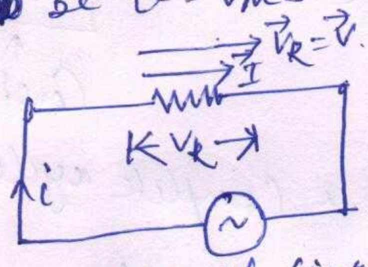
A.C. Through Resistance, Inductance and Capacitance. AC CIRCUIT:

In each case, we are given the equation of the alternating voltage as $e = E_m \sin\omega t$ and we will proceed to find the equation of the alternating current produced.

AC through pure Ohmic Resistance alone :→

Let the equation of the alternating voltage applied to the circuit of fig. be $v = V_m \sin\omega t$ or $V_m \sin\theta$ — ①



$v = V_m \sin\omega t$.

Fig. 1.1

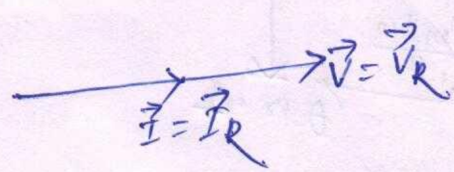
In the above circuit, $i = \dfrac{v}{R} = \dfrac{V_m \sin\omega t}{R} = \dfrac{V_m}{R}\sin\omega t$ — ②

Current i is maximum when $\sin\omega t$ is unity

$\therefore I_m = \dfrac{V_m}{R}$

Hence, eqⁿ ② becomes $i = I_m \sin\omega t$ where $I_m = \dfrac{V_m}{R}$. — ③

Comparing eqⁿ ① & ③, it is obvious that the supply voltage or voltage across the resistor & the circuit current or current flowing through the resistor are in phase with each other. The fact is illustrated in fig. 1.20(a) & (b).



(a)

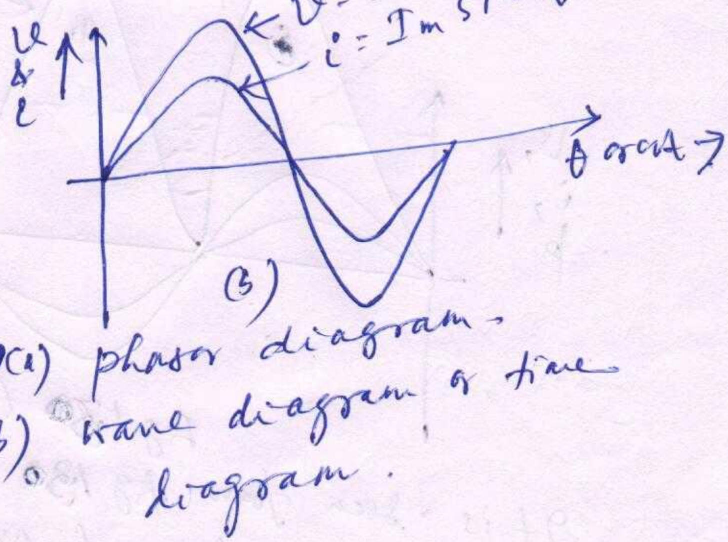


fig 1.20(a) phasor diagram.
(b) wave diagram or time diagram.

Power :→

Instantaneous power, $p = vi$

$$= (V_m \sin\omega t)(I_m \sin\omega t)$$

$$= V_m I_m \sin^2\omega t$$

$$= V_m I_m \sin^2\theta \quad (\because \omega t = \theta). \quad -(4)$$

Now, $\cos 2\theta = 1 - 2\sin^2\theta \Rightarrow \sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$

$$\therefore p = \frac{V_m I_m}{2}(1-\cos 2\theta) = \underbrace{\frac{V_m I_m}{2}}_{\text{Const. part.}} - \underbrace{\frac{V_m I_m}{2}\cos 2\theta}_{\text{Fluctuating part.}}$$

Average power over a complete cycle, $P = \frac{1}{2\pi}\int_0^{2\pi} p \, d\theta$

$$\Rightarrow P = \frac{1}{2\pi}\int_0^{2\pi} \frac{V_m I_m}{2}(1-\cos 2\theta)\, d\theta$$

$$= \frac{1}{2\pi} \times \frac{V_m I_m}{2}\left[\int_0^{2\pi} d\theta - \int_0^{2\pi} \cos 2\theta \, d\theta\right]$$

$$= \frac{1}{2\pi} \times \frac{V_m I_m}{2}\left[\theta\Big|_0^{2\pi} - \frac{1}{2}\sin 2\theta\Big|_0^{2\pi}\right]$$

$$= \frac{1}{2\pi} \times \frac{V_m I_m}{2}\left[(2\pi - 0) - \frac{1}{2}(\cancel{\sin 4\pi}^{\,0} - \cancel{\sin 0}^{\,0})\right]$$

$$= \frac{1}{2\pi} \times \frac{V_m I_m}{2}\left[(2\pi - 0)\right]$$

$$= \frac{V_m I_m}{4\pi} \times 2\pi = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} = VI$$

$$\therefore \boxed{P = VI} \quad (5)$$

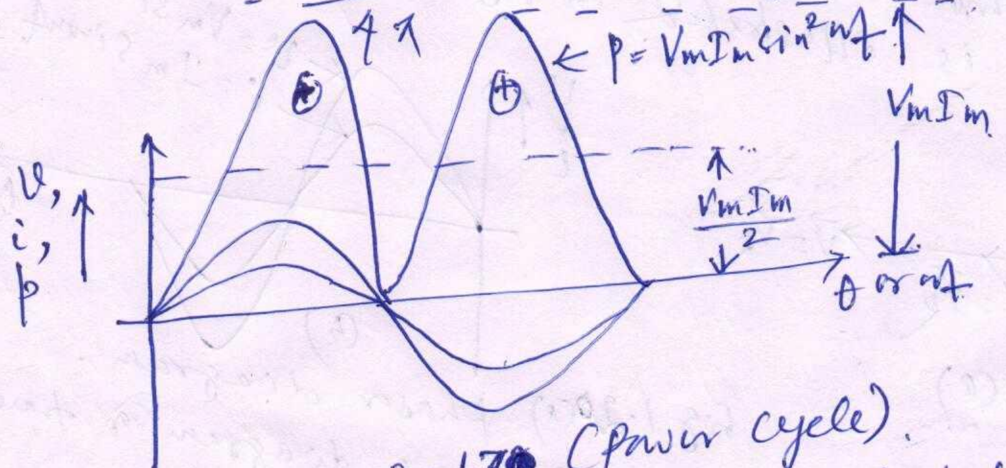
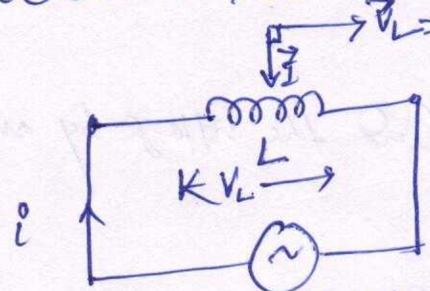


Fig 1.30 (power cycle).

It is seen from fig 1.30 that no part of the power

AC through pure inductance alone :→



$v = V_m \sin \omega t$

Fig. 1.4

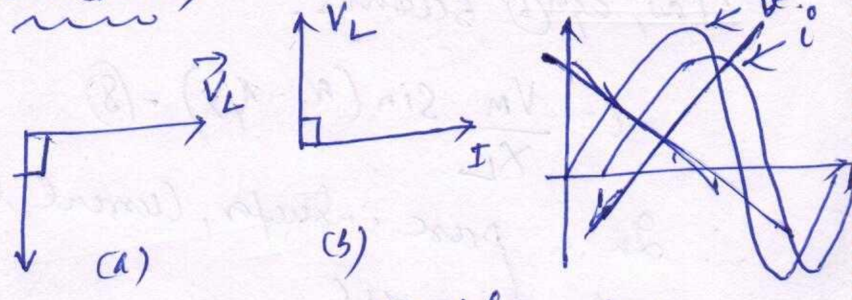


Fig 1.5 (a) & (b) phasor diagram.

Let the applied voltage be $v = V_m \sin \omega t$

Whenever an alternating voltage is applied to a pure inductor, a back emf is produced in the inductor, due to self-inductance of the coil which opposes the change in current through the coil at every step.

$$\therefore \; v_L = v = L\frac{di}{dt}$$

$$\Rightarrow V_m \sin \omega t = L\frac{di}{dt}$$

$$\Rightarrow di = \frac{V_m}{L} \sin \omega t \, dt$$

$$\Rightarrow \int di = \int \frac{V_m}{L} \sin \omega t \, dt$$

$$\Rightarrow i = \frac{V_m}{\omega L}(-\cos \omega t)$$

$$= \frac{V_m}{\omega L} \sin(\omega t - \pi/2) \quad \text{--- (6)}$$

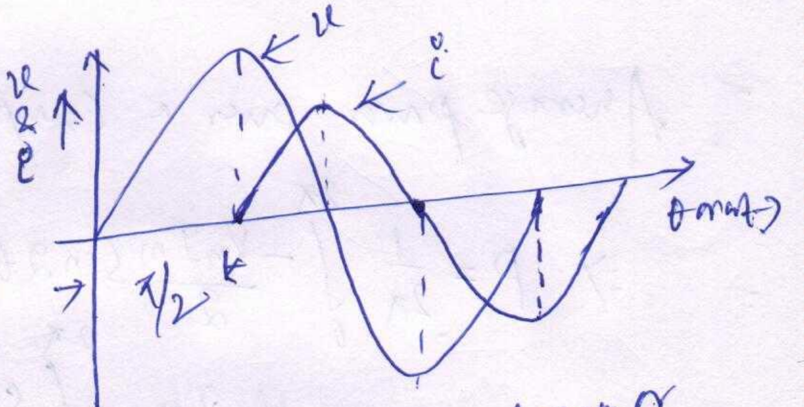


Fig. 1.6 (Time diagram or wave diagram)

Maximum value of i is

$I_m = \dfrac{V_m}{\omega L}$ when $\sin(\omega t - \pi/2)$ is unity.

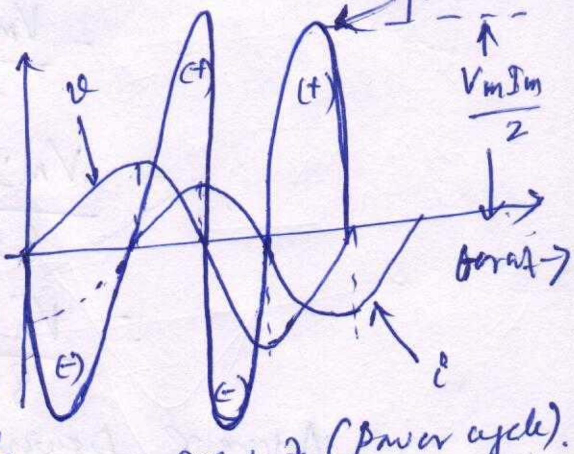


Fig 1.7 (Power cycle).

Now, eqn (6) can be written as

$$i = I_m \sin(\omega t - \pi/2) \quad \text{--- (7)} \quad \text{where, } I_m = \frac{V_m}{\omega L}$$

Here, ωL plays the part of resistance & is called as inductive reactance X_L of the coil. $\therefore \; \boxed{X_L = \omega L = 2\pi f L}$

Now, eqn (6) becomes.

$$i = \frac{V_m}{X_L} \sin(\omega t - \pi/2) \quad -(8)$$

$\therefore$ In a pure inductor, current lags behind the voltage by an angle $90°$ or $\pi/2^c$.

Power.

Instantaneous power, $p = v \times i = V_m \sin\omega t \times I_m \sin(\omega t - \pi/2)$

$$= V_m I_m \sin\omega t . \sin(\omega t - \pi/2)$$

$$= -V_m I_m \sin\omega t . \cos\omega t = \frac{-V_m I_m}{2} \sin 2\omega t.$$

$$= \frac{-V_m I_m \sin 2\theta}{2}$$

Average power over a complete cycle, $P = \frac{1}{2\pi}\int_0^{2\pi} p\, d\theta.$

$$\Rightarrow P = \frac{1}{2\pi}\int_0^{2\pi} -\frac{V_m I_m}{2} \sin 2\theta\, d\theta$$

$$= -\frac{V_m I_m}{4\pi}\int_0^{2\pi} \sin 2\theta\, d\theta$$

$$= -\frac{V_m I_m}{4\pi} \times \frac{-1}{2}\left[\cos 2\theta\right]_0^{2\pi}.$$

$$= \frac{V_m I_m}{8\pi} \cdot \left[\cos 4\pi - \cos 0\right]$$

$$= \frac{V_m I_m}{8\pi} (1-1) = 0.$$

$\therefore$ Average power taken by a pure inductor over a complete cycle is zero & the fact is illustrated in fig. 1.7.

A.C. through pure Capacitance alone.

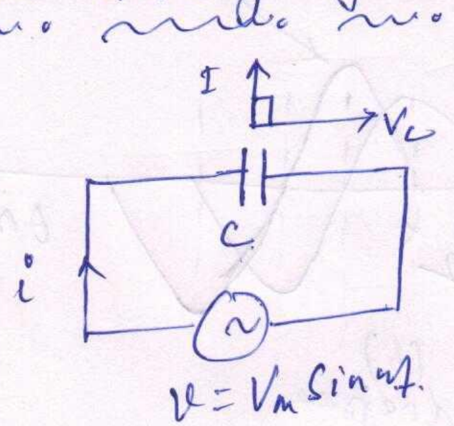


Fig 1.8 (Ckt. diagram)

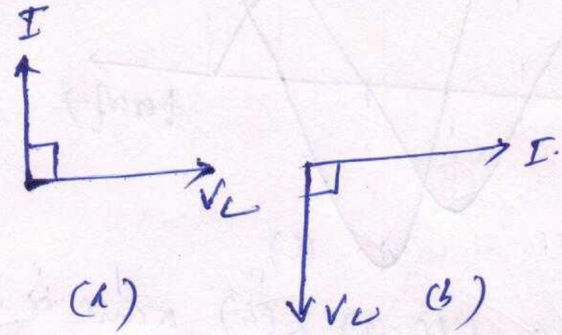


Fig 1.9. (a) & (b) phasor diagram.

Let v = p.d. across the capacitor at any instant.

q = charge of the capacitor at that instant

& C = capacitance of the capacitor.

Then, $q = Cv = C V_m \sin \omega t$.

Now, $i = \frac{dq}{dt} = \frac{d}{dt}(C V_m \sin \omega t) = \omega C V_m \cos \omega t$

$\Rightarrow i = \frac{V_m}{(1/\omega C)} \cos \omega t = \frac{V_m}{(1/\omega C)} \sin(\omega t + \pi/2)$ — (9)

$\frac{1}{\omega C}$ is known as the capacitive reactance of the capacitor (X_C). $\therefore X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$. Its unit is Ω.

Now, eqn (9) can be written as $i = \frac{V_m}{X_C} \sin(\omega t + \pi/2)$

Maximum current is $I_m = \frac{V_m}{X_C}$ when $\sin(\omega t + \pi/2)$ is unity.

$\therefore i = I_m \sin(\omega t + \pi/2)$ — (10) where, $I_m = \frac{V_m}{X_C}$.

From eqn (10), it is clear that, current in a capacitor leads the voltage across the capacitor by an angle 90° as depicted in fig 1.9 & fig. 1.10

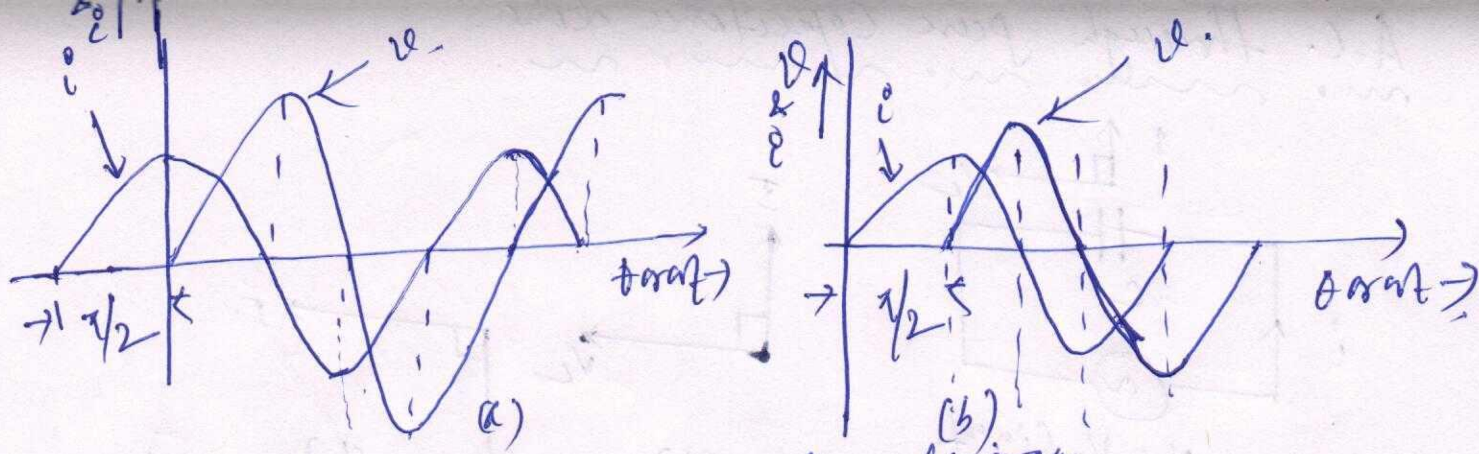


Fig. 1.10 (a) & (b) wave or time diagram.

Power:-

Instantaneous power:-

Instantaneous power, $p = v \times i = V_m \sin\omega t \times I_m \sin(\omega t + \pi/2)$

$= V_m I_m \sin\omega t . \cos\omega t$

$= \frac{V_m I_m}{2} \sin 2\omega t$

$= \frac{V_m I_m}{2} \sin 2\theta$

Average power over a complete cycle, $P = \frac{1}{2\pi}\int_0^{2\pi} p\, d\theta$

$\Rightarrow P = \frac{1}{2\pi}\int_0^{2\pi} \frac{V_m I_m}{2} \sin 2\theta\, d\theta = \frac{V_m I_m}{4\pi}\int_0^{2\pi} \sin 2\theta\, d\theta$

$\Rightarrow P = -\frac{V_m I_m}{8\pi}\left[\cos 2\theta\right]_0^{2\pi} = -\frac{V_m I_m}{8\pi}(\cos 4\pi - \cos 0)$

$\Rightarrow P = -\frac{V_m I_m}{8\pi}(1-1) = 0.$

From 1.11, it is clear that the average power taken by the capacitor over a complete cycle is zero.

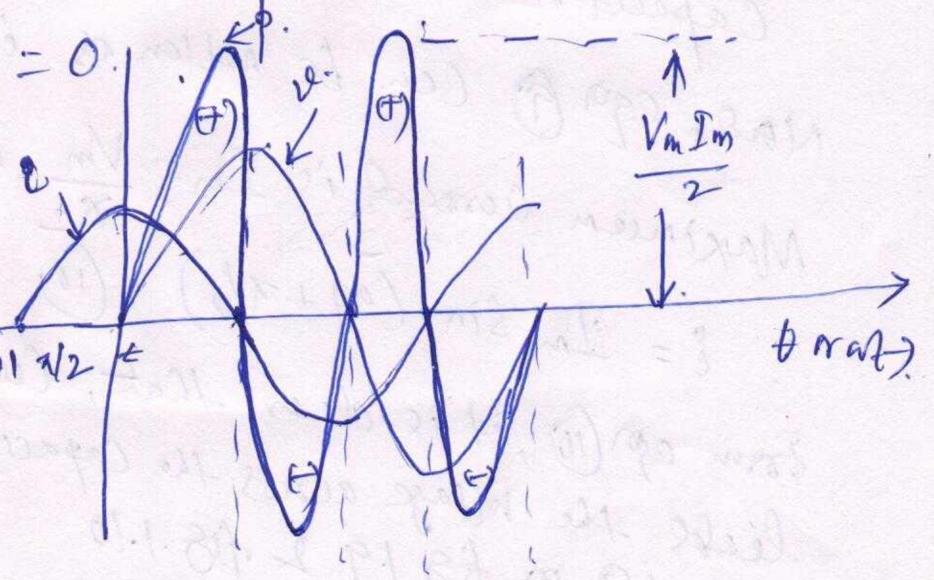
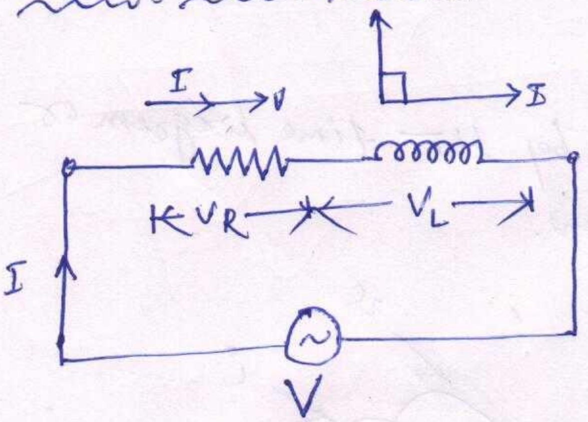
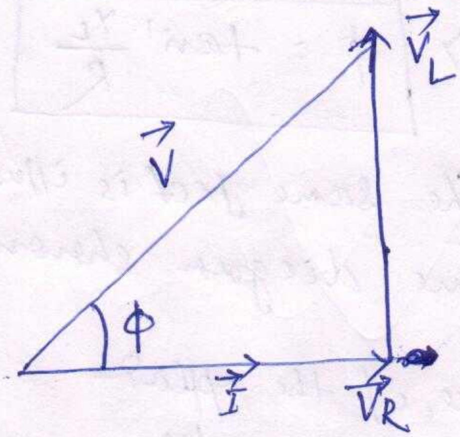


Fig 1.11

R–L SERIES CIRCUIT





Let, V = RMS value of the applied voltage

I = RMS value of the circuit current.

V_R = voltage drop across the resistance $R = IR$

V_L = Voltage drop across the inductance, $L = IX_L$

The phasor diagram of the circuit representing various voltages are shown in the phasor diagram shown above.

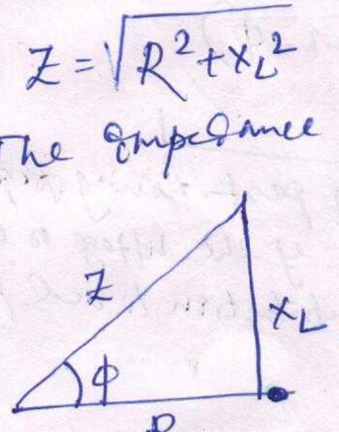
Now, from the phasor diagram, the magnitude of the applied voltage 'V' is given by: $V = \sqrt{V_R^2 + V_L^2} = \sqrt{(IR)^2 + (IX_L)^2}$

$\Rightarrow V = I\sqrt{R^2 + X_L^2} = IZ$

where, Z is known as the 'Impedance' of the circuit

& $Z = \sqrt{R^2 + X_L^2}$

The impedance triangle is shown in the fig. given below.



'Z'. Can be written in rectangular form as

$$\boxed{Z = R + jX_L}$$

& in polar form as.

$Z = Z\angle\phi$.

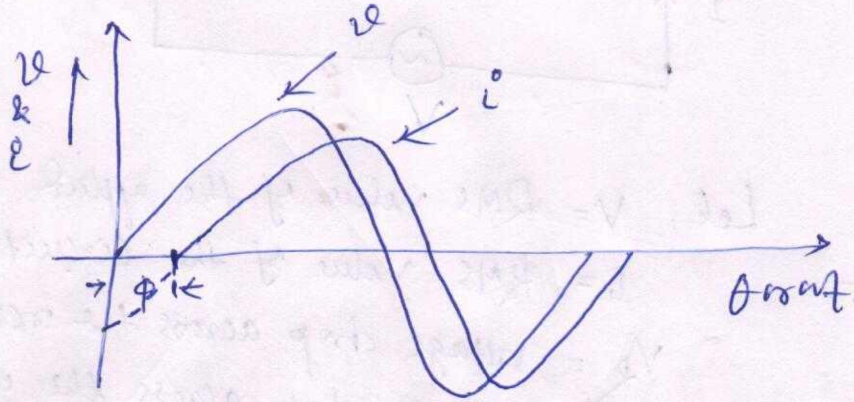
From the phasor diagram of the circuit, it is clear that current I lags behind the applied voltage by angle ϕ such that.

$$\tan\phi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R}$$

$$\Rightarrow \boxed{\phi = \tan^{-1}\frac{X_L}{R}}$$

The same fact is illustrated by the time diagram or wave diagram shown below.

Hence, if the applied voltage is given by $v = V_m \sin\omega t$, then the equation of the ckt. current will be



$$\boxed{i = I_m \sin(\omega t - \phi)}$$

where, $\boxed{I_m = \frac{V_m}{Z}}$

Power :→

Instantaneous power, $p = (V_m \sin\omega t)(I_m \sin(\omega t - \phi))$

$$\Rightarrow p = V_m I_m \sin\omega t \sin(\omega t - \phi)$$

$$= \frac{V_m I_m}{2}\left[\cos\phi - \cos(2\omega t - \phi)\right]$$

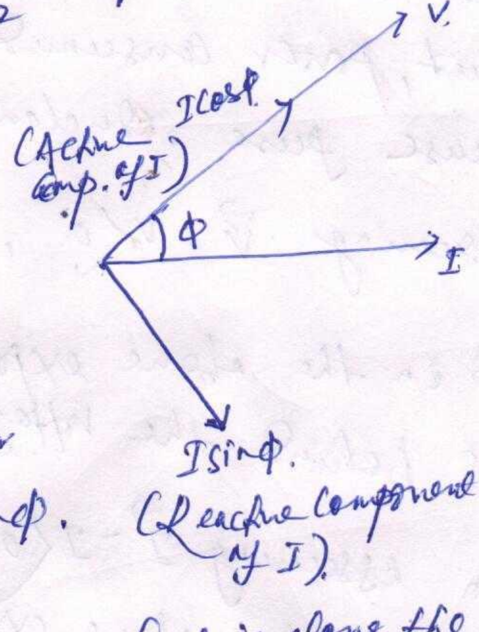
$$= \underbrace{\frac{V_m I_m \cos\phi}{2}}_{\substack{\text{Constant part} \\ \text{(Contributes to real power)}}} - \underbrace{\frac{V_m I_m}{2}\cos(2\omega t - \phi)}_{\substack{\text{Fluctuating part having frequency} \\ \text{twice that of the voltage or current.} \\ \text{(No contribution to real power)}}}$$

Average power over a complete cycle, $P = \frac{1}{2\pi}\int_0^{2\pi} p \, d\theta$.

$$\Rightarrow P = \frac{1}{2\pi}\int_0^{2\pi} \frac{1}{2} V_m I_m [\cos\phi - \cos(2\omega t - \phi)] \, d\theta.$$

$$= \frac{V_m I_m}{2}\cos\phi = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\phi = VI\cos\phi.$$

$$\therefore \boxed{P = VI\cos\phi}$$



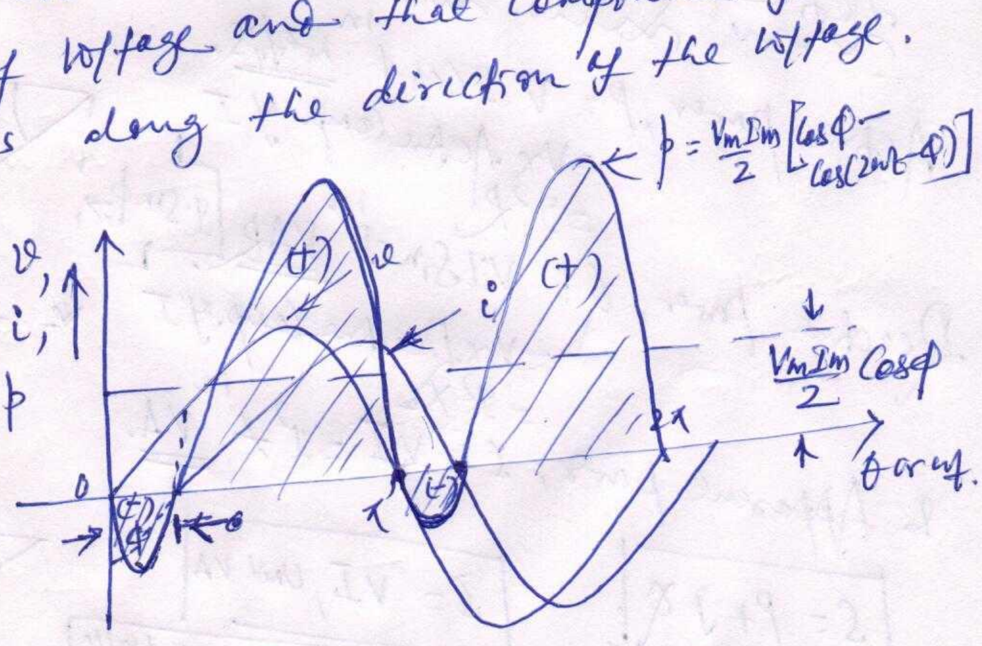
In the fig. I has been resolved into two components: one along the direction of V i.e. $I\cos\phi$ & another perpendicular to the direction of V i.e. $I\sin\phi$.

Now, $P = VI\cos\phi$

$= V \times$ (that component of I which is along the direction of the voltage).

Hence the power in a R-L series circuit is given by the product of voltage and that component of current which is along the direction of the voltage.

It should be noted that in an AC circuit, the product of voltage & current gives Volt-Amps (VA) and not the true power or real power in watts.



The term '$\cos\phi$' is known as the power factor (pf) of the circuit. From the impedance triangle, $\boxed{\cos\phi = \frac{R}{Z}}$

$$P = VI\cos\phi = VI\frac{R}{Z} = \frac{V}{Z} I R = I^2R$$

From the above equation, it is clear that, in R-L circuit, power consumed is only due to the resistance because pure inductance does not consume any power.

Assuming $\vec{V} = V\angle 0^\circ$, then, $\vec{I} = \frac{\vec{V}}{\vec{Z}} = \frac{V\angle 0}{Z\angle\phi} = \frac{V}{Z}\angle -\phi$

$-\phi$ in the above expression indicates that current lags behind the voltage.

Again assuming $\vec{I} = I\angle 0$, then, $\vec{V} = \vec{I}\vec{Z} = I\angle 0 \cdot Z\angle\phi = IZ\angle\phi$

$+\phi$ in the above expression indicates that current leads ahead of the voltage. ~~by~~

Active, Reactive and Apparent power:→

Active power, $P = VI\cos\phi$ in watts

$= V \times$ Active comp. of I

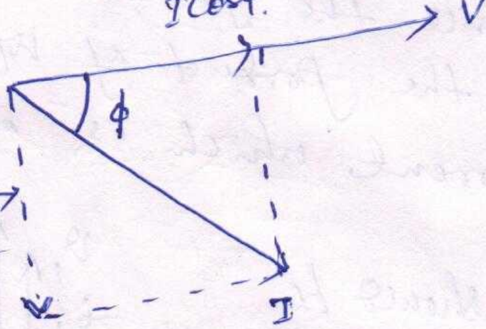
$= I^2R$

Reactive power, $Q = VI\sin\phi$ in VAR

$= V \times$ Reactive comp. of I

$= I^2X_L$

& Apparent power, $S = VI = I^2Z$ in VA.



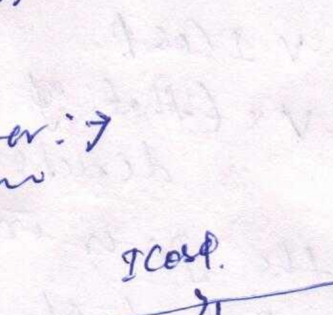
$\boxed{S = P + jQ}$

$\boxed{|S| = \sqrt{P^2 + Q^2}}$

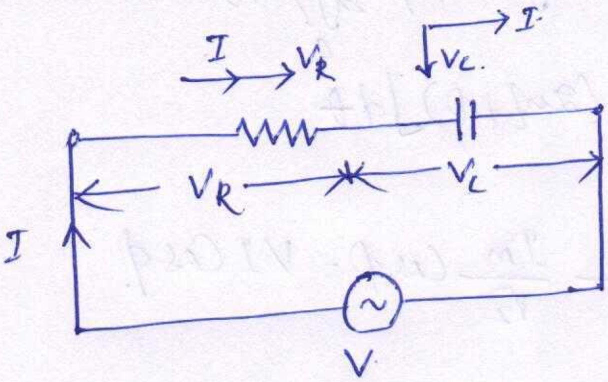
$\boxed{S = VI,\ \text{Unit VA}}$

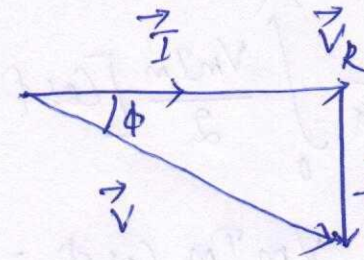
$\boxed{P = VI\cos\phi,\ \text{Unit watts}}$

$\boxed{Q = VI\sin\phi,\ \text{Unit VAR}}$

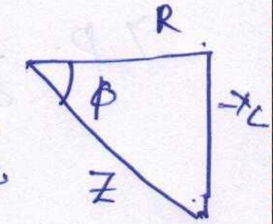


R-C Series Circuit.





(Voltage triangle.)



(Impedance triangle).

Applied voltage, $V = \sqrt{(V_R)^2 + (-V_C)^2}$

$= \sqrt{(IR)^2 + (-IX_C)^2}$

$= I\sqrt{R^2 + X_C^2}$

$= IZ$

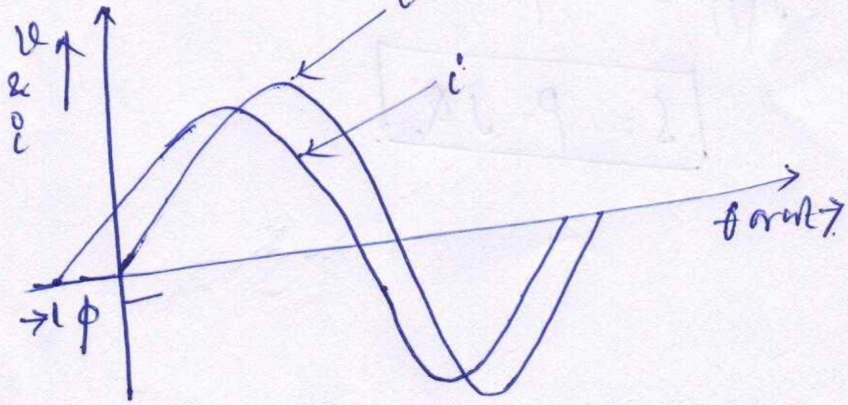
where, Z is known as impedance of the circuit & $\boxed{Z = \sqrt{R^2 + X_C^2}}$

$$\boxed{Z = R - jX_C = Z\angle -\phi}$$

& $\tan\phi = \frac{-V_C}{V_R} = \frac{-IX_C}{IR} = \frac{-X_C}{R}$ $\therefore \boxed{\phi = \tan^{-1}\frac{-X_C}{R}}$

Current leads ahead of the voltage by an angle ϕ.

Hence, if $\boxed{v = V_m \sin \omega t}$, then $\boxed{i = I_m \sin(\omega t + \phi)}$



Power:-

Instantaneous power, p

$= v\,i$

$= (V_m \sin\omega t)\left[I_m \sin(\omega t + \phi)\right]$

$= V_m I_m \sin\omega t \cdot \sin(\omega t + \phi)$

$= \frac{V_m I_m}{2}\left[\cos\phi - \cos(2\omega t + \phi)\right]$

$= \underbrace{\frac{V_m I_m}{2}\cos\phi} - \underbrace{\frac{V_m I_m}{2}\cos(2\omega t + \phi)}$

Pulsating part having frequency twice ... current & does not contribute

Average power over a complete cycle, $P = \frac{1}{2\pi}\int_0^{2\pi} p\,d\theta$

$$\Rightarrow P = \frac{1}{2\pi}\int_0^{2\pi} \frac{V_m I_m}{2}\left[\cos\phi - \cos(2\omega t + \phi)\right] d\theta$$

$$= \frac{V_m I_m}{2}\cos\phi = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}\cos\phi = VI\cos\phi$$

$\therefore \boxed{P = VI\cos\phi}$

power factor, $\boxed{\cos\phi = \frac{R}{Z}}$

If $\vec{V} = V\angle 0$, then, $\vec{I} = \frac{V\angle 0}{Z\angle -\phi} = \frac{V}{Z}\angle\phi$ (current leads the voltage)

If $\vec{I} = I\angle 0$, then, $\vec{V} = \vec{I}Z = (I\angle 0)(Z\angle -\phi) = IZ\angle -\phi$ (voltage lags the current).

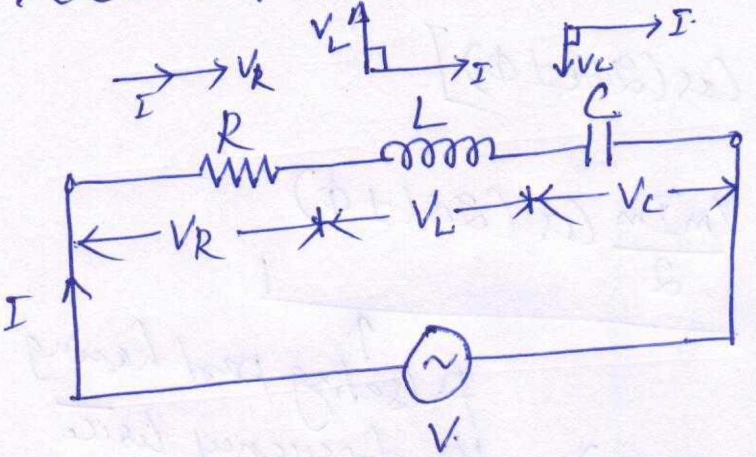
Active power or True power or Real power, $P = VI\cos\phi$
$= I^2R$ watts.

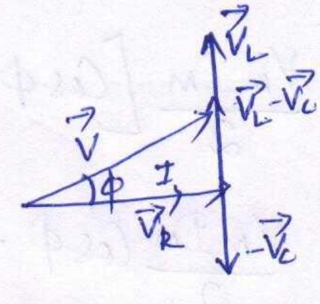
Reactive power, $Q = VI\sin\phi = (IZ)\,I \times \frac{X_C}{Z} = I^2 X_C$. VAR.

Apparent power, $S = VI = (IZ)I = I^2 Z$ VA.

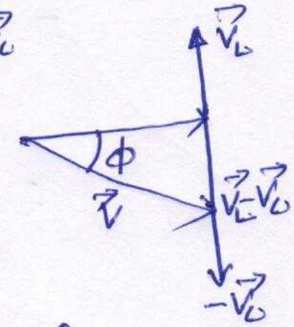
$\boxed{S = P - jQ}$

R-L-C Series Circuit





$(X_L > X_C$ or $V_L > V_C)$.



$(X_L < X_C$ or $V_L < V_C)$.

Applied voltage, $V = \sqrt{(V_R)^2 + (V_L - V_C)^2}$

$$= \sqrt{(IR)^2 + I^2(X_L - X_C)^2}$$

$$= I\sqrt{R^2 + (X_L - X_C)^2} = I\sqrt{R^2 + X^2}$$

(X is the net reactance of the circuit).

$$= IZ$$

where, Z is known as the impedance of the circuit

$$\& \; Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\boxed{Z = R + j(X_L - X_C) = Z\angle \pm \phi}$$

when $X_L > X_C$, ϕ is +ve.
& when $X_L < X_C$, ϕ is -ve.

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{I(X_L - X_C)}{IR} = \frac{X_L - X_C}{R} = \frac{X}{R}.$$

If $v = V_m \sin \omega t$, then, $i = I_m \sin(\omega t \pm \phi)$

when $X_L < X_C$ (for $+$), when $X_L > X_C$ (for $-$).

Power:-

Instantaneous power, $p = v\,i$

$$= (V_m \sin \omega t)\,[I_m \sin(\omega t \pm \phi)]$$

$\Rightarrow p = V_m I_m \sin\omega t . \sin(\omega t \pm \phi)$

$= \frac{V_m I_m}{2}\left[\cos\phi - \cos(2\omega t \pm \phi)\right]$

$= \underbrace{\frac{V_m I_m}{2}\cos\phi}_{\substack{\text{Constant part} \\ \text{(Contributes to real power)}}} - \underbrace{\frac{V_m I_m}{2}\cos(2\omega t \pm \phi)}_{\substack{\text{Pulsating part having the frequency twice that of the voltage or current.} \\ \text{(does not contribute to real power)}}}$

Average power, over a complete cycle, $P = \frac{1}{2\pi}\int_0^{2\pi} p\, d\theta$

$\Rightarrow P = \frac{1}{2\pi}\int_0^{2\pi} \frac{V_m I_m}{2}\left[\cos\phi - \cos(2\omega t \pm \phi)\right] d\theta$

$= \frac{V_m I_m}{2}\cos\phi = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\phi = VI\cos\phi.$

$\therefore \boxed{P = VI\cos\phi}$ & power factor, $\boxed{\cos\phi = \frac{R}{Z}}$

Notes:

Case-I when $X_L > X_C$, then $V_L > V_C$ & ϕ is +ve. It is an inductive circuit & current lags behind the voltage. The net reactance, $X = X_L - X_C$ is +ve.

Case-II when $X_L < X_C$, then $V_L < V_C$ & ϕ is -ve. It is a capacitive circuit & current leads ahead of the voltage. The net reactance, $X = X_L - X_C$ is -ve.

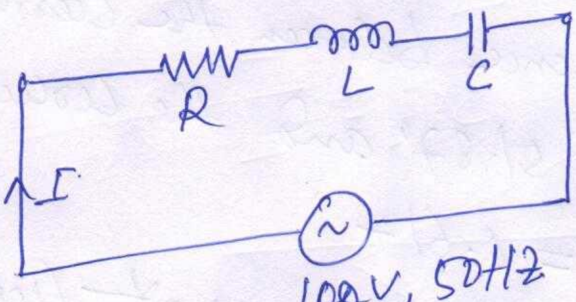
Active power $P = VI\cos\phi = I^2R$ (watts) & Apparent power, $P = VI$ (VA).

Reactive power $Q = VI\sin\phi$ (VAR)

Problem :→ (Problem on RLC Series circuit).

A circuit having a resistance of 12 Ω, an inductance of 0.15 H and a capacitance of 100 μF in series is connected across a 100 V, 50 Hz supply. Calculate: (a) the impedance, (b) the current, (c) the voltage across R, L & C, (d) the phase difference between the current and the supply voltage & (e) the active, reactive and apparent power of the circuit.

Sol :→



Inductive reactance, $X_L = \omega L = 2\pi f L = 2\times\pi\times 50\times 0.15$

$\Rightarrow X_L = 47.12\ \Omega$

Capacitive reactance, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

$\Rightarrow X_C = \frac{1}{2\times\pi\times 50\times 100\times 10^{-6}} = 31.83\ \Omega$.

(a) $Z = R + j(X_L - X_C) = 12 + j(47.12 - 31.83)$

$= 12 + j15.29 = 19.43\angle 51.87$

$\therefore$ Value of impedance of the circuit is $19.43\ \Omega$

(b) Let $\vec{V} = V\angle 0^\circ$

$\therefore \vec{I} = \frac{\vec{V}}{Z} = \frac{100\angle 0^\circ}{19.43\angle 51.87} = 5.14\angle -51.87$

$\therefore$ Circuit current $= 5.14\,A$.

(c) voltage across R, $V_R = IR = 5.14 \times 12 = 61.68\,V$.

voltage across L, $V_L = IX_L = 5.14 \times 47.12 = 242.2\,V$.

voltage across C, $V_C = IX_C = 5.14 \times 31.83 = 163.6\,V$.

(d) $Z = 19.43\angle 51.87$

$\therefore$ phase difference between the current and the supply voltage is 51.87° and the circuit power factor.

$\cos\phi = \cos 51.87 = 0.617$

e) power in complex form $S = V \cdot I^* = (100\angle 0^\circ)(5.14\angle 51.87)$

$\Rightarrow S = 514\angle 51.87 = 317.37 + j404.32 = P + jQ$.

$\therefore$ Active power, $P = 317.37\,W$,

Reactive power, $Q = 404.32\,VAR$

& Apparent power, $S = 514\,VA$.

Alternatively.

$P = VI\cos\phi = 100 \times 5.14 \times 0.617 = 317.138\,W$.

$= I^2R = (5.14)^2 \times 12 = 317\,W$.

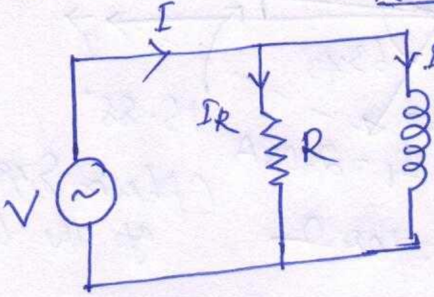
$Q = VI\sin\phi = 100 \times 5.14 \times \sin(51.87) = 404.32\,VAR$

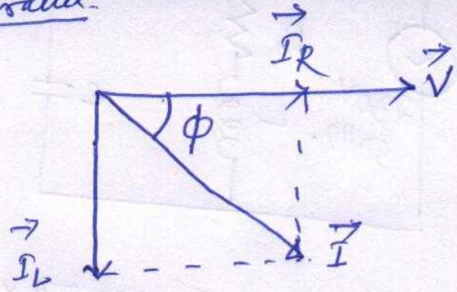
$= I^2X = (5.14)^2 \times \cancel{47.12}\ 15.29 = 404\,VAR$

$S = VI = 100 \times 5.14 = 514\,VA$ or $S = I^2Z = (5.14)^2 \times 19.43 = 513.33\,VA$.

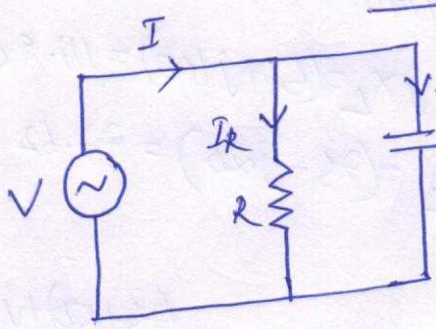
SIMPLE PARALLEL CIRCUITS:-

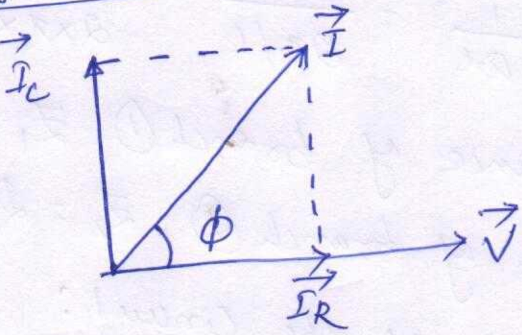
R&L in parallel



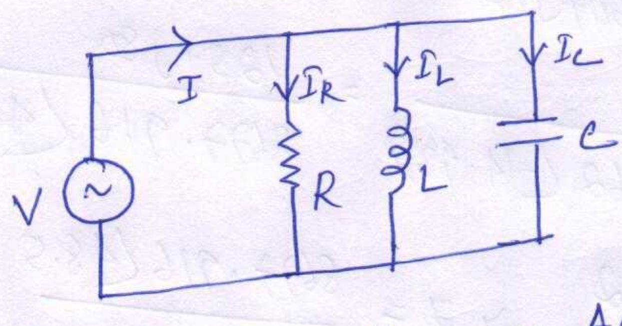


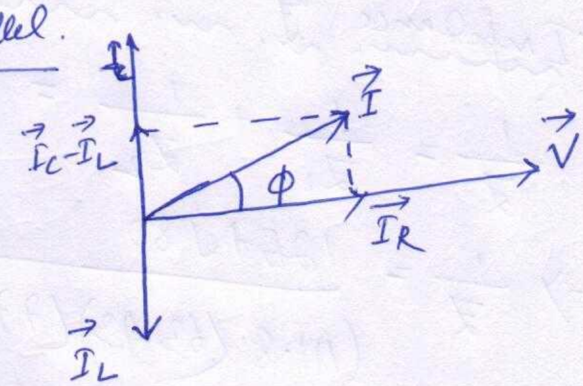
R&C in parallel





R, L & C in parallel





SOLUTIONS OF PARALLEL AC CIRCUITS.

Problem:-> A coil of resistance 50Ω and inductance of $0.318H$ is connected in parallel with a circuit comprising a 75Ω resistor in series with a $159\,\mu F$ capacitor. The resulting circuit is connected to a 230V, 50Hz AC supply. Calculate (a) the circuit impedance, resistance & reactance

(b) p.f of the circuit and pf of each branch.

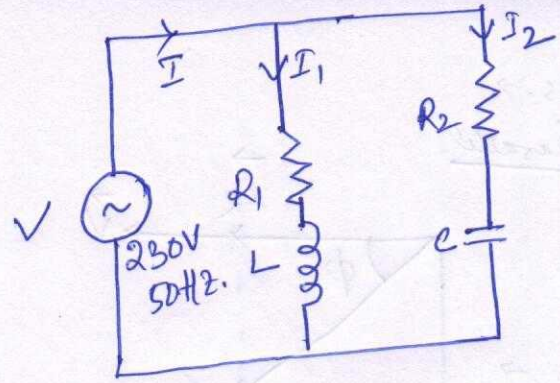
(c) the supply current and current in each branch.

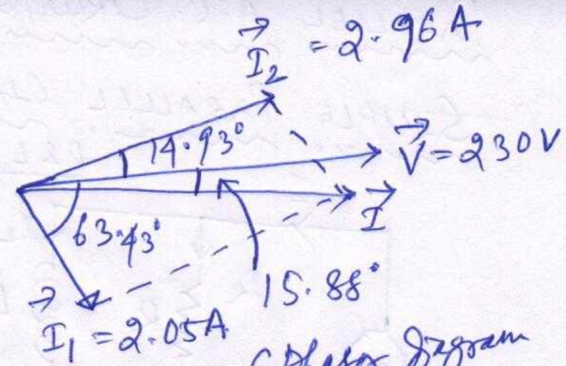
(d) Active, reactive and apparent power of the circuit

(e) Active, reactive and apparent power of each branch.

Draw the phasor diagram of the circuit.

Sol:→





(phasor diagram of the circuit).

$X_L = \omega L = 2\pi f L = 2\times\pi\times 50\times 0.318 = 100\,\Omega$

$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\times\pi\times 50\times 159\times 10^{-6}} = 20\,\Omega.$

Impedance of branch ①, $Z_1 = R_1 + jX_L = (50+j100) = 111.8\angle 63.43°$

Impedance of branch ②, $Z_2 = R_2 - jX_C = (75-j20) = 77.62\angle -14.93°$

(a) Impedance of the circuit:

$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{50+j100} + \frac{1}{75-j20} = \frac{(75+50)+j(100-20)}{(50+j100)(75-j20)}$

$\Rightarrow \frac{1}{Z} = \frac{125+j80}{(111.8\angle 63.43)(77.62\angle -14.93°)} = \frac{125+j80}{8677.916\angle 48.5}$

$\Rightarrow \frac{1}{Z} = \frac{148.4\angle 32.62}{8677.916\angle 48.5} \Rightarrow Z = \frac{8677.916\angle 48.5}{148.4\angle 32.62}$

$\Rightarrow Z = 58.47\angle 15.88 = 56.24 + j16. = R + jX$

∴ Impedance of the circuit $= 58.47\,\Omega$

Resistance of the circuit, $R = 56.24\,\Omega$

Reactance of the circuit, $X = 16\,\Omega$.

(b) Pf of the circuit & pf of each branch :→

$Z = 58.47 \angle 15.88$

$\therefore$ phase angle between the applied voltage (V) & circuit current 'I' is $\phi = 15.88^\circ$

$\therefore$ pf of the circuit $= \cos\phi = \cos 15.88 = 0.96$ (lagging).

$Z_1 = 111.8 \angle 63.43^\circ$

$\therefore$ pf of the branch 1, $\cos\phi_1 = \cos 63.43^\circ = 0.4473$ (lagging)

$Z_2 = 77.62 \angle -14.93^\circ$

$\therefore$ pf of the branch 2, $\cos\phi_2 = \cos(-14.93^\circ) = 0.966$ (leading).

(c) Supply current & current in each branch :→

Let $\vec{V} = V + j0 = V\angle 0 = 230\angle 0$

$\therefore$ Supply current, $\vec{I} = \dfrac{\vec{V}}{Z} = \dfrac{230\angle 0}{58.47\angle 15.88} = 3.93 \angle -15.88$

$\therefore$ Magnitude of supply current $I = 3.93$ A.

$\vec{I_1} = \dfrac{\vec{V}}{Z_1} = \dfrac{230\angle 0}{111.8\angle 63.43} = 2.05 \angle -63.43^\circ$

$\therefore$ Magnitude of current in branch 1, $I_1 = 2.05$A.

$\vec{I_2} = \dfrac{\vec{V}}{Z_2} = \dfrac{230\angle 0}{77.62\angle -14.93} = 2.96 \angle 14.93^\circ$

$\therefore$ Magnitude of current in branch 2, $I_2 = 2.96$ A.

(d) Active, Reactive and Apparent power of the circuit :→

Complex power of the circuit, $S = \vec{V} \cdot \vec{I^*}$

$\Rightarrow S = (230\angle 0^\circ)(3.93\angle 15.88)$

$= 904 \angle 15.88 = 869.5 + j\,247.35 = P + jQ.$

$\therefore$ Active power, $P = 869.5\,W$

Reactive power, $Q = 247.35$ VAR (lagging VAR).

Apparent power, $S = 904$ VA

Alternatively,

$P = VI\cos\phi = I^2R$, $Q = VI\sin\phi = I^2X$,

$S = VI = I^2Z = \sqrt{P^2 + Q^2}$

(e) Active, Reactive & Apparent power of each branch :→

For branch (1)

Complex power, $S_1 = \vec{V} \cdot \vec{I_1^*} = (230\angle 0^\circ)(2.05\angle 63.43^\circ)$

$\Rightarrow S_1 = 471.5 \angle 63.43 = 210.89 + j\,421.70 = P_1 + jQ_1$

$\therefore P_1 = 210.89\,W$, $Q_1 = 421.70$ VAR (lagging VAR) & $S = 471.5$ VA

Alternatively

$P_1 = VI_1\cos\phi_1 = I_1^2R_1$, $Q_1 = VI_1\sin\phi_1 = I_1^2X_L$ & $S = VI_1 = I_1^2Z_1 = \sqrt{P_1^2 + Q_1^2}$

For branch (2)

Complex power, $S_2 = \vec{V} \cdot \vec{I_2^*} = (230\angle 0^\circ)(2.96\angle -14.93^\circ) = 680.8\angle -14.9$

$= 657.81 - j\,175.4 = P_2 - jQ_2$

$\therefore P_2 = 657.81\,W$, $Q_2 = -175.4$ VAR (leading VAR), $S_2 = 680.8$ VA

$\ldots(VI_2\sin\phi_2) = -(I_2^2X_C)$ &

SERIES PARALLEL AC CIRCUIT :→

Problem :→ Determine the current drawn by the following circuit when a voltage of 200V is applied across the same. Also determine the following:

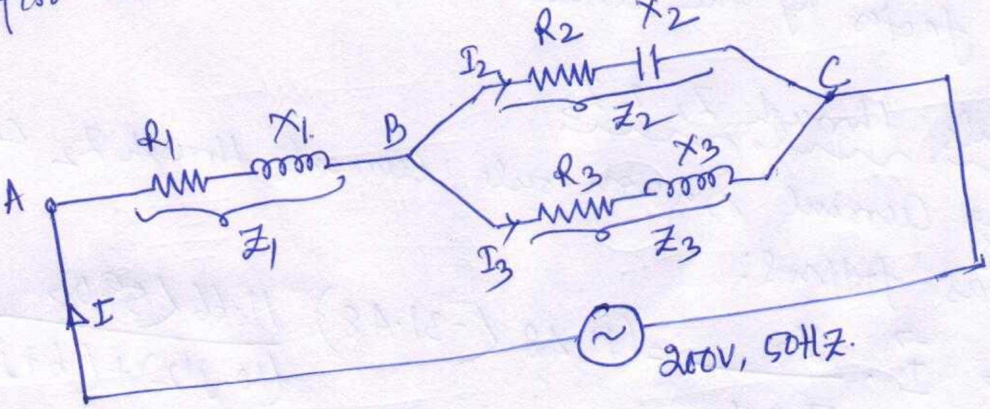
i) power factor of the circuit

ii) Currents through Z_2 & Z_3.

iii) power factors of the branches comprising of Z_1, Z_2 & Z_3.

iv) Active, Reactive and Apparent power of the circuit & branches comprising of Z_1, Z_2 & Z_3.

Also draw the phasor diagram of the circuit.



Soln:→ $Z_1 = R_1 + jX_1 = 4+j6 = 7.21\angle 56.31$

$Z_2 = R_2 - jX_2 = 10-j12 = 15.62\angle -50.2$

$Z_3 = R_3 + jX_3 = 6+j10 = 11.66\angle 59.03$

Equivalent impedance between B & C:

$$\frac{1}{Z_{BC}} = \frac{1}{Z_2} + \frac{1}{Z_3} = \frac{1}{10-j12} + \frac{1}{6+j10} = \frac{(6+j10)+(10-j12)}{(10-j12)(6+j10)}$$

$$\Rightarrow \frac{1}{Z_{BC}} = \frac{16-j2}{(15.62\angle -50.2)(11.66\angle 59.03)} = \frac{16.12\angle -7.12}{182.13\angle 8.83}$$

$$\Rightarrow Z_{BC} = \frac{182.13\angle 8.83}{16.12\angle -7.12} = 11.3\angle 15.95 = 10.86+j3.1$$

$\therefore$ Impedance of the Circuit, $Z = Z_1 + Z_{BC} = (4+j6) + (10.86 + j3.1)$

$\Rightarrow Z = 14.86 + j9.1 = 17.42 \angle 31.48$
$= R + jX$

Let, $\vec{V} = V + j0 = V\angle 0$
$= 200 + j0 = 200\angle 0$

$\therefore$ Current drawn by the Circuit, $\vec{I} = \dfrac{\vec{V}}{Z} = \dfrac{200\angle 0}{17.42\angle 31.48} = 11.48\angle -31.48$

i) Power factor of the Circuit.

$Z = 17.42 \angle 31.48 = |Z| \angle \phi$

$\therefore \phi = 31.48$ (Angle between V & I)

$\therefore$ Power factor of the Circuit, $\cos\phi = \cos 31.48 = 0.853$ (Lagging).

ii) Currents through Z_2 & Z_3

Using Current division rule, currents through Z_2 can be found as follows:

$$\vec{I_2} = \vec{I} \cdot \frac{Z_3}{Z_2 + Z_3} = (11.48\angle -31.48)\,\frac{11.66\angle 59.03}{(10-j12)+(6+j10)}$$

$$\Rightarrow \vec{I_2} = (11.48\angle -31.48)\,\frac{11.66\angle 59.03}{16-j2} = (11.48\angle -31.48)\,\frac{11.66\angle 59.03}{16.12\angle -7.12}$$

$$= 8.3\angle 34.67^\circ$$

Similarly, Current through Z_3 can be found as follows:

$$\vec{I_3} = \vec{I} \cdot \frac{Z_2}{Z_2 + Z_3} = (11.48\angle -31.48)\,\frac{15.62\angle -50.2}{16.12\angle -7.12} = 11.12\angle -74.56$$

Alternatively, $\vec{I_3} = \vec{I} - \vec{I_2}$ (Applying KCL at point B).

$\Rightarrow \vec{I_3} = (11.48\angle -31.48) - (8.3\angle 34.67)$
$= (9.8 - j6) - (6.82 + j4.72) = 2.98 - j10.72 = 11.12\angle -74.46$

Altsnate Method to find I_2 & I_3.

$$\vec{V}_{BC} = \vec{V} - \vec{V}_{AB}$$

$$\vec{V}_{AB} = \vec{V}_1 = \vec{I} \cdot \vec{Z}_1 = (11.48\angle -31.48)(7.21\angle 56.31) = 82.77\angle 24.83$$

$$\therefore \vec{V}_{BC} = (200\angle 0) - (82.77\angle 24.83) = (200 + j0) - (75.12 + j\,34.75)$$

$$= 124.88 - j\,34.75 = 129.62\angle -15.55$$

$$\therefore \vec{I}_2 = \frac{\vec{V}_{BC}}{Z_2} = \frac{129.62\angle -15.55}{15.62\angle -50.2} = 8.3\angle 34.65$$

$$\& \ \vec{I}_3 = \frac{\vec{V}_{BC}}{Z_3} = \frac{129.62\angle -15.55}{11.66\angle 59.03} = 11.12\angle -74.58$$

iii) Power factors of the branches comprising of Z_1, Z_2 & Z_3

$Z_{AB} = Z_1 = 7.21\angle 56.31$

$\therefore$ Angle between V_{AB} & I, $\phi_1 = 56.31$ & $\cos\phi_1 = \cos 56.31 = 0.55$ (lagging).

$Z_2 = 15.62\angle -50.2$

$\therefore$ Angle between V_{BC} & I_2, $\phi_2 = -50.2$ & $\cos\phi_2 = \cos(-50.2) = 0.64$ (leading).

$Z_3 = 11.66\angle 59.03$

$\therefore$ Angle between V_{BC} & I_3, $\phi_3 = 59.03$ & $\cos\phi_3 = \cos 59.03 = 0.514$ (lagging).

iv) Active, Reactive & Apparent power:→

For the Circuit:

$$S = \vec{V}\vec{I}^* = (200\angle 0)(11.48\angle 31.48) = 2296\angle 31.48$$

$$= 1958 + j1199 = P + jQ.$$

$P = 1958\,W$, $Q = 1199$ (VAR, lagging), $S = 2296\,VA$.

Alternatively,

$P = VI\cos\phi$ (OR) $P = I^2R$

$= 200 \times 11.48 \times 0.853$ $= (11.48)^2 \times 14.86$

$= 1958\,W$ $= 1958\,W.$

$Q = VI\sin\phi$ (OR) $Q = I^2X$

$= 200 \times 11.48 \times \sin(31.48)$ $= (11.48)^2 \times 9.1$

$= 1199$ VAR, (Lagging) $= 1199$ VAR (Lagging)

$S = VI$ (OR) $S = I^2Z$ (OR) $S = \sqrt{P^2+Q^2}$

$= 200 \times 11.48$ $= (11.48)^2 \times 17.42$ $= \sqrt{(1958)^2 + (1199)^2}$

$= 2296\,VA$ $= 2296\,VA$ $= 2296\,VA.$

For the branches

(a) For branch AB comprising of Z_1.

$S_1 = \vec{V_1} \cdot \vec{I^*} = (82.77\angle 24.83)(11.48\angle 31.48)$

$= 950.2\angle 56.31 = 527 + j790 = P_1 + jQ_1$

$\therefore P_1 = 527\,W$, $Q = 790$ VAR (lagging), $S_1 = 950.2\,VA$

Alternatively,

$P_1 = V_1 I\cos\phi_1 = 82.77 \times 11.48 \times \cos 56.31 = 527\,W$ (OR) $P_1 = I^2R_1 = (11.48)^2 \times 4 = 527\,W.$

$Q_1 = V_1 I\sin\phi_1 = 82.77 \times 11.48 \times \sin 56.31 = 790$ VAR, (lagging)

(OR)

$Q_1 = I^2X_1 = (11.48)^2 \times 6 = 790$ VAR, (lagging).

$S_1 = V_1 I = 82.77 \times 11.48$ (OR) $S_1 = I^2Z_1$ (OR) $S = \sqrt{P_1^2 + Q_1^2}$

$= 950.2\,VA$ $= (11.48)^2 \times 7.21$ $\Rightarrow S = \sqrt{(527)^2 + (790)^2}$

$= 950.2\,VA$ $= 950.2\,VA$

(b) For branch comprising of Z_2:

$$S_2 = \vec{V}_{BC} \cdot \vec{I_2^*}$$

$$\vec{V}_{BC} = 129.62 \angle -15.55$$

$\vec{V}_{BC}$ Can also be found as follows:

$$\vec{V}_{BC} = \vec{I_2} \cdot Z_2 \text{ or } \vec{V}_{BC} = \vec{I_3} \cdot Z_3$$

Now, $S_2 = (129.62 \angle -15.55)(8.3 \angle -34.67)$

$= 1076 \angle -50.2 = 689 - j827 = P_2 - jQ_2$

$= 1076 \angle -50.2 = 689 - j827 = P_2 - jQ_2$ & $S_2 = 1076$ VA.

$\therefore P_2 = 689$ W, $Q_2 = -827$ VAR, (leading) & $S_2 = 1076$ VA.

Alternatively,

$P_2 = V_{BC} I_2 \cos\phi_2$

$= 129.62 \times 8.3 \times \cos(-50.2)$

$= 689$ W

(OR) $P_2 = I_2^2 R_2$

$= (8.3)^2 \times 10$

$= 689$ W.

$Q_2 = V_{BC} I_2 \sin\phi_2$

$= 129.62 \times 8.3 \times \sin(-50.2)$

$= -827$ VAR (leading)

(OR) $Q_2 = -(I_2^2 X_2)$

$= -(8.3^2 \times 12)$

$= -827$ VAR (leading).

$S_2 = V_{BC} I_2 = 129.62 \times 8.3$

$= 1076$ VA

(OR) $S_2 = I_2^2 Z_2$

$= (8.3)^2 \times 15.62$

$= 1076$ VA

(OR) $S_2 = \sqrt{P_2^2 + Q_2^2}$

$\Rightarrow S_2 = \sqrt{689^2 + 827^2}$

$= 1076$ VA.

(c) For branch comprising of Z_3:

$S_3 = \vec{V}_{BC} \cdot \vec{I_3^*} = (129.62 \angle -15.55)(11.12 \angle 74.56)$

$= 1441 \angle 59.01 = 742 + j1235$.

$\therefore P_3 = 742$ W, $Q = 1235$ VAR, (lagging), $S_3 = 1441$ VA.

Alternatively,

$P_3 = V_{BC} I_3 \cos\phi_3 = 129.62 \times 11.12 \times \cos 59.03$ (OR) $P_3 = I_3^2 R_3$

$= 742\,W.$ $= (11.12)^2 \times 6$

$= 742\,W.$

$Q_3 = V_{BC} I_3 \sin\phi_3 = 129.62 \times 11.12 \times \sin 59.03$ (OR) $Q_3 = I_3^2 X_3$

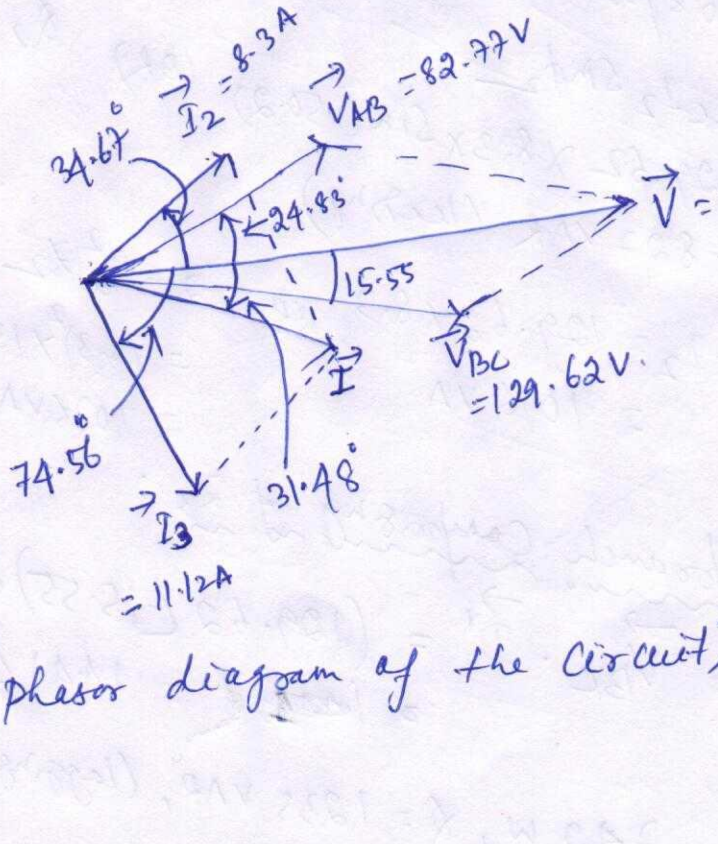
$= 1235\,VAR$ (lagging) $= (11.12)^2 \times 10$

$= 1235\,VAR$ (lagging).

$S_3 = V_{BC} I_3 = (129.62 \times 11.12)$ (OR) $S_3 = I_3^2 Z_3$ (OR) $S_3 = \sqrt{P_3^2 + Q_3^2}$

$= 1441\,VA$ $= (11.12)^2 \times 11.66$ $= \sqrt{(742)^2 + (1235)^2}$

$= 1441\,VA$ $= 1441\,VA.$



(phasor diagram of the circuit).

ADMITANCE, CONDUCTANCE & SUSCEPTANCE :→

IN DC circuits, when two or more resistance are connected in parallel, the equivalent resistance R is given by:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$$

IN DC work, the reciprocal of resistance is known as conductance. It is represented by the symbol 'G' and the unit of conductance is 'Siemens' (S). Hence, the above equation can be written as

$$G = G_1 + G_2 + \cdots$$

where G is the equivalent conductance & $G_1, G_2, \ldots$ are the conductances of individual branches.

However, in a.c. work the conductance is the reciprocal of resistance only when the circuit possesses no reactance.

IN AC circuits, when two or more impedances are connected in parallel, the equivalent impedance 'Z' is given by:

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \cdots$$

IN a.c. work, the reciprocal of impedance is known as admittance and is represented by the symbol Y. The unit of admittance is Siemens (S). Hence, the above equation can be written as

$$Y = Y_1 + Y_2 + \cdots$$

where Y is the equivalent admittance & $Y_1, Y_2, \ldots$ are the admittances of individual branches.

Let us consider two cases as follows:

Case-I When the net reactance is inductive.

$$Z = R + jX$$

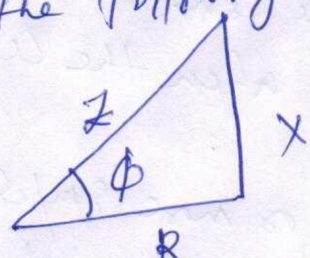
$$Y = \frac{1}{Z} = \frac{1}{R+jX} = \frac{R-jX}{R^2+X^2} = \frac{R}{R^2+X^2} - j\frac{X}{R^2+X^2}$$

$$\Rightarrow Y = G - jB = G + j(-B)$$

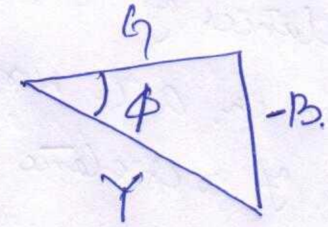
where, $G = \text{Conductance} = \frac{R}{R^2+X^2} = \frac{R}{Z^2}$

& $B = \text{Susceptance} = \frac{X}{R^2+X^2} = \frac{X}{Z^2}$

Notes:→ Inductive reactance $(+jX)$ is positive but the inductive susceptance $(-jB)$ is negative as shown on the following fig.



(Impedance triangle)



(Admittance triangle).

Case-II When the net reactance is capacitive

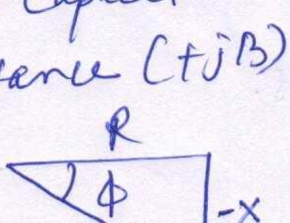
$$Z = R - jX$$

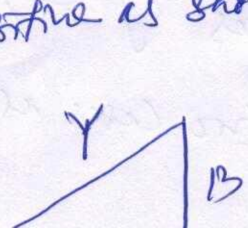
$$Y = \frac{1}{Z} = \frac{1}{R-jX} = \frac{R+jX}{R^2+X^2} = \frac{R}{R^2+X^2} + j\frac{X}{R^2+X^2}$$

$\Rightarrow Y = G + jB$, where, $G = \text{Conductance} = \frac{R}{R^2+X^2} = \frac{R}{Z^2}$

& $B = \text{Susceptance} = \frac{X}{R^2+X^2} = \frac{X}{Z^2}$

Notes:→ Capacitive reactance $(-jX)$ is negative but the capacitive susceptance $(+jB)$ is positive as shown in the following fig.





Now, let us Consider two particular cases as follows:

Case-I If the Circuit has no reactance, i.e., if $X = 0$, then

$$G = \frac{1}{R}$$

which means the reciprocal of resistance is Conductance in this Case.

Case-II If the circuit has no resistance, i.e., if $R = 0$ then

$$B = \frac{1}{X}$$

which means the reciprocal of reactance is susceptance in this case.

Notes:→ Inverting a Complex number changes the sign of the j term and it is for this reason, the inductive reactance is +ve whereas the inductive susceptance is −ve and similarly, the capacitive reactance is −ve whereas the capacitive susceptance is +ve.

Units of admittance, Conductance & susceptance are Siemens (S).

$$\vec{I} = \frac{\vec{V}}{Z}$$

$$\Rightarrow \boxed{\vec{I} = \vec{V} Y}$$

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