

Number System

- ① Positional Number System
- ② Non positional Number System.

There are various types of positional number system, but generally 4 types of number system are used in digital computer system.

- ① Decimal
- ② Binary
- ③ Octal
- ④ Hexadecimal

Base or Radix of a number system

The base or Radix of a number system is defined as the no. of different symbols (digits or characters) used in that number system.

(1) Decimal Number System

The base or Radix of this system is 10.

i.e. 0, 1, 2, ..., 9.

EX: $(756.8)_{10} = 7 \times 10^2 + 5 \times 10^1 + 6 \times 10^0 + 8 \times 10^{-1}$

relationship between 10 and 9

$$[\dots 10^2 \ 10^1 \ 10^0 \ 10^{-1} \ 10^{-2} \dots]$$

(2) Binary Number System

Base or Radix is 2

i.e. 0 and 1.

Ex: $(1001101)_2$

The representation is

$$[\dots 2^3 2^2 2^1 2^0 \cdot 2^{-1} 2^{-2} \dots]$$

$$(1001.10)_2 = 1 \times 2^0 + 0 \times 2^1 + 0 \times 2^2 + 1 \times 2^3 + 1 \times 2^{-1} + 0 \times 2^{-2}$$

$$= (1+8) + 2^{-1}$$

$$= (9.5)_{10}$$

3) Octal Number System

Base or Radix is 8

i.e. 0, 1, 2, 3, 4, 5, 6, 7

It is represented as

$$[\dots 8^2 8^1 8^0 \cdot 8^{-1} 8^{-2} 8^{-3} \dots]$$

4) Hexadecimal number system

Here Base or Radix is 16.

i.e. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10(A), 11(B), 12(C), 13(D), 14(E), 15(F)

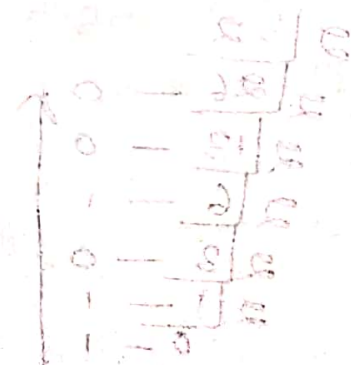
Ex: 35H

It is represented as

$$[\dots 16^2 16^1 16^0 \cdot 16^{-1} 16^{-2} 16^{-3} \dots]$$

→ It is also known as alpha numeric number system.

$$2(001011) = 01(52)$$



Relationship between different Number System.

<u>Decimal</u>	<u>Binary</u>	<u>Octal</u>	<u>Hexadecimal</u>
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Number System Conversion

Decimal to Binary

$(52.75)_{10}$

$$\begin{array}{r|l} 2 & 52 \\ \hline 2 & 26 - 0 \\ 2 & 13 - 1 \\ 2 & 6 - 1 \\ 2 & 3 - 1 \\ 2 & 1 - 1 \\ & 0 - 0 \end{array}$$

$$(52)_{10} = (110100)_2$$

$$0.75 \times 2 = 1.50$$

$$0.50 \times 2 = 1.00$$

$$(0.75)_{10} = (0.11)_2$$

$$(52.75)_{10} = (110100.11)_2$$

Decimal to Octal

$$(496.45)_{10} = ()_8$$

$$\begin{array}{r} 8 \overline{) 496} \\ \underline{62} \\ 8 \overline{) 62} \\ \underline{7} \\ 8 \overline{) 7} \\ \underline{0} \end{array} \begin{array}{l} - 0 \\ - 6 \\ - 7 \end{array} \uparrow$$

$$(496)_{10} = (760)_8$$

$$0.45 \times 8 = 3.60$$

$$0.60 \times 8 = 4.80$$

$$0.80 \times 8 = 6.40$$

$$0.40 \times 8 = 3.20$$

$$(0.45)_{10} = (0.346)_8$$

$$(496.45)_{10} = (760.346)_8$$

Decimal to Hexadecimal

$$(496.45)_{10} = ()_{16}$$

$$\begin{array}{r} 16 \overline{) 496} \\ \underline{31} \\ 16 \overline{) 31} \\ \underline{1} \\ 16 \overline{) 1} \\ \underline{0} \end{array} \begin{array}{l} - 0 \\ - 15(F) \\ - 1 \end{array} \uparrow$$

$$0.45 \times 16 = 7.20$$

$$0.20 \times 16 = 3.20$$

$$0.20 \times 16 = 3.20$$

$$(496.45)_{10} = (1F0.733)_{16}$$

Binary to decimal

$$\begin{aligned}(110100.11)_2 &= 0 \times 2^0 + 0 \times 2^1 + 1 \times 2^2 + 0 \times 2^3 + 1 \times 2^4 \\ &\quad + 1 \times 2^5 + 1 \times 2^{-1} + 1 \times 2^{-2} \\ &= 4 + 16 + 32 + \frac{1}{2} + \frac{1}{4} \\ &= (52.75)_{10}\end{aligned}$$

Octal to decimal

$$\begin{aligned}(760.346)_8 &= 0 \times 8^0 + 6 \times 8^1 + 7 \times 8^2 + 3 \times 8^{-1} + 4 \times 8^{-2} \\ &\quad + 6 \times 8^{-3} \\ &= 48 + 448 + \frac{3}{8} + \frac{4}{64} + \frac{6}{512} \\ &= 496 + 0.375 + 0.062 + 0.011 \\ &= (496.45)_{10}\end{aligned}$$

Hexadecimal to decimal

$$\begin{aligned}(1F0.733)_{16} &= 0 \times 16^0 + 15 \times 16^1 + 1 \times 16^2 + 7 \times 16^{-1} \\ &\quad + 3 \times 16^{-2} + 3 \times 16^{-3} \\ &= 0 + 240 + 256 + 0.43 + 0.011 + 0.0007 \\ &= (496.45)_{10}\end{aligned}$$

Binary to octal

$$\left(\begin{array}{ccc} 110 & 100 & 110 \\ \hline 6 & 4 & 6 \end{array} \right)_2 = (64.6)_8$$

Octal to Binary

$$\begin{aligned}(67.73)_8 &= (\quad)_2 \\ &= (110111.111011)_2\end{aligned}$$

Binary to Hexadecimal

$$\left(\frac{1011}{B} \frac{1010}{A} \cdot \frac{0011}{3} \right)_2 = (BA.3)_{16}$$

Hexadecimal to Binary

$$(BA.3)_{16} = ()_2$$
$$\downarrow$$
$$(10111010.0011)_2$$

Hexadecimal to Octal

The hexadecimal number is $(2A.B)_{16} = ()_8$

$\downarrow \downarrow \downarrow$
2 10 11

$$= \left(\frac{00010}{05} \frac{1010}{2} \cdot \frac{101100}{54} \right)_2$$
$$(052.54)_8$$

Octal to Hexadecimal

$$(37.62)_8 = ()_{16}$$

$\downarrow$

$$\frac{00011111}{1F} \cdot \frac{11001000}{C8}$$

$$(1F.C8)_{16}$$

Binary Arithmetic

4 types of binary arithmetics are present

- ① Addition
- ② Subtraction
- ③ Multiplication
- ④ Division

(1) Addition

$$0 + 0 = 0$$

$$0 + 1 = 1$$

$$1 + 0 = 1$$

$$1 + 1 = 10$$

↓ sum
Carry

$$(1011)_2 + (1110)_2 =$$

$$\begin{array}{r} 10 \\ + 11 \\ \hline 11 \end{array} \quad \begin{array}{r} 11 \\ + 1 \\ \hline 100 \end{array}$$

Hexadecimal to Binary

$$(11001)_2$$

Carry

(2) Subtraction

$$0 - 0 = 0$$

$$1 - 0 = 1$$

$$1 - 1 = 0$$

$$0 - 1 = 1 \text{ with borrow } 1$$

$$(11011.11)_2 - (10101.01)_2 = (00110.10)_2$$

$$\begin{array}{r} 11011.11 \\ 10101.01 \\ \hline 00110.10 \end{array}$$

(3) Multiplication

$$0 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 0 = 0$$

$$1 \times 1 = 1$$

$$(111)_2 \times (101)_2 = (100011)_2$$

$$\begin{array}{r}
 111 \\
 101 \\
 \hline
 10001 \\
 111 \\
 \hline
 (100011)_2
 \end{array}$$

4) Division :

$$(101101)_2 \div (110)_2 =$$

$$\begin{array}{r}
 110 \overline{) 101101} \\
 \underline{1010} \\
 110 \\
 \underline{1001} \\
 0110 \\
 \underline{110} \\
 0
 \end{array}$$

Complement

complements are used in digital computer for simplifying the subtraction operation and for logical manipulation.

1's complement

1's complement of (101) is 010

2's Complement

2's complement of 101 is 011

$$\begin{array}{r}
 010 \\
 + 1 \\
 \hline
 011
 \end{array}$$

2's complement = 1's complement + 1

Subtraction by compliments method

EX Set - I

$$\begin{array}{r} 0110 \\ - 0011 \\ \hline + (0011) \end{array}$$

EX Set - II

$$\begin{array}{r} 0011 \\ - 0110 \\ \hline - (0011) \end{array}$$

Step-1 First write the 1st number.

Step-2 Find the 1's complement of 2nd number.

Step-3 Add both the numbers.

$$\begin{array}{r} 0110 \\ 1100 \\ \hline 10010 \\ \text{Carry} \end{array}$$

Step-4 If carry is there, then the answer is the

Step-5 Remove the carry from the left side & add that to the result.

$$\begin{array}{r} 0010 \\ + 1 \\ \hline + (0011) \end{array}$$

EX Set - II

Follow the first 3 steps as discussed above.

$$\begin{array}{r} 0011 \\ 1001 \\ \hline 1100 \end{array}$$

Step-4 If carry is ^{not} there then answer is -ve.

Step-5 Find out 1's complement of result and represent with -ve sign.

$$\text{ans} = - (0011)$$

2's complement method of subtraction.

EX-1

$$\begin{array}{r} 0110 \\ - 0011 \\ \hline \end{array}$$

Step-1 write the 1st number.

Step-2 find the 2's complement of 2nd number.
 $1100 + 1 = 1101$

Step-3 Add both the numbers.

$$\begin{array}{r} 0110 \\ + 1101 \\ \hline 10011 \\ \downarrow \\ \text{Carry} \end{array}$$

Step-4 If carry is there, then remove the carry and represent by +ve sign.

$$\text{ans} = + (0011)$$

EX-2

Step-1

$$00011$$

Step-2 1's of 2nd no.

$$\begin{array}{r} 1001 \\ + 1 \\ \hline 1010 \end{array}$$

2's

Step-3

$$\begin{array}{r} 0011 \\ 1010 \\ \hline 1101 \end{array}$$

Step-4

It carry is not there the result is ~~ve~~ -ve

Step-5

Find out 2's Complement of result & represent with -ve sign.

$$\begin{array}{r} \text{1's} \quad 0010 \\ \quad \quad +1 \\ \hline \text{2's} \quad - (0011) \end{array}$$

Sign magnitude

$$\begin{array}{ll} \text{+ve } 0, 1100 & (+12) \\ \text{-ve } 1, 1100 & (-12) \end{array}$$

one extra bit is required for sign representation.

Logic gate and Boolean ~~Expression~~ Algebra

A ~~gate~~ logic gate is an electronic circuit which takes one or more inputs and give one output.

NOT, AND, OR $\rightarrow$ Basic logic gates.

NAND and NOR $\rightarrow$ Universal logic gates.

Exclusive-OR or EX-OR or XOR
Exclusive-NOR or XNOR $\rightarrow$ special gates.

NOT Gate / Inverter Gate



$\bar{x}$ or x' or $(x+0)$ or $(0+x)$

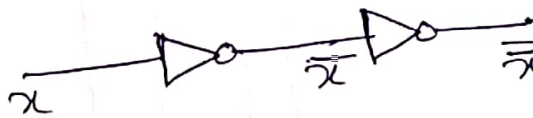
Truth Table

Input (x)

0
1

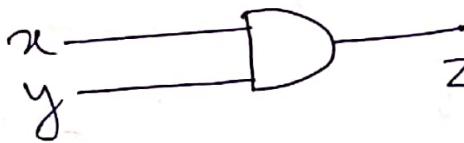
output ($\bar{x}$ or x')

1
0



$\bar{\bar{x}} = x$

AND Gate



$z = x \cdot y$

x

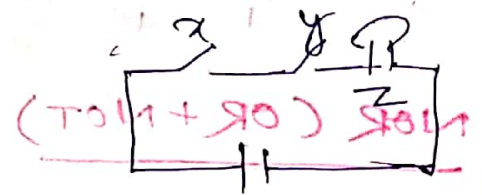
0
0
1
1

y

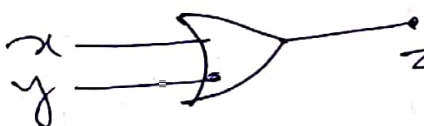
0
1
0
1

$z = x \cdot y$

0
0
0
1



OR Gate



$z = x + y$

x

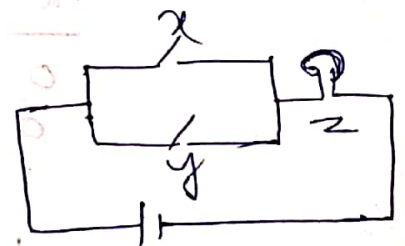
0
0
1
1

y

0
1
0
1

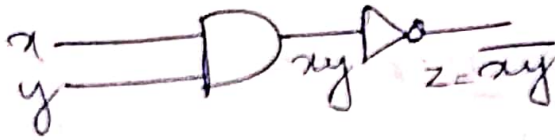
$z = x + y$

0
1
1
1



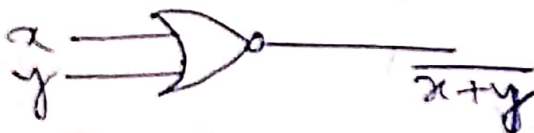
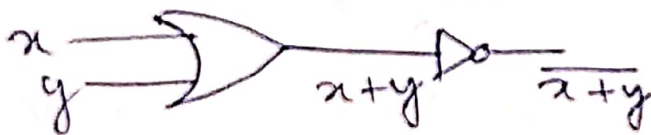
→ Binary arithmetic output depends upon the no. of input but binary logic output always one digit i.e. 0 or 1.

NAND Gate (AND + NOT)



<u>x</u>	<u>y</u>	<u>$z = \overline{xy}$</u>
0	0	1
0	1	1
1	0	1
1	1	0

NOR (OR + NOT)



<u>x</u>	<u>y</u>	<u>$z = \overline{x+y}$</u>
0	0	1
0	1	0
1	0	0
1	1	0

EX-OR Gate

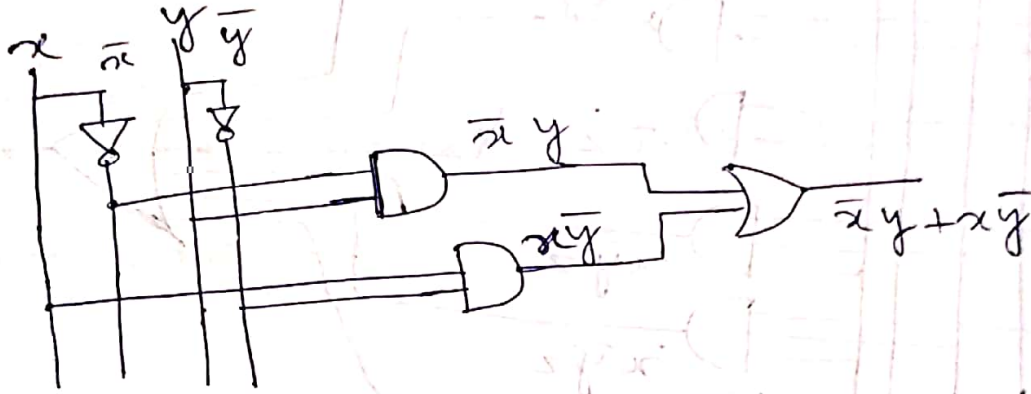
$$z = \bar{x}y + x\bar{y}$$



$$z = \bar{x}y + x\bar{y}$$

$$= x \oplus y$$

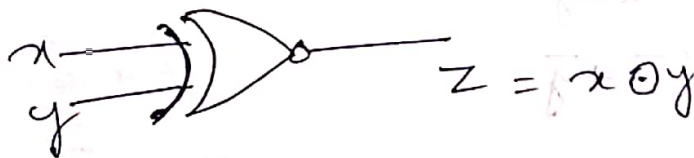
x	y	$z = \bar{x}y + x\bar{y}$
0	0	0
0	1	1
1	0	1
1	1	0



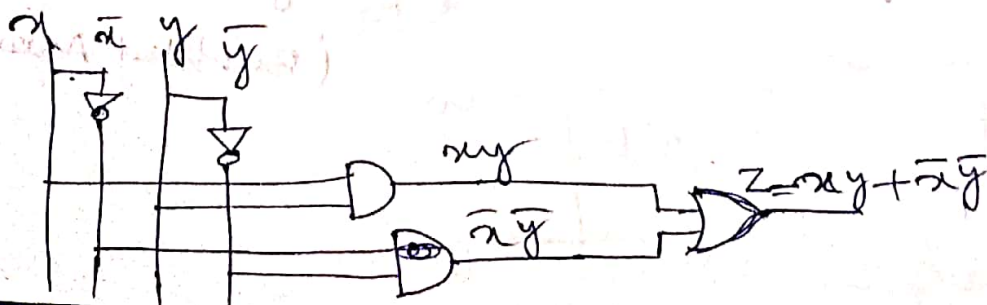
EX-NOR Complement of X-OR

$$z = \overline{\bar{x}y + x\bar{y}} = \overline{x \oplus y}$$

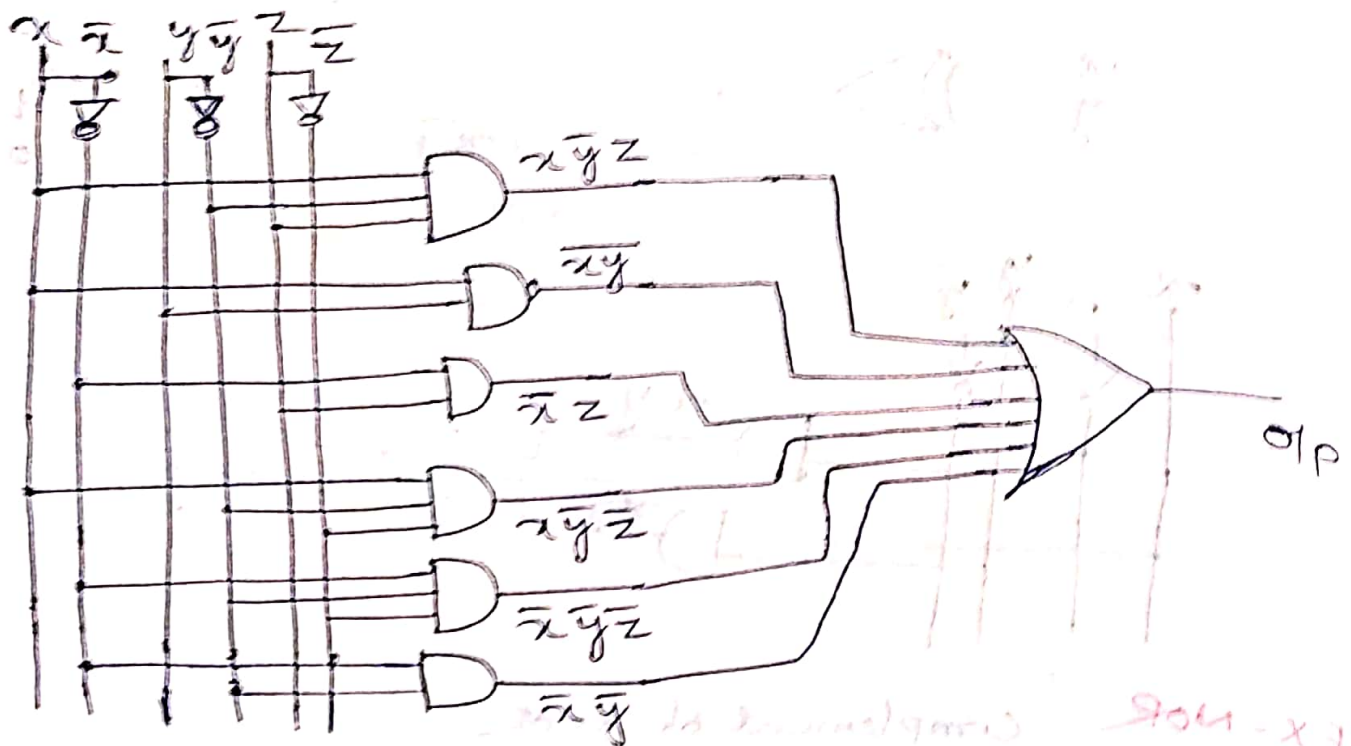
$$= xy + \bar{x}\bar{y}$$



x	y	$z = xy + \bar{x}\bar{y}$
0	0	1
0	1	0
1	0	0
1	1	1



Q1) $f(x, y, z) = x\bar{y}z + \bar{x}y + \bar{x}z + x\bar{y}\bar{z} + \bar{x}y\bar{z} + \bar{x}\bar{y}\bar{z}$
 Implement the function by logic gates.



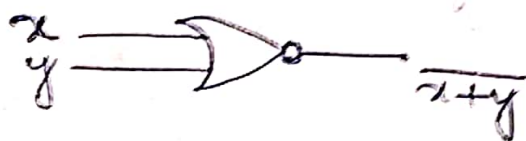
Demorgan's Law/Theorem

two theorems are there

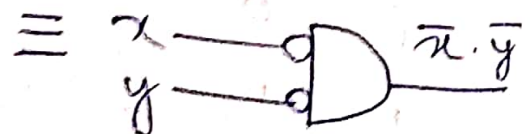
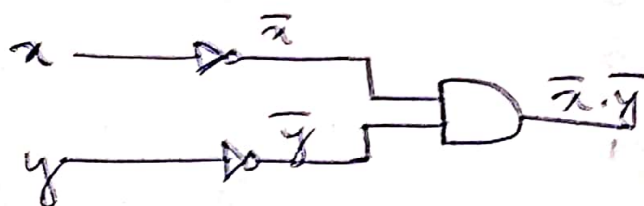
$$1) \overline{x+y} = \bar{x} \cdot \bar{y}$$

$$2) \overline{x \cdot y} = \bar{x} + \bar{y}$$

L.H.S



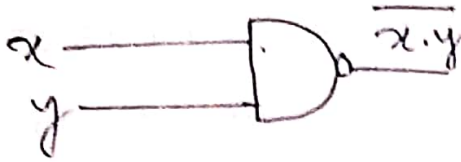
R.H.S



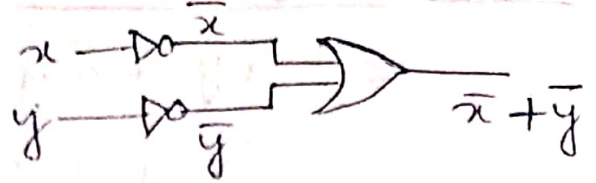
(Bubbled AND gate)

2)

L.H.S



$\equiv$



$\equiv$



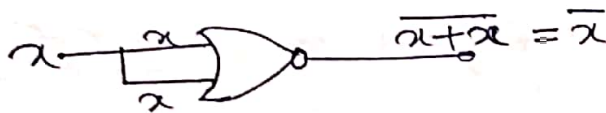
Universal gates

NAND & NOR Gates are called universal gates.

1) NAND as NOT :-



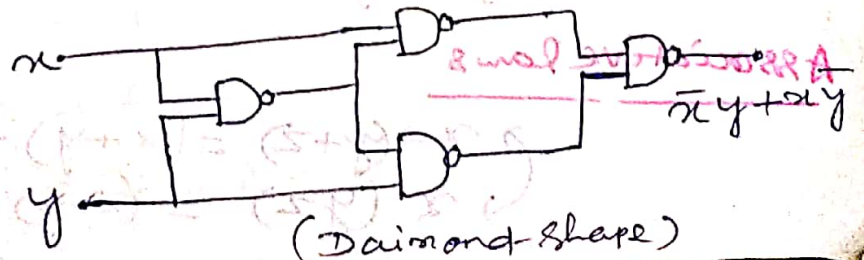
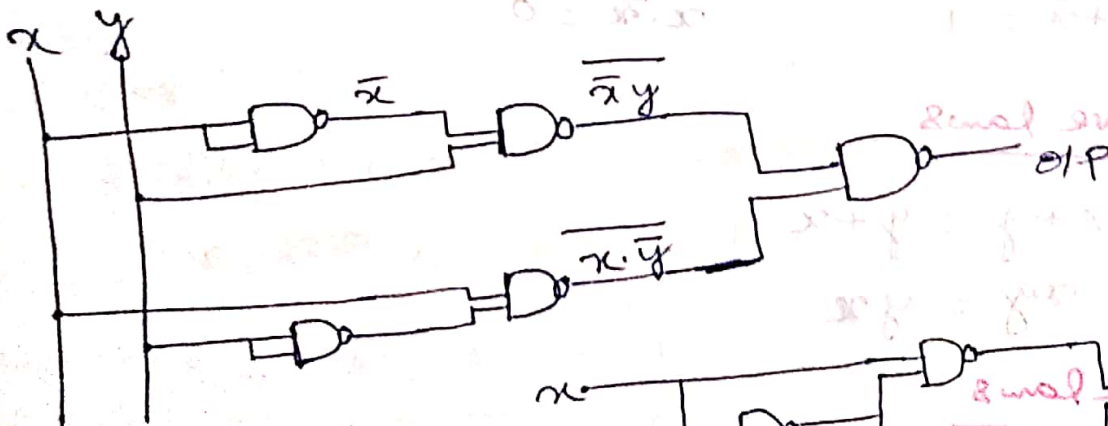
2) NOR as NOT :-



Ex-OR using NAND

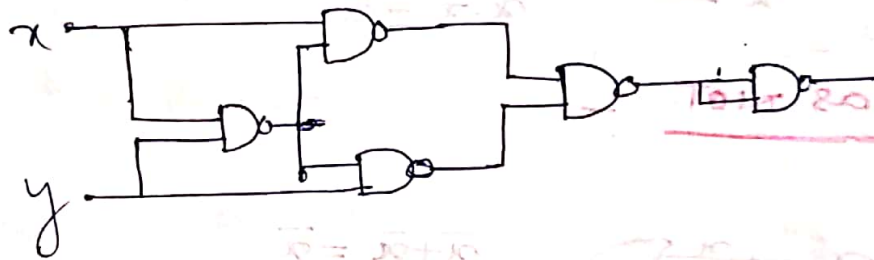
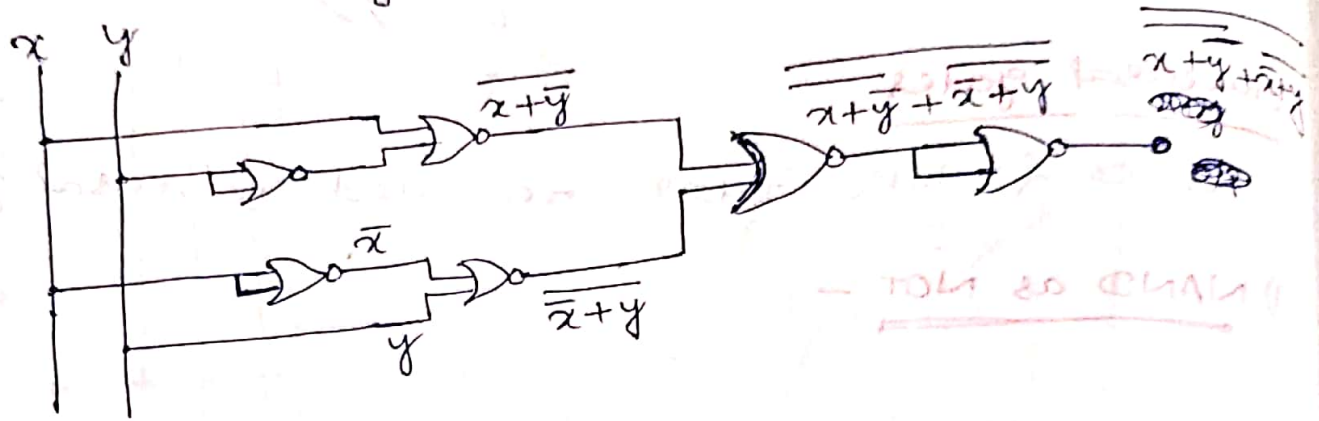
$$z = \overline{x}y + x\overline{y}$$

$$= \overline{\overline{x}y} \cdot \overline{x\overline{y}}$$



EX-OR using NOR gate

$$\begin{aligned} Z &= \overline{\overline{x}y} + \overline{x\overline{y}} \\ &= \overline{\overline{x+y}} + \overline{\overline{x+y}} \\ &= \overline{\overline{x+y}} + \overline{\overline{x+y}} \end{aligned}$$



Boolean Algebra

OR laws

$$\begin{aligned} x+0 &= x \\ x+1 &= 1 \\ x+x &= x \\ x+\overline{x} &= 1 \end{aligned}$$

AND laws

$$\begin{aligned} x \cdot 0 &= 0 \\ x \cdot 1 &= x \\ x \cdot x &= x \\ x \cdot \overline{x} &= 0 \end{aligned}$$

Commutative laws

$$\begin{aligned} x+y &= y+x \\ xy &= yx \end{aligned}$$

Associative laws

$$\begin{aligned} x+(y+z) &= (x+y)+z \\ x \cdot (y \cdot z) &= (x \cdot y) \cdot z \end{aligned}$$

Distributive law :-

$$x(y+z) = xy + xz$$

$$x + yz = (x+y)(x+z)$$

$$x + \bar{x}y = x + y$$

$$x(\bar{x}+y) = xy$$

Demorgan's Rule/Law

$$\overline{x+y} = \bar{x} \cdot \bar{y}$$

$$\overline{xy} = \bar{x} + \bar{y}$$

Q1/ $f = x\bar{y} + \bar{x}y + xy$

$$= x(y + \bar{y}) + \bar{x}y$$

Standard representation of Boolean Expression

$$= x + \bar{x}y$$

$$= x + y$$

Q1/ $f = xy\bar{z} + xyz + \bar{x}y + xy$

Ans :- $f = xy(\bar{z} + z) + y(\bar{x} + x)$

$$= xy + y$$

$$= y(x + 1)$$

$$= y$$

BPUT 2008

Q1/ Simplify the Boolean expression ①

$$f = \overline{AB + AC} + A\bar{B}C$$

ans :- $f = \overline{AB + AC} + A\bar{B}C$

$$= \overline{A(B+C)} + A\bar{B}C$$

$$= \bar{A} + \overline{(B+C)} + A\bar{B}C$$

$$\begin{aligned}
 &= \bar{A} + \bar{B} \cdot \bar{C} + A\bar{B}C \\
 &= \bar{A} + \bar{B}(\bar{C} + AC) \\
 &= \bar{A} + \bar{B}\{(\bar{C} + A)(\bar{C} + C)\} \\
 &= \bar{A} + \bar{B}(\bar{C} + A) \\
 &= \bar{A} + \bar{B}\bar{C} + A\bar{B} \\
 &= \bar{A} + A\bar{B} + \bar{B}\bar{C} \\
 &\Rightarrow \bar{A} + \bar{B}(\bar{A} + \bar{C}) \\
 &= \bar{A} + \bar{B} + \bar{B}\bar{C} \\
 &= \bar{A} + \bar{B}(1 + \bar{C}) \\
 &= \bar{A} + \bar{B} \text{ (ans)}
 \end{aligned}$$

Standard representation of Boolean Expression

x	y	min term 'm' (SOP)	max term 'M' (POS)
0	0	$\bar{x} \cdot \bar{y} = m_1$	$x + y = M_1$
0	1	$\bar{x} \cdot y = m_2$	$x + \bar{y} = M_2$
1	0	$x \cdot \bar{y} = m_3$	$\bar{x} + y = M_3$
1	1	$x \cdot y = m_4$	$\bar{x} + \bar{y} = M_4$

1 → min term
0 → max term

→ No. of min terms = No. of max terms = 2^n

→ min terms = complement of max terms & vice versa

ex: $\frac{x}{0}$ $\frac{y}{0}$ $\frac{F}{0}$

0	0	0
0	1	1
1	0	1
1	1	0

$$SOP = \bar{x}y + x\bar{y}$$

$$POS = (x+y) \cdot (\bar{x} + \bar{y})$$

ex:

$\frac{x}{0}$	$\frac{y}{0}$	$\frac{z}{0}$	$\frac{F}{0}$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$$SOP = \bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}z + xyz$$

Standard form means all variables will be present in complement or in the same form.

$$= m_1 + m_2 + m_5 + m_7$$

$$= \sum m(1, 2, 5, 7)$$

POS

$$F(x, y, z) = (x+y+z) \cdot (\bar{x} + \bar{y} + \bar{z}) \cdot (\bar{x} + \bar{y} + \bar{z}) \cdot (\bar{x} + \bar{y} + \bar{z})$$

$$= M_0 \cdot M_3 \cdot M_4 \cdot M_6$$

$$= \Pi M(0, 3, 4, 6)$$

Q1/ $F(x, y, z) = \sum m(1, 2, 4, 7)$. what is it's equivalent canonical form or $\Pi M = ?$

$$\Pi M(0, 3, 5, 6)$$

Q1/ ~~what~~ $f(x, y, z) = \sum m(0, 1, 2, 3, 4, 5, 6, 7)$
equivalent canonical form?

ans :- does not exist.

$$Q1/ F(x, y, z) = \sum m(0, 1, 2, 3, 4, 5, 6, 7) = 1$$

$$Q1/ F(x, y, z) = \Pi M(0, 1, 2, 3, 4, 5, 6, 7) = 0$$

→ Sum of all the minterms of a given boolean equation is unity.

→ The product of all the maxterms of a given boolean equation is equal to zero.

SOP

$$F(x, y, z) = xy + \bar{x}y\bar{z} + \bar{x}yz$$

Convert into standard SOP.

$$= xy(z + \bar{z}) + \bar{x}y\bar{z} + \bar{x}yz$$

$$= xyz + x\bar{y}\bar{z} + \bar{x}y\bar{z} + \bar{x}yz$$

$$\begin{matrix} 111 & 110 & 010 & 011 \end{matrix}$$

$$= \sum m(2, 3, 6, 7)$$

POS $f(x, y, z) = (x+y) \cdot (\bar{x} + \bar{y} + z)$

convert into standard POS

$$= (x+y+z \cdot \bar{z}) (\bar{x} + \bar{y} + z)$$

$$= \underset{0 \ 0 \ 0}{(x+y+z)} \underset{0 \ 0 \ 1}{(x+y+\bar{z})} \underset{1 \ 1 \ 0}{(\bar{x} + \bar{y} + z)}$$

$$= \pi M (0, 1, 6)$$

$$= \sum m (2, 3, 4, 5, 7)$$

Q1/ $F = xy + yz + \bar{x}\bar{y}\bar{z}$. Implement the function in SOP.

Soln: $F = xy(z + \bar{z}) + yz(x + \bar{x}) + \bar{x}\bar{y}\bar{z}$

$$= xyz + xy\bar{z} + xyz + \bar{x}yz + \bar{x}\bar{y}\bar{z}$$

$$= \underset{111 \ 0}{xyz} + \underset{110}{xy\bar{z}} + \underset{011}{\bar{x}yz} + \underset{000}{\bar{x}\bar{y}\bar{z}}$$

$$= \sum m (0, 3, 6, 7)$$

Q1/ Implement the function in POS.

$$(x+y)(y+z)(z+x)$$

ans:- $(x+y+z \cdot \bar{z}) \cdot (y+z+x \cdot \bar{x}) \cdot (z+x+y \cdot \bar{y})$

$$= (x+y+z)(x+y+\bar{z}) \cdot (\bar{x}+y+z)(\bar{x}+y+z) \cdot (x+\bar{y}+z)$$

$$= (x+y+z)(x+y+\bar{z})(\bar{x}+y+z)(x+\bar{y}+z)$$

Q1/ $F(x, y, z) = \sum m (0, 1, 6, 7)$ simply using SOP concept.

$$= 000 + 001 + 110 + 111$$

$$= \bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z + xy\bar{z} + xyz$$

$$= \bar{x}\bar{y}(\bar{z} + z) + xy(\bar{z} + z)$$

$$= \bar{x}\bar{y} + xy$$

K-map (Karnaugh-map)

No. of cells = 2^n (n = no. of variable)

Two variable K-map

$x \backslash y$	0 $\bar{y}$	1 y
0 $\bar{x}$	0	1
1 x	2	3

① $F = \bar{x}y + x\bar{y}$
 $= \sum m(1, 2)$

$x \backslash y$	0	1
0	0	1
1	1	0

② $f(x, y) = \bar{x}y + x\bar{y} + x\bar{y}$

$= \bar{x} + \bar{y}$

$x \backslash y$	0 $\bar{y}$	1 y
0 $\bar{x}$	1	1
1 x	1	0

③ $f(x, y) = \bar{x}y + x\bar{y} + xy$

$= x + y$

$x \backslash y$	$\bar{y}$	y
$\bar{x}$ 0	0	1
x 1	1	1

3-Variable K-map

$x \backslash yz$	$\bar{y}\bar{z}$ 00	$\bar{y}z$ 01	$y\bar{z}$ 11	yz 10
$\bar{x}$ 0	0	1	3	2
x 1	4	5	7	6

Q11 $f(x, y, z) = \sum m(2, 3, 4, 5)$

$x \backslash yz$	$\bar{y}\bar{z}$ 00	$\bar{y}z$ 01	$y\bar{z}$ 11	yz 10
$\bar{x}$ 0	0	0	1	1
x 1	1	1	0	0

$$= \bar{x}y + x\bar{y}$$

Q11 $f(x, y, z) = \sum m(2, 3, 5, 7)$

$x \backslash yz$	$\bar{y}\bar{z}$ 00	$\bar{y}z$ 01	$y\bar{z}$ 11	yz 10
$\bar{x}$ 0	0	0	1	1
x 1	0	1	1	0

$$= \bar{x}y + xz$$

Assignment

① $F(x, y, z) = \sum m(1, 2, 3, 4, 5, 7)$

② $F(x, y, z) = \sum m(0, 2, 4, 5, 6, 7)$

①

		yz	$\bar{y}\bar{z}$	$\bar{y}z$	$y\bar{z}$	$y\bar{z}$	$y\bar{z}$
			00	01	11	10	
$\bar{x}$	0	0	1	1	1	1	
x	1	1	1	1	1	0	

$$= z + \bar{x}y + x\bar{y}$$

②

		yz	$\bar{y}\bar{z}$	$\bar{y}z$	$y\bar{z}$	$y\bar{z}$	$y\bar{z}$
			00	01	11	10	
$\bar{x}$	0	1	0	0	0	1	
x	1	1	1	1	1	1	

$$= x + \bar{z}$$

→ one pair eliminates 1 variable.
 one quad " 2 variables.
 one octate " 3 variables.

Pair 2^① → eliminate
 quad 2^②
 octate 2^③

4-Variables K-map

		CD	$\bar{C}\bar{D}$	$\bar{C}D$	$C\bar{D}$	CD	$\bar{C}\bar{D}$
			00	01	11	10	
$\bar{A}\bar{B}$	00	0	1	3	2		
$\bar{A}B$	01	4	5	7	6		
AB	11	12	13	15	14		
$A\bar{B}$	10	8	9	11	10		

Q1/ $F(A, B, C, D) = \sum m(0, 2, 4, 5, 6, 8, 9, 10, 11, 12, 13)$

	CD	$\overline{A}\overline{D}$ 00	$\overline{C}D$ 01	$C\overline{D}$ 11	$C\overline{D}$ 10
AB					
$\overline{A}\overline{B}$	00	1	0	0	1
$\overline{A}B$	01	1	1	0	1
AB	11	1	1	0	0
$A\overline{B}$	10	1	1	1	1

$= \overline{A}\overline{B} + B\overline{C} + \overline{A}D$

Q1/ $F(A, B, C, D) = \sum m(0, 1, 2, 3, 8, 9, 10, 11, 6, 7, 14, 15)$

	CD	$\overline{C}\overline{D}$ 00	$\overline{C}D$ 01	$C\overline{D}$ 11	$C\overline{D}$ 10
AB					
$\overline{A}\overline{B}$	00	1	1	1	1
$\overline{A}B$	01	0	0	1	1
AB	11	0	0	1	1
$A\overline{B}$	10	1	1	1	1

$= \overline{B} + C$

Q1/ $f(A, B, C, D) = \prod M(1, 3, 5, 6, 11, 13, 15)$

AB \ CD	00	01	11	10
00	1	0	0	1
01	1	0	1	0
11	1	0	0	1
10	1		0	1

$= (A + B + \overline{D})$

$(\overline{B} + \overline{C} + \overline{D})$

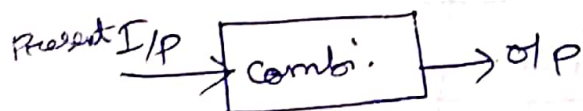
$(\overline{A} + \overline{C} + \overline{D})$

$(A + \overline{B} + \overline{C} + D)$

Digital circuits

classified into 2 groups

Combinational ckt



EX :- Half Adder, FA,
Half Subtractor, FS,
Encoder, Decoder,
MUX, DEMUX, ROM

→ The ckt which consist
of only logic gates is
Combinational ckt.

Combinational ckt

① Half Adder

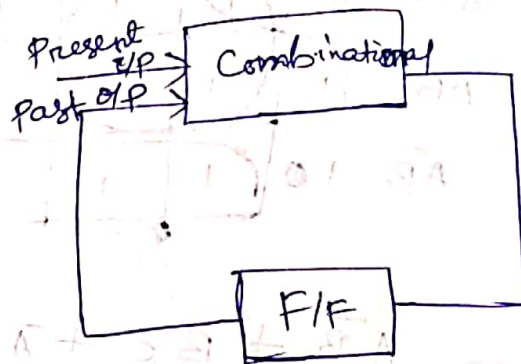
x	y	<u>S</u>	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Forc Sum

x \ y	0	1
0	0	1
1	1	0

$$S = \bar{x}y + x\bar{y}$$

Sequential ckt



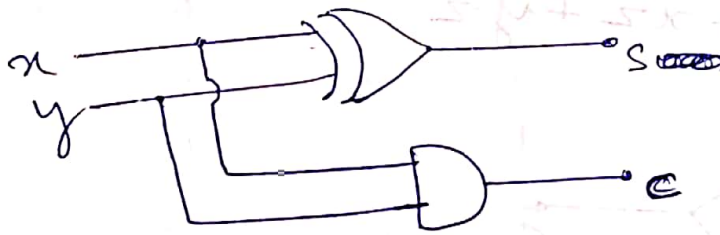
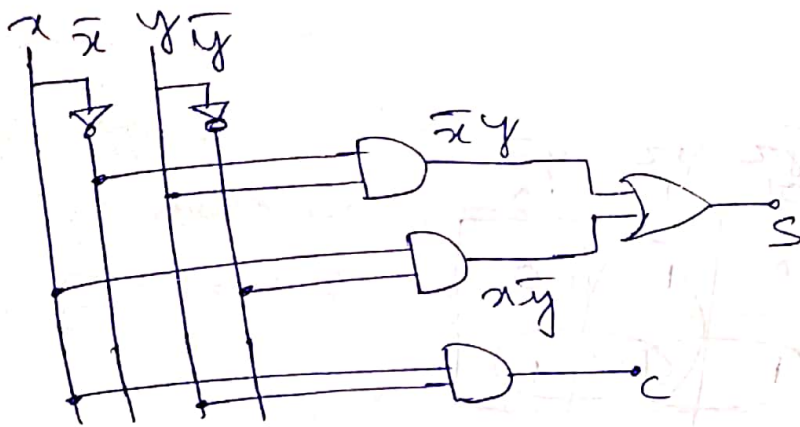
EX :- Counter, Register,
R/W memory etc.

→ consist of logic gates
& memory elements.

Forc Carry

x \ y	0	1
0	0	0
1	0	1

$$C = xy$$



Full Adder

x	y	z	S	C
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Full Sum

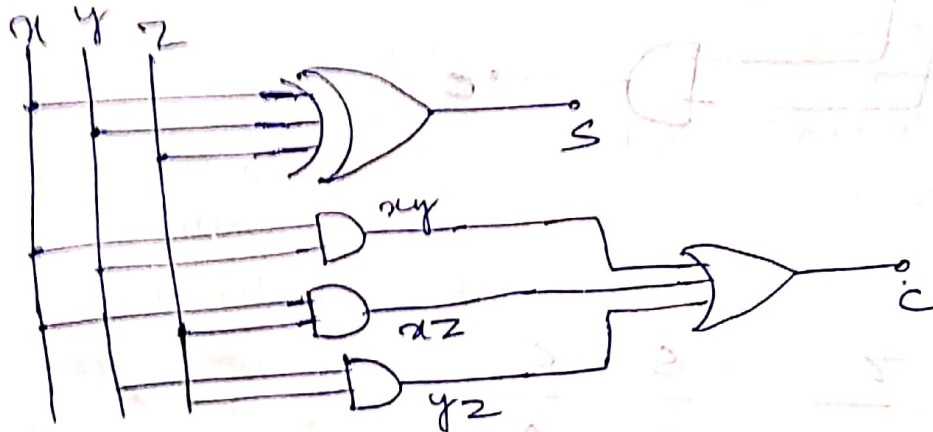
$x \backslash yz$	00	01	11	10
$x=0$	0	1	0	1
$x=1$	1	0	1	0

$$\begin{aligned}
 &= \bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}\bar{z} + xyz \\
 &= x \oplus y \oplus z
 \end{aligned}$$

For Carry

	$y\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}z$	$y\bar{z}$	$y\bar{z}$
x	0	0	0	1	0
$\bar{x}$	0	0	1	1	0
x	1	0	1	1	1

$$= xy + xz + yz$$

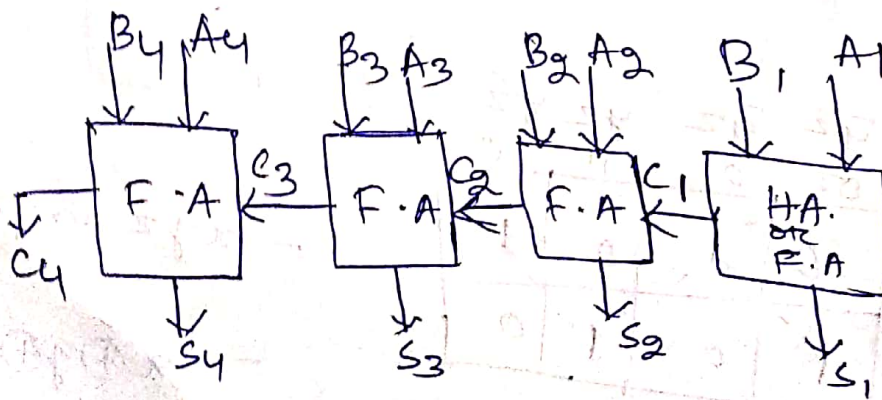


Four bit Binary Parallel Adder

$$\begin{array}{r} A = \begin{array}{cccc} A_4 & A_3 & A_2 & A_1 \\ 1 & 0 & 1 & 1 \end{array} \\ B = \begin{array}{cccc} B_4 & B_3 & B_2 & B_1 \\ 1 & 1 & 0 & 1 \end{array} \\ \hline 110001 \end{array}$$

$$\begin{array}{cccc} C_3(1) & C_2(1) & C_1(1) & \\ A_4 & A_3 & A_2 & A_1 \\ 1 & 0 & 1 & 1 \\ B_4 & B_3 & B_2 & B_1 \\ 1 & 1 & 0 & 1 \end{array}$$

$$\begin{array}{cccc} C_4 & 1 & 0 & 0 & 0 \\ S_4 & S_3 & S_2 & S_1 \end{array}$$



Subtractor

It is two types

- ① Half Subtractor
- ② Full Subtractor

① Half Subtractor

<u>x</u>	<u>y</u>	<u>D</u>	<u>B</u>
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

For Difference

x \ y	0	1
0	0	1
1	1	0

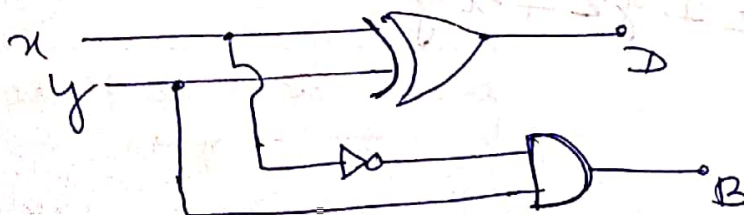
$$= \bar{x}y + x\bar{y}$$

$$= x \oplus y$$

For Borrow

x \ y	0	1
0	0	1
1	0	0

$$B = \bar{x}y$$



full subtractor :-

x	y	z	D	B
0	0	0	0	0
0	0	1	1	1
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

for Difference

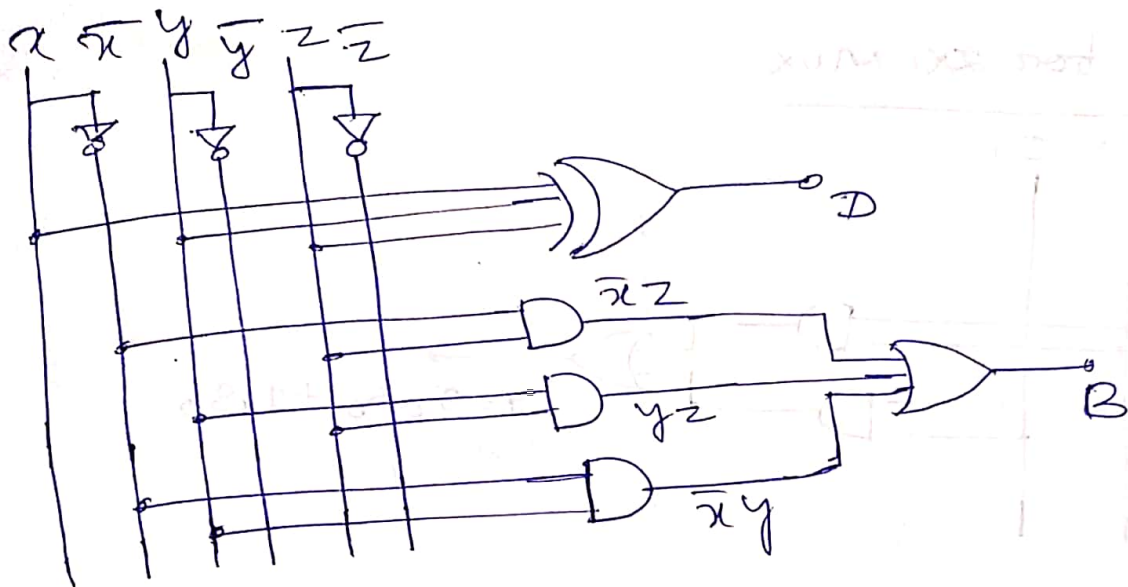
x \ yz	00	01	11	10
$\bar{x}$ 0	0	1	0	1
x 1	1	0	1	0

$$D = x \oplus y \oplus z$$

for Borrow

x \ yz	00	01	11	10
0	0	1	1	1
1	0	0	1	0

$$= \bar{x}z + yz + \bar{x}y$$



sum of minterms

sum of minterms