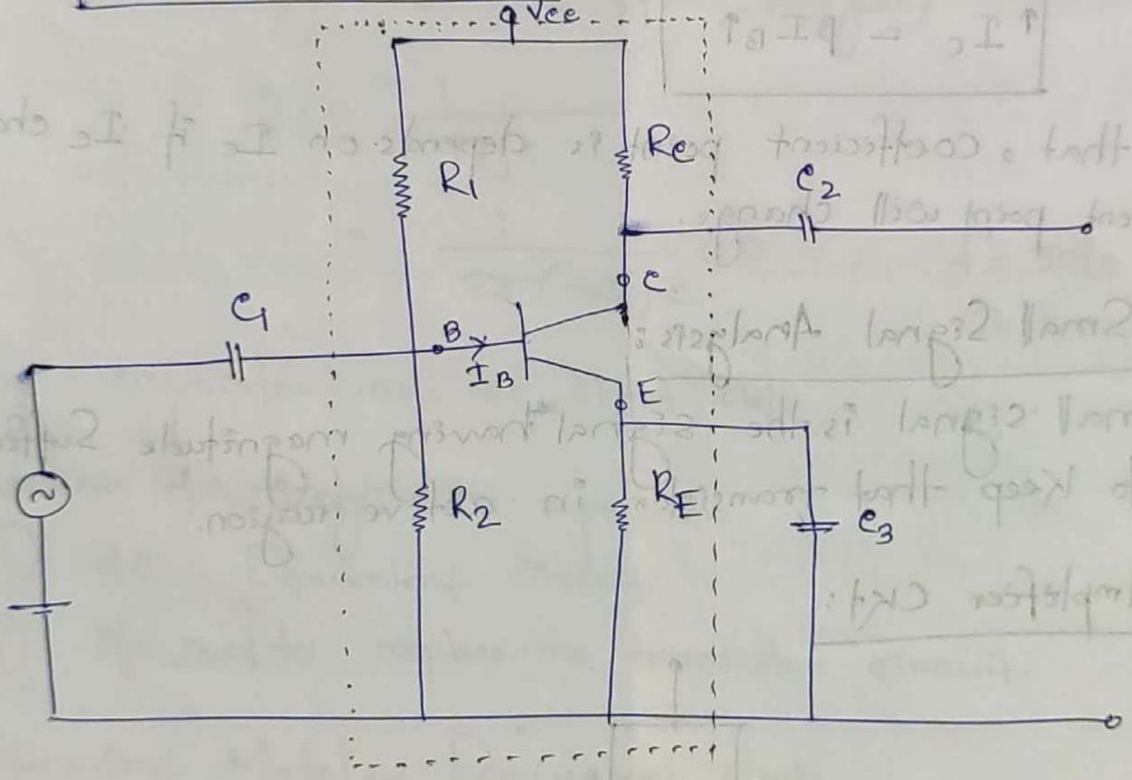
Small Signal Analysis

Law frequency Analysis or Small signal model
High frequency model or Power Amplifier

Total Response of Transistor = dc Analysis + Ac Analysis

$$\boxed{X_C = \frac{1}{2\pi f c}}$$



Voltage divider CKt

Ac Response of transistor:

Ac response of the transistor is represented in two signal analysis that is

(i) Small signal Analysis or low frequency Analysis
(ii) High frequency Analysis or power Amplifier.

Small Signal Analysis (low frequency Analysis)

In low frequency analysis or small signal analysis I_B will be small I_C will not change much. So no change in Q. (operating point).

High frequency Analysis or large signal Analysis or Power Amplifier.

In large signal or High frequency analysis the base current I_B will increase so I_C will increases, because,

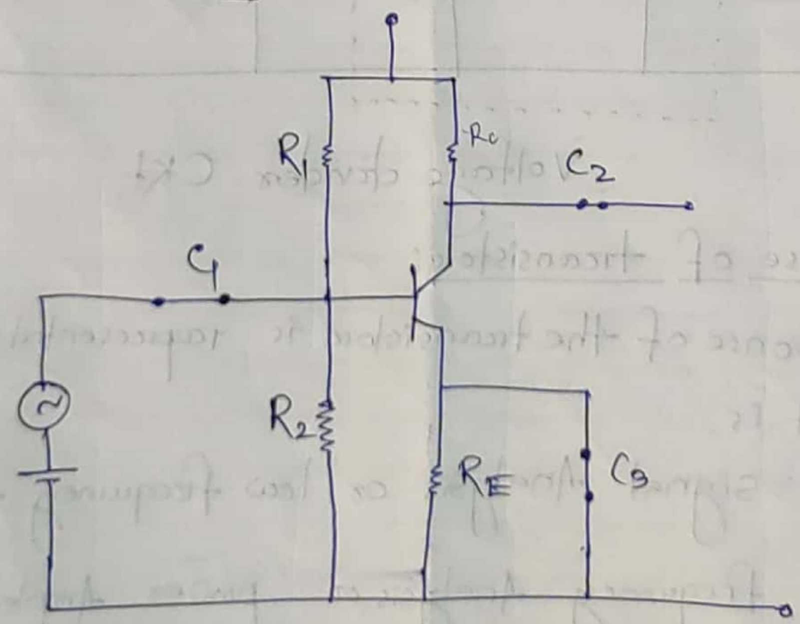
$$\boxed{\uparrow I_C = \beta I_B \uparrow}$$

After that, coefficient point is depends on I_C if I_C changes Coefficient point will change.

Def^n Small Signal Analysis:

The small signal is the signal having magnitude sufficiently small to keep that transistor in active region.

BJT Amplifier Ckt:



C_1 & C_2 are coupling capacitor

C_3 is bypass capacitor.

In this circuit Capacitor are behaves as a short circuit

because, $\boxed{X_C = \frac{1}{2\pi f c}}$

and c = High value.

In dc Analysis, $f=0$, $X_C = \frac{1}{2\pi(0)c} = \infty$ (D.C. biasing)

So, capacitor will be open circuit.

In Ac Analysis (Small Signal Analysis)

$$X_C = \frac{1}{2\pi f c}$$

$$= \frac{1}{2\pi(50Hz)c} \simeq 0$$

$f = 50Hz$
C = large value.

So, capacitor acts as short ckt.

In this Ac Analysis we represent

(i) Ac Equivalent Circuit

(ii) We need to replace the transistor circuit.

Equivalent Model or Equivalent Ckt:

Equivalent model is combination of ckt element (like voltage source, current source, resistors, capacitors etc) properly choosen to best approximates the actual behaviour of devices under specific operating point.

There are 3 models commonly used in the small signal Ac analysis of transistor networks.

They are:

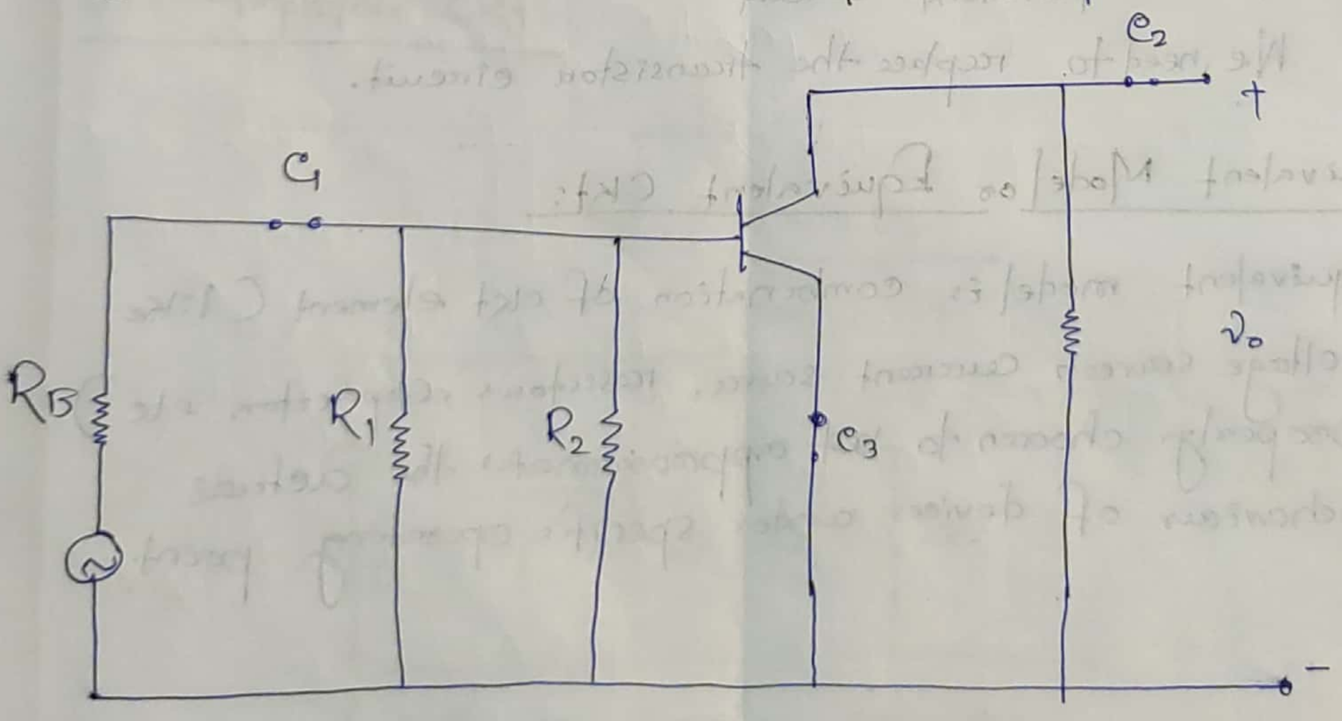
1. r_e (dynamic emitter resistance model)
2. Hybrid model
3. Hybrid π model.

Ac Equivalent Ckt of BJT Amplifier:

Step-1: Ac Eq ckt is obtained by setting all dc sources equal to zero and replace them by short circuit.

Step-2: Replace all the capacitors by short circuit
Remove all bypass elements by short circuit.

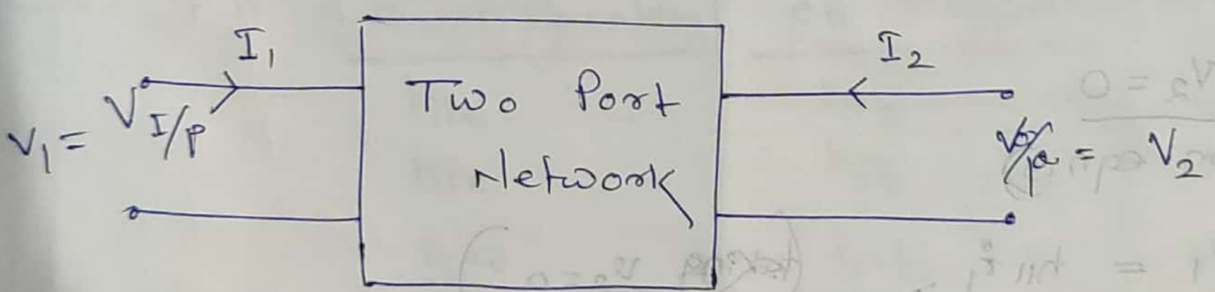
Step 3: Redrawn the Network, removing all the elements which are short circuit in step1 and step2.



Hybrid Model of transistor: (Calculation of h-Parameter)

* Hybrid model is eqv. model of transistor in small signal analysis.
* imp — Hybrid model is widely used in earlier before the popularity of Re model.
* Incase of Hybrid model the parameters are defined in general terms for any operating condition.
* Incase of Re model the parameters are defined by the actual operating conditions.

Z-Parameter & Y-Parameter ⇒ Vacuum tube.



* For Hybrid model ckt we consider V_1 and I_2 dependent quantities.
* V_2 and I_1 independent quantities.

V_1 is dependent on $f(I_1, V_2)$

$V_1 = f_1(I_1, V_2)$

$I_2 = f_2(I_1, V_2)$

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$$dv_1 = \frac{\partial v_1}{\partial I_1} dI_1 + \frac{\partial v_1}{\partial v_2} dv_2 \quad \text{--- (1)}$$

$$dI_2 = \frac{\partial I_2}{\partial I_1} . dI_1 + \frac{\partial I_2}{\partial v_2} . dv_2 \quad \text{--- (2)}$$

Simply we can write.

AC current i

AC voltage v

$$v_1 = h_{11} dI_1 + h_{12} dv_2 \quad \text{--- (3)}$$

$$i_2 = h_{21} dI_1 + h_{22} dv_2 \quad \text{--- (4)}$$

The Hybrid parameters of common base, common emitter and common collector is same.

Let, $v_2 = 0$

from eqn (3)

$$v_1 = h_{11} i_1 \quad (\text{taking } v_2 = 0)$$

$$h_{11} = \frac{v_1}{i_1} \quad (\text{impedance parameter})$$

From eqn (4)

$$i_2 = h_{21} i_1 \Rightarrow$$

$$h_{21} = \frac{i_2}{i_1}$$

Taking $v_2 = 0$

(Current gain)

Make $i_1 = 0$

$$v_1 = h_{12} . v_2$$

$$h_{12} = \frac{v_1}{v_2} \bigg| i_1 = 0$$

$i_2 = h_{22} v_2$

$h_{22} = \left.\frac{i_2}{v_2}\right| \; i_1 = 0$

h_{11} = is Called input impedance parameter when output
(h_i) voltage . v_2 is short ckted.

h_{21} = is Called Forward current gain when O/p voltage
(h_f) v_2 is short circuited.

h_{12} = is Called Reverse voltage gain when i/p Current is
(h_r) open circuited.

h_{22} = is Called Admitance when i/p current O/c
(h_o)

when it is

CE configuration	CB configuration	CC configuration
h_{ie}	h_{ib}	h_{ie}
h_{fe}	h_{fb}	h_{fe}
h_{re}	h_{rb}	h_{re}
h_{oe}	h_{ob}	h_{oe}

For Example:

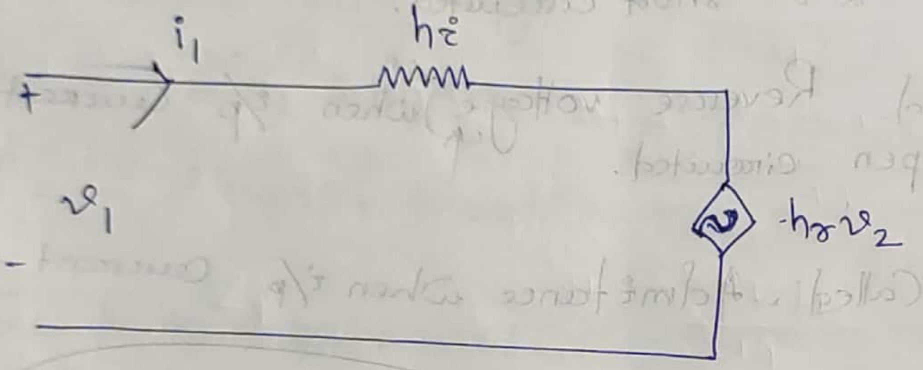
(i) input impedance of CE : h_{ie}

(ii) - Reverse voltage gain of CE : h_{re}

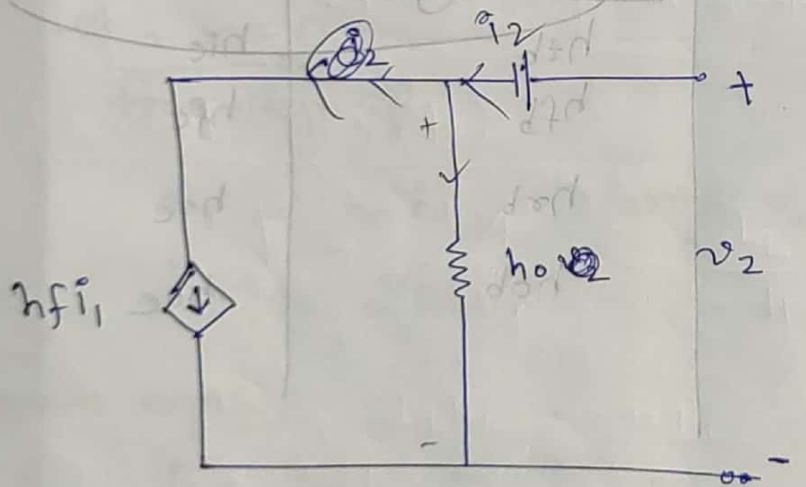
Eqn ③ & ④ can be written as.

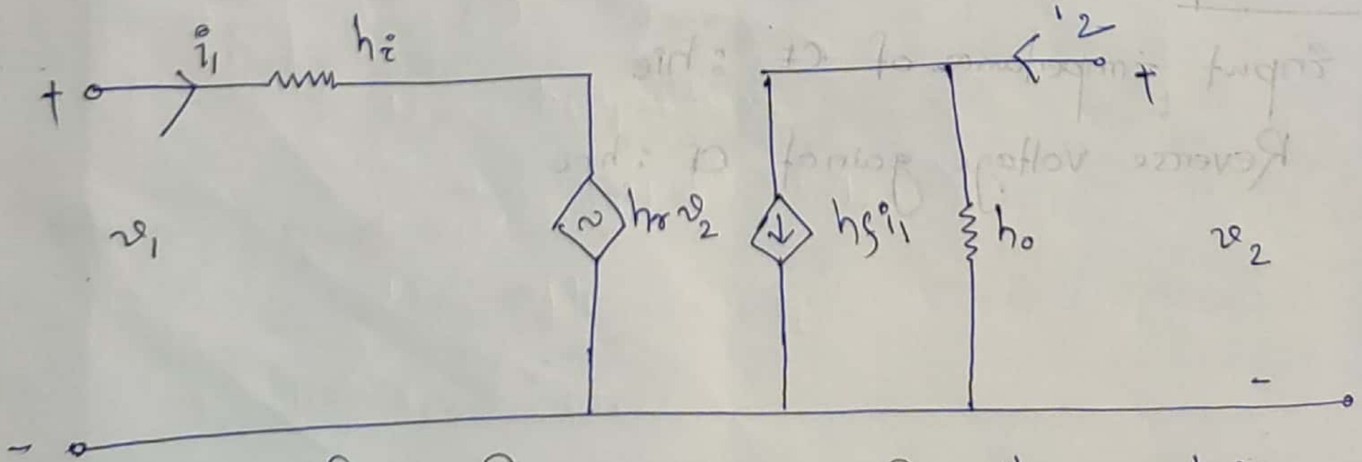
$$\boxed{\begin{aligned} v_1 &= h_{11} i_1 + h_{12} v_2 \Rightarrow h_i i_1 + h_r v_2 \quad \text{—⑤} \Rightarrow \text{KVL} \\ i_2 &= h_{21} i_1 + h_{22} v_2 \Rightarrow h_f i_1 + h_o v_2 \quad \text{—⑥} \Rightarrow \text{KCL} \end{aligned}}$$

$$v_1 - h_i i_1 - h_r v_2 = 0$$



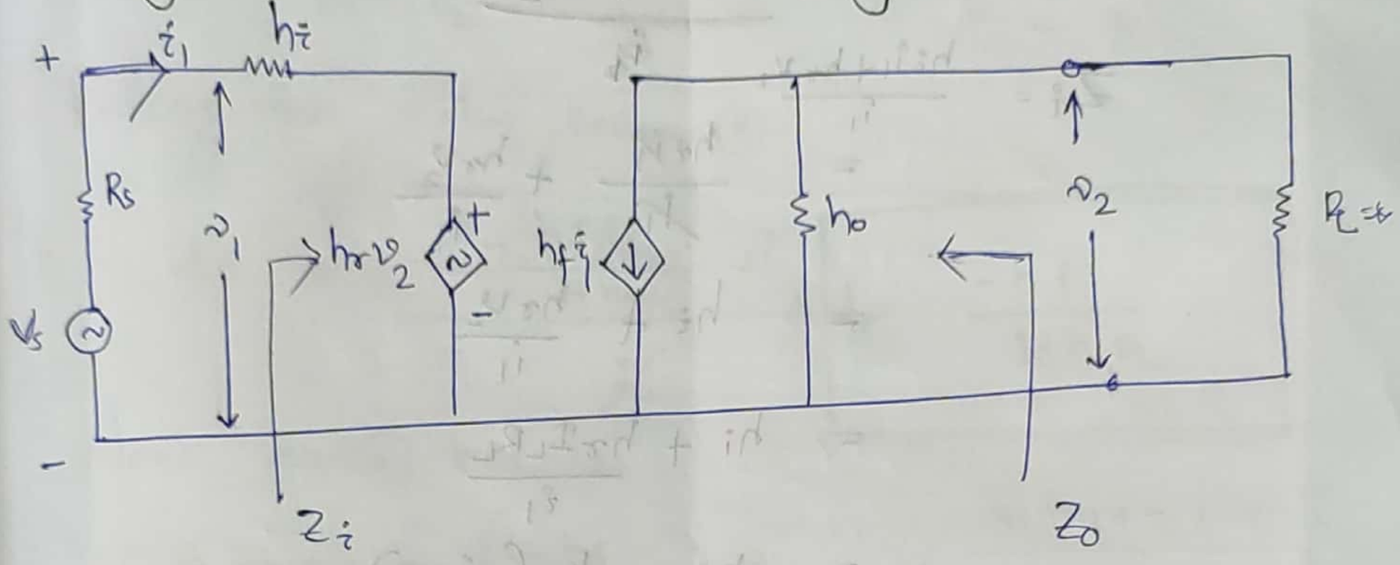
$$-i_2 + h_f i_1 + h_o v_2 = 0$$





This is final Equivalent Ckt for Hybrid Model.

Analysis of transistor Amplifier using h-parameters.



Z_i = input impedance.

Z_o = output impedance.

Current gain (A_i)

voltage gain (A_v)

power gain (A_p)

Current gain $(A_i) = \dfrac{i_{o/p}}{i_{i/p}}$

voltage gain $(A_v) = \dfrac{V_{o/p}}{V_{i/p}}$

$$\boxed{A_p = A_i \times A_v}$$

Input impedance:

$v_1 = Z_i i_1$

$h_i = Z_i = \dfrac{v_1}{i_1}$

From eqn (5) $v_1 = h_i i_1 + h_r v_2$

$$Z_i = \frac{h_i i_1 + h_r v_2}{i_1}$$

$$= \frac{h_i \cancel{i_1}}{\cancel{i_1}} + \frac{h_r v_2}{i_1}$$

$$= h_i + \frac{h_r v_2}{i_1}$$

$$= h_i + \frac{h_r I_L R_L}{i_1}$$

$$= h_i + \frac{h_r(-i_2 R_L)}{i_1}$$

$$\therefore Z_i = h_i - \frac{h_r i_2 R_L}{i_1}$$

$$\therefore \boxed{Z_i = h_i + A_i h_r R_L} \text{ — (2)} \quad \left[\because \frac{i_2}{i_1} = -A_i\right]$$

Current Gain (A_i):

$$A_i = \frac{i_{o/p}}{i_{i/p}} = \frac{I_L}{i_1} = \frac{-i_2}{i_1} \text{ — (4)}$$

$$v_2 = I_L R_L = -i_2 R_L$$

From equation (6)

$$i_2 = h_f i_1 + h_o v_2$$

$$= h_f i_1 + h_o(-i_2 R_L)$$

$$i_2 = h_f i_1 - h_o i_2 R_L$$

$$i_2 + i_2 h_o R_L = h_f i_1$$

$$i_2(1 + h_o R_L) = h_f i_1$$

$$i_2 = \frac{h_f i_1}{1+h_o R_L}$$

Putting this value in eqn (A)

$$A_i = \frac{\frac{-h_f \cancel{i_1}}{1+h_o R_L}}{\cancel{i_1}} \Rightarrow \boxed{A_i = \frac{-h_f}{1+h_o R_L}}$$

Now eqn (a) becomes

$$Z_i \Rightarrow h_i + h_r \left(\frac{-h_f}{1+h_o R_L}\right) \cdot R_L$$

$$Z_i \Rightarrow h_i - \frac{h_f h_r}{\frac{1}{R_L} + h_o}$$

Voltage gain (A_v):

$$A_v = \frac{V_{o/p}}{V_{i/p}}$$

$$= \frac{v_2}{v_1} = \frac{-i_2 R_L}{v_1}$$

$$= \frac{A_i i_1 R_L}{v_1}$$

$$= \frac{\left(\frac{-h_f}{1+h_o R_L}\right) R_L}{Z_i}$$

$A_i = \frac{-i_2}{i_1}$

$-i_2 = A_i i_1$

$$A_v = \frac{-h_f R_L}{h_i + (h_i h_o - h_r h_f) R_L}$$

$$\boxed{A_v = \frac{-h_f R_L}{h_i + \Delta h R_L}}$$

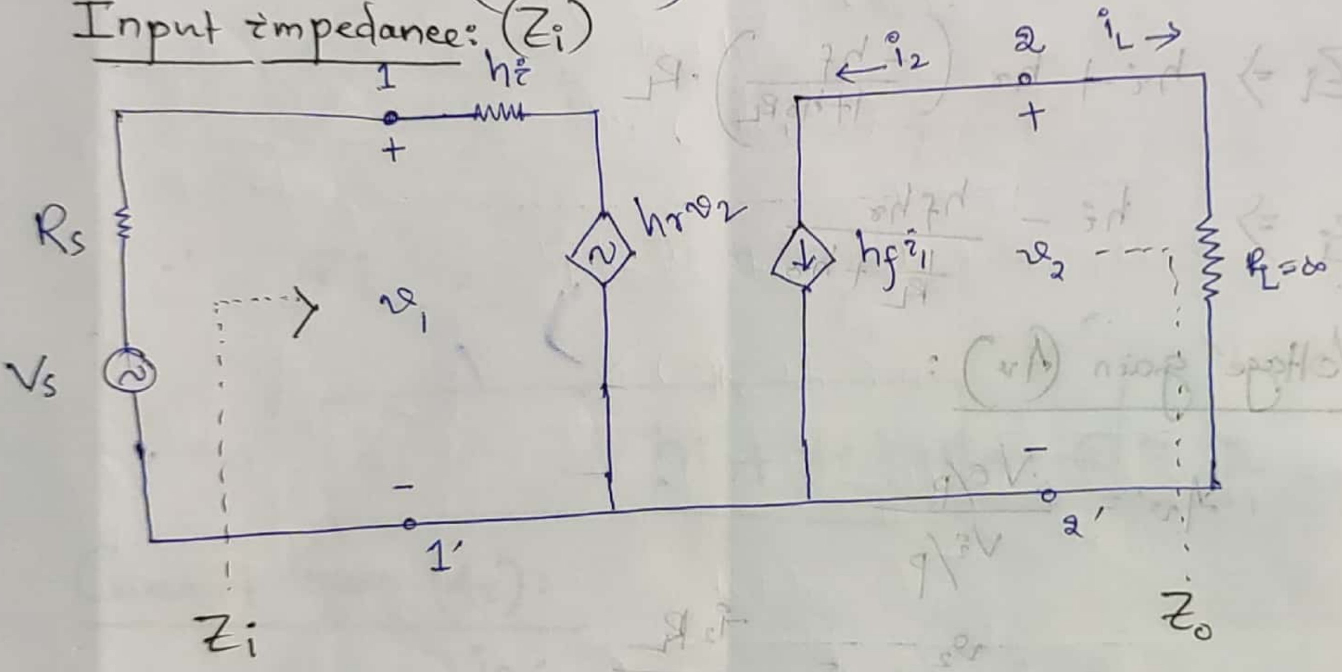
Power Gain (A_p):

$$A_p = A_v A_i$$

$$= \frac{-h_f R_L}{h_i + \Delta h R_L} \times \frac{-h_f}{1 + h_o R_L}$$

$$= \frac{h_f^2 R_L}{(h_i + \Delta h R_L)(1 + h_o R_L)}$$

Input impedance: (Z_i)



$$v_1 = i_1 \times Z_i$$

$$Z_i = \frac{v_1}{i_1}$$

$$\boxed{\begin{aligned} v_1 &= h_i i_1 + h_r v_2 \quad \text{---} (5) \\ i_2 &= h_f i_1 + h_o v_2 \quad \text{---} (6) \end{aligned}}$$

$$v_1 = h_i i_1 + h_r(-i_2 R_L)$$

$$\frac{v_1}{i_1} = h_i - h_r \frac{i_2}{i_1} R_L \qquad \left(\because A_i = \frac{-i_2}{i_1}\right)$$

$$\frac{v_1}{i_1} = h_i + h_r A_i R_L$$

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$$Z_i = h_i + h_r\left(\frac{-h_f}{1+h_oR_L}\right)R_L \quad \left(\because A_i = \frac{-h_f}{1+h_oR_L}\right)$$

$$\therefore \boxed{Z_i = h_i - \frac{h_r h_f}{\frac{1}{R_L}+h_o}}$$

Output impedance: (Z_o)

To calculate the output impedance we need to short circuit input source and we need to open circuit output terminal

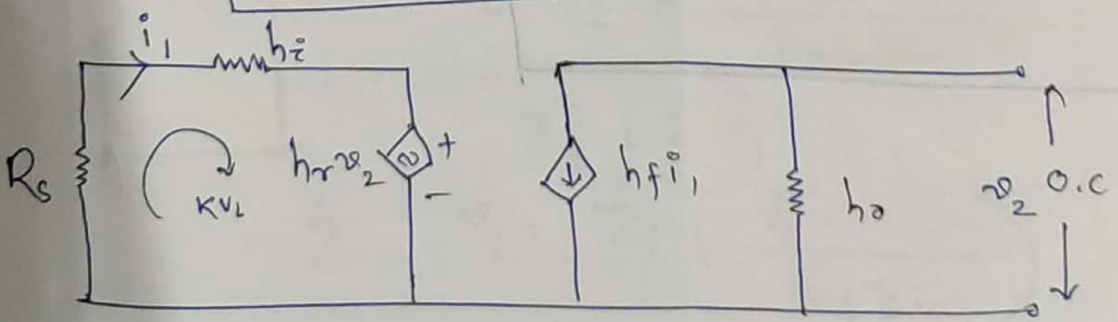
i.e. $V_s = 0$ & O.C. $R_L = \infty$.

$$Z_o = \frac{v_2}{i_2}$$

From eqn ⑥

$$i_2 = h_f i_1 + h_o v_2$$

$$\boxed{Z_o = \frac{v_2}{h_f i_1 + h_o v_2}} \quad \text{——— (i)}$$



Applying KVL,

$$R_s i_1 + h_i i_1 + h_r v_2 = 0$$

$\Rightarrow i_1 (R_s + h_i) + h_r v_2 = 0$

$$\Rightarrow i_1 = \frac{-h_r v_2}{R_s + h_i} \quad \text{(ii)}$$

Putting the value of i_1 in eqn (i)

$$Z_o = \frac{v_2}{h_f\left(-\frac{h_r v_2}{R_s + h_i}\right) + h_o v_2}$$

$$Z_o = \frac{v_2 (R_s + h_i)}{-h_f h_r v_2 + h_o v_2 R_s + h_o v_2 h_i}$$

$$= \frac{\cancel{v_2}(R_s + h_i)}{\cancel{v_2}\{(h_o h_i - h_f h_r) + R_s h_o\}}$$

$$= \frac{R_s + h_i}{h_o h_i - h_f h_r + R_s h_o}$$

$$\therefore \boxed{Z_o = \frac{R_s + h_i}{\Delta h + h_o R_s}} \quad (\because h_o h_i - h_f h_r = \Delta h)$$

Overall Voltage gain (A_{vs}):

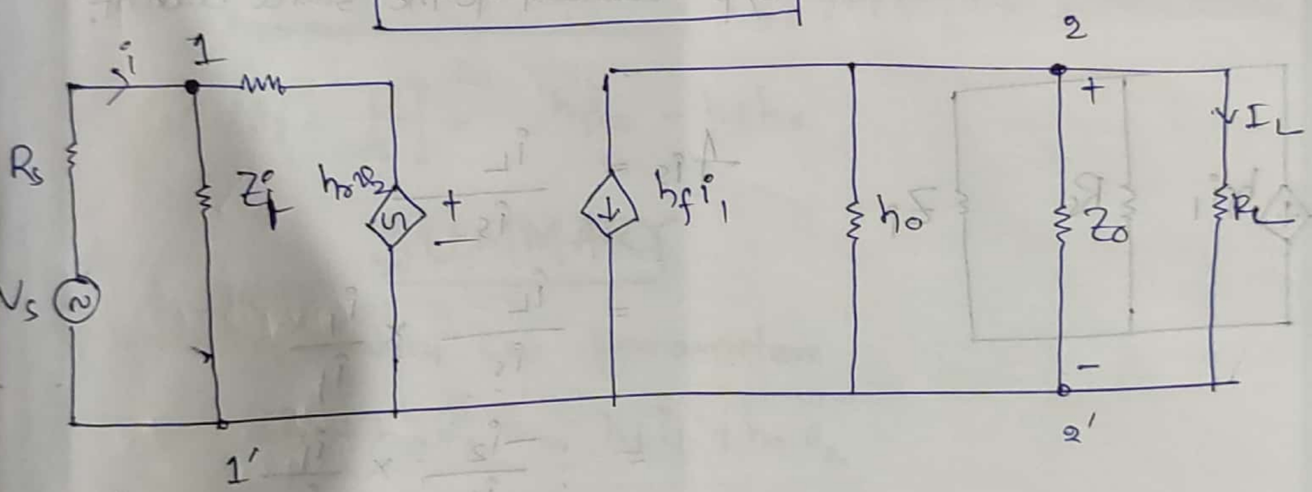
The ratio of output voltage to the source voltage.

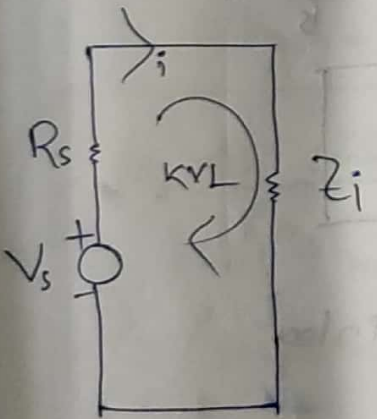
$$A_{vs} = \frac{V_2}{V_s}$$

$$= \frac{V_2}{V_s} \times \frac{V_1}{V_1}$$

$$= \frac{V_2}{V_1} \times \frac{V_1}{V_s} = A_v \cdot \frac{V_1}{V_s}$$

$$\Rightarrow \boxed{A_{vs} = A_v \cdot \frac{V_1}{V_s}}$$





$$-V_s + iR_s + iZ_i = 0$$

$$\Rightarrow V_s = i(R_s + Z_i)$$

$$\Rightarrow i = \frac{V_s}{R_s + Z_i}$$

$$Z_i \cdot i = v_1$$

$$\Rightarrow v_1 = Z_i\left(\frac{v_s}{R_s + Z_i}\right)$$

$$\Rightarrow \frac{v_1}{v_s} = \frac{Z_i}{R_s + Z_i}$$

$$\boxed{A_{vs} = A_v \cdot \left(\frac{Z_i}{R_s + Z_i}\right)}$$

$$\boxed{A_{vs} = A_v\left(\frac{Z_i}{R_s + Z_i}\right)}$$

If the voltage source is ideal,

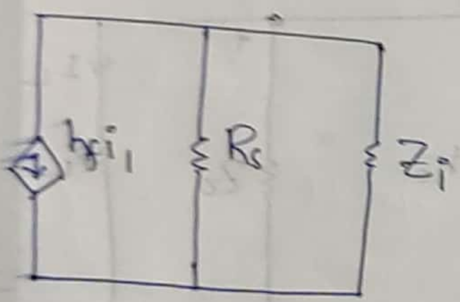
$R_s = 0$

$$\Rightarrow A_{vs} = A_v \times \frac{Z_i}{0 + Z_i} = A_V$$

$$\therefore \boxed{A_{vs} = A_v}$$

Overall Current Gain (A_{is}):

It is the ratio betn o/p current to the source current.

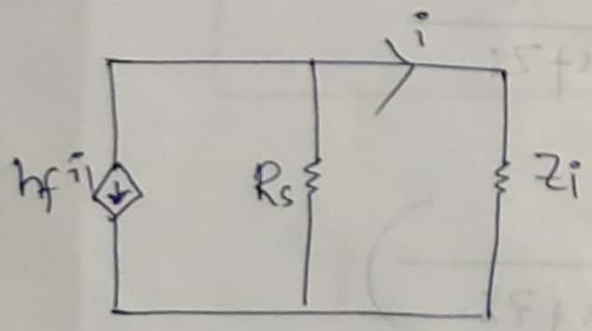


$$A_{is} = \frac{i_L}{i_s}$$

$$= \frac{i_L}{i_s} \times \frac{i_1}{i_1}$$

$$= \frac{-i_2}{i_1} \times \frac{i_1}{i_s}$$

$$\boxed{A_{is} = A_i \times \frac{i_1}{i_s}}$$



Applying KVL in loop.

$$i_1 = \frac{i_s R_s}{Z_i + R_s}$$

Ideal case $R_s = \infty$

$$A_{is} = A_i$$

Proof for $\Delta h = h_o h_i - h_f h_r$

From eqn (5) and (6)

$$v_1 = h_i i_1 + h_r v_2$$

$$i_2 = h_f i_1 + h_o v_2$$

$$\underbrace{\begin{pmatrix} v_1 \\ i_2 \end{pmatrix}}_{\text{Dependent}} = \begin{pmatrix} h_i & h_r \\ h_f & h_o \end{pmatrix} \underbrace{\begin{pmatrix} v_2 \\ i_1 \end{pmatrix}}_{\text{independent}}$$

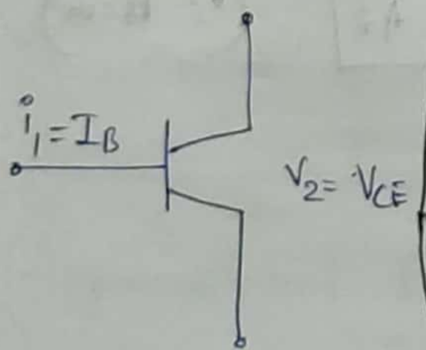
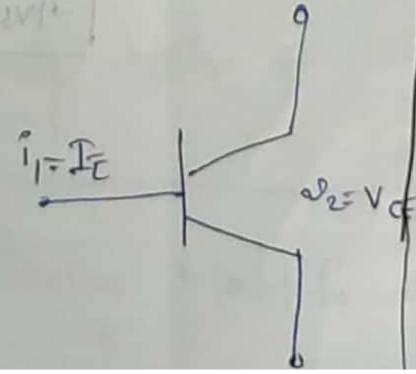
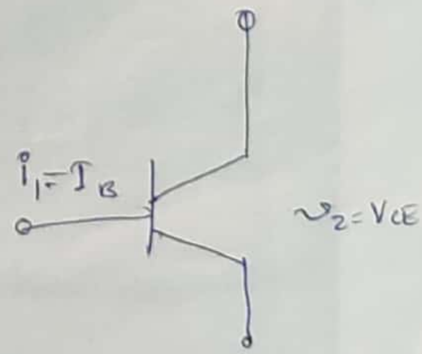
$$\Delta h = |h| = h_i h_o - h_f h_r$$

SUMMARY

Hybrid Equivalent ckt parameters.

$$v_1 = h_{11} i_1 + h_{12} v_2 = h_i i_1 + h_r v_2$$

$$i_2 = h_{21} i_1 + h_{22} v_2 = h_f i_1 + h_o v_2$$

For CE Configuration	For CB Configuration	For CC Configuration
$v_1 = h_{ie} i_1 + h_{re} v_2$	$v_1 = h_{ib} i_1 + h_{rb} v_2$	$v_1 = h_{ie} i_1 + h_{re} v_2$
$i_2 = h_{fe} i_1 + h_{oe} v_2$	$i_2 = h_{fb} i_1 + h_{ob} v_2$	$i_2 = h_{fe} i_1 + h_{oe} v_2$
		

Analysis of Transistor Amplifiers using h parameters:

(i) Current Gain $(A_i) = \dfrac{-h_f}{1+h_o R_L}$

(ii) Voltage Gain $(A_v) = \dfrac{-h_f R_L}{h_i + \Delta h R_L}$

(iii) Power Gain $(A_p) = (A_i \times A_v) = \dfrac{h_f^2 R_L}{(h_i + \Delta h R_L)(1+h_o R_L)}$

(iv) input impedance $(Z_i) = h_i - \dfrac{h_f h_r}{\frac{1}{R_L} + h_o}$

(v) Output impedance $(Z_o) = \dfrac{v_2}{h_f\left(\dfrac{-h_r v_2}{R_s + h_i}\right) + h_o v_2}$

$= \dfrac{h_i + R_s}{\Delta h + h_o R_s}$

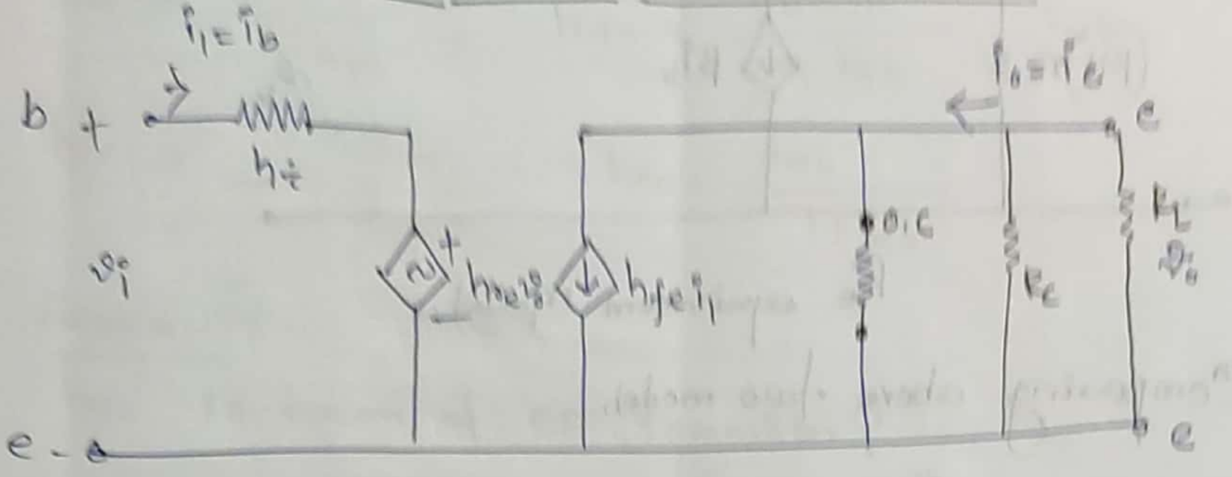
(vi) Over all voltage gain $(A_{vs}) = A_v \dfrac{Z_i}{R_s + Z_i} = A_v$

(vii) Overall Current Gain $(A_{is}) = A_i \times \dfrac{R_s}{R_s + Z_i}$

In ideal case,

$\boxed{A_{is} = A_i}$ $(\because R_s = \infty)$

Approximate Hybrid Equivalent Model : (For CE transistor)



h_r and h_o

$1/h_o \Rightarrow$ O/p imp.

$1/h_o$ to R_L

$\frac{1}{h_o} >> R_L$

So, we neglect R_L

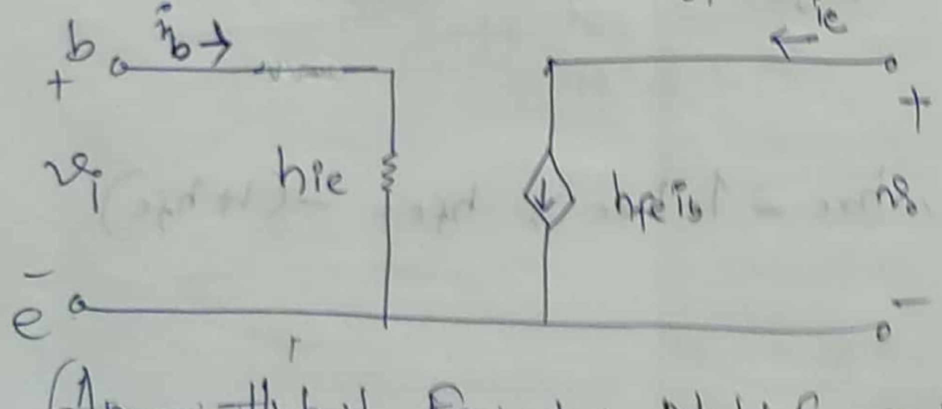
$\frac{1}{h_o} >> R_e \| R_L$

$R_{eq} <$ smallest (R_e, R_L)

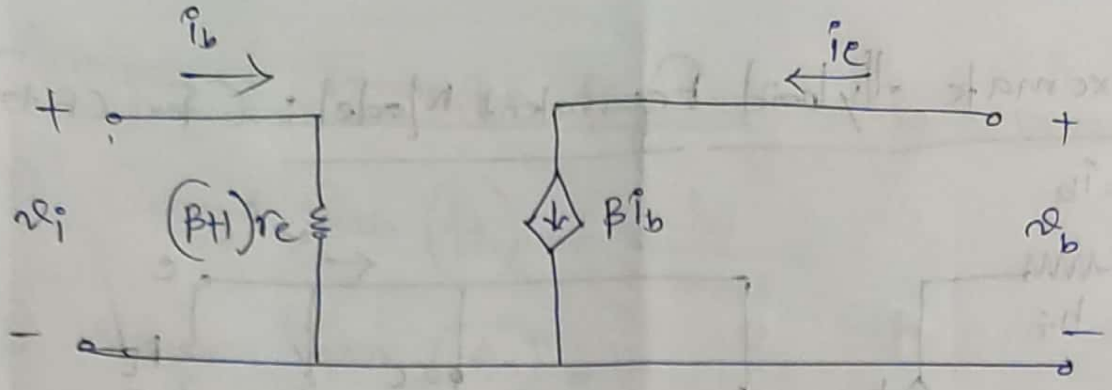
$h_r = \frac{v_i}{v_o} \simeq 0$

$h_{re} v_o = 0$

So, we s.c ckt $h_{re} v_o$.



(Approximate Hybrid Equivalent Model for CE transistor)



r_e equivalent Model.

Comparing above two model.

$$h_{ie} = (\beta+1) r_e = \beta r_e$$

$$h_{fe} = \beta$$

Conversion of h-parameters:

CE → CB

$$h_{ib} = \frac{h_{ie}}{1+h_{fe}}$$

$$h_{rb} = \frac{h_{ie} h_{oe}}{1+h_{fe}} - h_{re}$$

$$h_{fb} = \frac{-h_{fe}}{1+h_{fe}}$$

$$h_{ob} = \frac{h_{oe}}{1+h_{fe}}$$

CE → CC

$h_{ic} = h_{ie}$, $h_{rc} = 1 - h_{re}$, $h_{fc} = -(1+h_{fe})$

$h_{oc} = h_{oe}$

CB → CE

$h_{ie} = \frac{h_{ib}}{1+h_{fb}}$, $h_{re} = \frac{h_{ib} h_{ob}}{1+h_{fb}} - h_{rb}$

$h_{fe} = \frac{-h_{fb}}{1+h_{fb}}$, $h_{oe} = \frac{h_{ob}}{1+h_{fb}}$

Problem. 1

h_{re} in terms of CB transistor?

Ans: $h_{re} = \frac{h_{ib} h_{rb}}{1+h_{fb}} - h_{rb}$

Problem-2 : Convert h_{rc} in terms of CB transistor.

Ans: $h_{rc} \to$ CB

$h_{rc} = 1 - h_{re}$

$= 1 - \frac{h_{rb} \cdot h_{ob}}{1+h_{fb}} - h_{rb}$

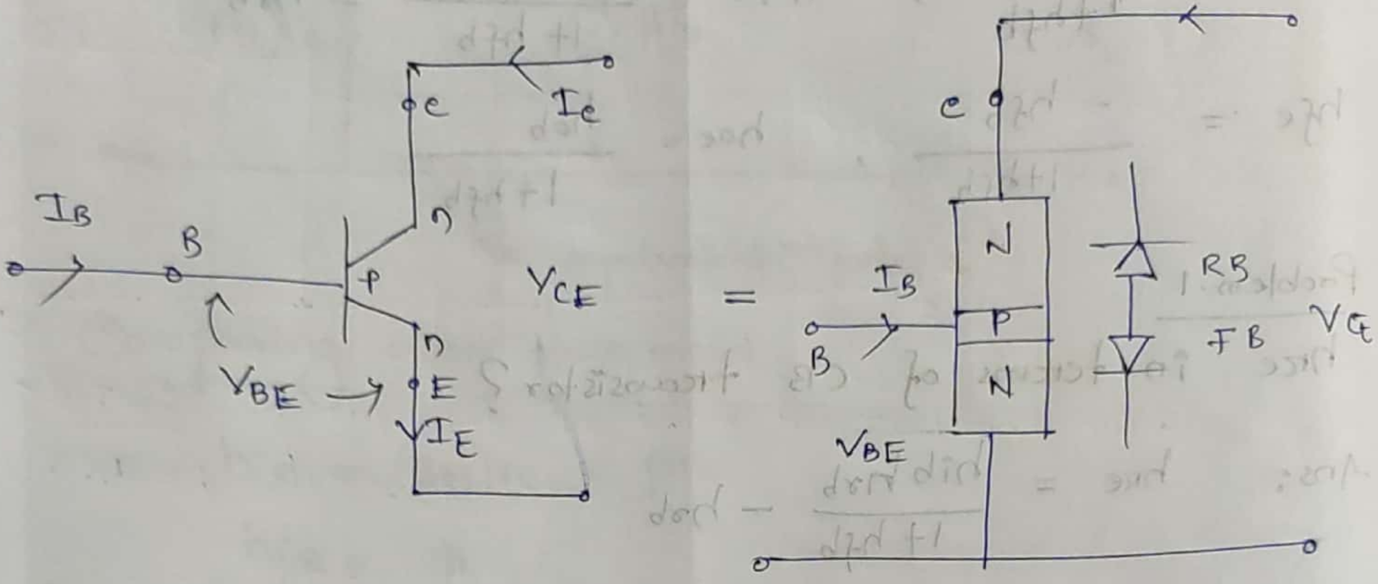
Problem-3 : Convert h_{fe} in terms of CB transistor.

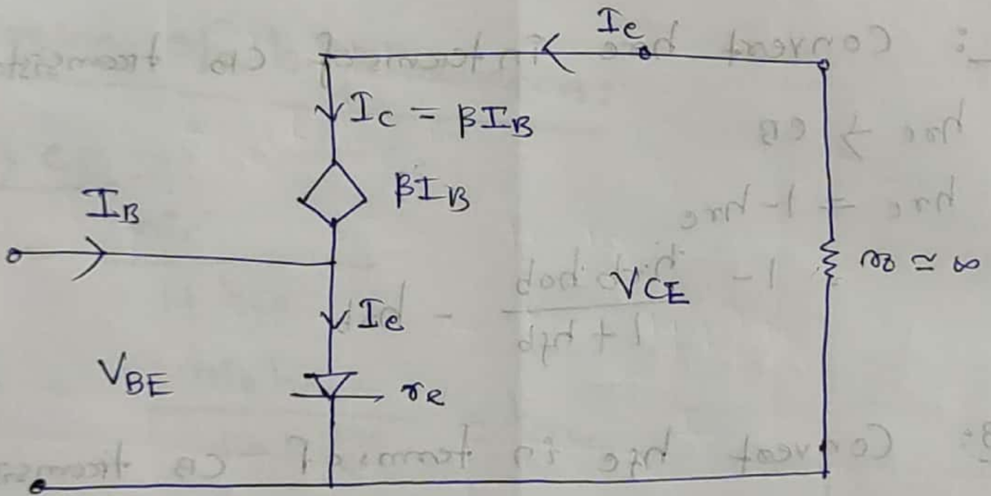
$h_{fe} = -(1+h_{fe})$

$= -\left(1 + \frac{-h_{fb}}{1+h_{fb}}\right)$

$= -1 + \frac{h_{fb}}{1+h_{fb}}$

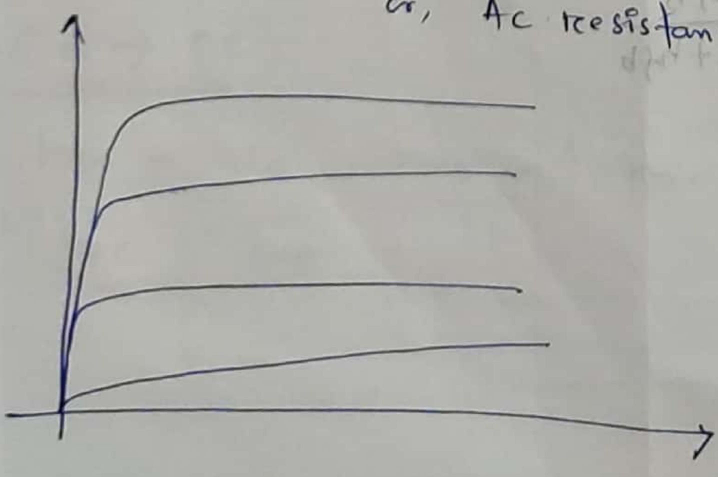
r_e transistor model for CE configuration:





$\boxed{r_e = r_d}$ (dynamic resistance or diode resista

Dynamic resistance $r_d = \dfrac{\Delta V_{BE}}{\Delta I_E}$
or, AC resistance.



$r_o = \dfrac{1}{\text{slope of current}} = \dfrac{1}{0} = \infty$

$$\boxed{I_D = I_s \left(e^{\frac{V_D}{\eta V_T}} - 1 \right)}$$

$I_s \Rightarrow$ Reverse Saturation Current.

differentiate w.r.t V_D.

$$\frac{dI_D}{dV_D} = \frac{d}{dV_D} I_s \left(e^{\frac{V_D}{\eta V_T}} - 1 \right)$$

$\eta = 1$ (for Ge)

$\eta = 2$ (for Si)

$\eta \simeq 1$ (for Si in Room temprature 27°C or 300K)

$$\Rightarrow \frac{1}{r_d} = I_s \frac{d}{dV_D} \left(e^{\frac{V_D}{\eta V_T}} - 1 \right)$$

$$\Rightarrow \frac{1}{r_d} = I_s \frac{e^{\frac{V_D}{V_T}}}{V_T}$$

$$\Rightarrow \frac{1}{r_d} = \frac{I_D + I_s}{V_T} = \frac{I_D}{V_T} \quad (\because I_D \gg I_s)$$

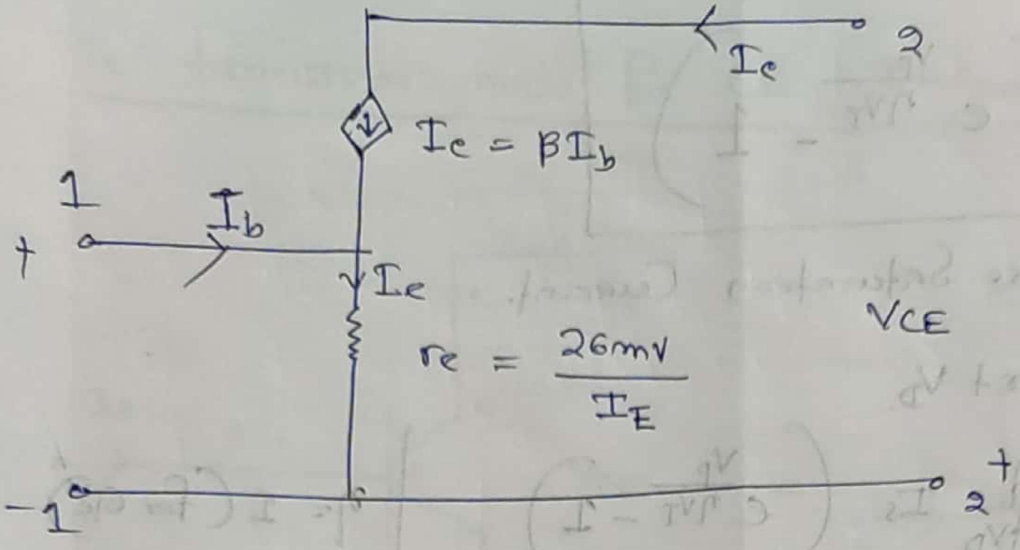
$$r_d = \frac{V_T}{I_D}$$

$V_T = 26mV$

$$r_d = \frac{26mV}{I_D}$$

$$= \frac{26mV}{I_E} \quad (\because I_D \simeq I_E)$$

$$\boxed{r_e = \frac{26mV}{I_E}}$$

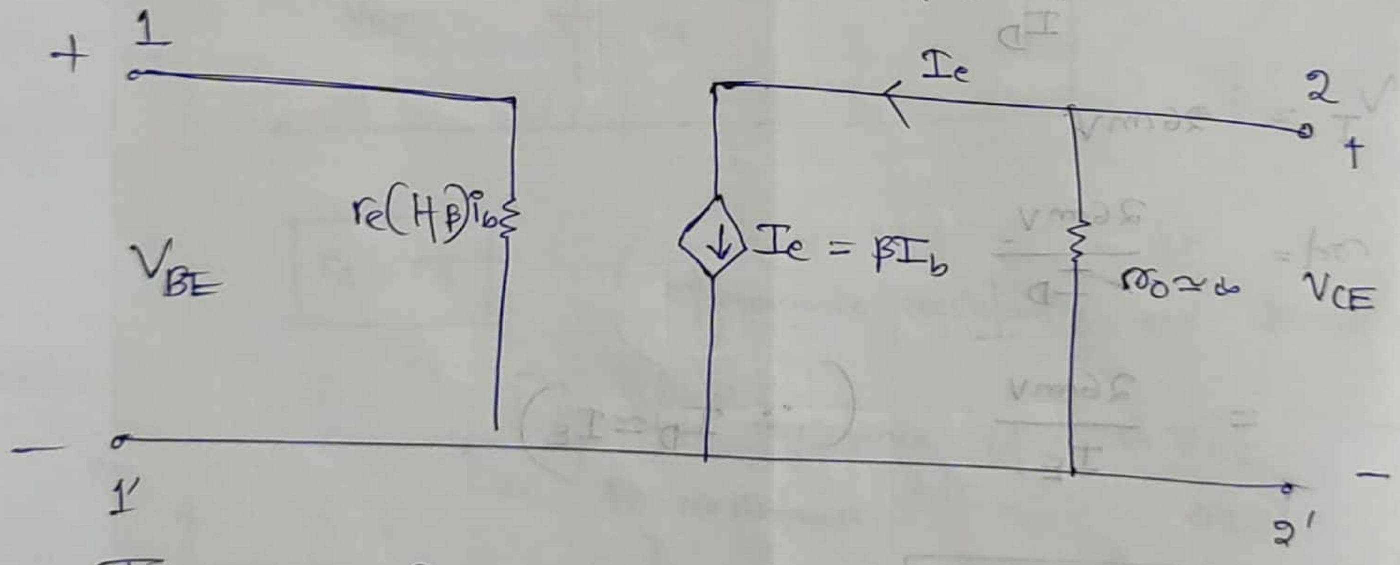


NOTE: To Calculate I_e value we need to draw dc equivalent CKt of transistor.

$$i_e = i_b + i_c$$
$$= i_b + \beta i_b$$
$$= (1+\beta)\, i_b$$

Voltage drop across r_e = $r_e \cdot i_e$

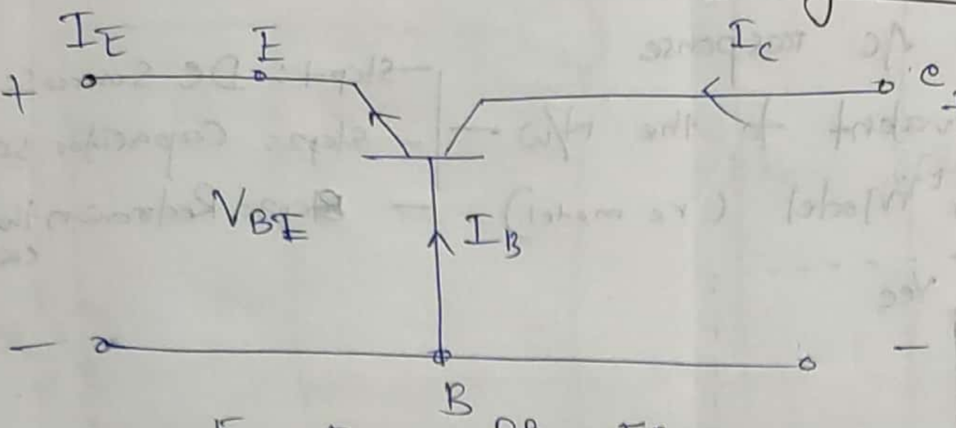
$$= r_e \cdot (1+\beta)\, i_b.$$

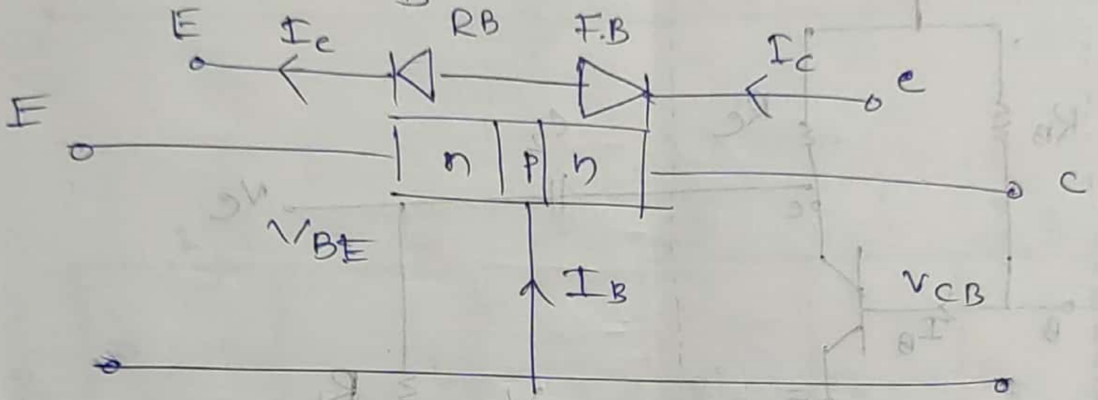


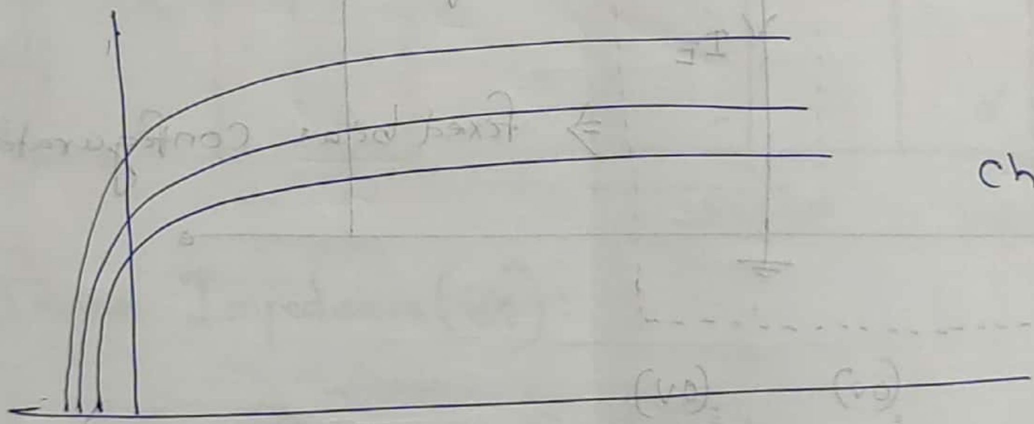
This is the final re model of CE configuration.

$$V_{BE} = r_e(1+\beta)$$
$$V_{BE} = r_e \beta$$

r_e transistor model for CB configuration:

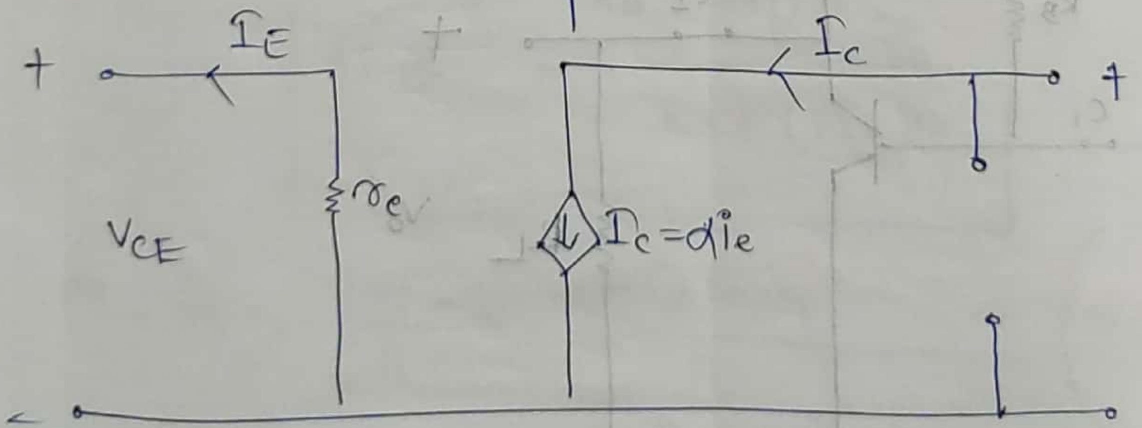






Characteristics of CB configuration,

$$r_o = \frac{1}{\text{slope}} = \infty$$

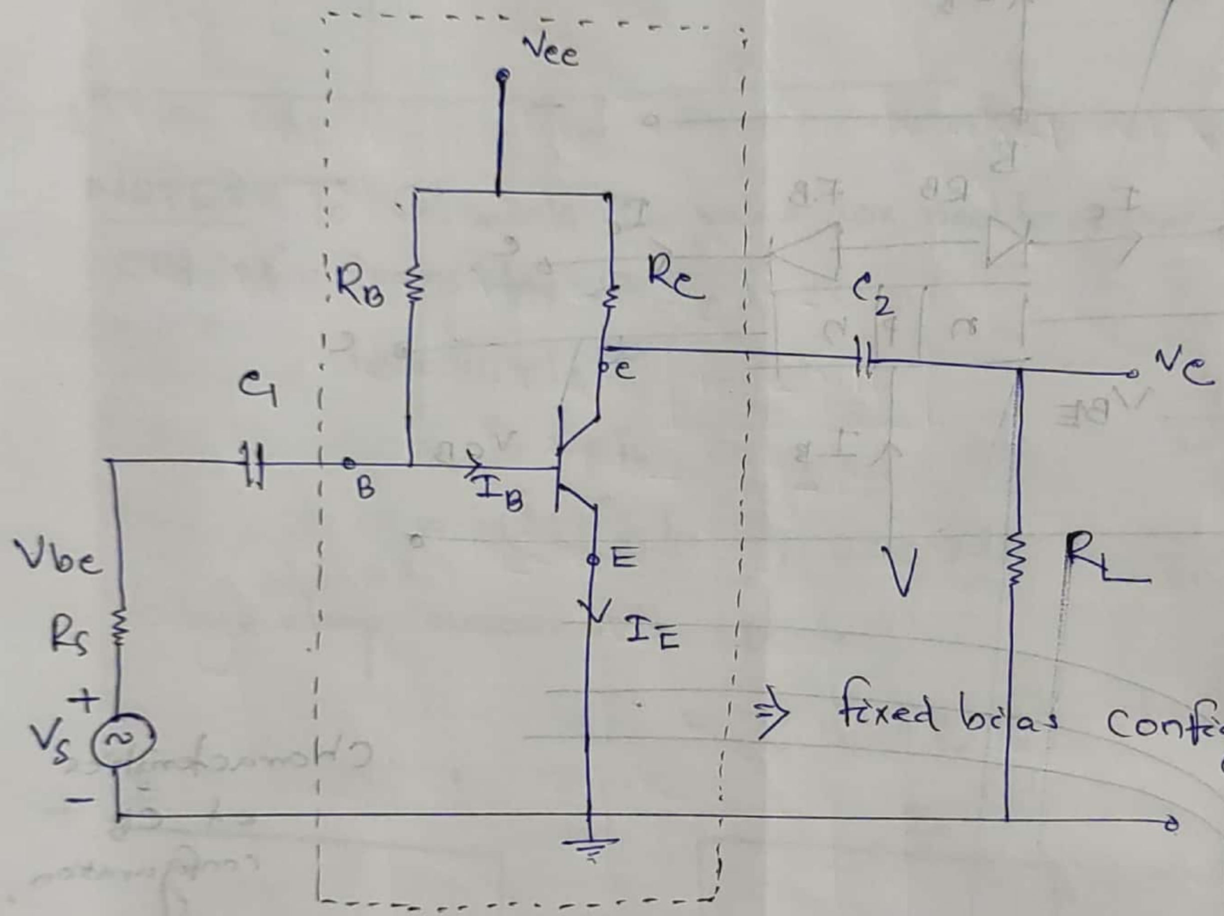


* r_e Transistor Model for CE fixed bias Configuration:

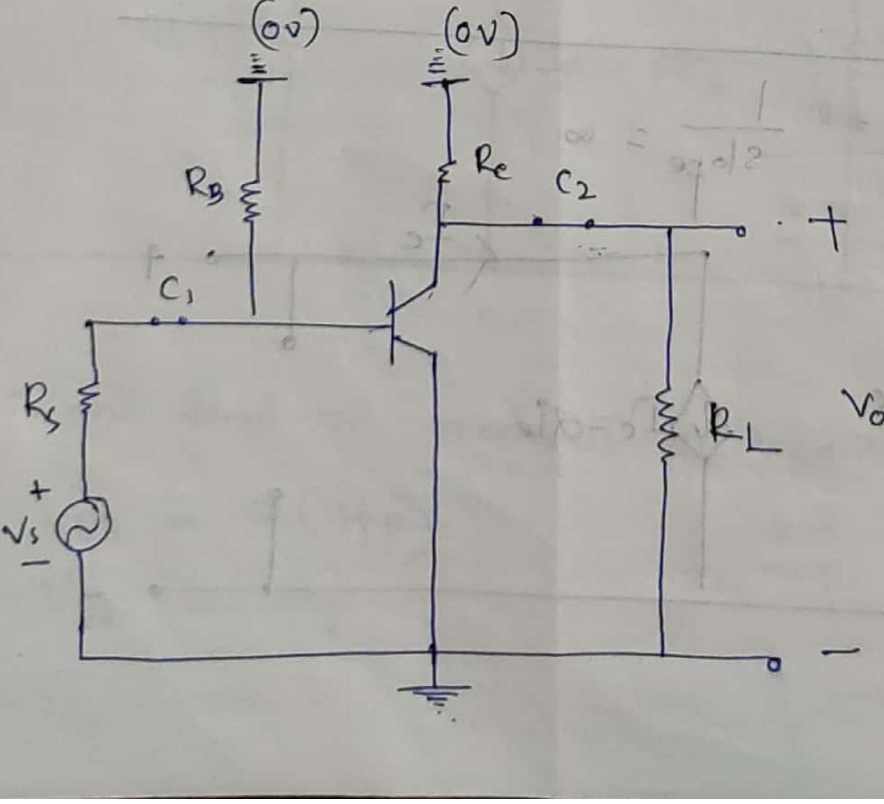
Small Signal Ac response

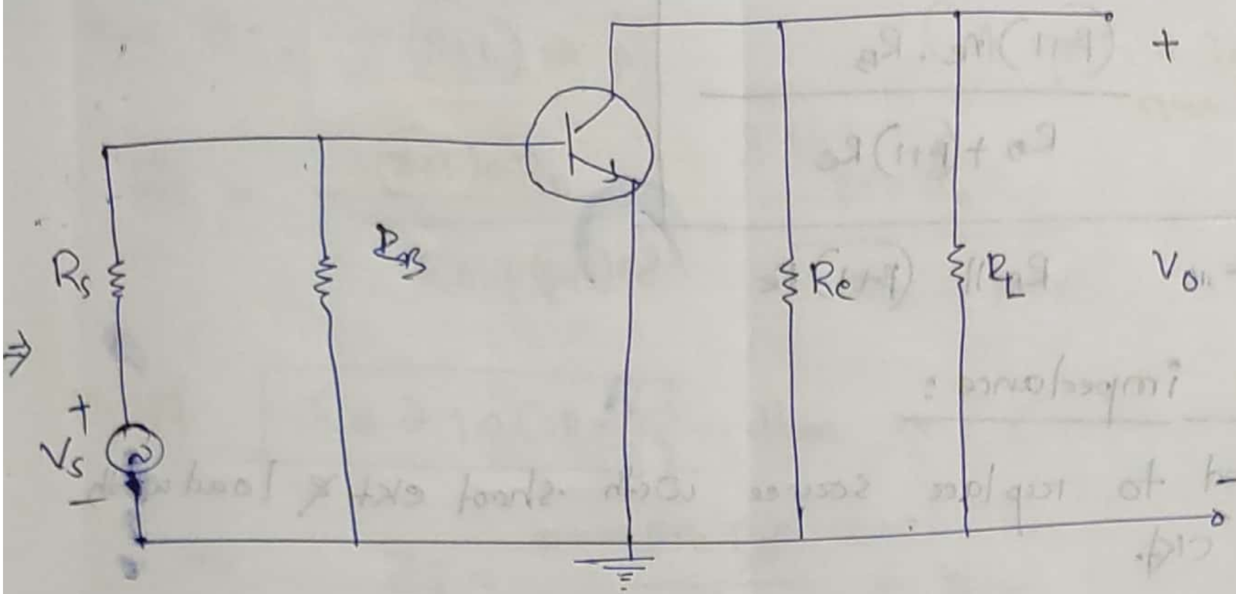
(i) Ac equivalent to the N/w — Step-1: Dc Source sc; Step2: Capacitor sc; Step3: Redrawn the ckt

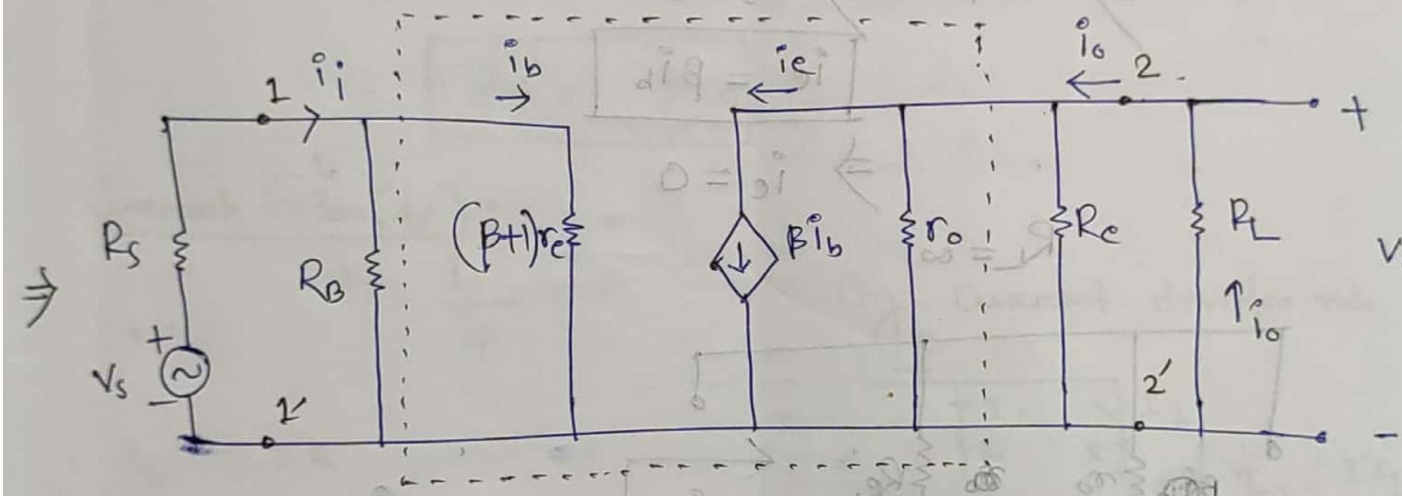
(ii) Transistor Model (r_e model)



⇒ fixed bias configuration.







(i) Input Impedance (Z_i):

$$Z_i = \frac{V_i}{i_1} = R_B \,\|\, (\beta+1) r_e$$

$$\boxed{Z_i = \frac{R_B (\beta+1) r_e}{R_B + (\beta+1) r_e}}$$

or, $Z_i = \frac{V_i}{i_i} = \frac{(\beta+1) r_e \cdot i_b}{i_1}$

$$= \frac{(\beta+1) r_e \cancel{i_1} R_B}{\cancel{i_i}\left(R_B + (\beta+1) r_e\right)}$$

By current divider rule:

$\therefore i_b = i_1 \times \frac{R_B}{R_B + (\beta+1) r_e}$

$$\boxed{Z_i = \frac{(\beta+1) r_e \cdot R_B}{R_B + (\beta+1) R_e}}$$

$$= R_B \parallel (\beta+1) r_e$$

Output impedance:

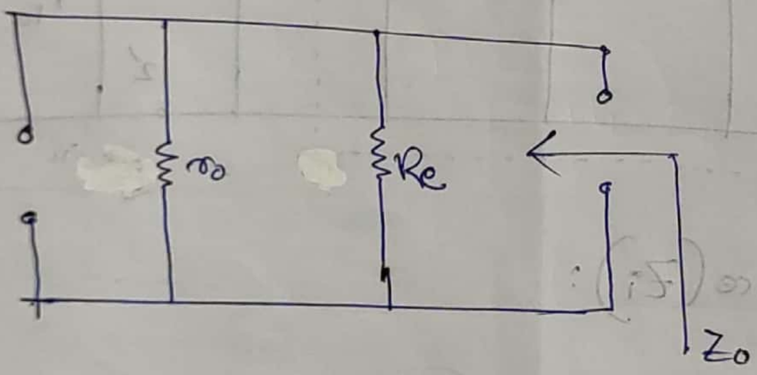
We need to replace source with short ckt & load with open ckt.

$V_s = 0 \Rightarrow V_{in} = 0 \Rightarrow i_b = 0$

$\boxed{i_c = \beta i_b}$

$\Rightarrow i_c = 0$

$R_L = \infty$



$$Z_o = \frac{V_o}{i_o} = r_o \parallel R_e = \frac{r_o R_e}{r_o + R_e}$$

r_o is a large resistance (in $M\Omega$)
(r_o, in case of CE configuration it is ∞)

r_o is greater than R_e

if $\boxed{r_o \geq 10 R_e}$, we can negleet R_e.

So, $Z_o = \frac{\cancel{r_o} R_e}{\cancel{r_o}} = R_e$. $\therefore \boxed{Z_o = R_e}$

for Z_i, $(\beta+1) \simeq \beta$ (As β is a large quantity)

$$Z_i = \frac{(\beta+1) r_e R_B}{R_B + (\beta+1) r_e} = \frac{\beta r_e R_B}{R_B + \beta r_e}$$

if $\boxed{R_B \geq 10(\beta+1) r_e}$ then r_e can be neglected

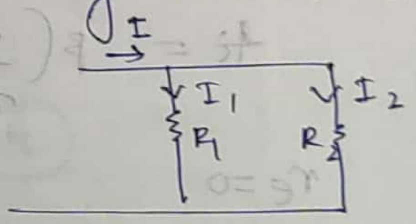
so, $$Z_i = \frac{\beta r_e \cancel{R_B}}{\cancel{R_B}} = \beta r_e$$

$$\therefore \boxed{Z_i = \beta r_e}$$

Current Gain (A_i):

$$A_i = \frac{i_o}{i_i}$$

By Current divider rule.



$$I_2 = \frac{I R_1}{R_1 + R_2}$$

$$I_1 = \frac{I R_2}{R_1 + R_2}$$

By C.D.R.

$$i_o = \frac{i_c (r_o \| R_e)}{r_o \| R_e + R_L}$$

$$A_i = \frac{\beta i_b (r_o \| R_e)}{r_o \| R_e + R_L} \bigg/ i_i \quad \text{— (A)} \quad (\because i_c = \beta I_b)$$

By CDR in i/p side

$$i_b = \frac{i_i R_B}{R_B + (\beta+1) r_e}$$

Now putting value of i_b in eqn (A)

$$\boxed{A_i = \beta\left(\frac{i_i R_B}{R_B + (\beta+1) r_e}\right)\left(\frac{r_o \| R_e}{r_o \| R_e + R_L}\right)}$$

$$\boxed{A_i = \beta\left(\frac{R_B}{R_B+(\beta+1)r_e}\right)\left(\frac{r_o \| R_C}{r_o \| R_C + R_L}\right)}$$

Case 1: $r_o \geq 10 R_C$

$r_o \| R_C = R_C$

$$A_i = \beta\left(\frac{R_B}{R_B+(\beta+1)r_e}\right)\left(\frac{R_C}{R_C+R_L}\right)$$

Case 2: R_B & $R_C = \infty$

$r_o < R_C$

$r_o \| R_L = r_o$

$$A_i = \beta\left(\frac{R_B}{R_B}\right)\left(\frac{r_o}{r_o+R_L}\right)$$

$$A_i = \beta\left(\frac{r_o}{r_o+R_L}\right)$$

Case: 3 $r_e = 0$

$$A_i = \beta\left(\frac{r_o \| R_C}{r_o \| R_C + R_L}\right)$$

Overall Current Gain:

$$A_{is} = \frac{i_o}{i_s} = A_i\left(\frac{R_s}{R_s+Z_i}\right)$$

$$\boxed{A_{is} = A_i\left(\frac{R_s}{R_s+Z_i}\right)}$$

Voltage Gain:

$$A_v = \frac{v_o}{v_i}$$

$$v_o = -i_o R_L$$

$$-i_o R_L - v_o = 0$$

$$V_o = -\beta I_b \left(\frac{r_o \| R_c}{r_o \| R_c + R_L} \right) R_L$$

$$v_i = (\beta+1) r_e \cdot i_b$$

$$\boxed{A_v = \frac{-\beta \left(r_o \| R_c \| R_L \right)}{(\beta+1) r_e}}$$

Case i: $\beta + 1 \simeq \beta$

$$A_v = \frac{-\cancel{\beta}\left(r_o \| R_c \| R_L \right)}{\cancel{\beta} r_e}$$

$$\boxed{A_v = -\frac{\left(r_o \| R_c \| R_L \right)}{r_e}}$$

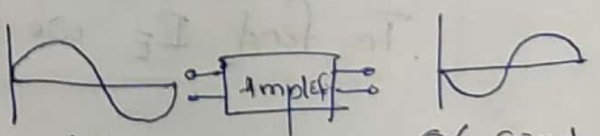
Case ii: $r_o \geq 10 R_c$

$$\boxed{A_v = -\frac{R_c \| R_L}{r_e}}$$

Case iii: $R_L = \infty \gg R_c$

$$\boxed{A_v = -\frac{R_c}{r_e}}$$

NOTE: -ve Sign of voltage gain is represent 180° phase shift.



i/p Signal

O/p Signal

Overall Voltage gain (A_{vs}):

$$A_{vs} = \frac{v_o}{V_s}$$

$$A_{vs} = A_v\left(\frac{Z_i}{Z_i + R_s}\right)$$

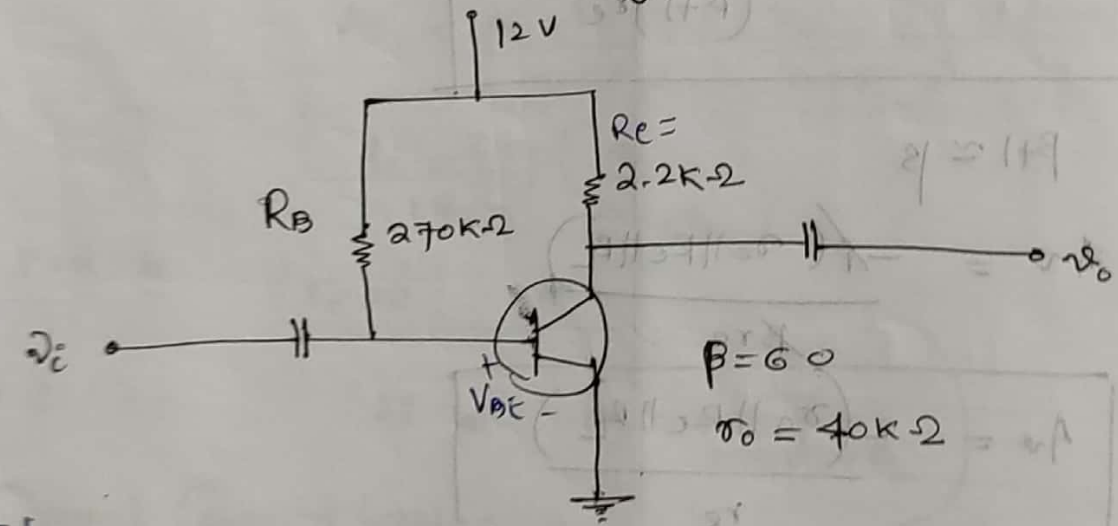
r_e model (Solved Problem 1)

1) For the network shown:

i) Determine input and output impedances.

ii) Determine voltage gain.

iii) Determine current gain.



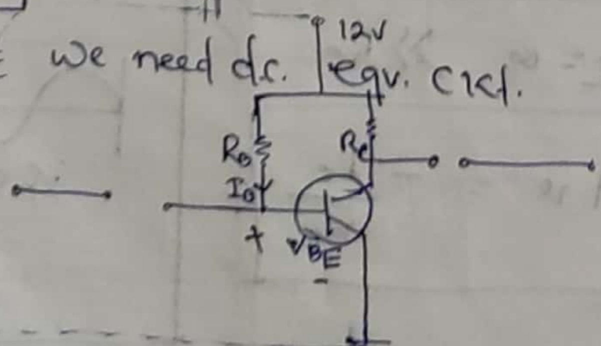
Sol.

$R_B = 270K\Omega$ $\quad$ $\beta = 60$ $\quad$ $V_{cc} = 12V$

$R_c = 2.2K\Omega$ $\quad$ $r_o = 40K\Omega$

$$\boxed{r_e = \frac{26mV}{I_E}}$$

To find I_E we need d.c. eqv. ckt.



on applying KVL:

$$12V - I_B R_B - V_{BE} = 0V$$

$$I_B = \frac{12V - 0.7V}{270k\Omega}$$

$$= 0.04185\ mA$$

$$I_B = 41.85\ \mu A$$

$$I_E = I_C + I_B$$

$$= \beta I_B + I_B$$

$$= (\beta+1) I_B$$

$$I_E = 61 \times 41.85\ \mu A$$

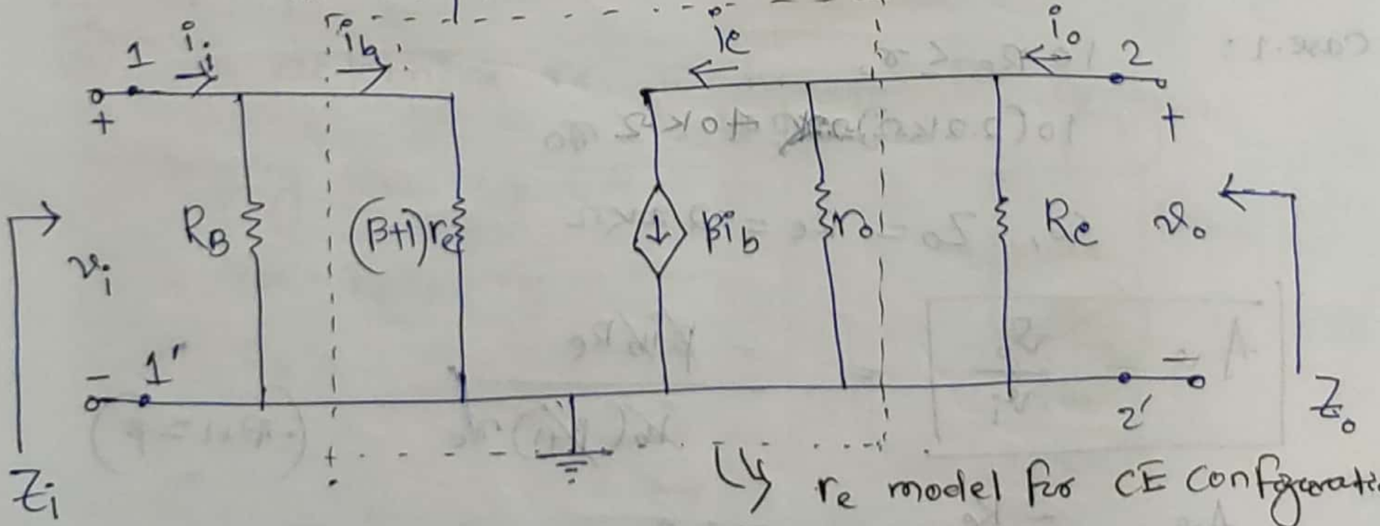
$$I_E = 2552.85\ \mu A = 2.553\ mA$$

$$r_e = \frac{26mV}{2.553mA}$$

$$\boxed{r_e = 10.18\Omega}$$

To Calculate the i/p and o/p impedance we required to two things

1. obtain the AC equ. ckt.
2. Replace the transistor with its Re equ. model.



r_e model for CE configuration

$$\boxed{Z_i = R_B \parallel (\beta+1) r_e}$$

$61 \times 10.18\,\Omega$

$\Rightarrow 621\,\Omega = 0.621\,k\Omega$

$$= \frac{R_B \cdot (\beta+1) r_e}{R_B + (\beta+1) r_e}$$

$$= \frac{(270\,k\Omega) \cdot (0.621\,k\Omega)}{270\,k\Omega + 0.621\,k\Omega} = 0.6195\,k\Omega \cong 619.5\,k\Omega$$

Case 1: $R_B \geq 10 \cdot (\beta+1) r_e$.

$$Z_i = (\beta+1) r_e$$

$$Z_i = (61)\, 10.18\,\Omega$$

$$= 0.621\,k\Omega$$

$$\boxed{Z_o = r_o \parallel R_e}$$

$$\boxed{Z_o = \frac{r_o R_e}{r_o + R_e}}$$

$$= \frac{40\,k\Omega \times 2.2\,k\Omega}{(40\,k\Omega + 2.2\,k\Omega)}$$

$$= 2.085\,k\Omega$$

Case.1: $10 R_e \leq r_o$

$10(2.2\,k\Omega) = 22\,k\Omega < r_o$

So, $Z_o = R_e = 2.2\,k\Omega$

$$\boxed{A_v = \frac{v_o}{v_i}} = \frac{-\cancel{\beta}\cancel{i_b} R_e}{\cancel{i_b}(\cancel{\beta+1}) r_e} \quad (\because \beta+1 \simeq \beta)$$

$$A_v = \frac{-R_e}{r_e} = -216.1$$

$$A_i = \frac{i_o}{i_i}$$

$$A_i = \beta\left(\frac{R_B}{R_B + (\beta+1)r_e}\right)\left(\frac{r_o \| R_C}{r_o \| R_C + R_L}\right)$$

② For the network of fig.

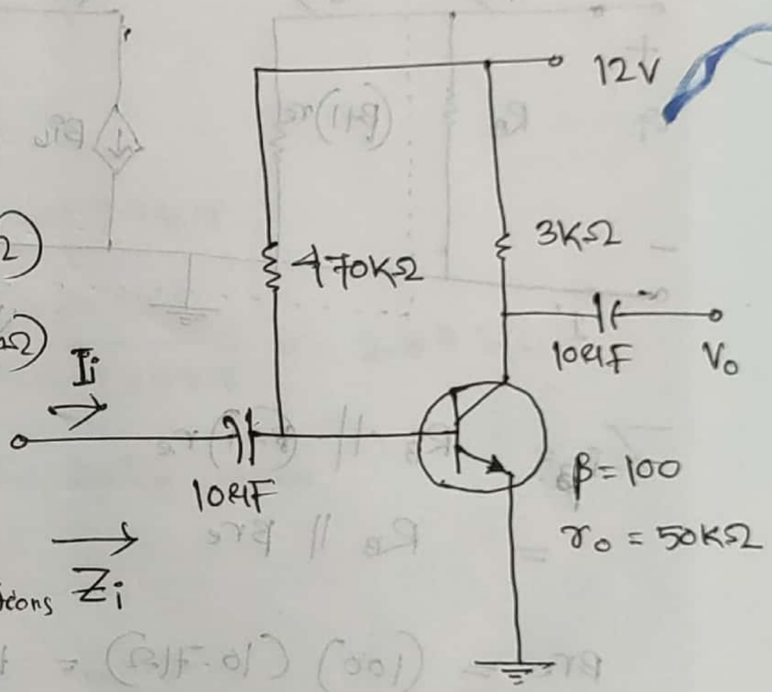
a. Determine r_e

b. Find Z_i (with $r_o = \infty\,\Omega$)

c. Calculate Z_o (with $r_o = \infty\,\Omega$)

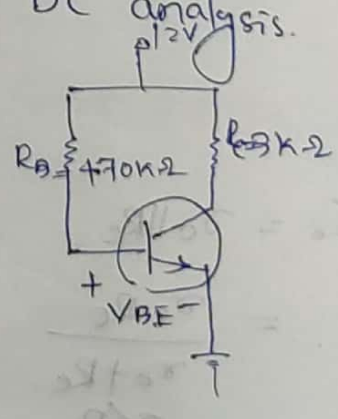
d. Determine A_v (with $r_o = \infty\,\Omega$)

e. Repeat part (c) and (d) including $r_o = 50\,k\Omega$ in all Calculations and compare results.



Solution:

ⓐ To determine r_e, we have to DC analysis.



Applying KVL i/p side.

$$V_{CC} - I_B R_B - V_{BE} = 0$$

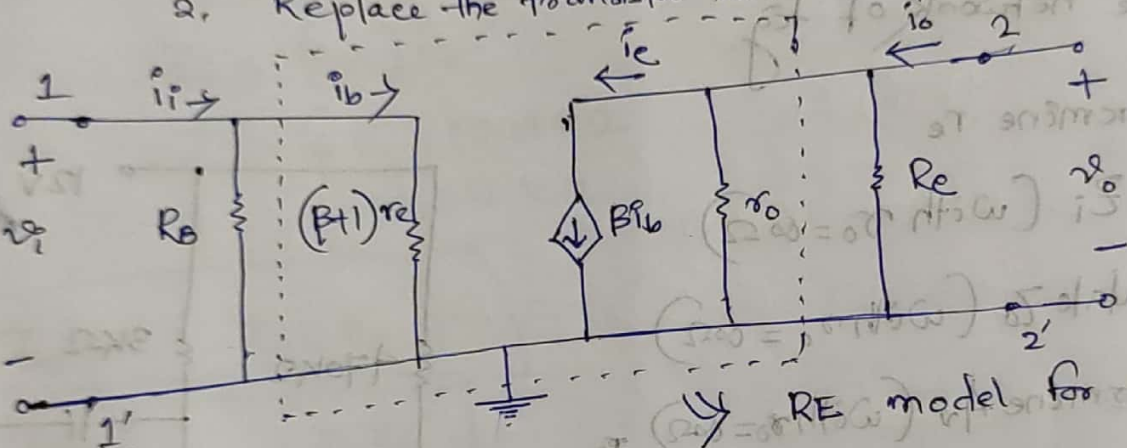
$$I_B = \frac{V_{CC} - V_{BE}}{R_B}$$

$$= \frac{12\,V - 0.7\,V}{470\,k\Omega} = 24.04\,\mu A$$

$I_E = (\beta+1) I_B$
$= (101)\cdot(24.04\mu A) = 2.428\ mA$

$\boxed{r_e = \dfrac{26mV}{I_E}} = \dfrac{26mV}{2.428mA} = 10.71\Omega$

b. For obtaining Z_i and Z_o we have to do two thing.
1. Draw AC eqv. ckt
2. Replace the transistor with Re eqv. model.



RE model for CE configuration

$Z_i = R_B \parallel (\beta+1) r_e$
$= R_B \parallel \beta r_e \quad (\because (\beta+1) r_e \simeq \beta r_e)$

$\beta r_e = (100)(10.71\Omega) = 1.071K\Omega$

$\therefore Z_i = \dfrac{R_B \cdot \beta r_e}{R_B + \beta R r_e} = \dfrac{470K\Omega \times 1.071K\Omega}{(470K\Omega + 1.071K\Omega)}$

$= 1.07K\Omega$

c. $Z_o = r_o \parallel R_e$
$= \dfrac{r_o R_c}{r_o + R_e} \quad (\because r_o = \infty)$
$= \dfrac{\cancel{r_o} R_e}{\cancel{r_o}(1 + \frac{R_e}{r_o})} = \dfrac{R_e}{1} = R_e \quad \therefore Z_o = R_e = 3K\Omega$

d. $$\boxed{A_v = -\frac{\beta(r_o \| R_C) \| R_L}{(\beta+1) r_e}}$$

Here, $r_o = R_L = \infty$ and $R_L \gg R_e$

So, $$\boxed{A_v = \frac{-R_e}{r_e}}$$

$$\Rightarrow A_v = \frac{-3k\Omega}{10.707 \times 10^{-3} k\Omega}$$

$$= -280.19$$

e. when, $r_o = 50k\Omega$.

$$Z_o = r_o \| R_e = 50k\Omega \| 3k\Omega$$

$$= \frac{(50 \times 3) k\Omega}{50k\Omega + 3k\Omega} = 2.83 \, k\Omega.$$

- when, $r_o = 50k\Omega$, if $r_o \geq 10 R_e$ then,

$$\boxed{A_v = -\frac{R_e \| R_L}{r_e}}$$

$$r_o \geq 10 \times 3k\Omega$$

$$\Rightarrow 50k\Omega > 3k\Omega.$$

So, $$A_v = -\frac{3k\Omega}{10.71 \, \Omega} = 0.28011$$

Emitter Follower & Darlington Amplifier

Emitter follower and Darlington amplifier are the most common examples for feedback amplifiers. These are the mostly used ones with a number of applications.

Emitter Follower:

Emitter follower circuit has a prominent place in feedback amplifiers. Emitter follower is a case of negative current feedback circuit. This is mostly used as a last stage amplifier in signal generator circuits.

The important features of Emitter Follower are-

It has high input impedance
It has low output impedance
It is ideal circuit for impedance matching.

All these ideal features allow many applications for the emitter follower circuit. This is a current amplifier circuit that has no voltage gain.

Construction:

The constructional details of an emitter follower circuit are nearly similar to a normal amplifier. The main difference is that the load R_L is absent at the collector terminal, but present at the emitter terminal of the circuit. Thus the output is taken from the emitter terminal instead of collector terminal.

The biasing is provided either by base resistor method or by potential divider method. The following (fig 1) shows the circuit diagram of an Emitter follower.

Scanned by CamScanner

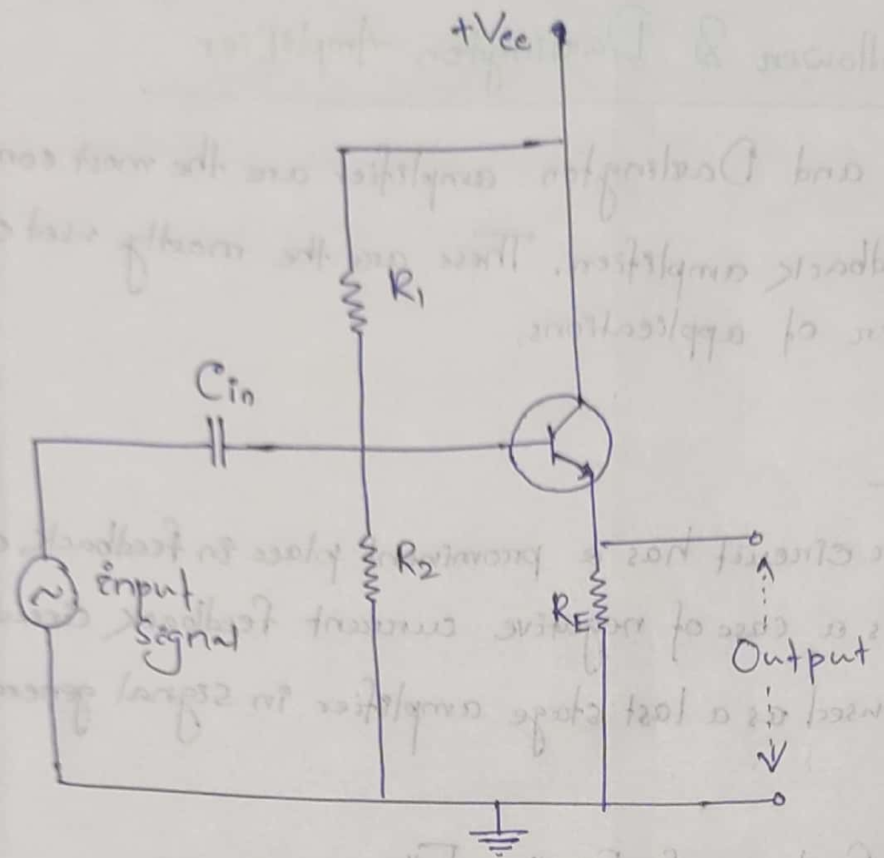


Fig.1 : Circuit diagram of an Emitter follower.

Operation:

The input signal voltage applied between base and emitter, develops an output voltage V_o across R_E, which is in the emitter section. Therefore,

$$V_o = I_E R_E$$

The whole of this output current is applied to the input through feedback. Hence,

$$V_f = V_o$$

As the output voltage developed across R_L is proportional to the emitter current, this emitter follower circuit is a current feedback circuit. Hence

$$\beta = \frac{V_f}{V_o} = 1$$

It is also noted that the input signal voltage to the transistor ($= V_i$) is equal to the difference of V_s and V_o i.e.,

$$V_i = V_s - V_o$$

Hence the feedback is negative.

Characteristics :

The major characteristics of the emitter follower are as follows –

No Voltage gain. In fact, the voltage gain is nearly 1.

Relatively high current gain and power gain.

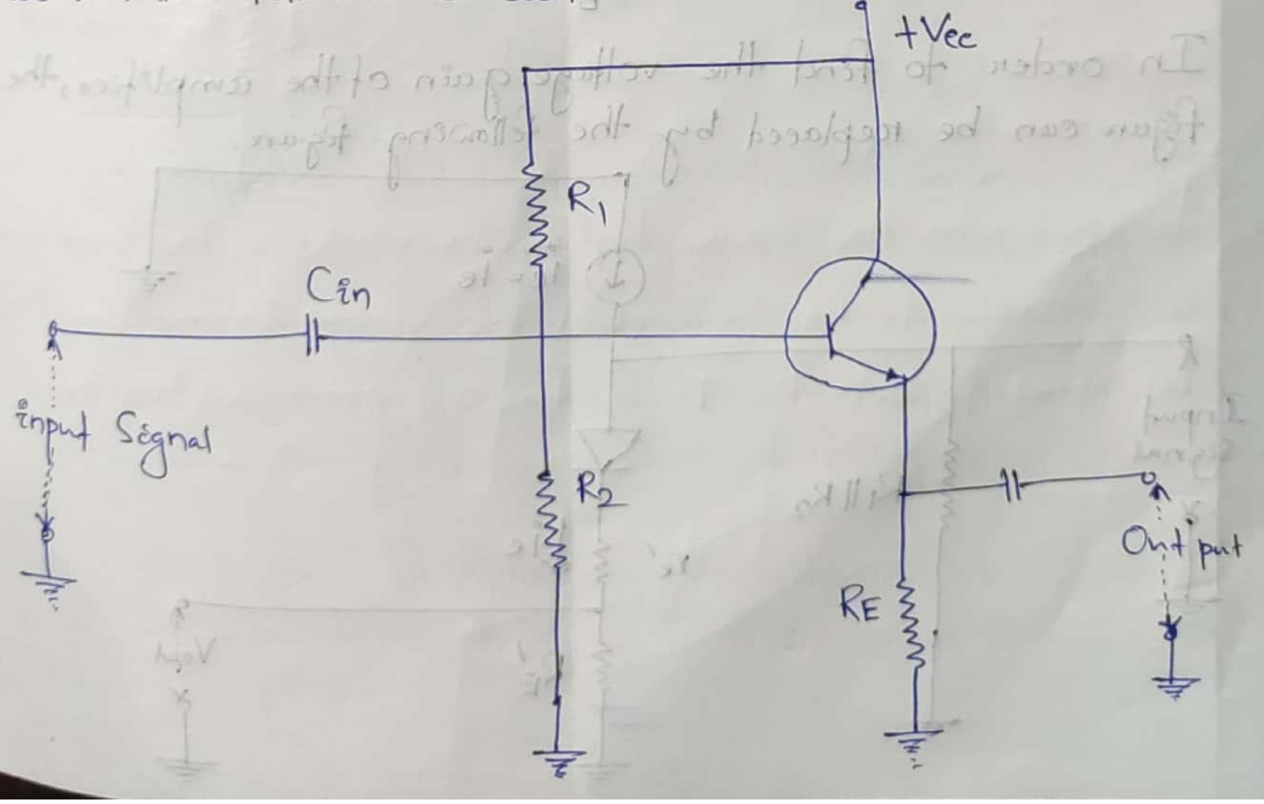
High input impedance and low output impedance

Input and Output ac voltages are in phase.

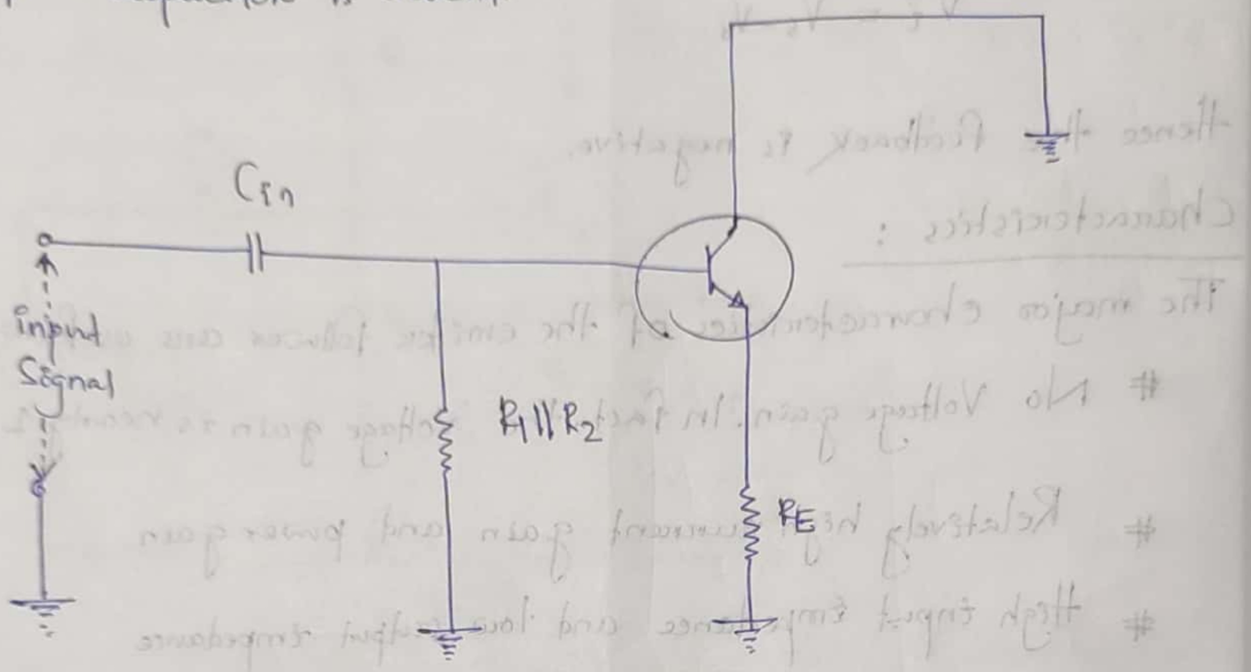
Voltage Gain of Emitter Follower :

As the Emitter Follower circuit is a prominent one, let us try to get the equation for the voltage gain of an emitter follower circuit.

Our Emitter follower circuit looks as follows –



If an AC equivalent circuit of the above circuit is drawn, it would look like the below one, as the emitter by pass capacitor is absent.



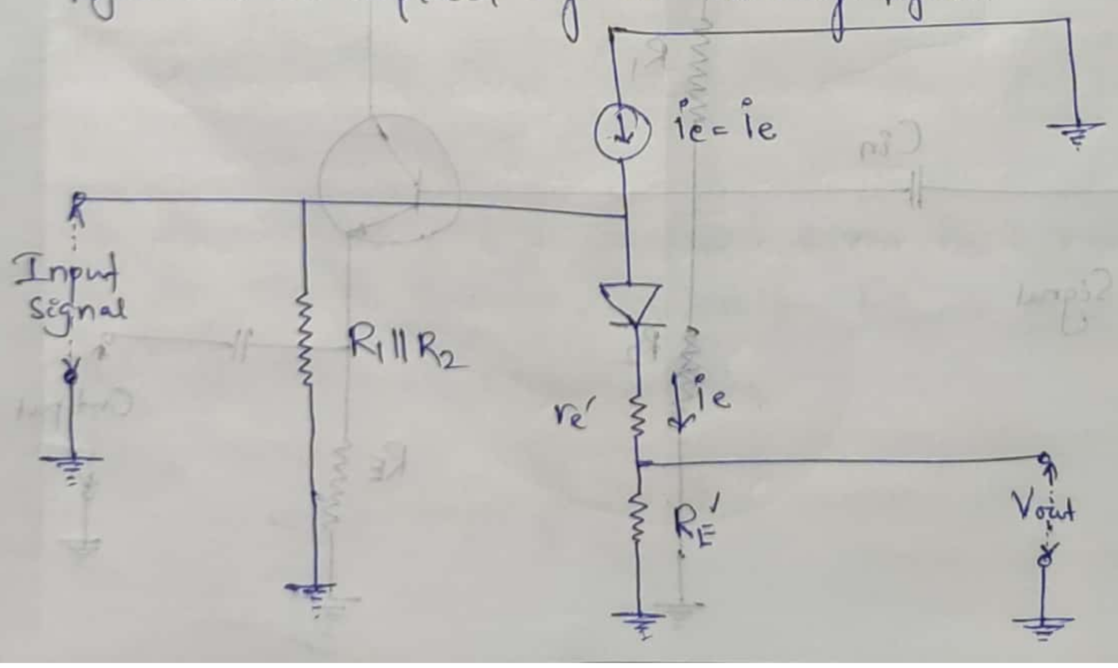
The AC resistance r_E of the emitter circuit is given by

$$r_E = r'_E + R_E$$

Where,

$$r'_E = \frac{25\,mV}{I_E}$$

In order to find the voltage gain of the amplifier, the above figure can be replaced by the following figure.



Note that input voltage is applied across the ac resistance of the emitter circuit i.e., $(r'_E + R_E)$. Assuming the emitter diode to be ideal, the output voltage V_{out} will be

$$V_{out} = i_e R_E$$

Input Voltage V_{in} will be

$$V_{in} = i_e (r_e' + R_E)$$

Therefore, the voltage Gain of emitter follower is

$$A_v = \frac{V_{out}}{V_{in}} = \frac{i_e R_E}{i_e (r_e' + R_E)} = \frac{R_E}{(r_e' + R_E)}$$

or

$$A_v = \frac{R_E}{r_e' + R_E}$$

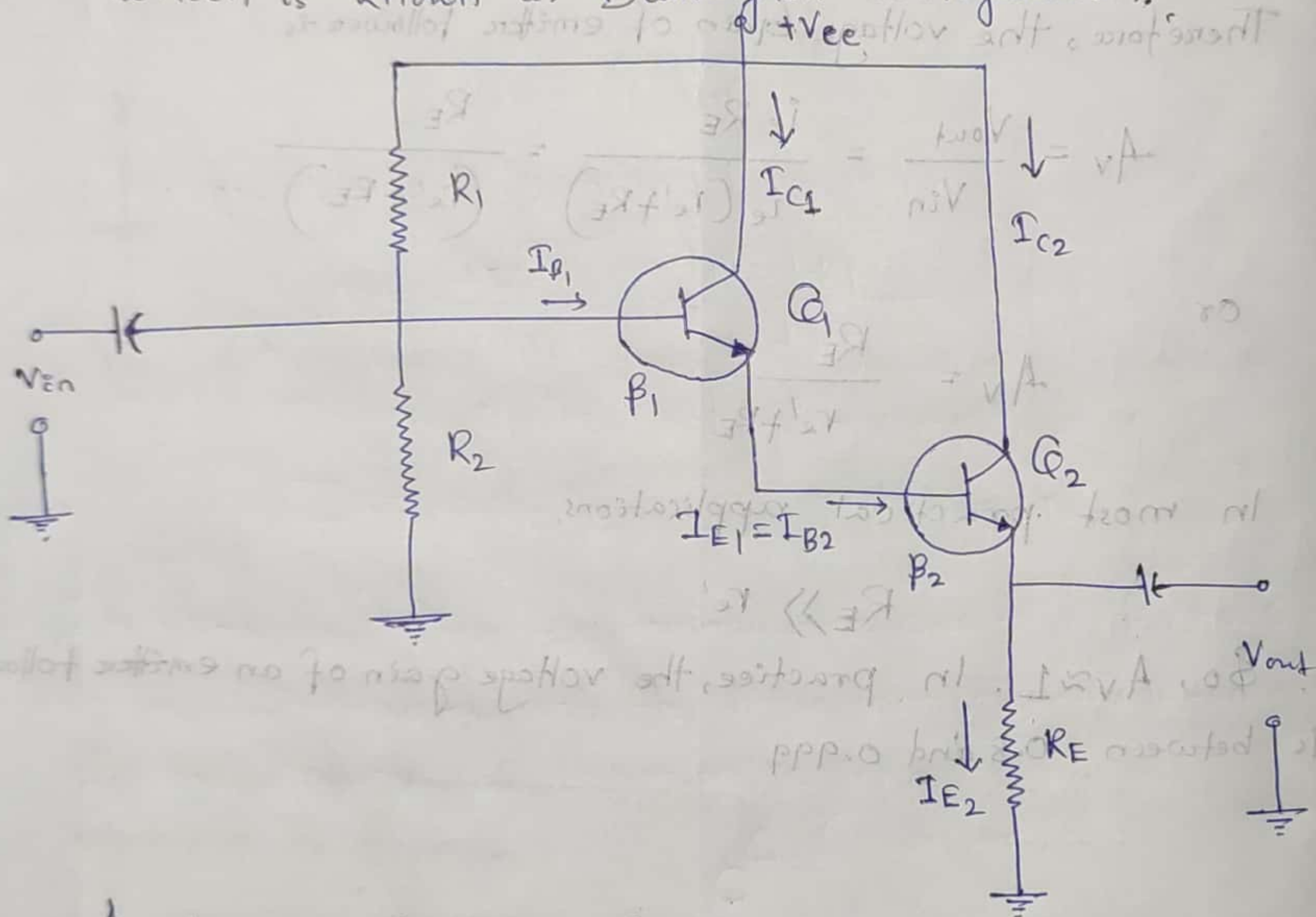
In most practical applications,

$$R_E \gg r_e'$$

So, $A_v \approx 1$. In practice, the voltage gain of an emitter follower is between 0.8 and 0.999

Darlington Amplifier :

The emitter follower circuit lacks to meet the requirements of the circuit current gain (A_i) and input impedance (Z_i). In order to achieve some increase in the overall values of circuit current gain and input impedance, two transistors are connected as shown in the following circuit diagram, which is known as Darlington Configuration.



As shown in the above figure, the emitter of the first transistor is connected to the base of of the second transistor. The Collector terminal of both the transistors are connected together.

Biasing Analysis:

Because of this type of connection, the emitter current of the first transistor will also be the base current of the second transistor. Therefore, the current gain of the pair is equal to the product of individual current gains i.e.

$$\beta = \beta_1 \beta_2$$

A high current gain is generally achieved with a minimum number of components.

As two transistors are used here, two V_{BE} drops are to be considered. The biasing analysis is otherwise similar for one transistor.

Voltage across R_2,

$$V_2 = \frac{V_{CC}}{R_1 + R_2} \times R_2$$

Voltage across R_E, $V_E = V_2 - 2V_{BE}$

Current through R_E, $I_{E2} = \frac{V_2 - 2V_{BE}}{R_E}$

Since the transistor are directly coupled.

$$I_{E1} = I_{B2}$$

Now, $I_{B2} = \frac{I_{E2}}{\beta_2}$

Therefore, $I_{E1} = \frac{I_{E2}}{\beta_2}$

Which means,

$$I_{E1} = I_{E1}\,\beta_2$$

We have,

$$I_{E1} = \beta_1 I_{B1} \quad \text{since } I_{E1} \cong I_{C1}$$

Hence, as

$$I_{E2} = I_{E1}\,\beta_2$$

We can Write

$$I_{E2} = \beta_1 \beta_2 I_{B1}$$

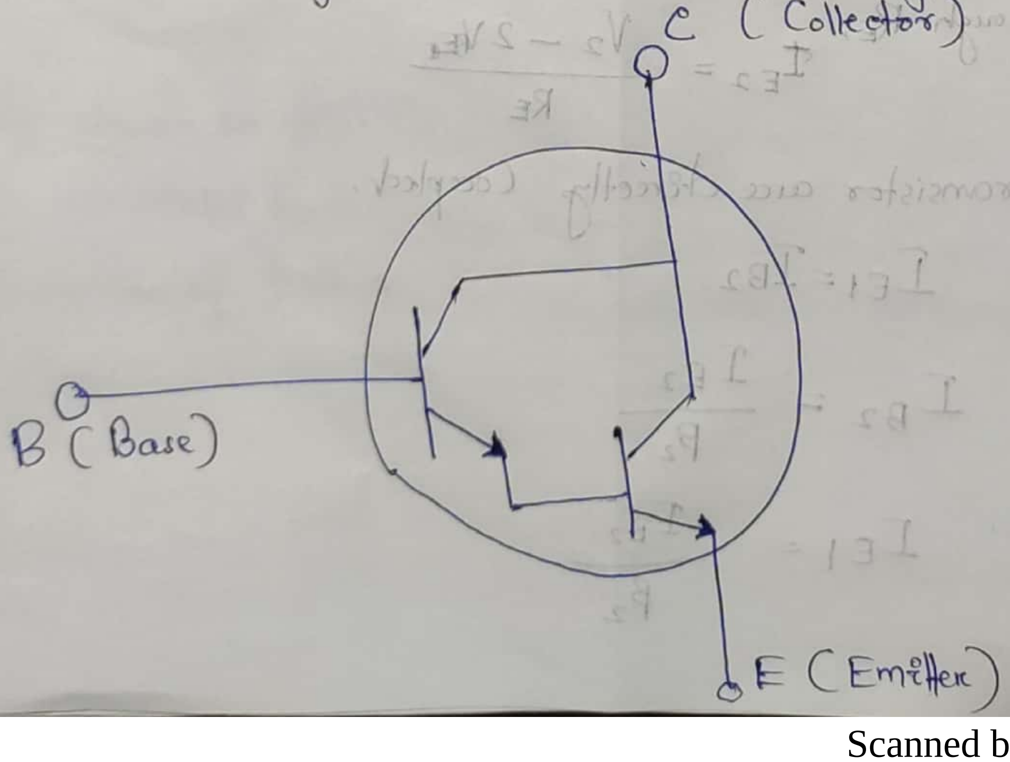
Therefore, Current Gain can be given as

$$\beta = \frac{I_{E2}}{I_{B1}} = \frac{\beta_1 \beta_2 I_{B1}}{I_{B1}} = \beta_1 \beta_2$$

Input impedance of the darlington amplifier is

$$Z_{in} = \beta_1 \beta_2 R_E \;\ldots\ldots\; \text{neglecting } r_e'.$$

In practice, these two transistors are placed in a single transistor housing and the three terminals are taken out of the housing as shown in the following figure.



This three terminal device can be called as Darlington transistor. The darlington transistor acts like a single transistor that has high current gain and high input impedance.

Characteristics:

The following are the important characteristics of Darlington amplifier.

Extremely high input impedance (MΩ)
Extremely high current gain (Several thousands)
Extremely low output impedance (a few Ω).

Since the characteristics of the Darlington amplifier are basically the same as those of the emitter followers the two circuits are used for similar applications.