Module-III

FLUID DYNAMICS

Introduction: when a fluid in motion various other fluid properties becomes significant, such as nature of flow of real fluid is complex.

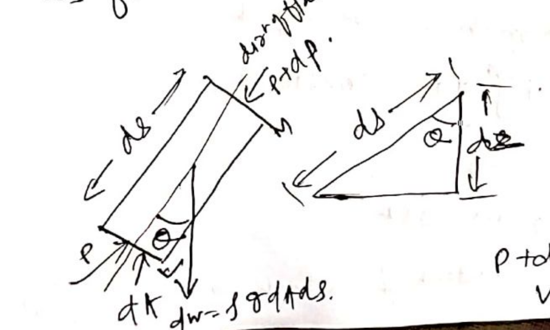
Kinematics & hydrokinematics - The science which deals with the geometry of motion of fluids with no reference to the forces causing motion is known as kinematics or hydrokinematics. Kinematics involves with the description of the motion of fluid in terms of space & time relationship.

Hydrokinetics - The science which deals with the action of forces in producing or changing motion of fluids known as hydrokinetics or simply kinetics.

The dynamics equation of fluid motion is obtained by applying Newton's 2nd law of motion to a fluid element considered as a free body. The fluid is assumed to be incompressible and nonviscous.

Euler's eqn of motion

Consider steady flow of an ideal fluid along the stream tube. Taking out a small element of fluid of crosssectional area dA & length ds from streamtube as free body from the moving fluid.



The fig shows a small element LM of fluid crosssection area dA & length ds.

Let P = Pressure on the element

P + dP = Pressure on element at M, and

V = velocity of the fluid element

The external forces tending to accelerate the fluid element in the dirn of stream line are as follows.

① Net pressure force in the dirn of flow is.

$$PdA - (P+dP)dA = -dP \cdot dA \quad \text{—— ①}$$

② Component of the weight of the fluid element in the dirn of flow

$$= -\rho g \, dA \, ds \cos\alpha$$

$$= -\rho g \, dA \, ds \left(\frac{dz}{ds}\right) \quad \left(\because \cos\alpha = \frac{dz}{ds}\right) \quad \text{—— ②}$$

$$= -\rho g \, dA \, dz$$

Mass of the fluid element $= \rho \, dA \, ds$ —— ③

The acceleration of the fluid element

$$a = \frac{dV}{dt} = \frac{dV}{ds} \cdot \frac{ds}{dt} = V \cdot \frac{dV}{ds} \quad \text{—— ④}$$

Now according to Newton's 2nd law of motion,

Force = mass × acceleration.

$$\therefore -dp \cdot dA - \rho g \, dA \, dz = \rho \, dA \, ds \cdot V \frac{dV}{ds}$$

Dividing both sides by ρdA, we get

$$-\frac{dP}{\rho} - g\,dz = V \cdot dV$$

$$\text{or} \quad \frac{dP}{\rho} + V dV + g dz = 0 \quad \text{—— ⑤}$$

This is the required Euler's equation for the motion, and is in the form of differential equation.

Integrating the above equation, we get

$$\frac{1}{\rho}\int dp + \int V dV + \int g dz = \text{constant}.$$

$$= \frac{1}{\rho} + \frac{V^2}{2} + gz = \text{const}.$$

Dividing both side by 'g' we get.

$$\boxed{\frac{P}{\rho g} + \frac{V^2}{2g} + z = \text{constant}}$$ → which is the Bernoulli's equation.

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Assumptions

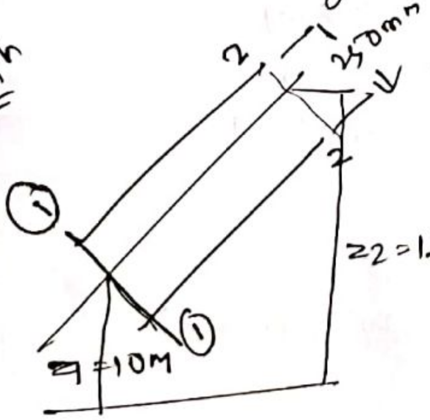
The following assumptions are made to derive Bernoulli's equation.

1) The liquid is ideal & incompressible.
2) The flow is steady and continuous.
3) The flow is along the stream i.e it is one dimensional.
4) The velocity is uniform over the section and is equal to the mean velocity.
5) The only forces acting on the fluid are the gravity force and pressure force.

Numerical related to Bernoulli's eqn

Q. In a smooth inclined pipe of uniform diameter 250 mm, a pressure of 50 kPa was observed at sec^n 1 which was at elevation 10 m. At another section 2 elevation 12 m, the pressure was 20 kPa and velocity was 1.25 m/s. Determine the direction of flow and the head loss between these two sections. The fluid in pipe is water. The density of water at 20°C and 760 mm Hg is 998 kg/m3

Soln



$P_2 = 20kPa$.

Given

$D = 250\,mm = 0.25\,m$

$P_1 = 50\,kPa = 50\times10^3\,N/m^2$

$Z_1 = 10\,M$, $Z_2 = 12\,M$.

$P_2 = 20\,kPa = 20\times10^3\,N/m^2$

$v_1 = v_2 = 1.25\,m/s$, $\rho = 998\,kg/m^3$

Total energy at 1-1

$$E_1 = \frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{50\times10^3}{998\times9.81} + \frac{1.25^2}{2\times9.81} + 10 = 15.187\,m.$$

· Total energy at 2-2

$$E_2 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 = \frac{20\times10^3}{998\times9.81} + \frac{1.25^2}{2\times9.81} + 12 = 14.122\,m$$

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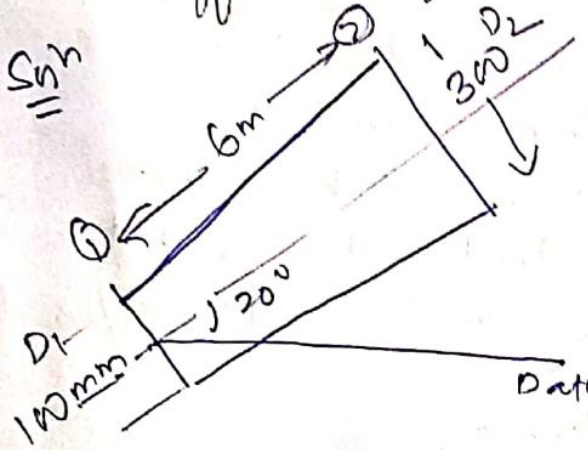
Loss of head = $h_L = E_1 - E_2 = 15.187 - 14.122 = 1.065$ m.

Direction of flow

Since $E_1 > E_2$, the dirn of flow from section 1-1 to sec 2-2

Q. A 6 m. long pipe is inclined at an angle of 20° with the horizontal. The smaller section of the pipe which is at lower level is of 100 mm diameter and larger section of pipe is of 300mm diameter, of the pipe is uniformly tapering and the velocity of at the smaller section 1.8 m/s. Determine the difference of pressure between two sections.

Soln



Given Length of pipe = $l = 6$m.

Angle of inclination = $\theta = 20°$

at Secn-1 $D_1 = 100$mm $= 0.1$ m

$A_1 = \pi/4\, D_1^2 = \pi/4\,(0.1)^2 = 0.00785$ m2

vel. $v_1 = 1.8$ m/s

Datum. $z = 0$

Let P_1 = press at 1

at Secn-2

$D_2 = 300$mm $= 0.3$ m

Area $A_2 = \pi/4 \times 0.3^2 = 0.0707$ m2

Datum $z_2 = 0 + 6 \sin\theta = 6 \sin 20° = 2.05$ m.

Let P_2 = Press at Secn-2,

From continuity equation, we know that

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1 v_1}{A_2} = \frac{0.00785 \times 1.8}{0.0707} = 0.2 \text{ m/s}$$

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Applying Bernoulli's eqn to both section of the pipe, we get

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

$$\frac{1}{\rho g}(P_1 - P_2) = \left(\frac{v_2^2}{2g} - \frac{v_1^2}{2g}\right) + (z_2 - z_1)$$

$$= \frac{1}{2g}\left((0.2\,m/s)^2 - 1.8^2\right) + (2.05 - 0) = 1.88$$

$$P_1 - P_2 = 9.81 \times 1.88 = 18.44\ kN/m^2$$

[i.e $W = \rho g$, for water $= 9.81\ kN/m^3$]

Practical Application of Bernoulli's equation

Bernoulli's equation is applied to the following measuring devices.

1) venturimeter
2) orifice meter
3) pitot tube.
4) syphon.

Venturimeter.

It is an instrument used to measure the rate of discharge in pipeline and often fixed permanently at different sections of the pipeline to know the discharge there.

The different types of venturimeter's are
i) horizontal venturimeter, ii) vertical ventrimeter.
iii) inclined venturimeter.

Horizontal ventrimeter - A venturimeter consists of the following three parts.

i) A short converging part.
ii) Throat and
iii) Diverging part.

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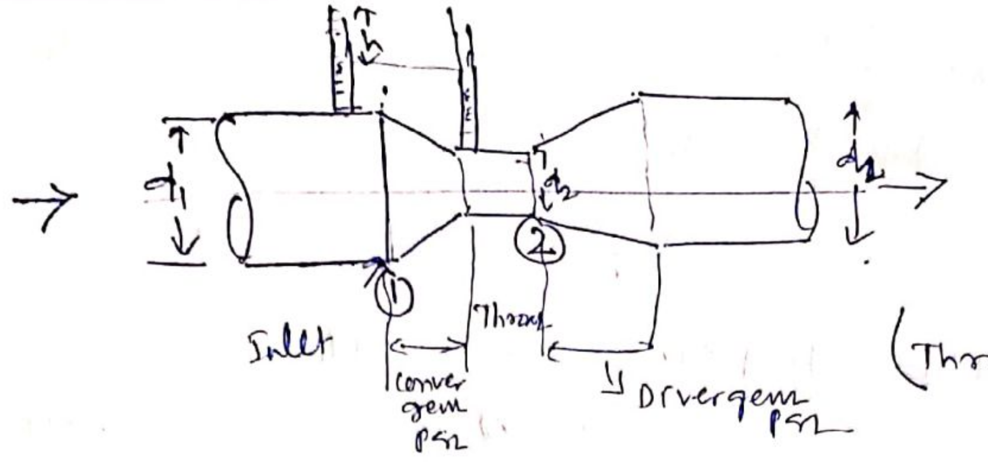
Expression for rate of flow



(Throat ratio $\frac{d_2}{d_1}$ varies $\frac{1}{4}$ to $\frac{1}{3}$)

The above fig. shows a venturimeter fitted in a horizontal pipe through which a fluid is flowing.

Let D_1 = dia at inlet 1

A_1 = Area at inlet ($\frac{\pi}{4} d_1^2$)

P_1 = pressure at secn (1)

V_1 = velocity of the fluid at secn 1

and D_2, A_2, P_2 & V_2 are the corresponding value at secn-2

Applying Bernoulli's equation at (1) & (2), we get

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

As pipe is horizontal $z_1 = z_2$

The equation is reduces to $\frac{P_1 - P_2}{\rho g} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$ —— (2)

But $\frac{P_1 - P_2}{\rho g} = h$, difference of press head at secn (1) & 2

Substituting in eqn (2)

$$h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} \quad \text{—— (3)}$$

Applying ~~Bernoulli's eqn~~ continuity equation at (1) & (2) we have

$A_1 V_1 = A_2 V_2$, $V_1 = \frac{A_2 V_2}{A_1}$

Substituting the value of V_1 in eqn (3), we get

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$$h = \frac{v_2^2}{2g} - \frac{\left(\frac{A_2 v_2}{A_1}\right)^2}{2g} = \frac{v_2^2}{2g}\left(1 - \frac{A_2^2}{A_1^2}\right)$$

$$\text{or } h = \frac{v_2^2}{2g}\left(\frac{A_1^2 - A_2^2}{A_1^2}\right) \quad \text{or } v_2^2 = 2gh\left(\frac{A_1^2}{A_1^2 - A_2^2}\right)$$

$$\text{or } v_2 = \sqrt{2gh\left(\frac{A_1^2}{A_1^2 - A_2^2}\right)} = \sqrt{2gh}\,\frac{A_1}{\sqrt{A_1^2 - A_2}}$$

Discharge $Q = A_2 V_2 = A_2 \frac{A_1}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh}$

$$\boxed{Q = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh}} \rightarrow$$

This above equation gives the discharge under ideal conditions is called theoritical discharge.

Actual discharge =

$$Q_{act} = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh}.$$

where C_d = co-efficient of discharge of venturimeter and the value is less than unity. (varies between 0.96 and 0.98)

* Due to variation of C_d venturimeters are not suitable for very low velocities.

value of 'h' by differential manometer.

Case-1 – Differential manometer containing a liquid heavier than the liquid flowing through the pipe.

$$h = y\left[\frac{S_{he}}{S_p} - 1\right]$$

where S_{he} = Sp. gravity of heavier liquid.

S_p = Sp. gravity of liquid flowing through pipe.

y = Difference of the heavier liquid column in U-Tube.

Case-2 - Differential manometer containing a liquid lighter than the liquid flowing through the pipe.

$$h = Y\left(1 - \frac{S_{ll}}{S_p}\right)$$

where
S_{ll} = Sp. gravity of lighter liquid.
S_p = Sp. gravity of liquid flowing through pipe.

Y = Difference of lighter liquid.

Problem related to ventimeter

A horizontal venturimeter with inlet diameter 200 mm & throat diameter 100mm is used to measure the flow of water. The pressure at inlet is 0.18 N/mm2 and the vacuum pressure at the throat is 280 mm of Hg. Find the rate of flow. Take Cd = 0.98.

Soln Given

Inlet dia = D_1 = 200mm = 0.2 m. , $A_1 = \frac{\pi}{4}(0.2)^2 = 0.0314 m^2$

Dia at Throat D_2 = 100mm = 0.1 M $A_2 = \frac{\pi}{4} \times (0.1)^2 = 0.00785 m^2$

Pressure at inlet $P_1 = 0.18 N/mm^2 = 180 KN/m^2$.

$$\frac{P_1}{\rho g} = \frac{180}{9.81} = 18.3 m.$$

vacuum pressure at throat

$$\frac{P_2}{\rho g} = -280 mm \text{ of Hg.} = -0.28 m \text{ of Hg}$$

$$= -0.28 \times 13.6 = -3.8 m \text{ of water.}$$

coefficient of discharge, Cd = 0.98.

Differential head = $h = \frac{P_1}{\rho g} - \frac{P_2}{\rho g} = 18.3 - (-3.8) = 22.1 m$

Hence Rate of flow = $Q = Cd \times \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh} = 0.98 \times \frac{0.0314 \times 0.00785}{\sqrt{(0.0314)^2 - 0.00785^2}} \times \sqrt{2 \times 9.81 \times 22.1}$

$$Q = 0.165 m^3/s$$

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Q-2 - A horizontal venturimeter with inlet & throat diameters 300mm & 100mm respectively is used to measure the flow of water. The pressure intensity at inlet is 130kN/m² while the vacuum pressure head at the throat is 350mm of mercury. Assuming that 3 percent of head is lost between the inlet & throat, find.

(i) The value of Cd (co-efficient of discharge) for the venturimeter.

(ii) Rate of flow.

Soln Given :- Inlet dia $D_1 = 300mm = 0.3\,m.$, $A_1 = \frac{\pi}{4} 0.3^2 = 0.07 m^2$

Throat dia $D_2 = 100\,mm = 0.1\,m$ $A_2 = \frac{\pi}{4} 0.1^2 = 0.00785 m^2$

Pressure at inlet $P_1 = 130\,kN/m^2$

Pressure head $\frac{P}{\rho g} = \frac{130}{9.81}$ $13.25\,m.$

Similarly pressure head at throat $= \frac{P}{\rho g} = -350$ mm of mercury

$= -0.35 \times 13.6\,m$ of water

$= -4.76\,m.$

(i) Co-efficient of discharge Cd

Differential head, $h = \frac{P_1}{\rho g} - \frac{P_2}{\rho g} = 13.25 - (-4.25) = 18.01\,m.$

Head lost $h_f = 3\%$ of $h = \frac{3}{100} \times 18.01 = 0.54\,m.$

Hence $C_d = \sqrt{\frac{h - h_f}{h}} = \sqrt{\frac{18.01 - 0.54}{18.01}} = 0.985.$

(ii) Rate of discharge. Q

$$Q = C_d \times \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \times \sqrt{2gh}.$$

$$= 0.985 \times \frac{0.07 \times 0.00785}{\sqrt{0.07^2 - 0.00785^2}} \times \sqrt{2 \times 9.81 \times 18.01}$$

$$= \frac{0.000541}{0.0956} \times 18.79 = 0.146\,m^3/s$$

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Vertical and inclined venturimeter

Vertical & inclined venturimeters are employed for measuring discharge on pipeline which are not horizontal.

Here $h = \left(\frac{P_1}{\rho g} - \frac{P_2}{\rho g}\right) + (z_1 - z_2)$

Q. A vertical venturimeter carries a liquid of relative density 0.8 & has inlet and throat diameter of 150mm & 75 mm respectively. The pressure connection at throat is 150 mm above that at the inlet. If the actual rate of flow is 40 lit/sec and $C_d = 0.96$. Calculate the pressure difference between inlet & throat in N/m2

Sol^n Given - sp. gravity = 0.8

$D_1 = 150mm = 0.15m$, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.15)^2 = 0.01767 m^2$

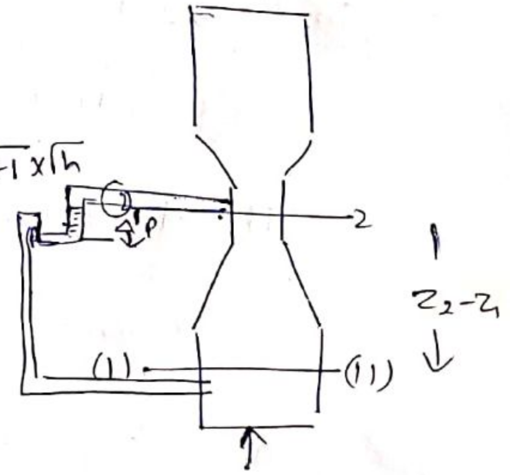
$D_2 = 75mm = 0.075m$, $A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4}(0.075)^2 = 0.00442 m^2$

$z_1 - z_2 = 150mm = 0.15m$,

$Q_{act} = 40 lit/sec = \frac{40}{1000} = 0.04 m^3/s$, $C_d = 0.96$

$$Q_{act} = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2}} \times \sqrt{2gh}$$

$$0.04 = 0.96 \times \frac{0.01767 \times 0.00442}{\sqrt{(0.01767)^2 - (0.00442)^2}} \sqrt{2 \times 9.81} \times \sqrt{h}$$



Solving $h = 4.247 m.$

As we know $h = \left(\frac{P_1}{\rho g} - \frac{P_2}{\rho g}\right) + (z_1 - z_2)$

$$2.247 = \left(\frac{P_1 - P_2}{\rho g}\right) - 0.15$$

$$P_1 - P_2 = \rho g(2.247 + 0.15) = (0.8 \times 1000 \times 9.81)(4.247 + 0.15)$$

$$= 34.51\ kN/m^2$$

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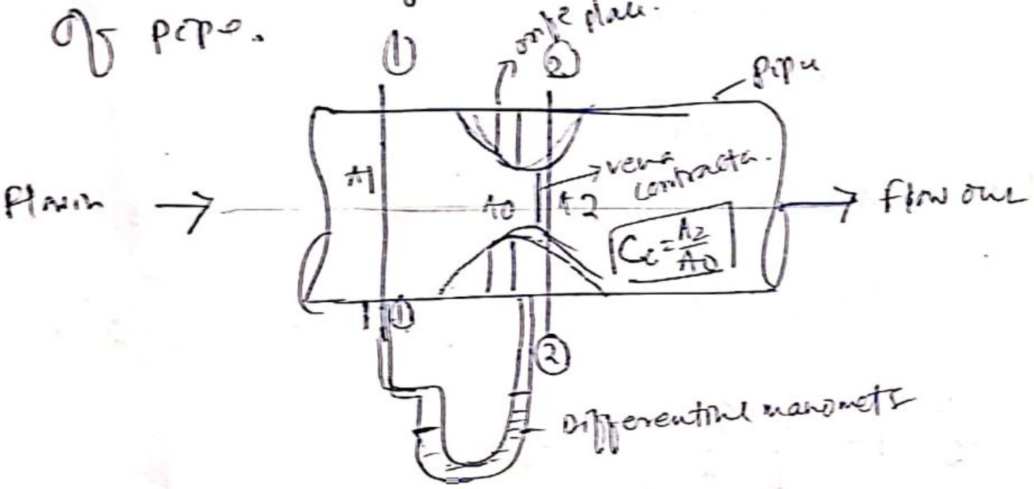
Orificemeter :-

Orificemeter is a device employed for measuring the discharge of fluid through a pipe. It also works on the same principle of a venturi meter.

It consists of a flat circular plate having a circular sharp edged hole (called orifice) concentric with pipe. The dia of orifice is 0·4 to 0·8 time the diameter of pipe.



Let A_1 = Area of pipe at section (1)

v_1 = velocity at section (1).

P_1 = Pressure at section (1), and

$A_2, v_2, \& P_2$ are corresponding values at section (2)

Applying Bernoulli's equation at (1) & (2)

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

$$\left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right) = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

& $h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$, where $h = \left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right)$ = differential pressure head

$$\frac{v_2^2}{2g} = h + \frac{v_1^2}{2g} \quad \text{———— (1)}$$

or $v_2 = \sqrt{2g\left(h + \frac{v_1^2}{2g}\right)} = \sqrt{2gh + v_1^2}$

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at Section (2) is at vena contracta & A_2 represents the area of venacontracta. If A_0 is the area of orifice then

$$C_c = \frac{A_2}{A_0} \quad , \text{ where } C_c = \text{coefficient of contraction}.$$

$$A_2 = A_0 C_c \text{ ——— } (2)$$

Using continuity equation, we get,

$$A_1 V_1 = A_2 V_2$$

$$V_1 = \frac{A_2 V_2}{A_1} = \frac{A_0 C_c V_2}{A_1} \text{ ——— } (3)$$

Substituting the value of V_1 in equation '1', we get.

$$V_2 = \sqrt{2gh + \frac{A_0^2 C_c^2 V_2^2}{A_1^2}} = 2gh \left(\frac{A_0}{A_1}\right)^2 C_2^2 V_2^2$$

$$\text{or } V_2^2 \left[1 - \left(\frac{A_0}{A_1}\right)^2 C_2^2\right] = 2gh$$

$$\therefore V_2 = \frac{\sqrt{2gh}}{1 - (A_0/A)^2 C_c^2}$$

Again the discharge, $Q = A_2 V_2 = A_0 C_c V_2$

$$= A_0 C_c \frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{A_0}{A_1}\right)^2 C_c^2}} \text{ ——— } (4)$$

The above expression is simplified by using

$$C_d = C_c \frac{\sqrt{1 - (A_0/A_1)^2}}{\sqrt{1 - (A_0/A_1)^2 C_c^2}}$$

where C_d = coefficient of discharge.

$$\text{or} \quad C_c = C_d \frac{\sqrt{1 - (A_0/A_1)^2 C_c^2}}{\sqrt{1 - (A_0/A_1)^2}}.$$

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Substituting the value of C_c in equation (4), we get

$$Q = A_0 C_d \frac{\sqrt{1-(A_0/A_1)^2 C_c^2}}{\sqrt{1-(A_0/A_1)^2}} \times \frac{\sqrt{2gh}}{\sqrt{1-(A_0/A_1)^2 C_c^2}}$$

$$= \frac{C_d A_0 \sqrt{2gh}}{\sqrt{1-(A_0/A_1)^2}} = \frac{C_d \cdot A_0 A_1 \sqrt{2gh}}{\sqrt{A_1^2 - A_0^2}}$$

$$\boxed{\text{ie} \quad Q = C_d \frac{A_0 A_1 \sqrt{2gh}}{\sqrt{A_1^2 - A_0^2}}}$$

The coefficient of discharge of orificemeter is much smaller than that of venturimeter.

Difference between venturimeter & orificemeter

venturimeter	orificemeter
i) venturimeter can be used for measuring the flow rate of all incompressible flows. (gases with low pressure variation as well as liquids)	i. It is used for measuring flow rate of liquids.
ii) venturimeters are used for pipelines only and the accelerated flow through the apparatus, is subsequently descelerated to original velocity at the outlet of the venturimeter	ii) In orificemeter the entire potential energy of the fluid is converted into KE, & the jet discharges freely into the open atmosphere.
iii) The flow velocity measured by noting the pressure difference between the inlet and throat of the venturimeter	iii) The discharge velocity is measured by using pitottube or by trajectory method.

Q. For an orifice meter, Diameter of the pipe = 240 mm.
Diameter of the orifice = 120 mm.
Specific gravity of oil = 0.88
Reading of differential manometer = 400 mm of Hg.
Coefficient of discharge = 0.64.
Determine the rate of flow.

Sol.

$D_1 = 240mm = 0.24m$, $A_1 = \frac{\pi}{4}(0.24)^2 = 0.0452 m^2$

$D_0 = 120mm = 0.12m \Rightarrow A_2 = \frac{\pi}{4}(0.12)^2 = 0.0113 m^2$

Differential head: $h = y\left(\frac{S_{h1}}{S_0} - 1\right) = 0.4\left[\frac{13.6}{0.88} - 1\right]$

$= 5.78 m$ of oil.

Discharge $Q = C_d \frac{A_1 A_0 \sqrt{2gh}}{\sqrt{A_1^2 - A_0^2}}$

$= 0.64 \times \frac{0.0113 \times 0.452 \times \sqrt{2 \times 9.81 \times 5.78}}{\sqrt{(0.0452)^2 - (0.0113)^2}}$

$= \frac{0.000353}{0.0437} = 0.08 m^3/s.$

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