Structural Analysis

Civil Engg.

Module - V

Analysis of Three hinged Arches :-

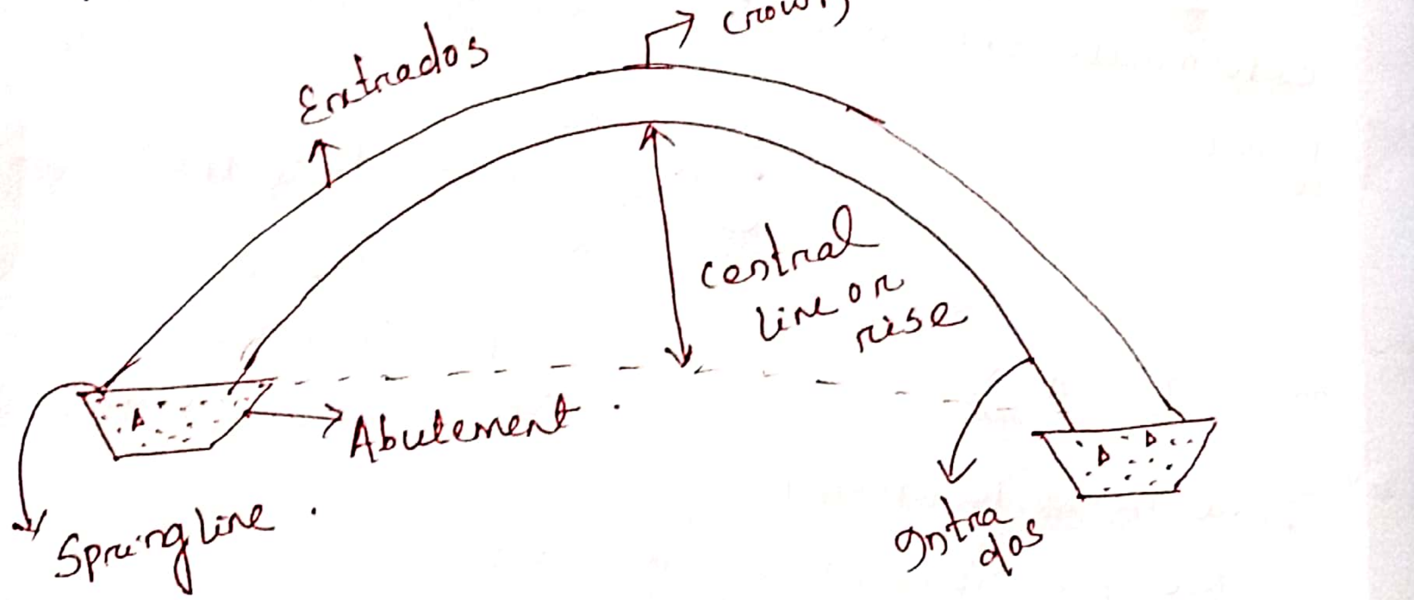
Arches can be used to reduce the bending moments in Long span structures.

→ Essentially an arch acts as an inverted cable. So it receives its Load mainly in compression although because of its rigidity.

→ It also resist some bending and shear depending upon how it is loaded and shaped.

Note

When an arch is subjected to uniformly distributed vertical loads then only compression forces will be resisted by the arch. under these condition the arch shape is called funicular arch. becoz no S.F and B.M occurs with in the arch.



Scanned with CamScanner

* Comparison between cable and Arch :-

Cable	Arch
→ Cable is a tension member	→ Arch is a Compression member.
→ A cable is flexible member and it changes its shape with different type and position of Loads.	→ A Arch is a rigid Structure and it doesn't changes its shape with different type and position of Loads
→ It can't resist any bending Moment hence B.M is zero every where.	→ Basically it is a Compression Member but it is often subjected to B.M and S.F of small Magnitude.

Types of Arches :-

There are 3 types of Arch.

(1) Three hinged arch :- A 3 hinged arch is statically determinate structure. There are four unknown reaction.

$r-(3+1)$

$\Rightarrow 4-4$

$\Rightarrow 0$

(So It is determinate)

Total no. of reaction $r = 4$

No. of equilibrium equation $= 3$

and one extra condition of equilibrium $(M_c = 0)$

due to internal hinge.

Scanned with CamScanner

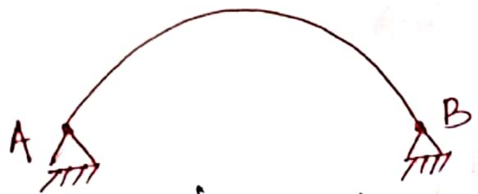
D_s = Total no. of reaction − No. of equilibrium equation

$= 4 - (3+1)$

$= 0$

(2) Two hinged arch :-

The two hinged arch is statically indeterminate structure of first degree.



for hinge support reaction = 2 (vertical, horizontal)

for Roller = 1 (vertical)

for fixed = 3 (vertical, horizontal, Moment)

Total no. of reaction = 4

equilibrium equation = 3

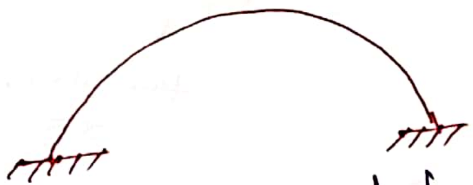
$D_s = r - e$

$= 4 - 3 = 1$

$\boxed{D_s = 1}$

→ So A two hinged arch is statically indeterminate of degree 1.

(3) Fixed Arch (Hingeless arch)



$r = 6$

$e = 3$

The fixed arch is statically indeterminate structure to 3rd degree.

r = No. of reaction = 6

e = equilibrium equation = 3

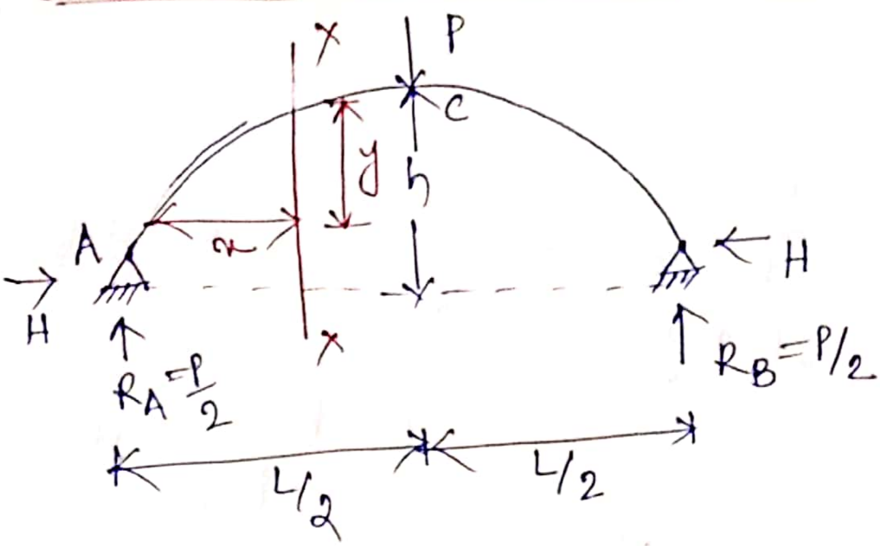
$D_s = r - e$

$= 6 - 3$

$\boxed{D_s = 3}$

Scanned with CamScanner

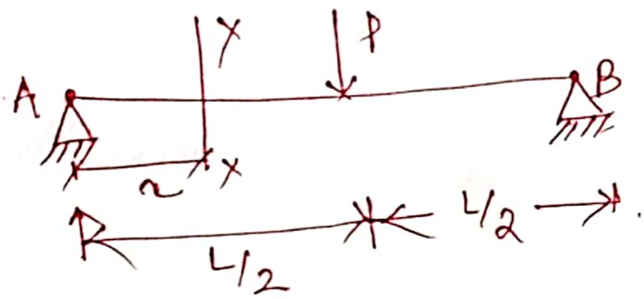
Comparison between Behaviour of Arch and Beam :-



$$M_x(\text{arch}) = R_A x - Hy$$

$$M_x = \frac{P}{2} x - Hy$$

$$M_x = M - Hy$$



$$M_x(\text{beam}) = R_A x$$

$$M_x = \frac{P}{2} x = M.$$

from the above analysis it is clear that the bending moment at any section in arch is smaller than that of beam by an amount of Hy.

Advantages of arch

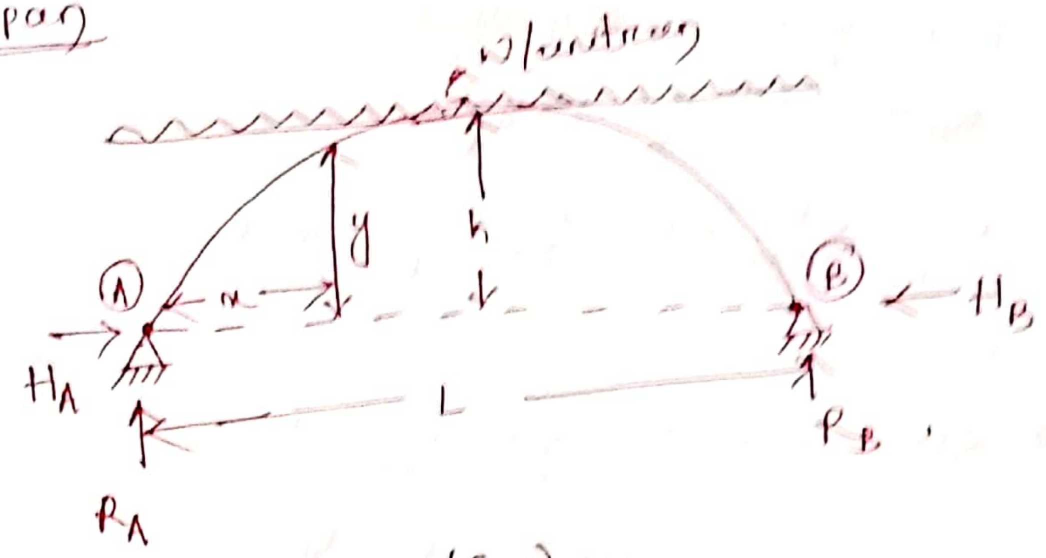
For same loading and same span, the bending moment in arch section is smaller than bending moment in beam section at same Location

Disadvantages of arch

Arches are very difficult to construct because of curved shape.

(3) Analyses of three Hinged Arches:

Three hinge parabolic Arch subjected to udl over entire span



$\Sigma f_x = 0 \quad H_A = H_B = H$ (say)

$\Sigma f_y = 0 \quad R_A + R_A = wl$

$\Sigma M_A = 0.$

$\Rightarrow R_B \times l - wL \times l/2 = 0$

$\Rightarrow R_B = wL/2$

$\Rightarrow R_A = wL/2$

Since there is a hinge at c hence ($M_c = 0$) (from left)

$\Rightarrow R_A \times L/2 - H \times h - w \times L/2 \times L/4 = 0$

$\Rightarrow \frac{wl}{2} \times \frac{l}{2} - H \times h - \frac{wL^2}{8} = 0$

$$\boxed{H = \frac{wL^2}{8h}}$$

Equation of parabolic Arch:

$$\boxed{y = \frac{4h}{L^2} x(L-x)}$$

Scanned with CamScanner

Bending Moment at any section

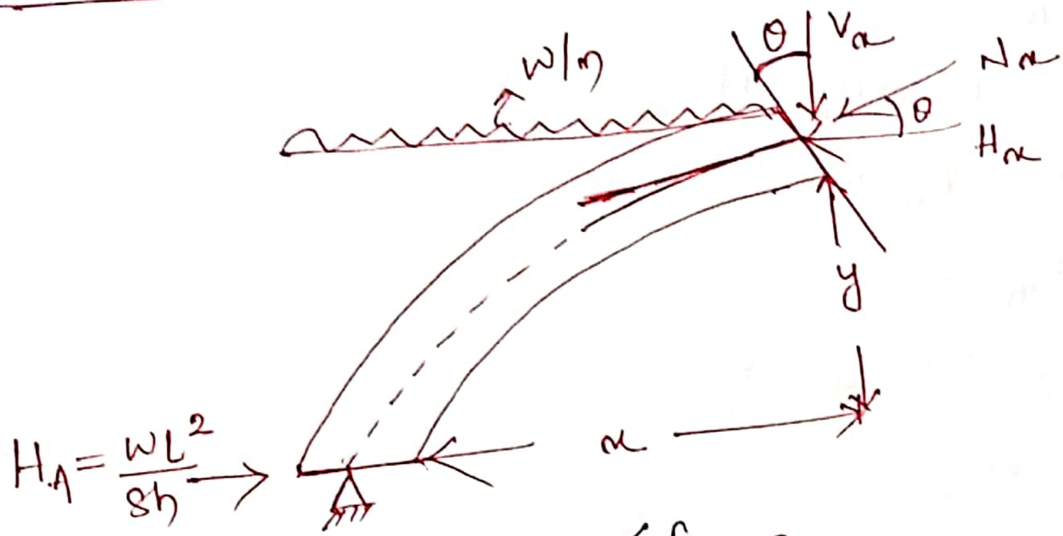
$M_x =$ Beam Moment $-$ H Moment

$$M_x = R_A x - wx\frac{x}{2} - H_A y$$

$$M_x = -\frac{wx^2}{2} \times \frac{wL}{2} \times x - \frac{wL}{8h} \times \frac{4h}{L^2} \times x \times (L-x)$$

$$\Rightarrow M_x = 0.$$

Radial shear and Normal thrust



$\Sigma f_x = 0$

$H_A - H_x = 0$

$$H_A = H_x = \frac{wL^2}{8h}$$

$\Sigma f_y = 0$

$R_A - wx = V_x$

$V_x = R_A - wx$

$$V_x = \frac{wL}{2} - wx \quad \text{——— (1)}$$

We know $\boxed{y = \frac{4h}{L^2} x(L-x)}$

and $$\tan\theta = \frac{dy}{dx}$$

$$= \frac{4h}{L^2}(L-2x)$$

Scanned with CamScanner

Putting value of V_x, $\tan\theta$ and y in equation (1)

$$S_x = \left(\frac{wL}{2} - wx\right)\cos\theta - \frac{wL^2}{8h}\sin\theta$$

$$= \cos\theta\left[\frac{wL}{2} - wx - \frac{wL^2}{8h}\tan\theta\right]$$

$$S_x = \cos\theta\left[\frac{wL}{2} - wx - \frac{wL^2}{8h}\times\frac{4h}{l^2}(L-2x)\right]$$

$$S_x = \cos\theta\left[\frac{wL}{2} - wx - \frac{wL}{2} + wx\right]$$

$(S_x = 0)$

Radial shear at x-x

$$S_x = V_x\cos\theta - H_x\sin\theta$$

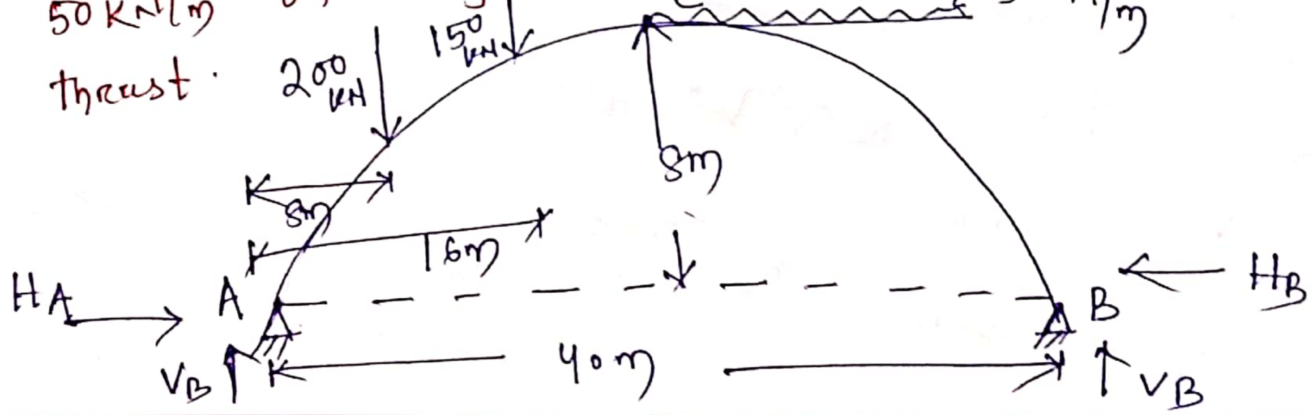
$$\boxed{S = V\cos\theta - H\sin\theta}$$

Normal thrust at x-x

$$\boxed{N_x = V_x\sin\theta + H_x\cos\theta}$$

Example:-

A 3 hinged arch of span 40m and rise 8m carries concentrated loads of 200 kN and 150 kN at a distance of 8m and 16m from the left end and an udl of 50 kN/m on the right half of the span. find horizontal thrust.



In arch all forces are inward (compressive) bcoz when we apply load from top the corner trying to open so H is always inward.

Step-1 Support reaction calculation

$\Sigma M_A = 0$

$\Rightarrow 200 \times 8 + 150 \times 6 + (50 \times 20)\left(20 + \frac{20}{2}\right) - V_B \times 40 = 0$

$\Rightarrow \boxed{V_B = 850 \text{kN}} \; (\uparrow)$

$\Sigma f_y = 0 \; (\uparrow)$

$V_A - 200 - 150 - (20 \times 50) + 850 = 0$

$\boxed{V_A = 500 \text{KN}}$

Step-2:- find horizontal thrust

Taking $BM_C = 0$. $\begin{bmatrix} \text{Sagging} = +ve \\ \text{Hogging} = -ve \end{bmatrix}$

Consider part (CA)

$500 \times 20 - 200 \times 12 - 150 \times 4 - H \times 8 = 0$

$\Rightarrow \boxed{H = 875 \text{kN}}$

$\boxed{H_A = H_B = 875 \text{kN}}$

Reaction $R = \sqrt{V^2 + H^2}$

$V_A = 500 \text{KN}$

$H_A = 875 \text{kN}$

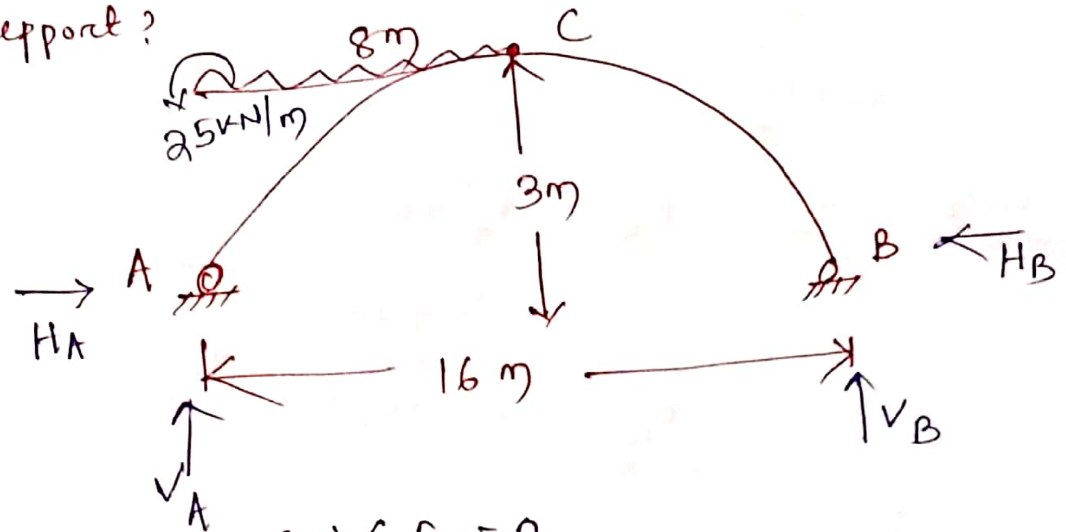
$R = \sqrt{V_A^2 + H_A^2}$

$= \sqrt{500^2 + 875^2} = 1007.7 \text{kN}.$

Scanned with CamScanner

Example:-

A 3 hinged parabolic arch carries a udl of 25 kN/m on the left half of the span. find reaction of supports, Bending Moment, Normal thrust, Radial shear at 4m from the left support?



$\Sigma F_x = 0$
$\Sigma F_y = 0$
$\Sigma M_z = 0.$

(a) $\Sigma F_x = 0$

$\Rightarrow H - H = 0$

(b) $\Sigma F_y = 0$

$V_A + V_B = 25 \times 8$

$V_A + V_B = 200$ kN ——— (1)

(c) $\Sigma M_z = 0$

$-V_B \times 16 + 25 \times 8 \times 4 = 0$

$\boxed{V_B = 50 KN}$

from equation (i) $V_A + V_B = 200$

$V_A + 50 KN = 200 KN$

$\boxed{V_A = 150 KN}$

Scanned with CamScanner

Horizontal thrust

To find horizontal thrust take $M_c = 0$.

$$\Sigma M_c = 0$$

$$-V_B \times 8 + H \times 3 = 0$$

$$\boxed{H = \frac{50 \times 8}{3}}$$

$$\boxed{H = 133.33 \text{ KN}}$$

Reaction at A

$$R_A = \sqrt{V_A^2 + H^2}$$

$$= \sqrt{150^2 + 133.33^2}$$

$$\boxed{R_A = 200.69 \text{ KN.}}$$

Reaction at B

$$R_B = \sqrt{V_B^2 + H^2}$$

$$= \sqrt{50^2 + 133.33^2} = 142.39 \text{ KN.}$$

$$\boxed{R_B = 142.39 \text{ KN}}$$

Bending moment at $x = 4m$

$$M_x = M - Hy$$

$$M_x = V_A \times 4 - 25 \times 4 \times 2 - Hy$$

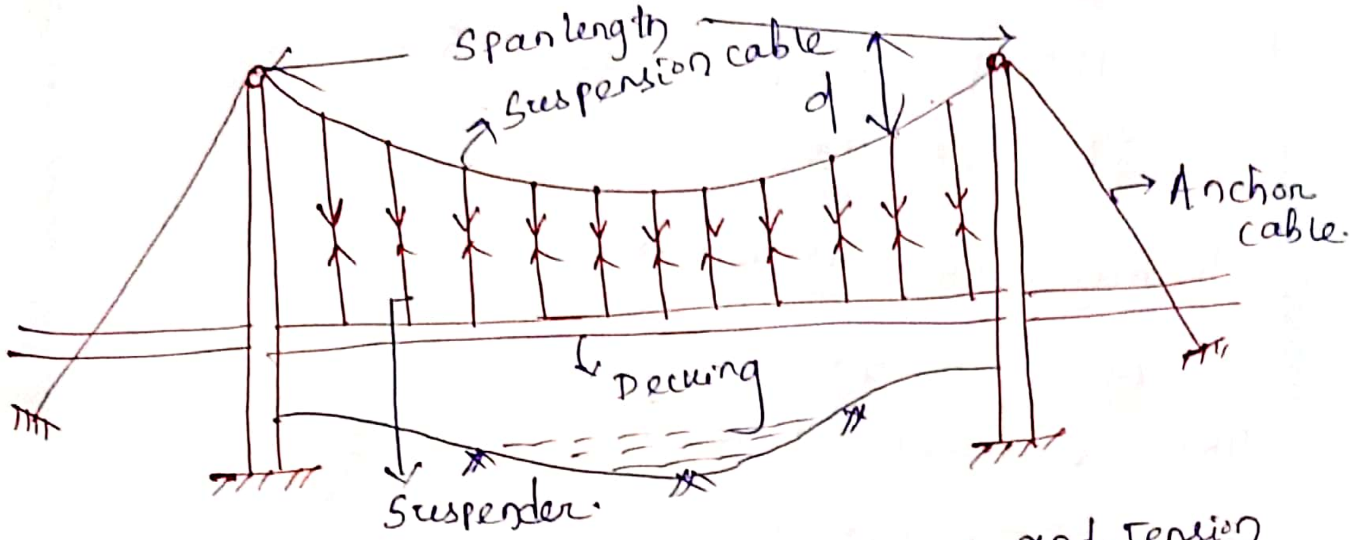
Equation of parabola $y = \frac{4hx}{l^2}(L - x)$

$$y_x = \frac{4 \times 3}{16^2} \times 4 \, (16 - \cancel{x}^{\,4})$$

$$\boxed{y = 2.25m}$$

Scanned with CamScanner

⑥ <u>Cables, Suspension Bridges</u>



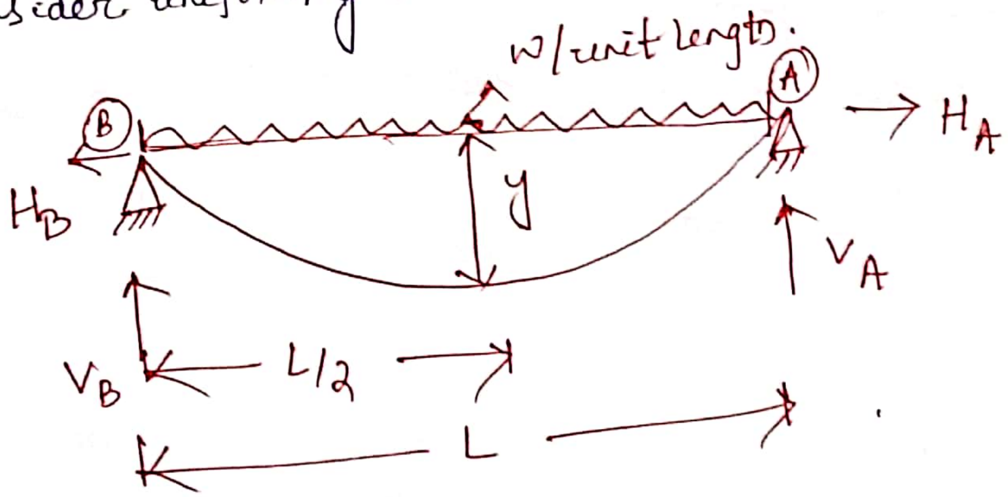
Tension will be maximum on corners and Tension will be minimum at centre in cables.

→ For suspension bridges, cables form an important structural element.

→ A suspension bridge consists of two cables on either side of the roadway which are stretched over the span to the bridges.

<u>Equation of the cable:-</u>

Consider uniformly distributed cable.



Putting value of y in $M_x = V_A \times 4 - 25 \times 4 \times 2 - Hy$

$M \Rightarrow 150 \times 4 - 25 \times 4 \times 2 - 133.33 \times 2.25$

$\boxed{M = 100 \text{ KNm}}$

Normal thrust at $x = 4m$

$N = V \sin\theta + H \cos\theta$

here V = Net vertical force.

$V = 150 - 25 \times x^{\to 4}$

$\boxed{V = 50 \text{ KN}}$

θ value:- $y = \frac{4h}{L^2} x(L - x)$

$\Rightarrow \tan\theta = \frac{4h}{L^2}(l - 2x)$

$= \frac{4 \times 3}{16^2}(16 - 2 \times 4)$

$\boxed{\theta = 20°33'}$

$N = V \sin\theta + H \cos\theta$

$\Rightarrow N = 50 \sin 20°33' + 133.33 \times \cos 20°33'$

$\boxed{N = 142.39 \text{ KN}}$

Radial shear at $x = 4m$

$R = V \cos\theta - H \sin\theta$

$= 50 \times \cos 20°33' - 133.33 \times \sin 20°33'$

$\boxed{R = 0.016 \text{ KN}}$

Scanned with CamScanner

y_c = sag of cable.

Vertical reaction at A and B

$$V_A = V_B = wL/2.$$

Horizontal reaction:-

$$H_A = H_B = H$$

H is determined by taking moments about c and equating it to zero. that is $M_C = 0$.

$$M_C = V_A \times L/2 - w \times L/2 \times L/4 - Hy_c = 0.$$

$$M_C = \frac{wL^2}{4} - \frac{wL^2}{8} - Hy_c = 0.$$

$$\boxed{H = \frac{wL^2}{8y_c}}.$$

To determine the shape of cable

$$M_x = V_A x - \frac{wx^2}{2} - Hy = 0.$$

$$= \frac{wL}{2} x - \frac{wx^2}{12} = \frac{wL^2}{8y_c} y$$

$$\boxed{y = \frac{4y_c}{L^2} x(L-x)} \Rightarrow$$ Equation of Parabola = Equation of cable.

Here 'x' is measured from left side support.

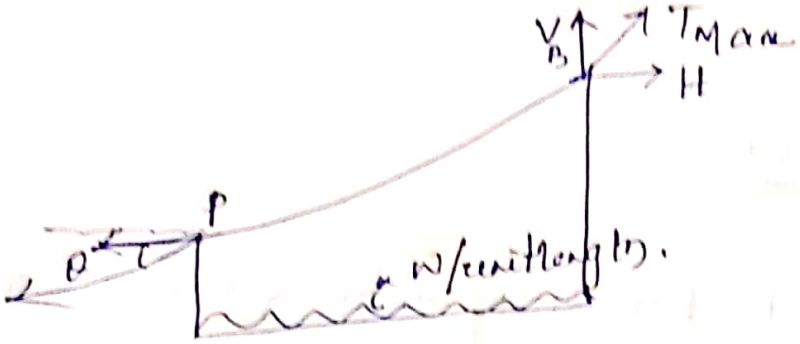
Horizontal tension in cables

$$\boxed{H = \frac{wL^2}{8y_c}}$$

Scanned with CamScanner

Tension in cable

The tension in cable varies along its length.



T_{max} is evaluated as $T_{max(A)} = \sqrt{V_A^2 + H_A^2}$

T_{max} at $B = \sqrt{V_B^2 + H_B^2}$

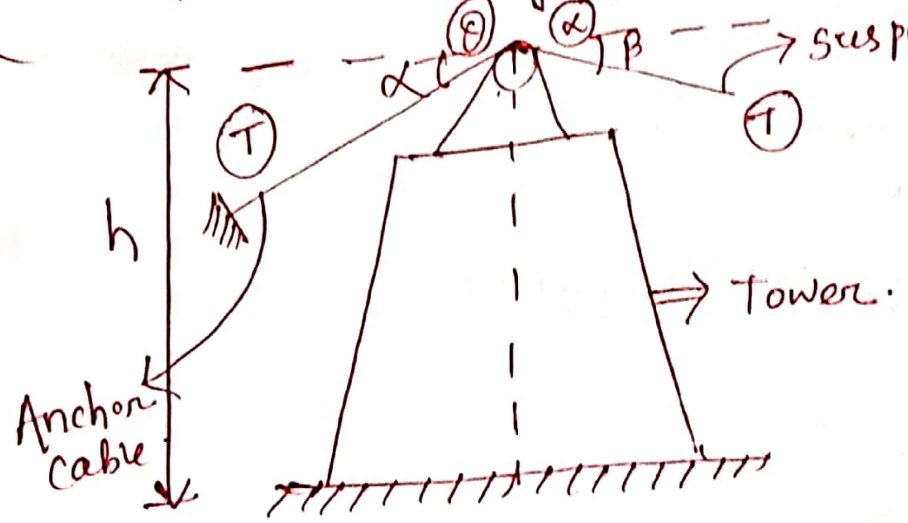
$$T_{max} = \left[\left(\frac{wL}{2}\right)^2 + \left(\frac{wL^2}{8y_c}\right)^2\right]^{1/2}$$

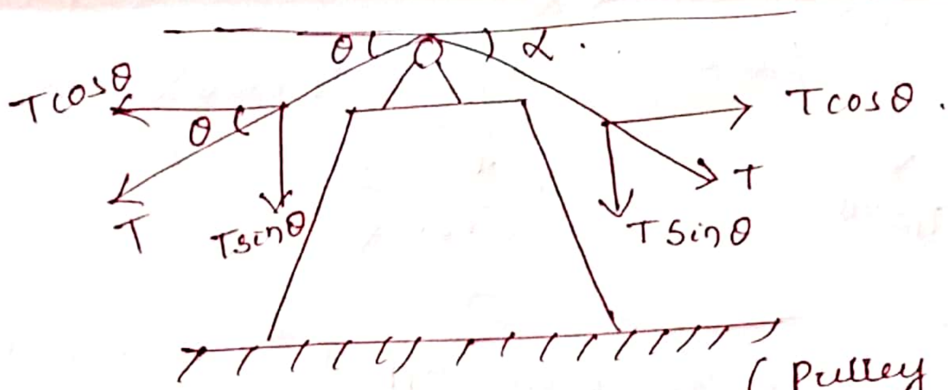
As we know $\left(V_A = wL/2 \quad H = wL^2/8y_c\right)$

$$= \frac{w}{2}\left[\left(1 + \frac{L^2}{16y_c^2}\right)\right]^{1/2}.$$

Case-1

(Cable passed over guide pulley at the support)





(Pulley support)

→ In pulley support both tension in anchor cable and suspension cable are same.

In vertical direction for pulley support

$T \sin\theta + T \sin\alpha$

$\Rightarrow \boxed{T\left[\sin\theta + \sin\alpha\right]}$.

In horizontal direction $T\cos\theta$ in one direction and $T\cos\alpha$ is in other direction.

In horizontal direction $\boxed{T\cos\theta - T\cos\alpha}$

Note

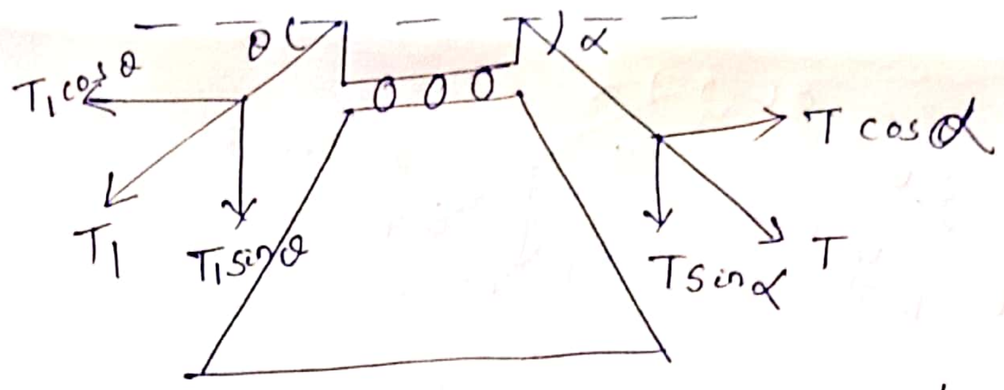
from right to left = +ve

from left to right = −ve

$\Rightarrow T\left[\cos\theta - \cos\alpha\right]$

Case-2:-

If cable passed or clamped to saddle carried on smooth Rollers on the top of the pier.



There are no loads for roller support in horizontal direction. due to roller support horizontal loads are cancelled out. only vertical loads are there.

In vertical direction

$= T_1 \sin\theta + T\sin\alpha$

Note:-
As Roller is frictionless, so it should n't consider horizontal load.
Therefore bending moment will n't occur at top.

$$\boxed{T_{max} = \sqrt{V_A^2 + H_A^2}} \text{ (In A-direction)}$$

$$T_{max} = \sqrt{V_B^2 + H_B^2} \text{ (In B-direction)}$$

As symmetrical ($V_A = V_B$)

$$\boxed{T_{min} = H}$$

* Tension always in the direction of cable.
* Tmin. always at centre and is equal to (H)

So $\boxed{T_{min} = H}$.

Scanned with CamScanner

(9)

Cable supported at same level

Arch

In Arch H force always inward

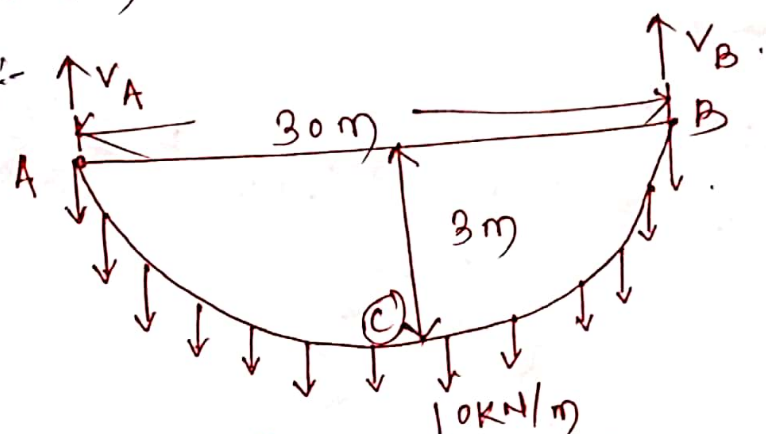
Cable

In cable always horizontal force H is outward.

Example

A suspension cable having supports at the same level, has a span of 30 m and a maximum dip of 3 m. The cable is loaded with u.d.L of 10 KN/m throughout its length. Find the maximum and minimum tension in the cable.

Solution:-



H is always away from the point.

Step-1 Calculation of support Reaction

$$\Sigma M_A = 0$$

$$\left[(10 \times 30) \times \frac{30}{2}\right] - V_B \times 30 = 0$$

$$\boxed{V_B = 150\ KN}$$

$\Sigma f_y = 0$ (upward +ve)

$V_A - (10\times30) + 150 = 0$

$\Rightarrow \boxed{V_A = 150 KN}$

$V_A = V_B = 150 KN$. As it is symmetrical Loading we can put $V_A = V_B = \frac{WL}{2} = 150 KN$.

$W = 10\times30 = 300 KN$.

Step-2 :- Horizontal thrust

Bending moment at C = 0 $\quad$ $\circlearrowright$ (+ve) $\quad$ $\circlearrowleft$ -ve

$BM_C = 0$

$\left(-10\times15\times\frac{15}{2}\right) + 150\times15 - H\times3 = 0$

$\boxed{H = 375KN}$

Direct formula $= H = wl^2/8h = \frac{30^2\times10}{8\times3} = 375 KN$

$\begin{bmatrix} w = 10 \\ L = 30m \end{bmatrix}$

$H_A = H_B = 375 KN$.

Maximum and Minimum tension in the cable

$T_{Max} = \sqrt{V_A^2 + H_A^2}$

$= \sqrt{150^2 + 375^2}$

$= 403.9 KN$.

$T_{Min.} = H$

$= 375 KN$.

$T_{Max} = 403.9 KN$.

Ans.

Scanned with CamScanner

(10) **Example:-**

A steel cable of 8 mm diameter is stretched across two poles 50 m apart. The central dip is 1 m at a temperature of 20°C. Assume unit weight of steel as 78 KN/m³ and thermal coefficient as 11×10^{-6} °C

(Not in course)

Determine

(i) stress intensity in the cable

(ii) Calculate Max.m tension.

Ans:- H = horizontal thrust

$$= \frac{WL^2}{8y_c} = \frac{0.196 \times 50^2}{8 \times 1}$$

given dip $(y_c) = 1$.

weight $(w) = A l y_c = \pi/4 \times 8^2 \times 50 \times 78 = 0.196$ kN $\times 10^{-6}$.

dia of cable = 8mm

$$V = \frac{WL}{2} = \frac{0.196}{2} = 0.098 \text{ KN}.$$

$$T_{max} = \sqrt{H^2 + V^2} = \sqrt{1.225^2 + 0.098^2} = 1.23 \text{ KN}.$$

Example on suspension cable with supports

A suspension cable of 150 m span and 15 m central dip carries a load of 2 KN/m. Calculate maximum and minimum tension in the cable, horizontal and vertical forces in each pier under the following conditions.

(i) Cable passes over a frictionless roller on the top of piers.

(ii) Cable is firmly clamped to saddles with backstay inclined at 30° with horizontal.

Solution

Horizontal Reaction $H = \frac{WL^2}{8y_c} = \frac{2\times150^2}{8\times15} = 375KN.$

given $w = 2 KN/m$

$l = 150m.$

$y_c = dip = 15m.$

vertical Reaction $V = wl/2 = \frac{2\times150}{2} = 150 KN.$

Max^m tension $T_{Max} = \sqrt{150^2 + 375^2} = 403.89 KN.$

Equation of cable $y = \frac{4y_c}{L^2} x (L-x)$

$\Rightarrow \frac{dy}{dx} = \frac{4y_c}{l^2}(l-2x)$

Slope of cable $\frac{dy}{dx} = \frac{4y_c}{l^2}$ at $x=0$

$\tan\theta = \frac{4\times150}{150^2} = 0.40$

$\theta = \tan^{-1}(0.4) = 21°48'$

(a) Horizontal force on pier

$H_1 = T_{Max}(\cos 21°48' - \cos 30°)$

$\theta = 21°48'$ $= 25.23KN.$

$\alpha = 30°$

vertical force $= T_{max}(\sin 21°48' + \sin 30°)$

$= 351.94 KN.$

(b) In this case $H_1 = 0.$

$T_2 = \cancel{403.89} \xrightarrow{} T_{Max} \times \frac{\cos 21°48'}{\cos 30°}$

$= 433 KN.$

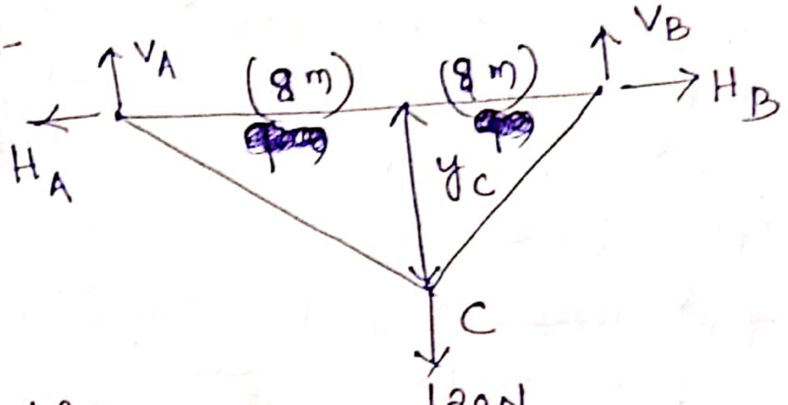
Vertical force $= 403.89 \times \sin 21°48' + 433 \sin 30° = 366.49 KN.$

Scanned with CamScanner

(11) Problem on cable subjected to point load

A light cable 18 m long is supported at two ends at the same level. The supports are 16 m apart. The cable supports 120 N load dividing the distance into two equal parts. Find the shape of the cable and tension in cable.

Solution :-



$V_A = V_B = \frac{120}{2} = 60\,N.$

Now length of the cable $L = \left[\left(\frac{L}{2}\right)^2 + y_c^2\right]^{1/2} \times 2$

$\Rightarrow 18 = \left[(8)^2 + y_c^2\right]^{1/2} \times 2$

$\Rightarrow y_c = 4.123\,m.$

Taking moment about c = 0

$H y_c - V_A \times l/2 = 0$

$\Rightarrow H \times 4.123 - 60 \times 8$

$\boxed{H = 116.42\,KN}$

Tension in the cable $= \sqrt{V_A^2 + H^2}$

$= \sqrt{60^2 + 116.42^2}$

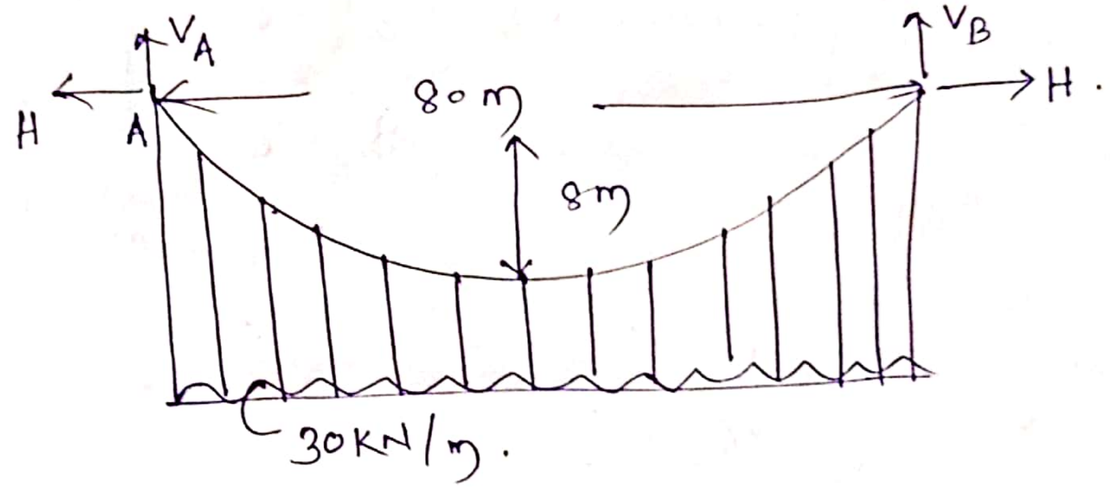
$= 130.97\,KN.$

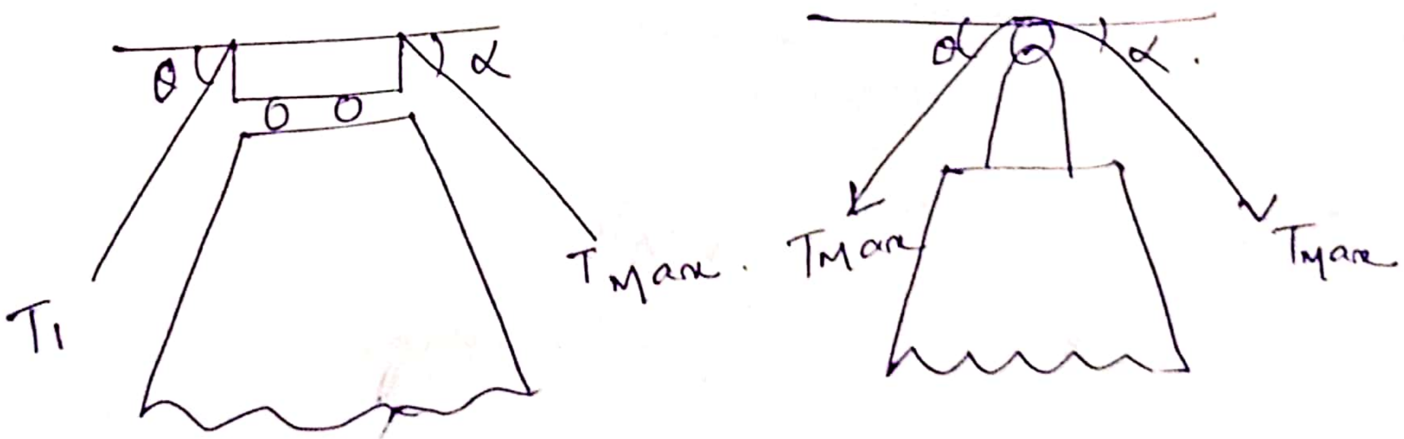
Scanned with CamScanner

Problem on uniformly distributed Load of cable with Pulley and Roller support :-

Q. A bridge cable is suspended from towers 80m apart and carries a load of 30 KN/m on the entire span. If the Maximum sag is 8m. calculate the Maximum tension in the cable. if the cable is supported by saddles which are stayed by wires inclined at 30° to the horizontal. Determine the forces acting on the towers. if the same inclination of back stay passes over pulley determine the forces on the towers.

Solution





(Forces on towers).

V_A = vertical force at A

$= WL/2 = \frac{30 \times 80}{2} = 1200\,KN.$

Taking moment about central point C

$H \times 8 - \frac{WL}{2} \times \frac{L}{2} + \frac{WL}{2} \times L/4 = 0$

$\Rightarrow H = \frac{WL^2}{8h} = \frac{30 \times 80^2}{8 \times 8} = 3000\,KN.$

Maximum tension occurs at support

$T_{max} = \sqrt{V_A^2 + H^2}$ $(V_A = V_B = 1200\,KN)$

$= \sqrt{1200^2 + 3000^2}$

$= 3231.1\,KN.$

$H = T_{max} \cos\theta$

$\theta = \cos^{-1}(H/T) = \cos^{-1}\left(\frac{3000}{3231.1}\right) = 21.8°.$

(i) If the cable is supported by saddle Roller

$\left.\begin{aligned} T_1 \cos\alpha &= T_{max} \cos\theta \\ T_1 \cos\theta &= T_{max} \cos\alpha \end{aligned}\right\}$ both same.

$\Rightarrow T_1 \times \cos 30° = 3231.80 \times \cos 21.8°$

$\boxed{T_1 = 3464.1\,KN}$

There is no horizontal force on the tower.

The vertical force on the tower is given by

$T_1 \sin\alpha + T_{max} \sin\theta$

$\Rightarrow 3464.1 \sin 30° + 3231.1 \sin 21.8°$

$= 2932.98\,KN.$

(ii) If the cable is on pulley, the vertical force on the tower is

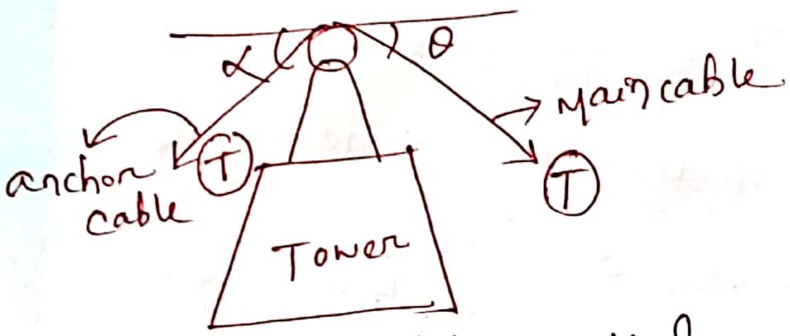
$$T_{Max}(\sin\alpha + \sin\theta)$$
$$= 3231.1(\sin 30^\circ + \sin 21.8^\circ)$$
$$= 2815.48\,KN.$$

Horizontal force on the tower

$$= T_{Max}(\cos\theta - \cos\alpha)$$
$$= 3231.1(\cos 21.8^\circ - \cos 30^\circ)$$
$$= 201.82\,KN.$$

Note formulae

(1) Guided pulley support



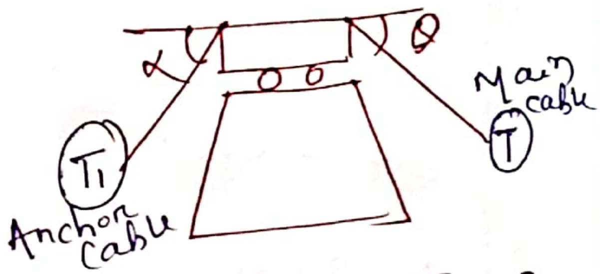
vertical load transmitted to tower $= \boxed{T(\sin\theta + \sin\alpha)}$

Horizontal load $= \boxed{T(\cos\theta - \cos\alpha)}$

Bending moment on the tower
= Horizontal force × Height of tower

$$\boxed{= T(\cos\theta - \cos\alpha) \times h}$$

(2) Roller support



$$T_1\cos\alpha = T\cos\theta$$
$$\Rightarrow T_1 = T\left(\frac{\cos\theta}{\cos\alpha}\right)$$

→ Saddle are friction less roller so no horizontal force.

$\boxed{\text{vertical force on tower} = T_1\sin\alpha + T\sin\theta}$

(13)

Suspension cable with three hinged stiffening girder :-

→ The shape of the suspension cable is same as the bending moment diagram for the simply supported beam.

→ As load passes over the bridge, the shape of the bending moment diagram changes with the position of loads and thus the profile of the cable also changes

→ Therefore to maintain the shape of the cable that is parabolic shape throughout the passage of loads, the moving loads are transferred to the suspension cable as uniformly distributed.

Objective of stiffening girder :-

This objective is obtained by providing stiffening girders which transmits the received load into the form of uniformly distributed load with the help of closely spaced hangers or suspenders.

→ For transmitting received loads, stiffening girder have to resist bending moment and shear force whereas dead load is transmitted to the suspension cable without creating Bending moment and shear force in the stiffening girder.

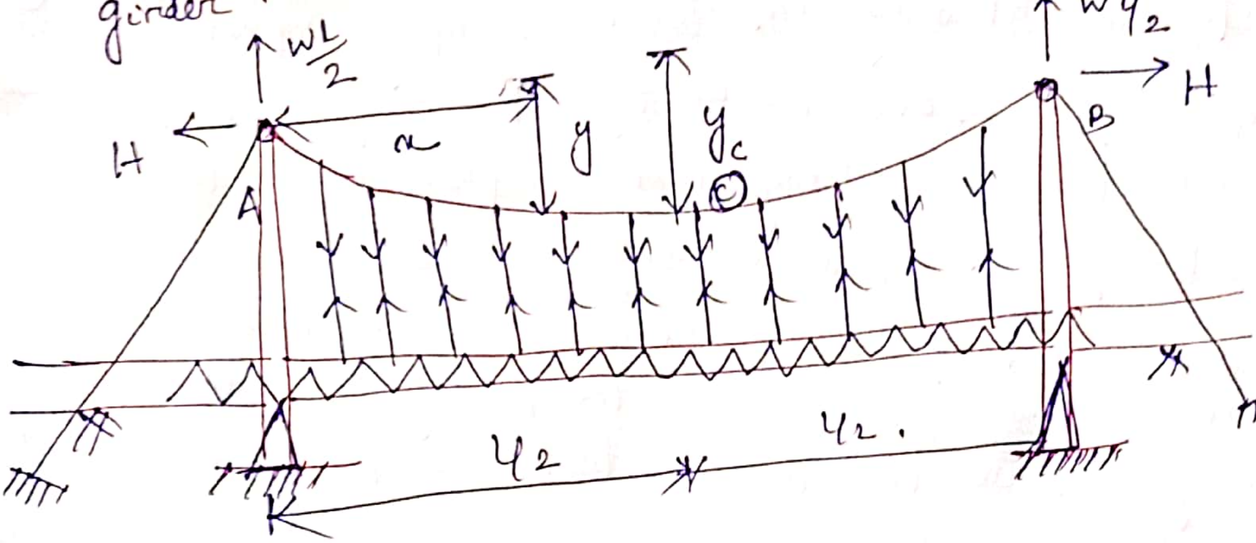
* In suspension cable bridge deck slab is provided & the deck slab is used for vehicles but in case of stiffening girder a girder is provided in place of deck slab for moving of vehicles.

→ The load transferred from the girder to the suspenders.

→ The depth of girder must be greater than 1m.

Scanned with CamScanner

The beam whose depth more than 1m is known as girder :



In cable the Load of deckslab is transferred to suspenders (respectively). That means at centre the transfered Load from deckslab to suspender is different from the support

In stiffening girder the loads from the girders are equally transferred to the suspenders.

→ The Load transfered from support and centre are same.

<u>Example</u> :-

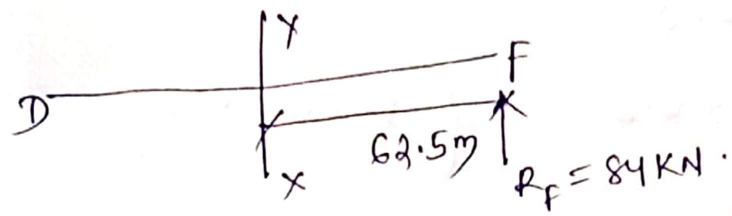
A suspension bridge of 250 m span has three hinged stiffening girder supported by a cable with a central dip of 25m, if four point loads of 150 kN each are placed at the distance of 20m, 30m, 40m and 50m from the left hand hinge, find the shearforce and bending moment in the girder at 62.5m from each end. Also find maximum tension in the cable ?

Bending moment $= M_x - Hy$

$= 15750 - 420 \times 18.75$

$= 7875 \text{ KNm}$

Bending moment at right end



Moment about $x = M_x = 84 \times 62.5 = 5250$ KNm

Bending moment $= M_x - Hy$

$= 5250 - 420 \times (18.75)$

$= -2625$ KNm

Shearforce calculation

Shearforce at 62.5m from eachend

$S.F = F_x - H\tan\theta$

$\tan\theta = \frac{dy}{dx}$ & $y = \frac{4hx}{l^2}(l-x)$.

By differentiating $y = \frac{4hx}{l^2}(l-x)$ we get

$= \frac{4h}{l^2}(l-2x)$

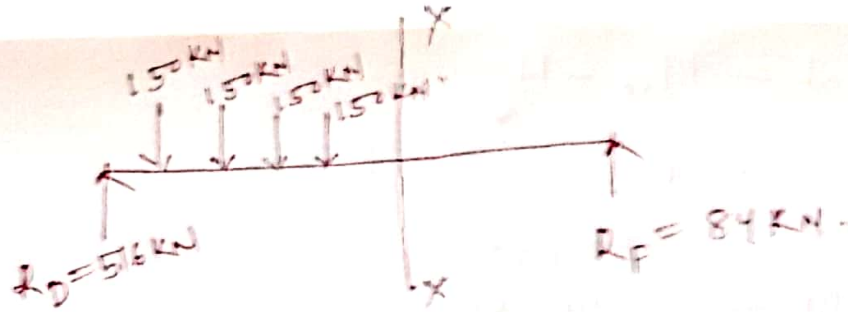
$= \frac{4 \times 25}{250^2}(250 - 2 \times 62.5)$

$L = 250$m
$h = 25$m
$x = 62.5$m

$\tan\theta = 0.2 \,(\pm)$

$\tan\theta = +0.2$ is taken for calculating left side

$\tan\theta = -0.2$ is taken for calculating Right side.



Shearforce at (F_x) = $516 - 150 \times 4 = -84$.

$S.F = F_x - H\tan\theta$

$= -84 - 420 \times 0.2$ $[\tan\theta = 0.2]$

$= -168$ kN $\rightarrow$ for left side.

<u>Shearforce calculating from right side</u>

$F_x = -84$. [$\tan\theta = -0.2$ for right side]

Shearforce $= F_x - H\tan\theta$

$= -84 - 420 \times (-0.2)$

$= 0$.

<u>Maximum tension in cable</u>

$T_{max} = \sqrt{R_A^2 + H^2}$

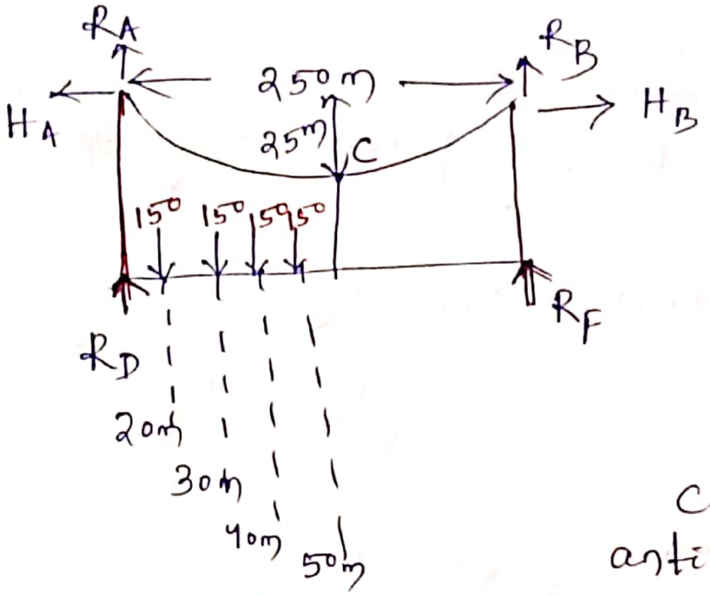
$= \sqrt{168^2 + 420^2}$

$= 452.35$ kN.

(Ans).

Scanned with CamScanner

15



clockwise = +ve
anticlockwise = −ve

$\Sigma M_A = 0$ (↻ +ve)

$\Rightarrow -R_F \times 250 + 150(20+30+40+50) = 0.$

$\Rightarrow \boxed{R_F = 84\,KN}$

$\Sigma f_y = 0 \Rightarrow R_D + 84 - 150 \times 4 = 0$

$\Rightarrow \boxed{R_D = 516\,KN}$

for Horizontal thrust $B.M_C = 0.$

$\Rightarrow 84 \times 125 - H \times 25 = 0$

$\Rightarrow \boxed{H = 420}.$

$H = \frac{wL^2}{8d}$ w = intensity of udl

$= \frac{w \times 250^2}{8 \times 25}$ $\Rightarrow \boxed{w = 1.344\,KN/m}$

Now convert the total girder into udl as $w = 1.344\,KN/m$

for udl $S.F = wL/2$ at both support

$R_A = R_B = \frac{wL}{2} = \frac{1.344 \times 250}{2} = 168\,KN.$

Scanned with CamScanner

$$\boxed{R_A = R_B = 168\ KN}$$

Step.2: Find Bending Moment in girder at 62.5m from each end

Bending moment $= M_x - Hy$

$$y = \frac{4hx}{l^2}(l-x) \qquad [h = 25m \text{ given}]$$

$$= \frac{4 \times 25 \times 62.5}{250^2}(250 - 62.5)$$

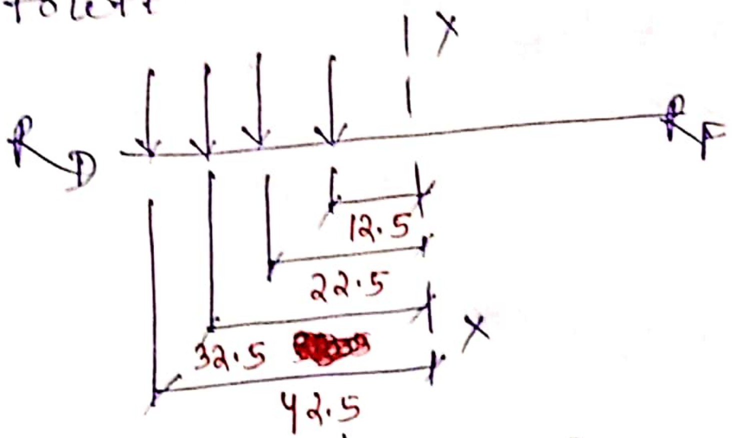
$$= 18.75m$$

$$\boxed{H = 420\ KN}$$

$$\boxed{y = 18.75m}$$

Bending moment at left end

Take a section x-x at a distance x from right to left.



$62.5 - 20 = 42.5\ m$

$62.5 - 30 = 32.5\ m$

$62.5 - 40 = 22.5\ m$

$62.5 - 50 = 12.5\ m$

$\rightarrow (62.5-20)$

$$\boxed{R_D = 516\ KN}$$

Bending moment about x

$$B.M_x = 516 \times 62.5 - 150(42.5 + 32.5 + 22.2 + 12.5)$$

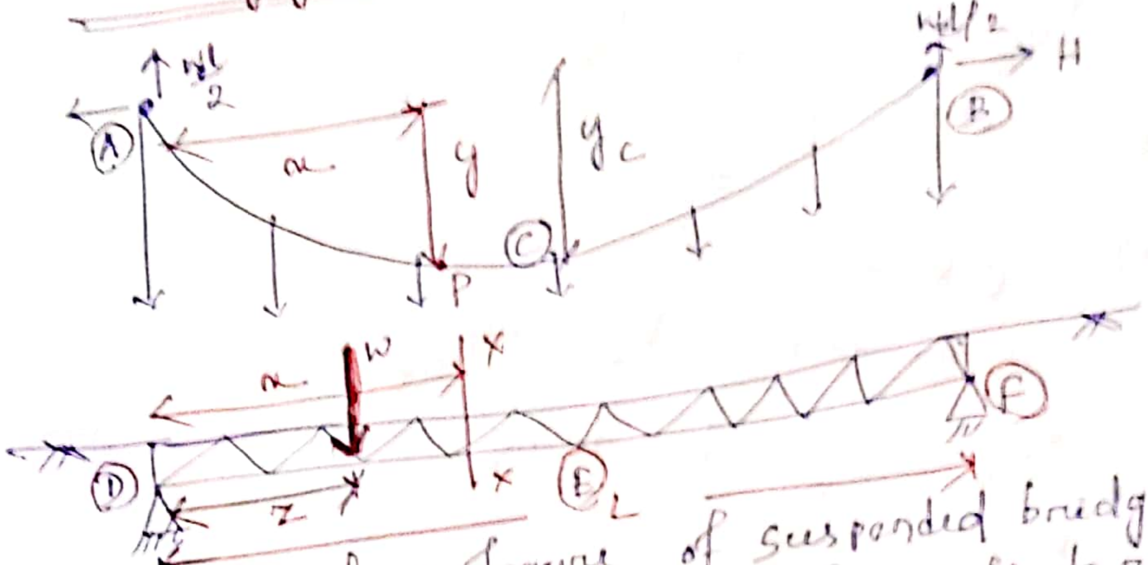
$$= 15750.$$

Scanned with CamScanner

(16)

Explanation

Stiffening girder with single concentrated load on it



Consider the above figure of suspended bridge on which concentrated load W is moving from left to right.

→ Here ACB is suspension cable and DEF is 3 hinged stiffening girder.

Let the load W be at a distance z from support D and consider a section at a distance x from A or D. The equation of shape

Cable:- Assume A as origin. The equation of cable is $y = \frac{4y_c}{L^2} x(L-x)$

Let the equivalent load (udl) acting on the cable be (w_e)

$\Rightarrow V_A = V_B = V = \frac{w_e L}{2}$

$\Rightarrow$ Horizontal force $H = \frac{w_e L^2}{8y_c}$

$\Rightarrow T_{max} = \sqrt{V^2 + H^2}$

Scanned with CamScanner

Consider a point $p(x, y)$ on the cable.

$$M_p = 0.$$

$$M_p = \frac{w_e L x}{2} - \frac{w_e x^2}{2} - Hy = 0$$

$$Hy = \frac{w_e l x}{2} - \frac{w_e x^2}{2}$$

Stiffening girder:

vertical force at D $= V_D = W\left(\frac{l - z}{l}\right)$ due to W

$$V_{D_2} = \frac{w_e l}{2}, \text{ due to } w_e.$$

where W = live load

w_e = dead load or udl.

Similarly $V_F = \frac{Wz}{l}$ due to W

$$V_{F_2} = \frac{w_e l}{2} \text{ due to } w_e.$$

* Bending Moment at a section with x metre from the left

$$M_x = \left\{V_D x - W(x - z)\right\} - \left\{\frac{w_e l x}{2} - \frac{w_e x^2}{2}\right\} \text{ for } z \leq x$$

$$M_x = (V_D x) - \left\{\frac{w_e l}{2}(x) - \frac{w_e x^2}{2}\right\} \text{ for } z \geq x$$

The second term in both the cases of bending moment above is equal to Hy

$$\boxed{M_x = \mu_x - Hy}$$

μ_x = Moment due to W at section x.

Scanned with CamScanner

Note

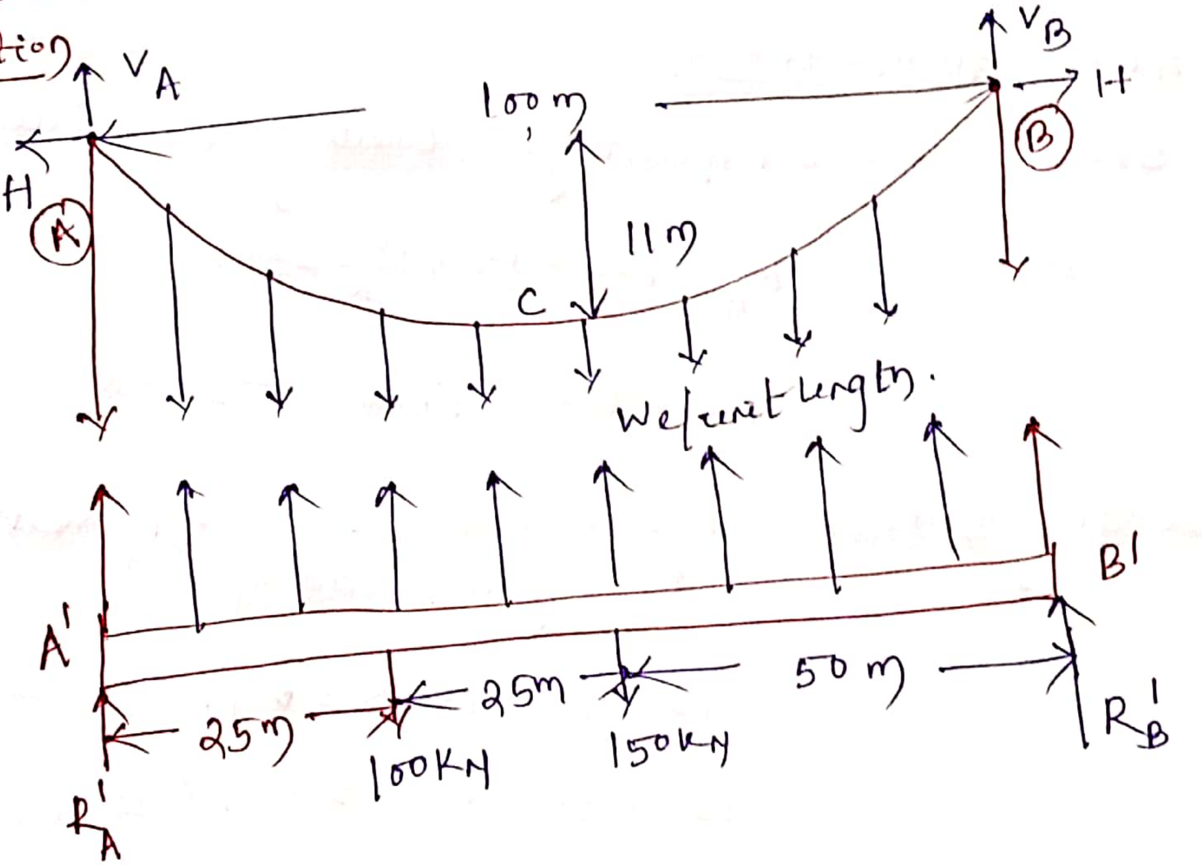
Due to w_e, the bending moment is $\frac{w_e x}{2}(l-x)$ at section x-x and Maximum Moment $\frac{w_e l^2}{8}$ at c.

→ The nature of this moment is hogging. The Shearforce is $-w_e\left(\frac{l}{2}-x\right)$.

Example:-

A 3 hinged stiffening girder of a suspension bridge of span 100 m is subjected to two point Load of 100 KN and 150 KN at a distance of 25m and 50m from the left end. find the shearforce and bending moment for the girder at a distance of 30m from the leftend. The supporting cable has a central dip of 11m. Also find the Maximum tension and its slope in the cable.

Solution



Consider girder

Let the reactions at A' and B' due to given loadings is R_A' and R_B' respectively.

Taking moment at A' and equating it to zero.

$$\Sigma M_A' = 0.$$

$$\Rightarrow R_B' \times 100 - 100 \times 25 - 150 \times 50 = 0$$

$$R_B' = 100\,KN.$$

$$\Sigma M_C' = 0 \quad (\text{where C is the midpoint})$$

Also $$R_B' \times \frac{L}{2} - \frac{W_e L^2}{8} = 0$$

$$\Rightarrow W_e = \frac{4 R_B'}{L} = \frac{4 \times 100}{100} = 4\,KN/m$$

$$R_A' = 250 - 100 = 150\,KN/m.$$

At 30m from left end

Shear force = S.F due to given Loading + S.F due to W_e

$$(S.F) = R_A' - 100 - W_e\left(\frac{100}{2} - 30\right)$$

$$= 150 - 100 - 4 \times 20 = -30\,KN.$$

Bending moment = Moment due to given Loading + Moment due to W_e

$$= 150 \times 30 - 100 \times 5 - 4 \times \frac{30}{2}(100-30)$$

$$= 150 \times 30 - 100 \times 5 - 4 \times \frac{30}{2}(100-30)$$

$$= -200\,KNm \text{ (hogging)}.$$

Scanned with CamScanner

Consider cable $V_A = \frac{w_e l}{2}$

$= \frac{4 \times 100}{2} = 200 KN.$

Horizontal reaction $H = \frac{w_e l^2}{8 y_c}$

$= \frac{4 \times 100^2}{8 \times 11} = 454.55 KN$

$T_{max} = \sqrt{H^2 + V_A^2} = \sqrt{454.55^2 + 200^2}$

$= \sqrt{200^2 + 454.55^2}$

$= 496.6 KN.$

Now slope to Horizontal is $T_{max} \cos\theta = H$

$\theta = \cos^{-1}\left(\frac{454.55}{496.6}\right)$

$= 23^\circ 45'$

(Ans).

Types of stiffening girder:-

Q:- what is stiffening girder?

A stiffening girder for a stayed cable bridge constructed of reinforced concrete or prestressed concrete is formed by a closed multicell box extending in the long direction of the bridge.

→ The girder or box is supported by inclined cables extending in at least one support plane along the long direction of the bridge.

Scanned with CamScanner

Q Why stiffening girders are necessary in the suspension bridges?

A:- Stiffening girders enable the suspension bridge decks to remain fairly level.

→ What ever be the live Load on deckslab the stiffening girder will convert and transmit the Load on the deck slab as a udl and thereby help the cable retain the parabolic shape during the passage of Loads.

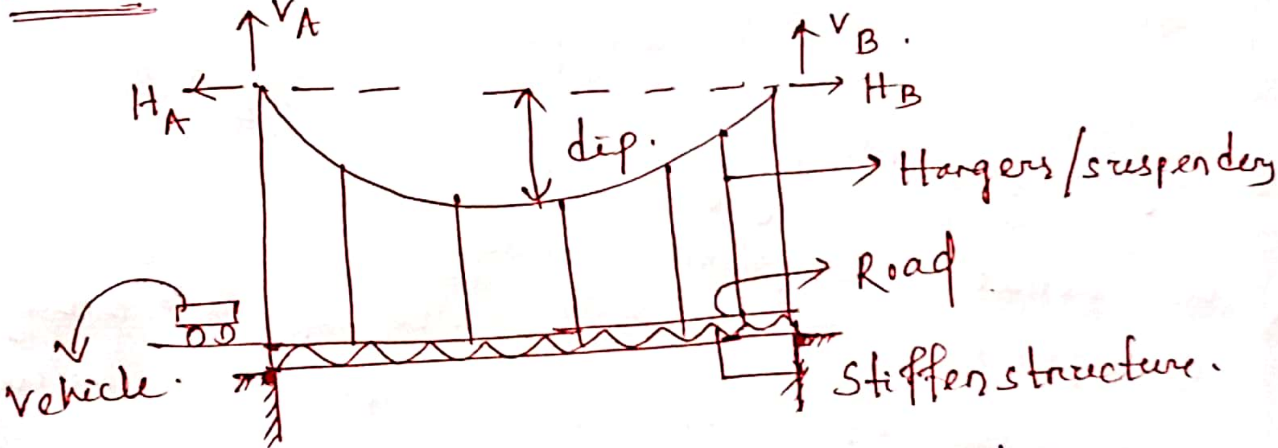
* There are two types of stiffening girder.

(1) 3-hinged stiffening girder.

(2) 2-hinged stiffening girder.

3-hinged stiffening girder :-

Basics



→ In this case we only consider live load. and B.M and S.F also calculated by using live load in this stiffening girder.

$$\boxed{B.M_x = \mu x - Hy}$$

$$\boxed{S.F_x = \underset{(S_b)}{S.F} - H\tan\theta}$$

Scanned with CamScanner

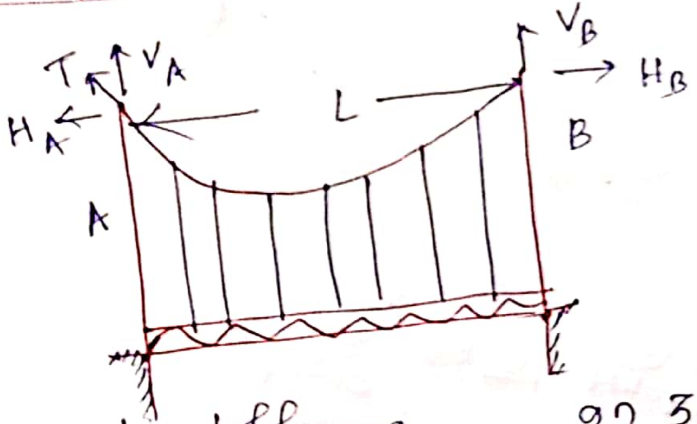
$\tan\theta = \frac{dy}{dx}$

$\Rightarrow y = \frac{4y_c}{l^2} x(l-x)$ → Equation of cable.

M_x = Bending moment due to external load

S_b = S.F due to external load.

<u>Two hinged stiffening girder</u>



In 2 hinged stiffening girder there are no central hinges.

→ It is indeterminate

($D_S = 1$)

In 3-hinged stiffening girder a central hinge is there.

→ It is determinate.

($D_S = 0$)

* Major difference in 2-hinged stiffening girder

$$\left[\begin{aligned} B.M_x &= M_x - Hy \\ S.F_x &= S_b - H\tan\theta \end{aligned}\right]$$

* In this case $\boxed{W = \frac{\text{Total Load}}{\text{Length of span}}} = \frac{D.L + L.L}{\text{Length of span}}$

Scanned with CamScanner

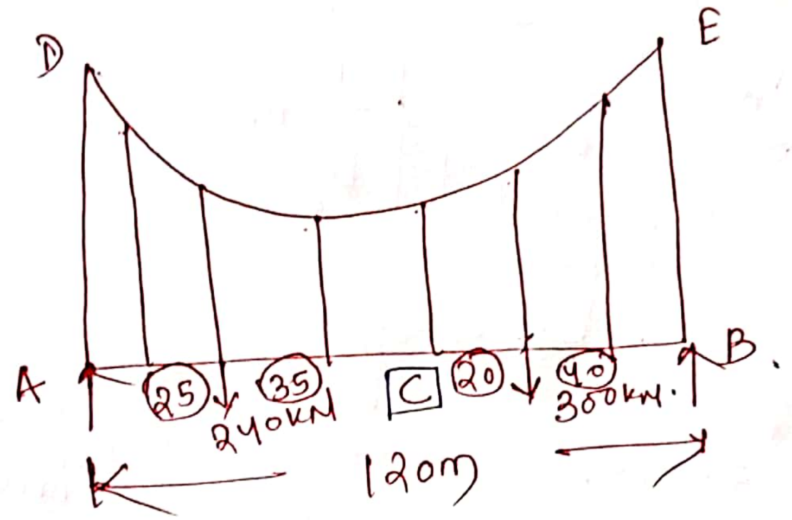
In this case total D.L and L.L (Live load) are considered.

for Dead Load = $D_1 \times$ length (as udL)

for Live Load = $L_1 \times \frac{L}{2}$ (as udL).

Another example

3-Hinged stiffening girder:



Q A 3-hinged stiffening girder of suspension bridge span 120m is subjected to two loads 240KN and 300KN at 25m, 80m from left. Find the S.F and B.M of the girder at a distance 40m from left end. Central dip of cable is 12m. Draw BMD and SFD of the bridge.

Solution Reaction Step-1

$\Sigma M @ A = 0$ ↺+ ↻(-)

$\Rightarrow (240 \times 25) + (300 \times 80) - (R_B \times 120) = 0.$

$R_B = 250 KN$ (↑)

$\Sigma F_y = 0$ (upward = downward)

$R_A + 250 = 240 + 300$

$\Rightarrow R_A = 290 KN$ (↑)

Step-2 :-

Moment @ c $= (\overset{R_A}{290} \times 60) - (240 \times 35)$

$= 9000 KNm.$

Step-3

Horizontal force (H) $= \frac{M@c}{h} = \frac{9000}{\cancel{h} 12} = 750 KN$

[h = dip = 12m given]

$V = \frac{WL}{2}$ = shear force.

udl on cable $= \frac{WL^2}{8} = M@c$

$\Rightarrow \frac{W \times 120^2}{8} = 9000$

$\Rightarrow W = 5 KN/m.$

Shear force $V = \frac{WL}{2} = \frac{5 \times 120}{2}$

$= 300 KN.$

$$\begin{bmatrix} V = 300 KN \\ H = 750 KN \end{bmatrix} \qquad \begin{bmatrix} R_A = 290 KN \\ R_B = 250 KN \end{bmatrix}$$

Scanned with CamScanner

Step-4

Tension in cable $T_{min} = H = 750\,KN$

$$T_{max} = \sqrt{V^2 + H^2}$$

$$= \sqrt{300^2 + 750^2} = 807.77\,KN$$

Step-5

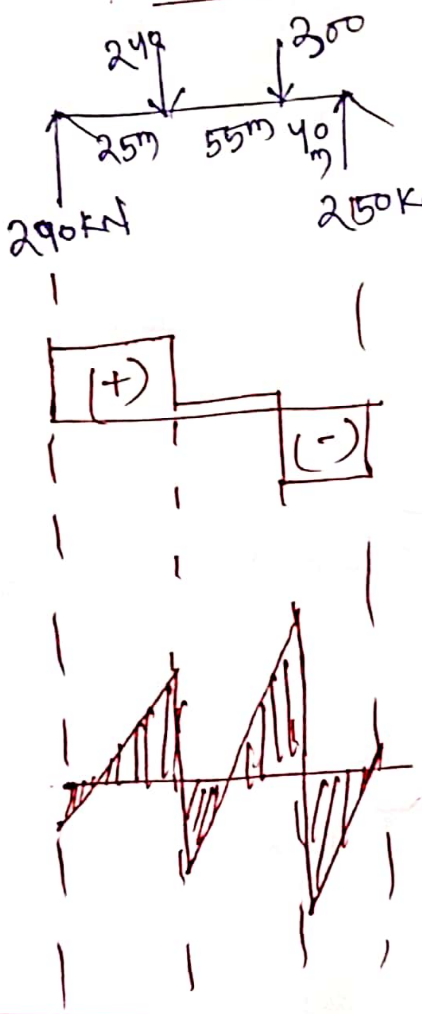
Shear force diagram

S.F is calculated on the basis girder and cable respectively.
or live load and dead Load
or actual Load and Load on cable.

* girder load = live Load = Actual Load

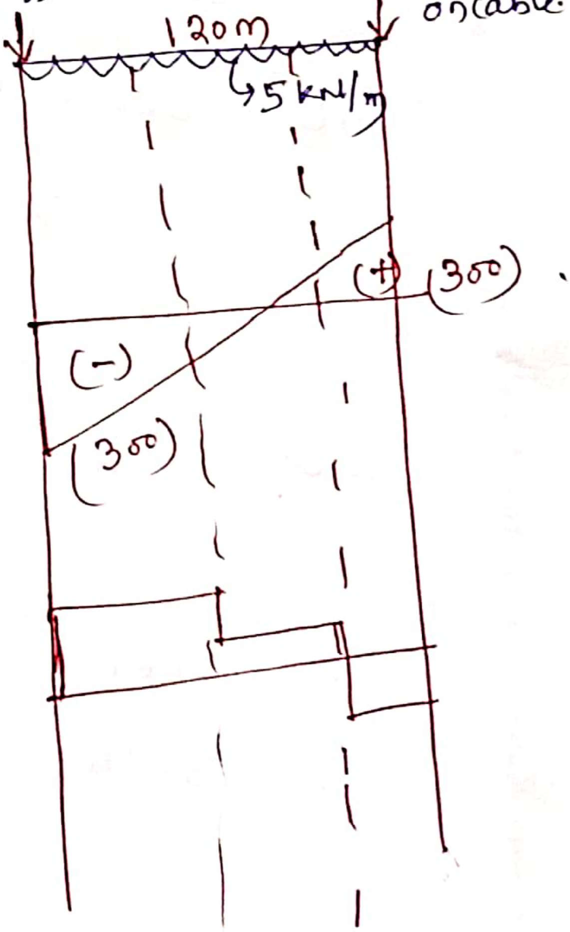
Cable Load = Dead Load = Load on cable.

Girder



Cable

assume a horizontal portion on cable.



Shear force @ 40m $= (S.F)_{girder} + (S.F)_{cable}$

$= (290-240) + (-300 + 5\times 40)$

$= -50\,KN.$

Bending moment @ 40m

$= (B.M)_{girder} + (B.M)_{cable}$

$= \left[(290\times 40) - (240\times 40 - 25)\right]$

$+ \left[(-300\times 40) + \left(5\times \frac{40^2}{2}\right)\right]$

$= 0\,KNm.$

Ans