel may be categorized broadly into:
  - Plain Mild steel bars
    - has well-defined yield point
    - Eg: *Fe250* – Yield strength= 250MPa; Ultimate strength= 412MPa; Min % elongation= 22%
  - High Yield Strength Deformed bars
    - Does not have well-defined yield point
    - these are cold-worked bars (involves stretching and twisting of mild steel bars)
    - Eg: *Fe415* – Yield strength= 415MPa; Ultimate strength= 485MPa; Min % elongation= 14.5%
    - Eg: *Fe500* – Yield strength= 500MPa; Ultimate strength= 545MPa; Min % elongation= 12%



- Characteristic strength of reinforcing steel =
    - *yield strength of steel*– for those with well-defined yield point (Fe250)
    - *0.20% Proof Stress* – for those without well-defined yield point (Fe415&Fe500)
- 0.2% Proof Stress is measured as shown below



Method of RCC design

A reinforced concrete structure should be designed to satisfy the following criteria-

i) Adequate safety, in items stiffness and durability

iii) Reasonable economy.

The following design methods are used for the design of RCC Structures.

a) The working stress method (WSM)

b) The ultimate load method (ULM)

c) The limit state method (LSM)

Working Stress Method (WSM)

This method is based on linear elastic theory or the classical elastic theory. This method ensured adequate safety by suitably restricting the stress in the materials (i.e. concrete and steel) induced by the expected working leads on the structures. The assumption of linear elastic behaviour considered justifiable since the specified permissible stresses are kept well below the ultimate strength of the material. The ratio of yield stress of the steel reinforcement or the cube strength of the concrete to the corresponding permissible or working stress is usually called factor of safety.

The WSM uses a factor of safety of about 3 with respect to the cube strength of concrete and a factor of safety of about 1.8 with respect to the yield strength of steel.

Ultimate state method (USM)

The method is based on the ultimate strength of reinforced concrete at ultimate load is obtained by enhancing the service load by some factor called as load factor for giving a desired margin of safety .Hence the method is also referred to as the load factor method or the ultimate strength method.

In the ULM, stress condition at the state of in pending collapse of the structure is analysed, thus using, the non-linear stress – strain curves of concrete and steel. The safely measure in the design is obtained by the use of proper load factor. The satisfactory strength performance at ultimate loads does not guarantee satisfactory strength performance at ultimate loads does not guarantee satisfactory serviceability performance at normal service loads.

Limit state method (LSM)

Limit states are the acceptable limits for the safety and serviceability requirements of the structure before failure occurs. The design of structures by this method will thus ensure that they will not reach limit states and will not become unfit for the use for which they are intended. It is worth mentioning that structures will not just fail or collapse by violating (exceeding) the limit states. Failure, therefore, implies that clearly defined limit states of structural usefulness has been exceeded.

Limit state are two types

i) Limit state of collapse

ii) Limit state of serviceability.

Limit states of collapse

The limit state of collapse of the structure or part of the structure could be assessed from rupture of one or more critical sections and from bucking due to elastic bending, shear, torsion and axial loads at every section shall not be less than the appropriate value at that section produced by the probable most unfavourable combination of loads on the structure using the appropriate factor of safely.

Limit state of serviceability

Limit state of serviceability deals with deflection and crocking of structures under service loads, durability under working environment during their anticipated exposure conditions resistance during service, stability of structures as a whole, fire etc.

Characteristic and design values and partial safety factor

1. Characteristic strength of materials.

The term characteristic strength‘ means that value of the strength of material below which not more than minimum acceptable percentage of test results are expected to fall. IS 456:2000 have accepted the minimum acceptable percentage as 5% for reinforced concrete structures. This means that there is 5% for probability or chance of the actual strength being less than the characteristic strength.

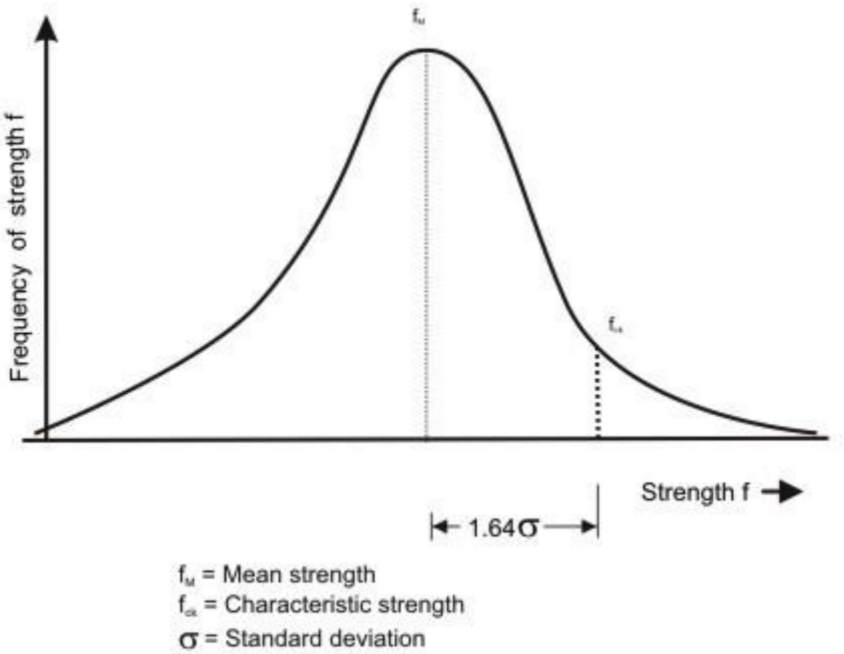


Figure shows frequency distribution curve of strength material (concrete or steel). The value of K corresponding to 5% area of the curve is 1.65.

The design strength should be lower than the mean strength (f_m)

Characteristic strength = Mean strength –K * standard deviation or

$f_k=f_m-KS_d$

Where, f_k=characteristic strength of the material

f_m=mean strength

K=constant =1.65

S_d=standard deviation for a set of test results.

The value of standard deviation (S_d) is given by

$$S_d = \sqrt{\frac{\sum \delta^2}{n-1}}$$

Where, δ=deviation of the individual test strength from the average or mean strength of n samples.

n= number of test results.

IS 456:2000 has recommended minimum value of n=30.

Characteristic strength of concrete

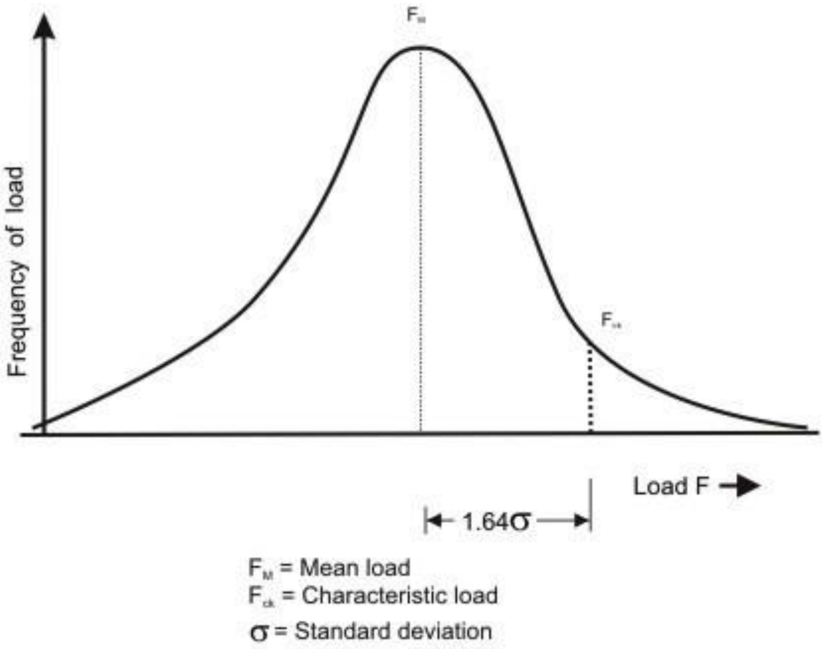
Characteristic strength of concrete is denoted by f_{ck} (N/mm^2) and its value is different for different grades of concrete e.g. M 15, M25 etc. In the symbol M used for designation of concrete mix, refers to the mix and the number refers to the specified characteristic compressive strength of 150 mm size cube at 28 days expressed in N/mm^2

Characteristic strength of steel

Until the relevant Indian Standard specification for reinforcing steel are modified to include the concept of characteristic strength, the characteristic value shall be assumed as the minimum yield stress or 0.2% proof stress specified in the relevant Indian Standard specification. The characteristic strength of steel designated by symbol f_y (N/mm^2)

Characteristic loads

The term Characteristic load means that values of load which has a 95% probability of not being exceeded during that life of the structure.



The design load should be more than average load obtained from statistic, we have

$F_k = F_m + KS_d$

Where, F_k=characteristic load;

F_m= mean load

K=constant=2.65;

Sd=standard deviation for the load.

Since data are not available to express loads in statistical terms, for the purpose of this standard, dead loads given in IS 875(Part-1), imposed loads given in IS 875(Part-2), wind loads. Given in IS 875 (Part-3), snow load as given in IS 875(Part-4) and seismic forces given in IS 1893 shall be assumed as the characteristic loads.

Design strength of materials

The design strength of materials (f_d) is given by

$$f_d = \frac{f_k}{\gamma_m}$$

Design loads

The design load (F_d) is given by.

$F_d = F_k . y_f$

y_f =partial safety factor appropriate to the nature of loading and the limit state being considered.

The design load obtained by multi plying the characteristic load by the partial safety factor for load is also known as factored load.

Partial safety factor (y_m) for materials

When assessing the strength of a structure or structural member for the limit state of collapse, the values of partial safety factor, y_m should be taken as 1.15 for steel.

Thus, in the limit state method , the design stress for steel reinforcement is given by

$f_y / y_{ms} = f_y/1.15=0.87f_y.$

According to IS 456:2000 for design purpose the compressive strength of concrete in the structure shall be assumed to be 0.67 times the characteristic strength of concrete in cube and partial safety factor y_{mc} =1.5 shall be applied in addition to this. Thus, the design stress in concrete is given by

$$0.67\ f_{ck} / y_{mc} = 0.67\ f_{ck} /1.5 = 0.446\ f_{ck}$$

Partial safety factor for loads

The partial safety factors for loads, as per IS 456:2000 are given in table below

Load combination	Limit State of collapse			Limit State of Serviceability		
	DL	LL	WL/EL	DL	LL	WL/EL
DL+IL	1.5	1.5	-	1.0	1.0	-
DL+WL	1.5 or 0.9*	-	1.5	1.0	-	1.0
DL+IL+WL	1.2	1.2	1.2	1.0	0.8	0.8

(* This value is to be considered when stability against overturning or stress reversal is critical)

LIMIT STATE METHOD

- The acceptable limit for safety and serviceability requirements before failure occurs is known as ***Limit State***.
- LSM involves ***underestimation of the material strength*** and ***overestimation of external loads***. For this, the method uses partial safety factor format.

The design of any structure should satisfy the following 2 conditions:

SAFETY

- With due consideration to strength, stability & structural integrity.
- If this condition is satisfied, the likelihood for "collapse" is acceptably low under service loads (usual or expected loads) as well as probable overloads (extreme winds, earthquake etc.)
- Collapse may occur due to:
  - Exceeding of strength of material or load bearing capacity of material.
  - Sliding
  - Overturning
  - Buckling
  - Fatigue
  - Fracture
- Limit states involved in collapse are called "Limit State of Collapse" or "Ultimate Limit State", which are defined for the following,
  - Flexure
  - Compression
  - Shear
  - Torsion

SERVICEABILITY

- Satisfactory performance of structure under service loads. Ensures no discomfort to the user
- If this condition is satisfied, the likelihood for "user discomfort" is acceptably low under service loads.
- User discomfort may occur due to:
  - Deflection

- Cracking
- Vibrations
- Durability
- Impermeability
- Thermal Insulation (or Fire resistance)

- Limit states involved in user comfort are called “Limit state of serviceability”, which are defined for,
  - Deflection
  - Cracking
  - Durability
  - Fire Resistance

Limit state of collapse in flexure

The behaviour of reinforced concrete beam sections at ultimate loads has been explained in detail in previous section. The basic assumptions involved in the analysis at the ultimate limit state of flexure (Cl. 38.1 of the Code) are listed here.

a) Plane sections normal to the beam axis remain plane after bending, i.e., in an initially straight beam, strain varies linearly over the depth of the section.

b) The maximum compressive strain in concrete (at the outermost fibre) E_C shall be taken as 0.0035 in bending.

c) The relationship between the compressive stress distribution in concrete and the strain in

concrete may be assumed to be rectangle, trapezoid, parabola or any other shape which results in prediction of strength in substantial agreement with the results of test. An acceptable stress-strain curve is given below in figure For design purposes, the compressive strength of concrete in the structure shall be assumed to be 0.67 times the characteristic strength. The partial safety factor y_m = 1.5 shall be applied in addition to this.

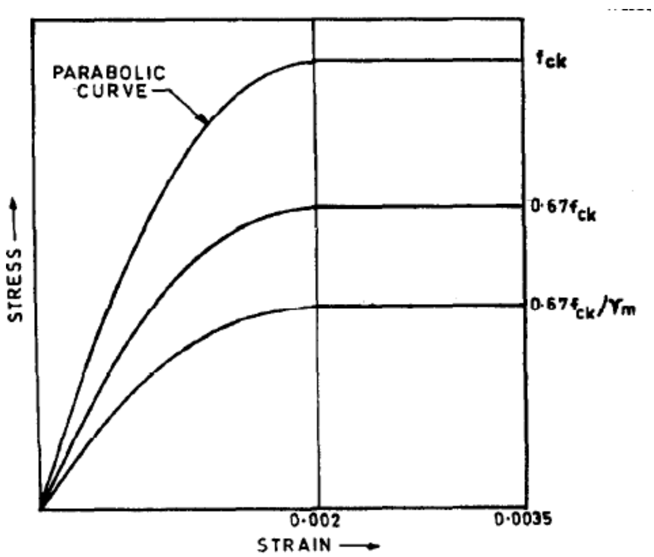


Figure 1.6 Stress-strain curve for concrete

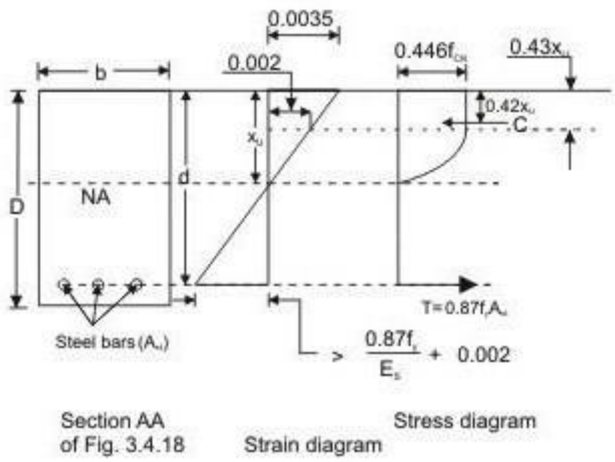
d) The tensile strength of the concrete is ignored.

e) The stresses in the reinforcement are derived from representative stress-strain curve for the type of steel used. Typical curves are given in figure 1.3. For design purposes the partial safety factor y_m equal to 1.15 shall be applied.

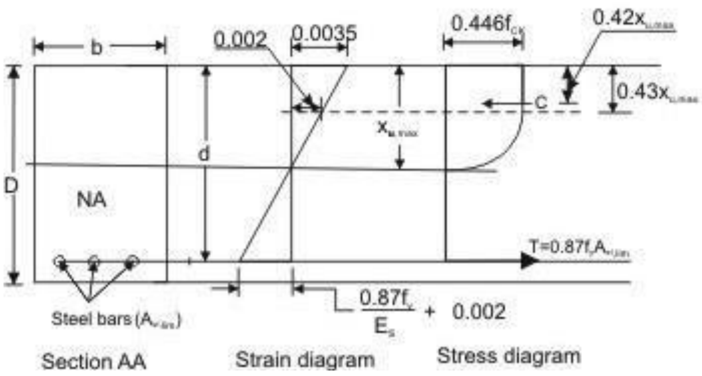
f) The maximum strain in the tension reinforcement in the section at failure shall not be less than:

$$\frac{f_y}{1.15E_s} + 0.002$$

Limiting Depth of Neutral Axis



Rectangular beam under flexure $x_u < x_{u,max}$



Rectangular beam under flexure $x_u = x_{u,max}$

Based on the assumption given above, an expression for the depth of the neutral axis at the ultimate limit state, x_u, can be easily obtained from the strain diagram in Fig. 1.8. Considering similar triangles,

$$\frac{x_u}{d} = \frac{0.0035}{0.0035 + \frac{0.87 f_y}{E_s} + 0.002}$$

According to IS 456:2000 cl no 38.1 (f) ,when the maximum strain in tension reinforcement is equal to $\frac{0.87 f_y}{E_s} + 0.002$, then the value of neutral axis will be $x_{u,max}$.

Therefore, $$\frac{x_{u,max}}{d} = \frac{0.0035}{0.0035 + \frac{0.87 f_y}{E_s} + 0.002} \quad (2)$$

The values of $x_{u,max}$ for different grades of steel, obtained by applying Eq. (2), are listed in table.

Table 1 Limiting depth of neutral axis for different grades of steel

Steel Grade	Fe 250	Fe 415	Fe 500
$x_{u,max} / d$	0.5313	0.4791	0.4791

The limiting depth of neutral axis $x_{u,max}$ corresponds to the so-called balanced section, i.e., a section that is expected to result in a 'balanced' failure at the ultimate limit state in flexure. If the neutral axis depth x_u is less than $x_{u,max}$, then the section is under-reinforced (resulting in a 'tension' failure); whereas if x_u exceeds $x_{u,max}$, it is over-reinforced (resulting in a 'compression' failure).

ANALYSIS OF SINGLY REINFORCED RECTANGULAR SECTIONS

- The Concrete Stress block (Compressive stress distribution in concrete at ultimate limit state) is analysed as follows:

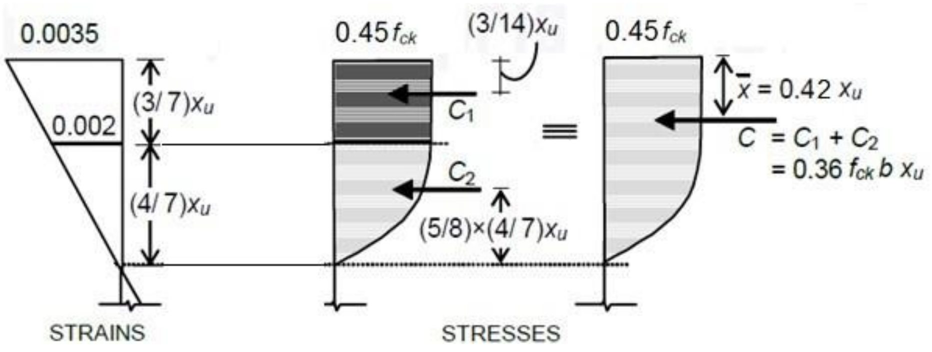


Fig. 1.9 Concrete stress-block parameters in compression

In order to determine the magnitude of *Cu* and its line of action, it is necessary to analyse the *concrete stress block* in compression. As ultimate failure of a reinforced concrete beam in flexure occurs by the crushing of concrete, for both under- and over-reinforced beams, the shape of the compressive stress distribution (stress block‘) at failure will be, in both cases, as shown in Fig. 1.9. The value of *Cu* can be computed knowing that the compressive stress in concrete is uniform at 0.447 f_{ck} for a depth of $3x_u$ / 7, and below this it varies parabolically over a depth of $4x_u$ / 7 to zero at the neutral axis

$$C_u = 0.447 f_{ck}\, \mathrm{b}\left[\frac{3x_u}{7} + \left(\frac{2}{3} x \frac{4x_u}{7}\right)\right]$$

$$\text{Therefore, } C_u = 0.361 f_{ck} b x_u$$

Also, the line of action of C_u is determined by the centroid of the stress block, located at a distance from the concrete fibres subjected to the maximum compressive strain. Accordingly, considering moments of compressive forces C_u, C_1 and C_2 about the maximum compressive strain location

$$(0.362 f_{ck} b x_u) \,\mathrm{x}\, \bar{x} = (0.447 f_{ck} b x_u)\left[\left(\frac{3}{7}\right)\left(\frac{1.5x_u}{7}\right) + \left(\frac{2}{3}\mathrm{x}\frac{4}{7}\right)\left(x_u - \frac{5}{8}\mathrm{x}\frac{4x_u}{7}\right)\right]$$

$$\text{Solving } \bar{x} = 0.416 x_u$$

Depth of Neutral Axis

- Compressive force C = 0.36 f_{ck} b x_u
- Tensile force T = 0.87 f_y A_{st}
- Lever arm (i.e., the perpendicular distance between line of action of compressive force and tensile force) z = d – 0.42 x_u
- For any given section, since the forces are in equilibrium, C = T

  Therefore, depth of neutral axis is obtained as

$$\Rightarrow \quad x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} \qquad \textbf{(valid only if } X_u \le X_{u,max}\textbf{)}$$

ULTIMATE MOMENT OF RESISTANCE

Accordingly, in terms of the concrete compressive strength,

$$M_{uR} = 0.361 f_{ck} b x_u (\mathrm{d} - 0.416\,\mathrm{x}_u) \quad \textit{for all } x_u$$

Alternatively, in terms of the steel tensile stress,

$$M_{uR} = f_{st}\,\mathrm{A}_{st} (\mathrm{d} - 0.416\,\mathrm{x}_u) \quad \textit{for all } x_u$$

With $f_{st}=0.87f_y$ for $x_u \le x_{u,\max}$

LIMITING MOMENT OF RESISTANCE

$$M_{u,\lim} = 0.361 f_{ck} b x_{u,\max} (\mathrm{d} - 0.416\,\mathrm{x}_{u,\max})$$

$$M_{u,\lim} = 0.361 f_{ck} \left(\frac{x_{u,\max}}{d}\right)\left(1 - \frac{0.416\,\mathrm{x}_{u,\max}}{d}\right) b d^2$$

Modes of failure: Types of section

A reinforced concrete member is considered to have failed when the strain of concrete in extreme compression fibre reaches its ultimate value of 0.0035. At this stage, the actual strain in steel can have the following values:

In balanced section,

The strain in steel and strain in concrete reach their maximum values simultaneously. The percentage of steel in this section is known as critical or limiting steel percentage. The depth of neutral axis (NA) is $X_u = X_{u, max}$

Under-reinforced section

An under-reinforced section is the one in which steel percentage (pt) is less than critical or limiting percentage (*pt*,lim). Due to this the actual NA is above the balanced NA and

$X_u < X_{umax}$

Over-reinforced section

In the over reinforced section the steel percentage is more than limiting percentage due to which NA falls below the balanced NA and $X_u > X_{u,max}$ . . Because of higher percentage of steel, yield does not take place in steel and failure occurs when the strain in extreme fibres in concrete reaches its ultimate value.

Design Type of Problems

The designer has to make preliminary plan lay out including location of the beam, its span and spacing, estimate the imposed and other loads from the given functional requirement of the structure. The dead loads of the beam are estimated assuming the dimensions *b* and *d* initially. The bending moment, shear force and axial thrust are determined after estimating the different loads. In this illustrative problem, let us assume that the imposed and other loads are given. Therefore, the problem is such that the designer has to start with some initial dimensions and subsequently revise them, if needed. The following guidelines are helpful to assume the design parameters initially.

(i) **Selection of breadth of the beam *b***

Normally, the breadth of the beam *b* is governed by: (i) proper housing of reinforcing bars and (ii) architectural considerations. It is desirable that the width of the beam should be less than or equal to the width of its supporting structure like column width, or width of the wall etc. Practical aspects should also be kept in mind. It has been found that most of the requirements are satisfied with *b* as 150, 200, 230, 250 and 300 mm. Again, width to overall depth ratio is normally kept between 0.5 and 0.67.

(ii) **Selection of depths of the beam *d* and *D***

The effective depth has the major role to play in satisfying (i) the strength requirements of bending moment and shear force, and (ii) deflection of the beam. The initial effective depth of the beam, however, is assumed to satisfy the deflection requirement depending on the span and type of the reinforcement. IS 456 stipulates the basic ratios of span to effective depth of beams for span up to 10 m as (Clause 23.2.1)

Cantilever 7

Simply supported 20

Continuous 26

For spans above 10 m, the above values may be multiplied with 10/span in metres, except for cantilevers where the deflection calculations should be made. Further, these ratios are to be multiplied with the modification factor depending on reinforcement percentage and type. Figures 4 and 5 of IS 456 give the different values of modification factors. The total depth *D* can be determined by adding 40 to 80 mm to the effective depth.

(iii) **Selection of the amount of steel reinforcement A_{st}**

The amount of steel reinforcement should provide the required tensile force *T* to resist the factored moment M_u of the beam. Further, it should satisfy the minimum and maximum percentages of reinforcement requirements also. The minimum reinforcement A_s is provided for creep, shrinkage, thermal and other environmental requirements irrespective of the strength requirement. The minimum reinforcement A_s to be provided in a beam depends on the f_y of steel and it follows the relation: (cl. 26.5.1.1a of IS 456)

$$\frac{A_s}{bd} = \frac{0.85}{f_y}$$

The maximum tension reinforcement should not exceed 0.04 *bD* (cl. 26.5.1.1b of IS 456), where *D* is the total depth.

Besides satisfying the minimum and maximum reinforcement, the amount of reinforcement of the singly reinforced beam should normally be 75 to 80% of $P_{t,lim}$. Moreover, in many cases, the depth required for deflection becomes more than the limiting depth required to resist *Mu*, *lim*. Thus, it is almost obligatory to provide more depth. Providing more depth also helps in the amount of the steel which is less than that required for *Mu*, *lim*. This helps to ensure ductile failure. Such beams are designated as under-reinforced beams.

(iv) **Selection of diameters of bar of tension reinforcement**

Reinforcement bars are available in different diameters such as 6, 8, 10, 12, 14, 16, 18, 20, 22, 25, 28, 30, 32, 36 and 40 mm. Some of these bars are less available. The selection of the diameter of bars depends on its availability, minimum stiffness to resist while persons walk over them during construction, bond requirement etc. Normally, the diameters of main tensile bars are chosen from 12, 16, 20, 22, 25 and 32 mm.

(v) **Selection of grade of concrete**

Besides strength and deflection, durability is a major factor to decide on the grade of concrete. Table 5 of IS 456 recommends M 20 as the minimum grade under mild environmental exposure and other grades of concrete under different environmental exposures also.

(vi) Selection of grade of steel

Normally, Fe 250, 415 and 500 are in used in reinforced concrete work. Mild steel (Fe 250) is more ductile and is preferred for structures in earthquake zones or where there are possibilities of vibration, impact, blast etc.

DESIGN FOR SHEAR

STRESSES IN HOMOGENOUS RECTANGULAR BEAMS

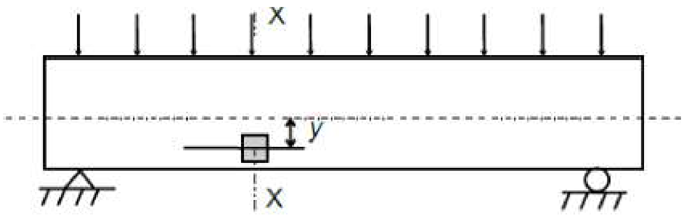
From basic mechanics of materials, it is known that the flexural (bending) stress f_x and the shear stress T at any point in the section, located at a distance y from the neutral axis, are given by:

$$f_x = \frac{M\,y}{I}$$

$$\tau = \frac{VQ}{Ib}$$

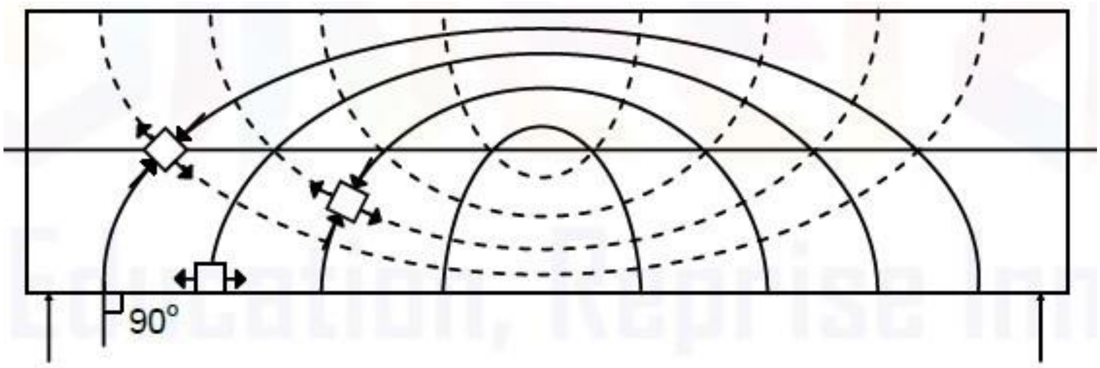
where *I* is the second moment of area of the section about the neutral axis, *Q* the first moment of area about the Neutral Axis of the portion of the section above the layer at distance *y* from the NA, and *b* is the width of the beam at the layer at which T is calculated.

Consider an element at a distance *y* from the Neutral Axis (NA).

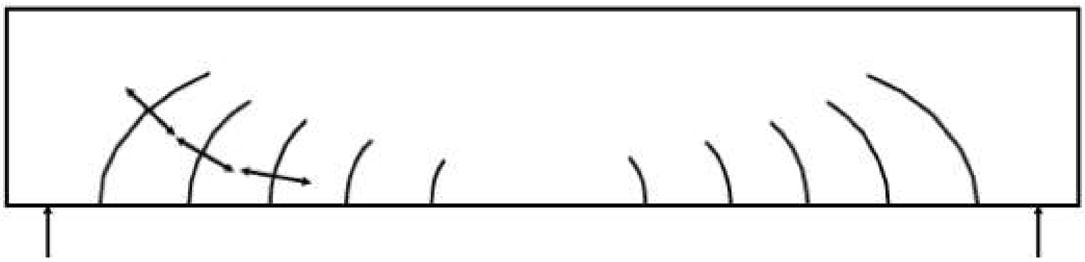


The combined flexural and shear stresses on that element can be resolved into equivalent *principal stresses* f_1 and f_2 acting on orthogonal planes.

As a result, the stress on the beam is depicted in terms of the *principal stress trajectories* as shown.



In a material like concrete which is weak in tension, tensile cracks would develop in a direction that is perpendicular to that of the principal tensile stress. Thus the compressive stress trajectories in the above figure indicate *potential* crack patterns, as shown below.

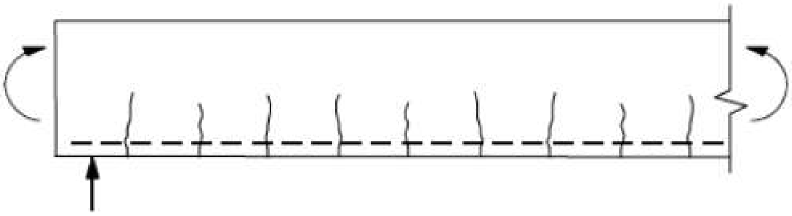


Potential crack patterns

MODES OF CRACKING

1) **Flexural cracks**

- Occurs in reinforced concrete beams of usual proportions, subjected to relatively high flexural stresses f_x and low shear stresses T.
- Maximum principal tensile stress occurs in the outer fibre at the bottom face of the concrete beam at the peak moment locations. As a result, cracks are formed, which are termed as *flexural cracks.*
- These are formed at 90^o from the extreme tension fibre towards neutral axis.
- These are controlled by the tension bars.

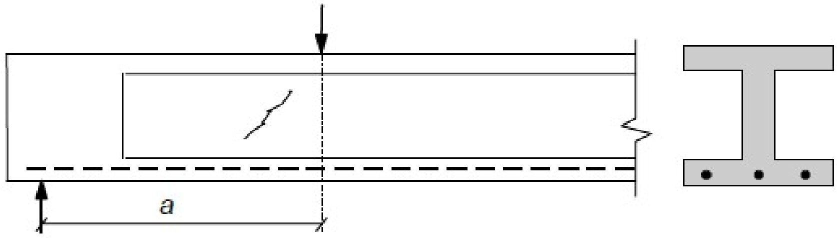


2) **Diagonal Tension Cracks / Web shear cracks**

Occurs in beams which are subjected to high shear stresses T (due to heavy concentrated loads) and relatively low flexural stresses f_x (such as, short–span beams which are relatively deep and have thin webs).

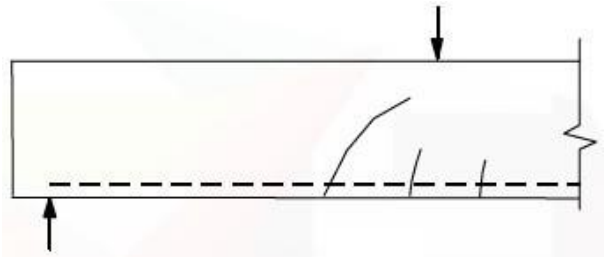
It is likely that the maximum principal tensile stress is located at the neutral axis level at an inclination $\alpha = 45^o$ (to the longitudinal axis of the beam)

- Cracks occur near the supports (where shear force is generally maximum) near neutral axis and inclined at 45^o to the longitudinal axis of the beam. These are termed as *web shear cracks* or *diagonal tension cracks*.
- These can be resisted by providing shear reinforcements or stirrups.



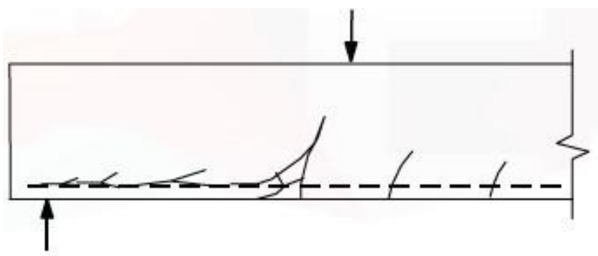
3) Flexure-Shear cracks

- When a ‘flexural crack’ occurs in combination with a ‘diagonal tension crack’, the crack is termed as a *flexure-shear crack*.
- Occurs in beam subjected to both flexure and shear.
- ***Note:*** The presence of shear stress reduces the *strength* of concrete in compression as well as tension. Accordingly, the tensile strength of the concrete in a reinforced concrete beam subjected to both flexure & shear will be less than that subjected to flexure only.
- Here, flexural crack usually forms first, and extends into a diagonal tension crack.



4) Secondary cracks / Splitting cracks

- The Flexure-Shear cracks may sometimes propagate along the tension reinforcement towards the support. These are referred to as *secondary cracks* or *splitting cracks*.
- These are attributed to
  - the *wedging action* of the tension bar deformations. As the tension reinforcement
  - the tension bars serve as dowels across the Flexure-Shear cracks. As the beam segments on either sides of the crack displaces, secondary cracks may propagate along tension bars. This is known as *dowel action*.



SHEAR PARAMETERS FOR DESIGN

1) Nominal Shear Stress

- For ***prismatic* members** of rectangular (or flanged) sections, the Code (Cl. 40.1) uses the term *nominal shear stress* T*v*, defined at the ultimate limit state, as

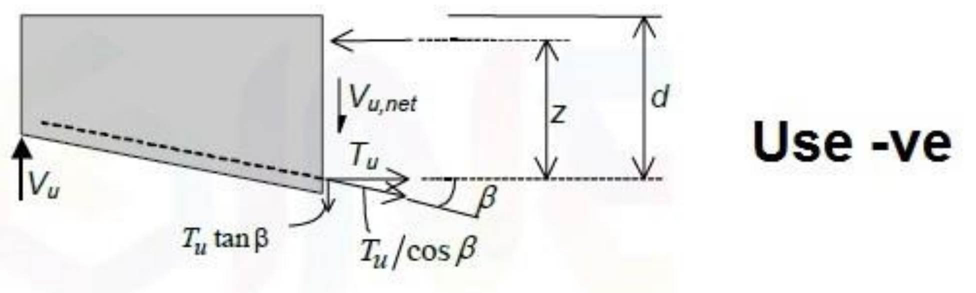
$$\tau_v = \frac{V_u}{bd}$$

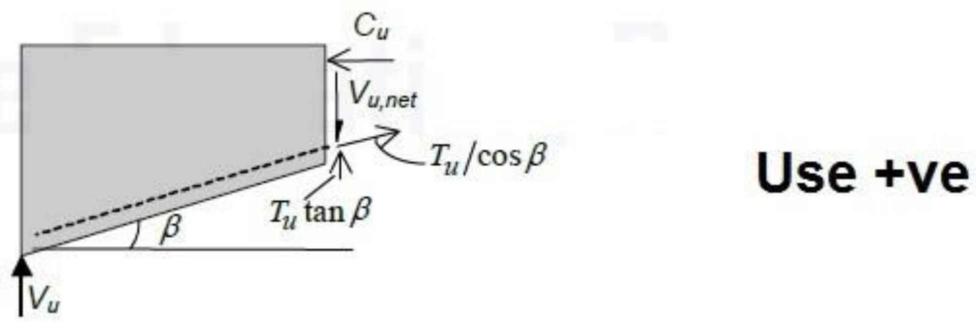
where V_u is the *factored* shear force at the section under consideration, b is the width of the beam (taken as the web width b_w in flanged beams), and d the effective depth of the section.

- In the case of **members with varying depth**, the nominal shear stress, defined above, needs to be modified, to account for the contribution of the vertical component of the flexural tensile force T_u which is inclined at an angle β to the longitudinal direction.
Accordingly, the *nominal shear stress* (Cl. 40.1.1 of the Code), is obtained as

$$\tau_v = \frac{V_u \pm \frac{M_u}{d}\tan\beta}{bd}$$

where V_u and M_u are the applied factored shear force and bending moment at the section under consideration. The negative sign applies where M_u *increases* in the same direction as the depth increases and the positive sign applies where M_u decreases in this direction, as shown below.

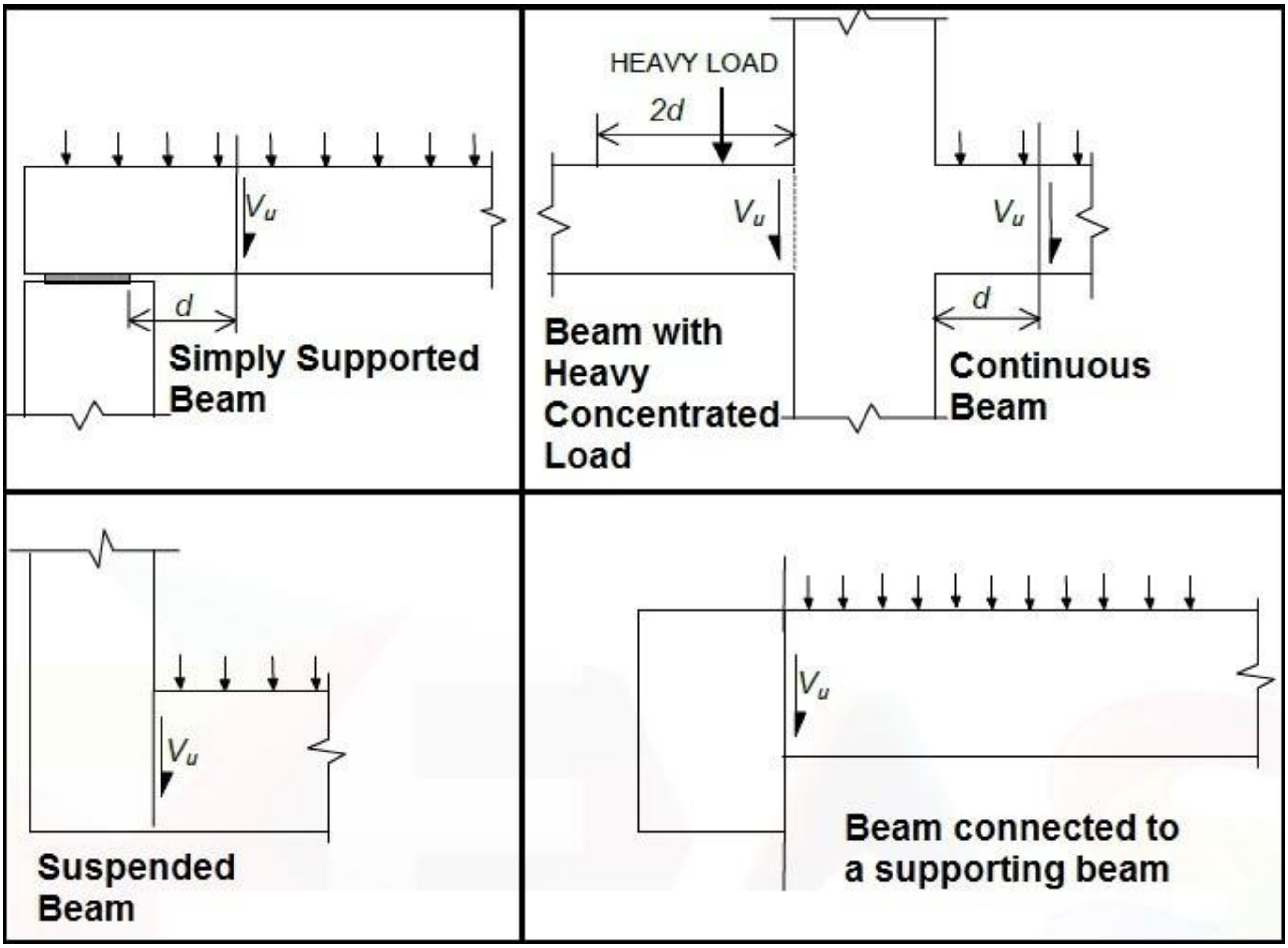




2) Critical Sections for shear

Critical sections are those sections at which shear force is maximum.

Location of critical sections for different cases are shown below. [Refer Cl. 22.6.2]



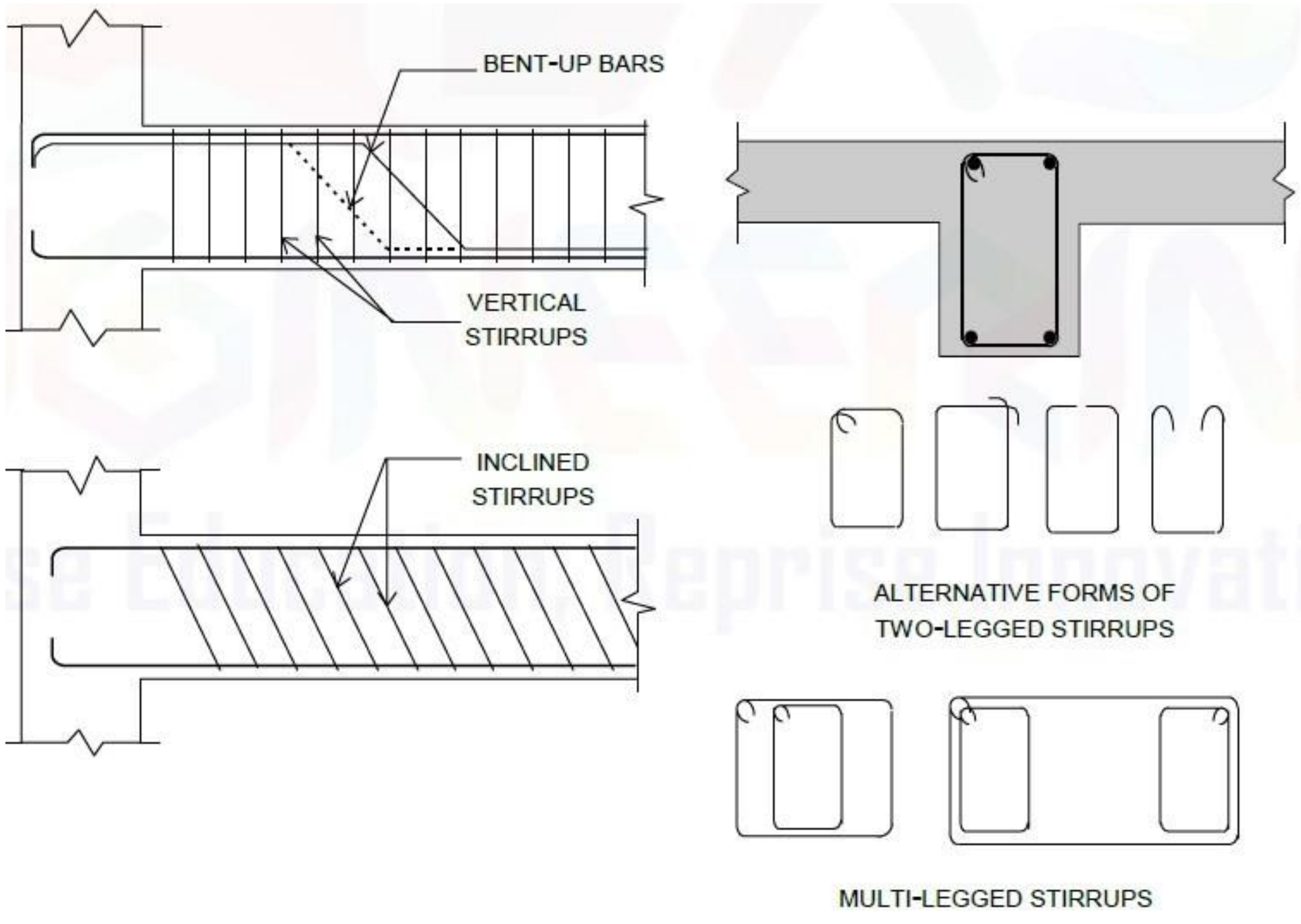
3) Design Shear Strength of Concrete in Beams (Tc)

- It is the average *shear strength* of concrete in reinforced concrete beams without shear reinforcement. It is the stress that corresponds to the load at which the first inclined crack develops.
- If the shear stress in beam Tv is less than Tc, shear reinforcements are not to be designed (only minimum shear reinforcements shall be provided). But if Tv > Tc, shear reinforcements shall be designed.
- Therefore, Tc is the safe limiting value below which the beam is safe even without shear reinforcement.
- Tc depends on grade of concrete (f_{ck}) and the percentage tension steel $p_t = 100A_{st}/(bd)$. The values of Tc are given in the Code (Refer Table 19).

- Shear strength of slabs is higher than that of beams, owing to small thickness. The thinner the slab, the greater is the increase in shear strength. The Code (Cl. 40.2.1.1) suggests an increased shear strength for slabs, equal to *k* Tc, where the multiplication factor *k* ranges between 1 and 1.3. **In general, slabs subjected to normal distributed loads satisfy the requirement Tv < Tc, and hence do not need shear reinforcement.**

4) Types of Shear Reinforcement

- *Shear reinforcement*, also known as *web reinforcement* may consist of any one of the following systems (Cl. 40.4 of the Code)
  a) stirrups perpendicular to the beam axis;
  b) stirrups inclined (at 45° or more) to the beam axis; and
  c) longitudinal bars bent-up (usually, not more than two at a time) at 45° to 60° to the beam axis, combined with stirrups.
- By far, the most common type of shear reinforcement is the *two-legged stirrup*, comprising a closed or open loop, with its ends anchored properly around longitudinal bars/stirrup holders (to develop the yield strength in tension). It is placed perpendicular to the member axis ('*vertical stirrup*'), and may or may not be combined with bent-up bars.
- Where *bent-up bars* are provided, their contribution towards shear resistance shall not be more than half that of the total shear reinforcement.



5) Limiting Ultimate Shear Strength of beam ($T_{c,max}$)

- The nominal shear stress (Tv) on the beam should not exceed the *limiting total shear strength of beam including shear reinforcement* ($T_{c,max}$).
- Such a limit is set to the shear stress in beam Tv because : if the shear reinforcement Provided in the section is excessive, failure may occur by crushing of concrete ***(known as shear-compression failure which occurs due to crushing of the reduced concrete section after formation of flexure-shear crack)***, even before yielding of shear reinforcements. Since this is a brittle fracture, such a failure is undesirable.

- Thus by limiting the shear stress in beam Tv to less than $T_{c,max}$, shear-compression failures can be prevented.
- Values of $T_{c,max}$ is given in Table 20 of IS456. It may also be obtained from the following approximate relation.

$$T_{c,max} \approx 0.62 \sqrt{f_{ck}}$$

- In the case of solid slabs, the Code (Cl. 40.2.3.1) specifies that Tv should not exceed 0.5 $T_{c,max}$.

6) Design of shear reinforcement

If Tv > Tc

- Design as per Cl. 40.4 of IS456
- Provide shear reinforcements in any of the following forms – vertical stirrups, Inclined Stirrups and Bent-up bars with stirrups
- Shear force to be resisted by stirrups $V_{us} = V_u - T_c\, b\, d$
- If vertical stirrups are used, center-to-center spacing of the stirrups along the length of the member, *Sv* is determined from:

$$V_{us} = 0.87 f_y A_{sv}\, d / s_v$$

Where A_{sv} is the cross-sectional area of stirrup legs or bent-up bars.

- For vertical stirrups, the maximum spacing between stirrups is limited as follows:

$$s_v \le \begin{cases} 0.75d \\ 300 \text{ mm} \end{cases} \text{ whichever is less}$$

- For inclined stirrups or a series of bars bent-up at different cross-sections:

$$V_{us} = \frac{0.87 f_y A_{sv} d}{s_v}(\sin\alpha + \cos\alpha)$$

- For single bar or single group of parallel bars, all bent-up at the same cross-section:

$$V_{us} = 0.87 f_y A_{sv} d \sin\alpha$$

where A_{sv} = total cross-sectional area of stirrup legs or bent-up bars within a distance *sv*,

s_v = spacing of stirrups or bent-up bars along the length of the member,

T_v = nominal shear stress,

T_c = design shear strength of concrete,

b = breadth of the member which for the flanged beams shall be taken as the breadth of the web *bw*,

f_y = characteristic strength of the stirrup or bent-up reinforcement which shall not be taken greater than 415 N/mm2,

α = angle between the inclined stirrup or bent-up bar and the axis of the member, not less than 45o, and

d = effective depth.

If Tv < Tc

- *If Tv < 0.5 Tc*

  No shear reinforcement is required.

- *If Tv > 0.5 Tc*

  The Code (Cl. 26.5.1.6) specifies a minimum shear reinforcement to be provided in the form of stirrups.

$$\frac{A_{sv}}{bs_v} \geq \frac{0.4}{0.87 f_y} \quad \text{when } \tau_v > 0.5\,\tau_c$$

$$\Rightarrow s_v \leq \frac{2.175 f_y A_{sv}}{b}$$

BOND

The bond between steel and concrete is very important and essential so that they can act together without any slip in a loaded structure. With the perfect bond between them, the plane section of a beam remains plane even after bending. The length of a member required to develop the full bond is called the anchorage length. The bond is measured by bond stress. The local bond stress varies along a member with the variation of bending moment.

Thus, a tensile member has to be anchored properly by providing additional length on either side of the point of maximum tension, which is known as Development length in tension. Similarly, for compression members also, we have Development length L_d in compression‘.

Accordingly, IS 456, cl. 26.2 stipulates the requirements of proper anchorage of reinforcement in terms of development length L_d only employing design bond stress τ_{bd}.

Design bond stress – values

The average bond stress is still used in the working stress method and IS 456 has mentioned about it in cl. B-2.1.2. However, in the limit state method of design, the average bond stress has been designated as design bond stress τ_{bd} and the values are given in cl. 26.2.1.1. The same is given below as a ready reference.

Table 5: τ_{bd} for plain bars in tension

Grade of concrete	M 20	M 25	M 30	M 35	M 40 and above
Design Bond Stress τ_{bd} in N/mm2	1.2	1.4	1.5	1.7	1.9

For deformed bars conforming to IS 1786, these values shall be increased by 60 per cent. For bars in compression, the values of bond stress in tension shall be increased by 25 per cent.

Development Length

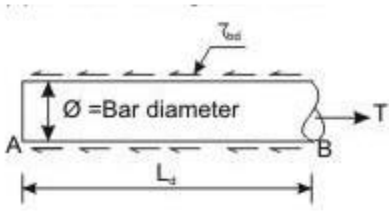


Figure 1.13 Development length of bar

Figure 1.13 shows the free body diagram of the segment AB of the bar. At B, the tensile force *T* is the tensile force trying to pull out the bar. It is necessary to have the resistance force to be developed by τ_{bd} for the length L_d to overcome the tensile force. Equating the two, we get

$$\pi \varphi (L_d) (\tau_{bd}) = (\pi \varphi^2 \sigma_s /4)$$

$$L_d = \frac{\phi \sigma_s}{4 \tau_{bd}}$$

The above equation is given in cl. 26.2.1 of IS 456 to determine the development length of bars. The example taken above considers round bar in tension. Similarly, other sections of the bar should have the required L_d as determined for such sections. For bars in compression, the development length is reduced by 25 per cent as the design bond stress in compression τ_{bd} is 25 per cent more than that in tension. Following the same logic, the development length of deformed bars is reduced by 60 per cent of that needed for the plain round bars. Tables 64 to 66 of SP-16 present the development lengths of fully stressed plain and deformed bars (when $\sigma_s = 0.87 f_y$) both under tension and compression. It is to be noted that the consequence of stress concentration at the lugs of deformed bars has not been taken into consideration.

Checking of Development Lengths of Bars in Tension

The following are the stipulation of cl. 26.2.3.3 of IS 456.

(i) At least one-third of the positive moment reinforcement in simple members and one-fourth of the positive moment reinforcement in continuous members shall be extended along the same face of the member into the support, to a length equal to $L_d/3$.

(ii) Such reinforcements of (i) above shall also be anchored to develop its design stress in tension at the face of the support, when such member is part of the primary lateral load resisting system.

(iii) The diameter of the positive moment reinforcement shall be limited to a diameter such that the L_d computed for $\sigma_s = f_d$ does not exceed the following:

$$(L_d)_{when\, \sigma_s = f_d} \le \frac{M_1}{V} + L_0$$

where M_1 = moment of resistance of the section assuming all reinforcement at the section to be stressed to f_d,

$f_d = 0.87 f_y$,

V = shear force at the section due to design loads,

L_o = sum of the anchorage beyond the centre of the support and the equivalent anchorage value of any hook or mechanical anchorage at simple support. At a point of inflection, L_o is limited to the effective depth of the member or 12θ, whichever is greater, and

θ = diameter of bar.

It has been further stipulated that *M1/V* in the above expression may be increased by 30 per cent when the ends of the reinforcement are confined by a compressive reaction.

Numerical problem of design of singly reinforced beam

A reinforced concrete beam is supported on two walls 250mm thick, spaced at a clear distance of 6m. The beam carries a super-imposed load of 9.8 KN/m. design the beam using M20 concrete and HYSD bars of Fe 415 grade.

SOLUTION

Now from experience, assume d=l/15=400mm

Therefore, overall depth= effective depth+ clear cover + diameter of stirrup +0.5(diameter of main reinforcement)

=400+25+8+0.5x20=443mm 450*mm*

Assume b=250mm

Therefore, try a trial section of dimension 250x450.

Load Calculation

Self-weight of beam (DL)= 0.25x0.45x1x25=2.8125 KN/m

Super-imposed load (LL)= 9.8 KN/m

Therefore, total load, *w* =(DL+LL)=(2.8125+9.8)=12.6125 KN/m

Design load, *wu* =1.5x *w*=18.9187 KN/m

Calculation of effective span

As per IS 456:2000, cl no 22.2 (a), the effective span of a simply supported beam is lesser of the following two.

Clear span+ the effective depth of beam or slab

Or centre to centre distance between supports.

Clear span =6m

Effective depth of beam, d=450-25-8-0.5x20=407mm

Therefore, clear span + effective depth of beam=(6+0.407)m=6.407m

Centre to centre distance between support=(6+0.25/2+0.25/2)m=6.25m

Lesser of two=6.25m

Therefore, effective span =6.25m

Calculation of BM and SF

$$\text{Maximum BM} = \frac{w_u l^2}{8} = \frac{18.9187x6.25^2}{8} = 92.376\,KN - m$$

$$\text{Maximum SF} = \frac{w_u l}{2} = \frac{18.9187x6.25}{2} = 59.12\,KN$$

Computation of effective depth, d

For M20 grade of concrete and Fe 415 grade of steel

$$M_u = 0.138 f_{ck} b d^2$$

$$\text{Therefore, d=}\sqrt{\frac{92.376x10^6}{0.138x20x250}} = 365.89\,mm$$

Now assumed depth was =407mm

Therefore, required< assumed

So, the section assumed is safe from bending moment point of view.

Since the available depth (407mm) is greater than required depth (365.89mm). So the section is under reinforced.

Calculation of steel reinforcement

The reinforcement for an under-reinforced section is given by

$$\text{Ast=}\frac{0.5 f_{ck}}{f_y}\left[1-\sqrt{1-\frac{4.6M_u}{f_{ck}bd^2}}\right]bd = \frac{0.5x20}{415}\left[1-\sqrt{1-\frac{4.6x92.376x10^6}{20x250x407^2}}\right](250x407)$$

A_{st}=740.88mm^2

$$\%\text{pt=}100\frac{740.88}{250x407} = 0.728\% \; < p_{t,\lim}$$

$$\text{Therefore, using 20 mm dia, no of bars required} = \frac{740.88}{\frac{\pi}{4}(20)} = 2.35 \simeq 3.$$

Provide 3nos 20 diameter bar.

Shear Reinforcement

As per IS 456:2000 Cl. No. 22.6.2, the critical section for shear is at a distance of d from the face of the support.

So, shear force at that distance, Vu =59.12-18.9187(0.25/2+0.407) = 49.05 KN.

$$\text{Nominal shear stress, } \tau_v = \frac{V_u}{bd} = \frac{49.05x10^3}{250x407} = 0.482\,N/mm^2$$

$$\%\ p_t \text{ at support} = 100x\frac{3x\frac{\pi}{4}(20)^2}{250x407} = 0.926\%$$

As per IS 456:2000, table 19, the design shear strength of concrete, for %pt=0.926 and M20 grade of concrete, τ_c =0.61 N/mm^2

Since $\tau_V<\tau_C$, no shear reinforcement is necessary. However, minimum shear reinforcement as per cl no 26.5.1.6 of IS 456:2000 should be provided.

$$\frac{A_{sv}}{bS_v} \geq \frac{0.4}{0.87 f_y} \Rightarrow S_v = \frac{2.175 A_{sv} f_y}{b}$$

As per cl no 26.5.1.5 of IS 456:2000, maximum spacing of shear reinforcement least of the following

(a) 0.75d or (b) 300mm

Hence provide 2-8 mm diameter @ 300mm c/c throughout the length of the beam.

Check for Development length

As per cl no 26.2.1 of IS 456:2000, the development length L_d is given by

$$L_d = \frac{\phi \sigma_s}{4 \tau_{bd}}$$

τ_{bd} =1.2 N/mm^2 for M20 grade of concrete.

For deformed bars conforming to Is 1786 these values shall be increased by 60%.

$$L_d = \frac{\phi \sigma_s}{(4\tau_{bd})x1.6} = \frac{20x0.87x415}{4x1.2x1.6} = 940.23\,mm$$

Now as per cl no 26.2.3.3 (c) of IS 456:2000, at a simple support and at points of inflection, positive moment tension reinforcement shall be limited to a diameter such that L_d computed for f_d does not exceed

$$\frac{M_1}{V} + L_0$$

The value of in the above expression M_1/V may be increased by 30% when the ends of the reinforcement are confined by a compressive reaction.

$$L_d \leq 1.3\frac{M_1}{V} + L_0$$

3 bars are available at supports.

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} = \frac{0.87x415x(3x314.15)}{0.36x20x250} = 189\,mm$$

$$\begin{aligned} M_1 &= 0.87 f_y A_{st}(d - 0.416 x_u) \\ &= 0.87x415x(3x314.15)(407 - 0.416x189) \\ &= 111.73\ KN - m \end{aligned}$$

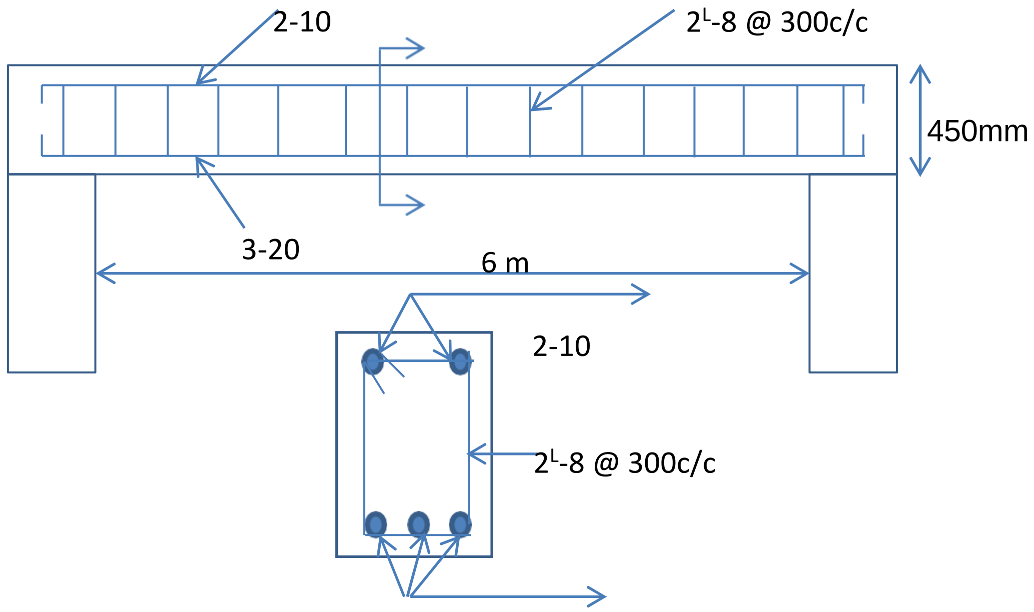
V=59.12 KN

L_0= greatest of effective depth of member or 12ϕ

=407mm

$1.3\frac{M_1}{V}+L_0=2.863m$

Therefore, $L_d \leq 1.3\frac{M_1}{V}+L_0$



Reinforcement Detailing

TORSION

Introduction

This lesson explains the presence of torsional moment along with bending moment and shear in reinforced concrete members with specific examples. The approach of design of such beams has been explained mentioning the critical section to be designed. Expressing the equivalent shear and bending moment, this lesson illustrates the step by step design procedure of beam under combined bending, shear and torsion. The requirements of IS 456 regarding the design are also explained. Numerical problems have been solved to explain the design of beams under combined bending, shear and torsion.

Approach of Design for Combined Bending, Shear and Torsion as per IS 456

As per the stipulations of IS 456, the longitudinal and transverse reinforcements are determined taking into account the combined effects of bending moment, shear force and torsional moment. Two impirical relations of equivalent shear and equivalent bending moment are given. These fictitious shear force and bending moment, designated as equivalent shear and equivalent bending moment, are separate functions of actual shear and torsion, and actual bending moment and torsion, respectively. The total vertical reinforcement is designed to resist the equivalent shear Ve and the longitudinal reinforcement is designed to resist the equivalent bending moment Me_1 and Me_2. These design rules are applicable to beams of solid rectangular cross-section. However, they may be applied to flanged beams by substituting b_w for b. IS 456 further suggests to refer to specialist literature for the flanged beams as the design adopting the code procedure is generally conservative.

Critical Section (cl. 41.2 of IS 456)

As per cl. 41.2 of IS 456, sections located less than a distance d from the face of the support is to be designed for the same torsion as computed at a distance d, where d is the effective depth of the beam.

Shear and Torsion

(a) The equivalent shear, a function of the actual shear and torsional moment is determined from the following impirical relation:

$$Ve = Vu + 1.6(Tu/b)$$

where Ve = equivalent shear,

Vu = actual shear,

Tu = actual torsional moment,

b = breadth of beam.

(b) The equivalent nominal shear stress ve τ is determined from:

$$\tau_{ve} = (V_e / bd)$$

However, τ_{ve} shall not exceed $\tau_{c\,max}$ given in Table 20 of IS 456 and Table 6.2 of Lesson 13.

(c) Minimum shear reinforcement is to be provided as per cl. 26.5.1.6 of IS 456, if the equivalent nominal shear stress τ_{ve} obtained from Eq does not exceed τ_c given in Table 19 of IS 456

(d) Both longitudinal and transverse reinforcement shall be provided as per cl. 41.4 if τ_{ve} exceeds τ_c given in Table 19 of IS 456 and is less than $\tau_{c\,max}$, as mentioned in (b) above.

Reinforcement in Members subjected to Torsion

(a) Reinforcement for torsion shall consist of longitudinal and transverse reinforcement

(b) The longitudinal flexural tension reinforcement shall be determined to resist an equivalent bending moment Me_1 as given below:

Me1 = Mu + Mt

where Mu = bending moment at the cross-section, and

Mt = (Tu/1.7) {1 + (D/b)}

where Tu = torsional moment,

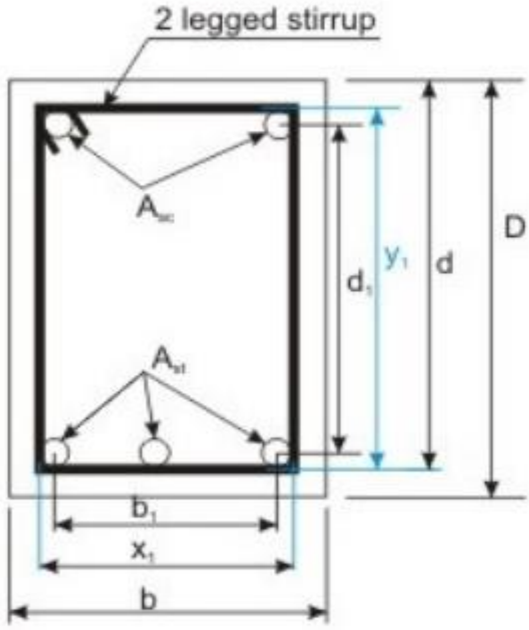
D = overall depth of the beam, and

b = breadth of the beam.

(c) The longitudinal flexural compression reinforcement shall be provided if the numerical value of Mt as defined above exceeds the numerical value of Mu. Such compression reinforcement should be able to resist an equivalent bending moment Me2 as given below:

Me_2 = Mt - Mu

The Me_2 will be considered as acting in the opposite sense to the moment Mu.



(d) The transverse reinforcement consisting of two legged closed loops enclosing the corner longitudinal bars shall be provided having an area of cross-section A_{sv} given below:

$$A_{sv} = \frac{T_u \, s_v}{b_1 \, d_1 (0.87 f_y)} + \frac{V_u \, s_v}{2.5 \, d_1 (0.87 f_y)}$$

However, the total transverse reinforcement shall not be less than the following:

$$A_{sv} \geq (\tau_{ve} - \tau_c) \, b \, s_v / (0.87 f_y)$$

where Tu = torsional moment,

Vu = shear force,

Sv = spacing of the stirrup reinforcement,

b_1 = centre to centre distance between corner bars in the direction of the width,

d_1 = centre to centre distance between corner bars,

b = breadth of the member,

fy = characteristic strength of the stirrup reinforcement,

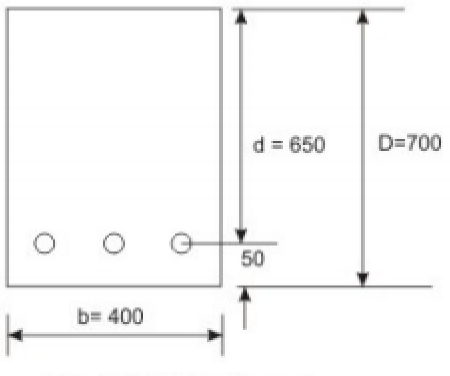
τ_{ve} = equivalent shear stress

and τ_c = shear strength of concrete as per Table 19 of IS 456.

Distribution of torsion reinforcement (cl. 26.5.1.7 of IS 456)

The transverse reinforcement shall consist of rectangular close stirrups placed perpendicular to the axis of the member. The spacing of stirrups shall not be more than the least of x_1, $(x_1 + y_1)/4$ and 300 mm, where x_1 and y_1 are the short and long dimensions of the stirrups Longitudinal reinforcements should be placed as close as possible to the corners of the cross-section.

Problem 1



Determine the reinforcement required of a ring beam of b = 400 mm, d = 650 mm, D = 700 mm and subjected to factored Mu = 200 kNm, factored Tu = 50 kNm and factored Vu = 100 kN. Use M 20 and Fe 415 for the design.

Solution 1

The solution of the problem is illustrated in seven steps below.

Step 1: Check for the depth of the beam

we have the equivalent shear

Ve = Vu + 1.6(Tu/b) = 100 + 1.6(50/0.4) = 300 kN

the equivalent shear stress

$$\tau_{ve} = (V_e / bd) = 300/(0.4)(0.65) = 1.154 \text{ N/mm}^2$$

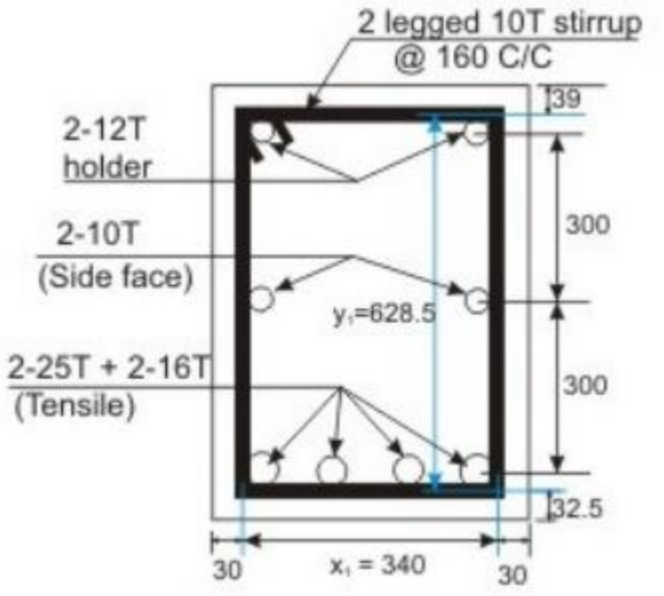
From (Table 20 of IS 456), $\tau_{c\,max} = 2.8$ N/mm^2 .

Hence, the section does not need any revision.

Step 2: Check if shear reinforcement shall be required.

Assuming percentage of tensile steel as 0.5, Table 19 of IS 456 gives c $\tau = 0.48$ N/mm^2 $< \tau_{ve} < \tau_{c\,max}$. So, both longitudinal and transverse reinforcement shall be required.

Step 3: Longitudinal tension reinforcement



: Longitudinal (tension & side face) reinforcement of Prob. 1

$$M_{e1} = M_u + M_t = M_u + (T_u/1.7)\{1 + (D/b)\}$$

$$= 200 + (50/1.7)\{1 + (700/400)\} = 200 + 80.88 = 280.88 \text{ kNm}$$

$$M_{e1}/bd^2 = (280.88)(10^6)/(400)(650)(650) = 1.66 \text{ N/mm}^2$$

From Table 2 of SP-16, corresponding to $Mu/bd^2 = 1.66$ N/mm^2, we have by linear interpolation pt = 0.5156.

So, $A_{st} = 0.5156(400)(650)/100 = 1340.56$ mm^2 .

Provide 2-25T and 2-16T = 981 + 402 = 1383 mm^2. This gives percentage of tensile reinforcement = 0.532, for which τ_c is 0.488 N/mm^2

Minimum percentage of tension reinforcement = (0.85/fy)(100) = 0.205 and the maximum percentage of tension reinforcement is 4.0. So, 2-25T and 2-16T bars satisfy the requirements

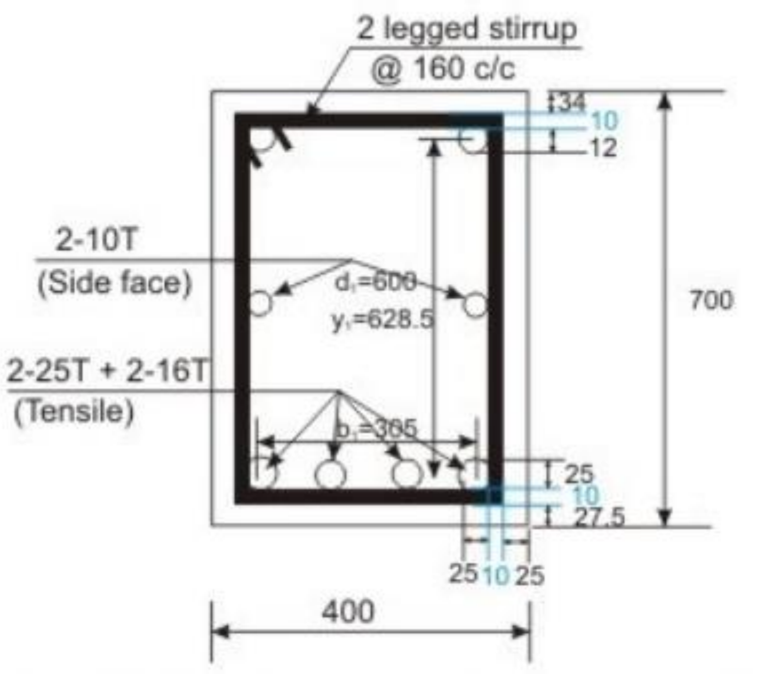
Step 4: Longitudinal compression reinforcement

Here, in this problem, the numerical value of Mt (= 80.88 kNm) is less than that of Mu (200 kNm). So, longitudinal compression reinforcement shall not be required.

Step 5: Longitudinal side face reinforcement

Side face reinforcement shall be provided as the depth of the beam exceeds 450 mm. Providing 2-10 mm diameter bars (area = 157 mm^2) at the mid-depth of the beam and one on each face the total area required, 0.1(400)(300)/100 = 120 mm^2 < 157 mm^2 . Hence o.k.

Step 6: Transverse reinforcement



Transverse reinforcement of Prob.1

Providing two legged, 10 mm diameter stirrups (area = 157 mm^2), we have

$d_1 = 700 - 50 - 50 = 600$ mm

$b_1 = 400 - 2(25 + 10 + 12.5) = 305$ mm

$$0.87 f_y \; A_{sv}/s_v \;=\; (T_u / b_1 \, d_1) \;+\; (V_u / 2.5 \, d_1)$$

Using the numerical values of Tu, b1, d1 and Vu,

$$0.87 f_y \; A_{sv}/s_v \;=\; 339.89 \text{ N/mm}$$

$$0.87 f_y \; A_{sv}/s_v \;\geq\; (1.154 - 0.48)\,400 \;\geq\; 269.6 \text{ N/mm}$$

we get for 2 legged 10 mm stirrups (A_{sv} = 157 mm2),

Sv = 0.87(415)(157)/339.89 = 166.77 mm

Step 7: Check for Sv

Figure 6.16.4 shows the two legged 10 mm diameter stirrups for which x_1 = 340 mm and y_1 = 628.5 mm. The maximum spacing Sv should be the least of x_1, $(x_1 + y_1)/4$ and 300 mm Here, x_1 = 340 mm, $(x_1 + y_1)/4$ = 242.12 mm. So, provide 2 legged 10 mm T stirrups @ 160 mm c/c.